

Bhubanananda Odisha School of Engineering, Cuttack
Department of Mechanical Engineering



SUBECT- ENGINEERING MECHANICS

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CH-1: FUNDAMENTALS OF ENGG. MECHANICS

Mechanics:-

It may be defined as the branch of Science which describes & predicts the condition of rest or motion of the bodies under the action of external force.

Classification:-

(1) Engg. Mechanics

(a) Static mechanics

(It deals with the forces & their effects while acting upon a body at rest.)

(b) Dynamic Mechanics

(It deals with the forces & their effects while acting upon a body in motion.)

(i) Kinematics

It is the branch of dynamics which deals with the body in motion without any reference of the forces which are responsible for the motion.

(ii) Kinetic

It is the branch of dynamics which deals with the body in motion with the reference of the forces which are responsible for the motion. * 2 terms are used in mech. Motion case
Rest case

Note:- Unit:- The standard measurement of physical quantity is called as unit

⇒ It is 2 type.

(a) Fundamental

Every physical quantity is measured in terms of some arbitrary but internationally accepted called Fundamental unit.

Ex:- Mass - kg, gm. | Current - ampere
Length - m, cm. | Temp - °C, K. etc

(b) Derived unit

The units which are derived from fundamental unit is known as derived unit.

Ex:- Area = $l \times b = m \times m = m^2$

Volume = $l \times b \times h = m \times m \times m = m^3$

Velocity = $\frac{s}{t} = \frac{\text{Distance}}{\text{Time}} = \text{m/sec}$ etc.

System of unit

- (1) CGS (Centimeter, gram, second are considered)
- (2) MKS (mass, Length, Time are considered)
- (3) FPS (Foot, Pound & second are considered)
- (4) S.I. (International standard unit are always accepted).

Ex:- Force = mass $\times$ Acceleration = $m \times a = \text{kg} \cdot \text{m/s}^2 = \text{N}$

Moment = Force $\times$ distance ($\perp$) = $\text{N} \cdot \text{m}$

Work = Joule.

Power = Joule/sec = Watt etc.

Body It may be defined as an object which has a definite shape & consisting of no. of particles.

Rigid Body:-

A body is called Rigid body if it does not undergoes any deformation when acted upon by a force system.

Elastic Body:-

A body is called elastic body if it undergoes & deformation when acted upon by a force system.

Scalar Quantities

The Physical Quantity whose only magnitude

Ex:- Length, mass, Time.

Vector Quantity

The Physical Quantity which has both magnitude & direction. Ex:- Velocity, Acceleration etc.

Mass:- It is defined as the total quantity of matter present in a body.

unit:- Kg, gm,

Weight:- It is defined as the force with which a body is attracted towards the centre of earth.

unit:- Newton. ($W = m \times g$)

Force It may be defined as an agent which ~~is attracted~~ changes or tends to change the state of body which is either at rest or in motion.

$\therefore$ Mathematically Force (F) = Mass $\times$ Acceleration

$$\Rightarrow \boxed{F = m \times a}$$

Unit $\rightarrow$ S.I (Newton) $\text{kg} \cdot \text{m/s}^2$ & C.G.S (Dyne) $\text{g} \cdot \text{cm/sec}^2$

How $1 \text{ N} = 10^5 \text{ dyne}$?

$$\begin{aligned} \text{We know } 1 \text{ N} &= \text{mass} \times \text{Acceleration} \\ &= 1 \text{ kg} \times 1 \text{ m/s}^2 \\ &= 1000 \text{ gm} \times 100 \text{ cm/s}^2 \\ &= 10^3 \times 10^2 (\text{gm} \cdot \text{cm/s}^2) \end{aligned}$$

$$\Rightarrow \boxed{1 \text{ N} = 10^5 \text{ dyne}}$$

How $1 \text{ kgf} = 9.8 \text{ N}$?

We know the force required to move a mass of 1 kg with an acceleration of 9.8 m/sec^2 known as kg force (kgf) .

$$\begin{aligned} 1 \text{ kgf} &= 1 \text{ kg} \times 9.8 \text{ m/s}^2 \\ &= 9.8 \text{ kg m/s}^2 \\ &= 9.8 \text{ N} \end{aligned}$$

$$\Rightarrow \boxed{1 \text{ kgf} = 9.8 \text{ N}}$$

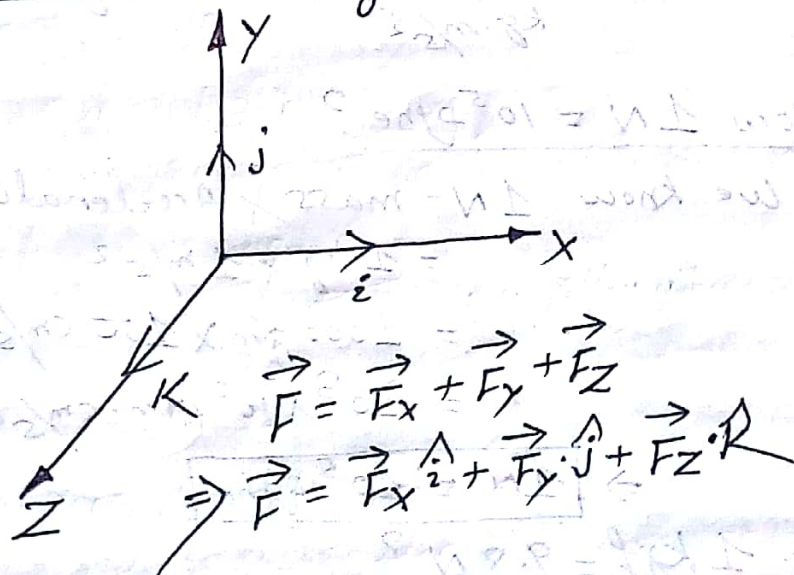
While considering the forces, one should know, following points.

- (i) The point of Application
- (ii) Direction
- (iii) Magnitude.

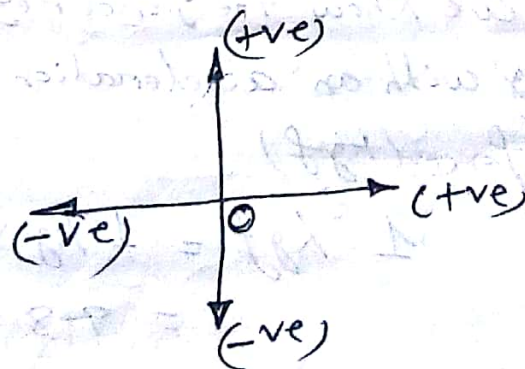
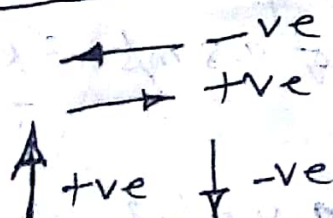
Different types of Force

- | | |
|----------------------|-----------------------|
| (1) Applied force | (2) Non-Applied force |
| (a) Axial force | (a) self weight force |
| (b) Tangential force | (b) Reaction force |
- Axial force $\left\{ \begin{array}{l} \text{Tensile} \\ \text{compressive} \end{array} \right.$

Representation of Force by vector



Sign convention



Characteristics of Force

- (1) The magnitude of Force (Ex: 2N, 5N, 10kgf)
- (2) Direction of the force
- (3) Nature of Force (Push or Pull)
(-ve) (+ve)
- (4) The point at which, force act on the body.

Effects of Force

(1) It may change the motion of the body. i.e. if a body is at rest, the force may set the body in motion, & if the body is already in motion, the force may accelerate it.

(2) It may retard the motion of the body.

(3) It may give rise to the internal stress to the body.

Principle of Transmissibility

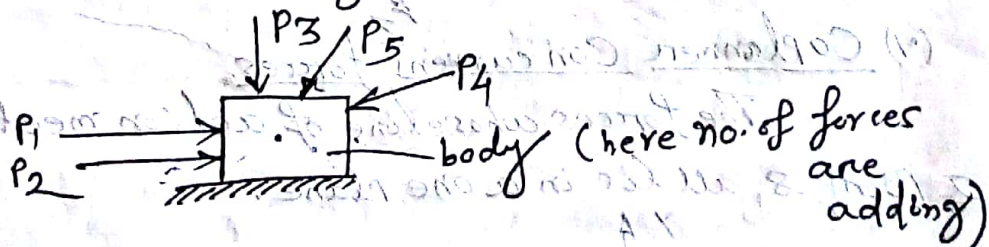
The force can be represented anywhere along the line of action without changing the effect of the forces on any rigid body.

Amount of
force
applied



Principle of Superposition

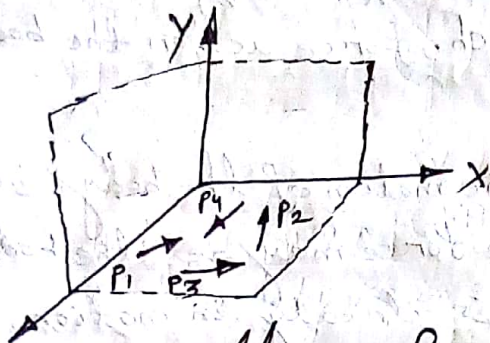
The Action of a given system of forces on a rigid body will never change if we add or subtract another system of forces in equilibrium



System of Forces

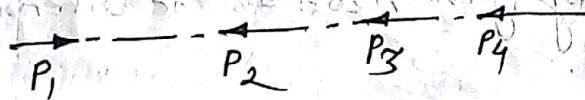
(i) Coplanar forces

The forces whose line of action lie in a common plane.



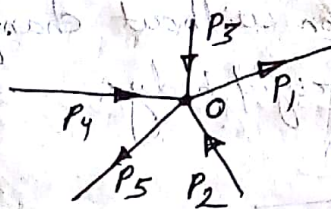
(ii) Collinear forces

The forces whose lines of action lie in a common straight line.



(iii) Con-current forces

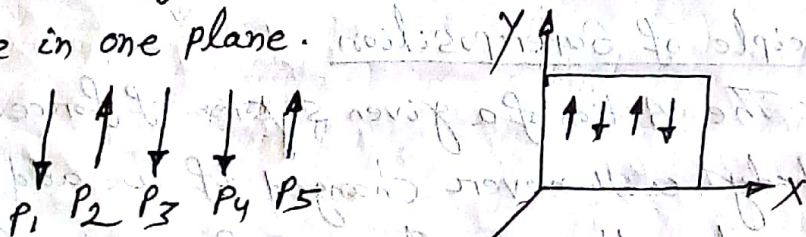
The forces whose line of action meet at one point.



EX:- Alloy wheel

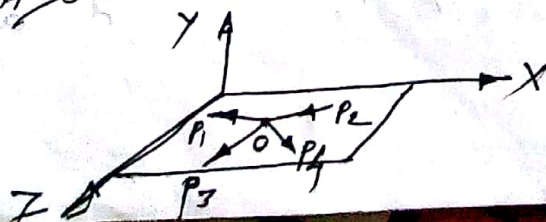
(iv) Coplanar Parallel forces

The forces whose line of action are parallel & lie in one plane.



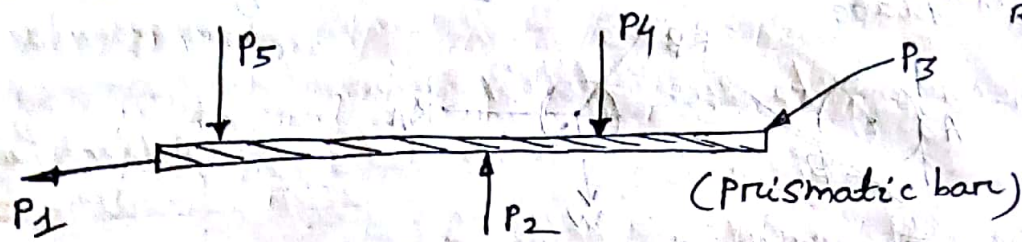
(v) Coplanar Con current forces

The forces whose line of action meet at a point & all lie in a one plane.



(vi) Coplanar & Non-concurrent forces

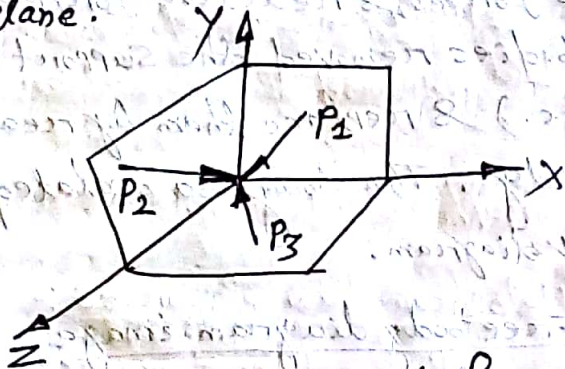
The forces whose lines of action lie in a plane, but never meet at a point. Example:- Beams in a plane or Roof.



(vii) Non coplanar concurrent force

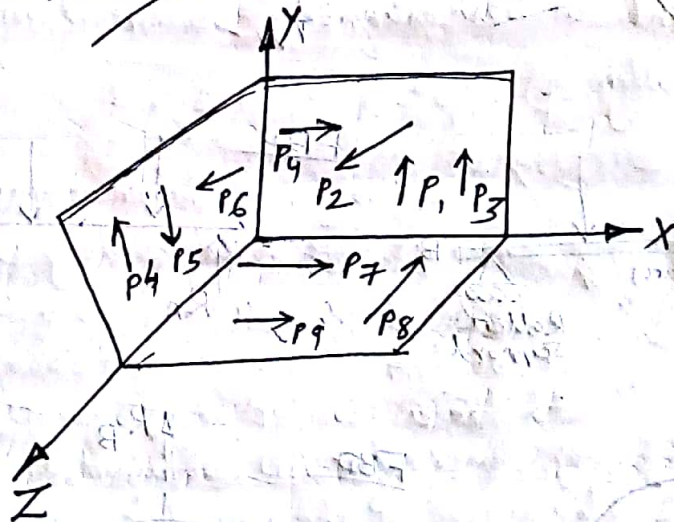
The forces whose line of action meet at a point, but never lie in a plane.

Ex:- Camera tripod stands.



(viii) Non-coplanar & Non concurrent forces

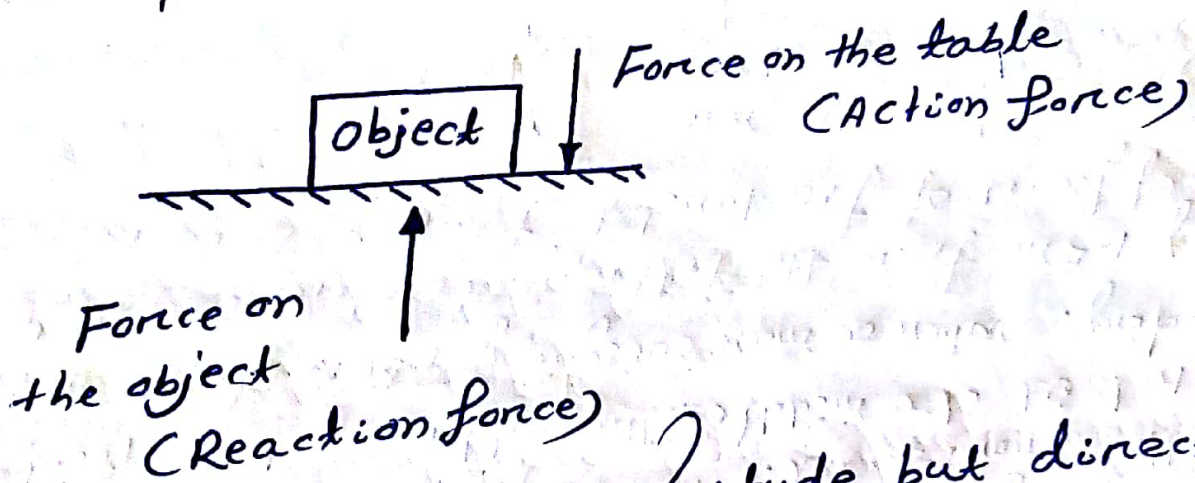
The forces whose lines of action do not meet at a point & never lie in a plane.



Action and Reaction Force

This comes under Newton's 3rd Law of motion which states that

"For every Action force there is an equal and opposite reaction force."



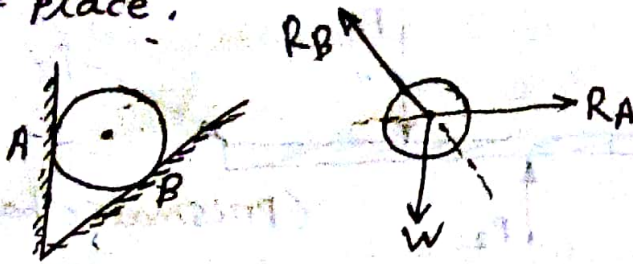
* Both have same magnitude but directions are different.

(1) The force which is acting in one direction is called Action force & equal force acting in opposite direction is called reaction force.

Reaction :-

The internal force of a body.
The reaction of a body is always perpendicular to the place.

Ex:-



Free body diagram (FBD)

When a body is in equilibrium is subjected to no. of forces & reactive forces, the equilibrium of such bodies removed the support (wall, floor, hinge etc.) & replace them by reaction which they exert body. The body so isolated system is called Free body diagram.

Some Free body diagram image

Fig-1

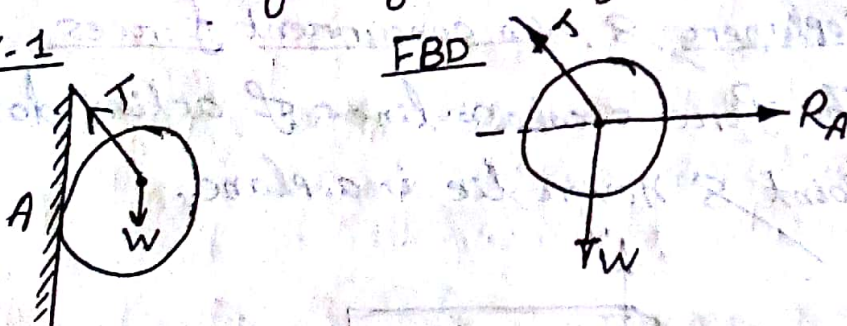


Fig-2

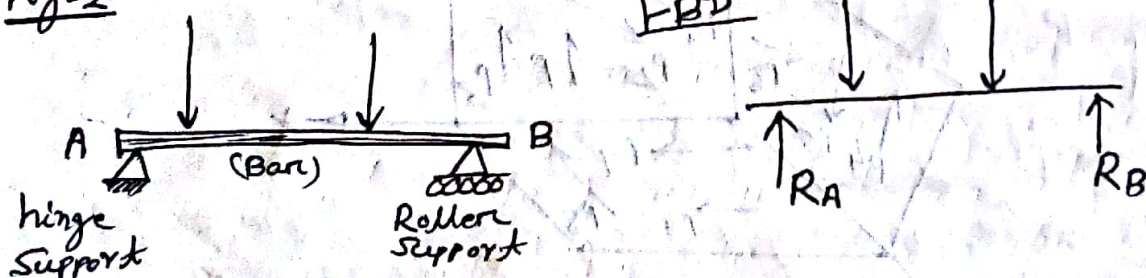


Fig-3

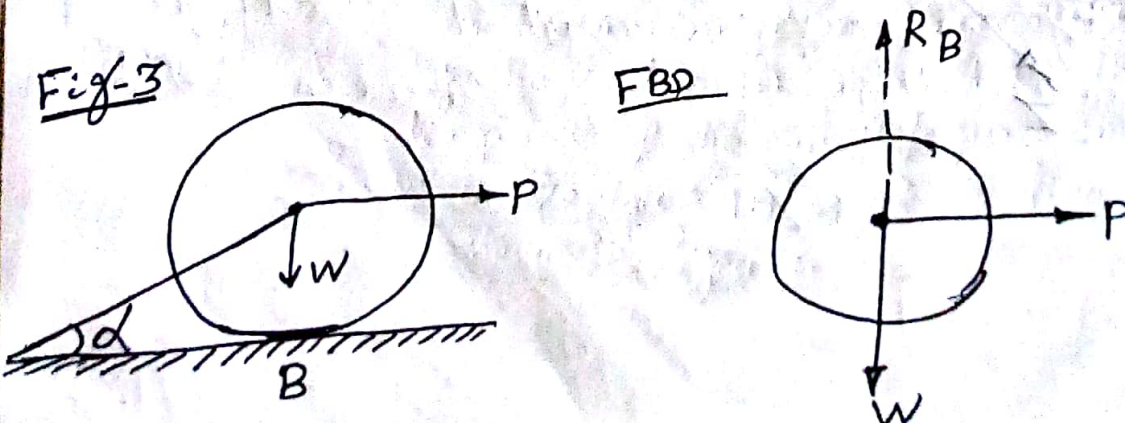


Fig-4

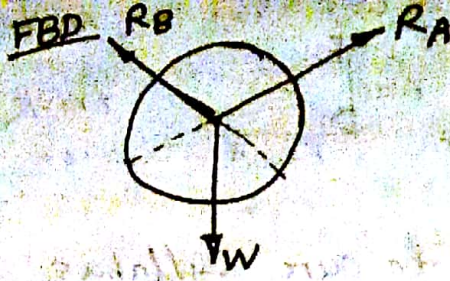
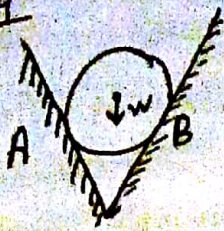
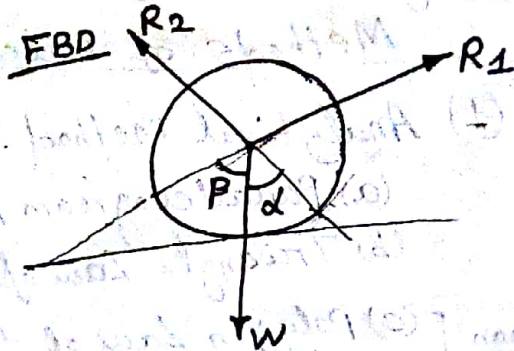
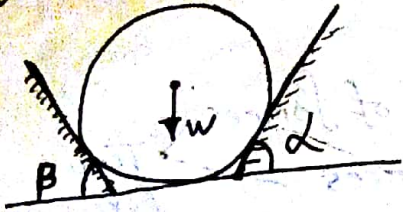


Fig-5:



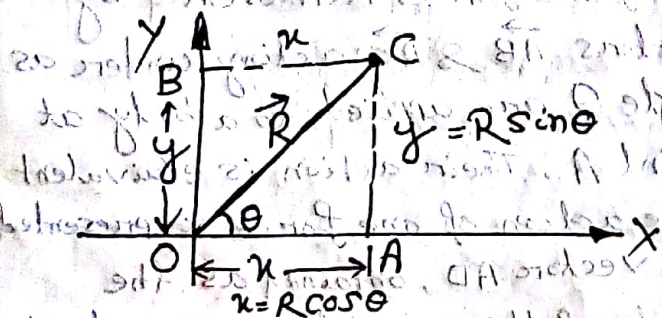
Resultant Force 2 mark Quest.

A single force which produces the same effect as a no. of forces acting together is called the resultant of these forces.

Resolution of Force

The process of splitting up the given forces into a no. of components without changing its effect on a body is called resolution of a force.

* Force is generally resolve along two mutually perpendicular direction.

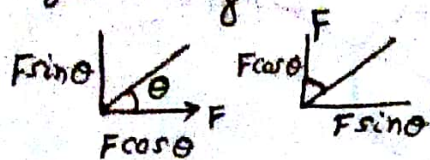


$$\text{In } \triangle OCA, \sin \theta = \frac{AC}{OC} = \frac{y}{R} \Rightarrow y = R \sin \theta$$

$$\cos \theta = \frac{OA}{OC} = \frac{x}{R} \Rightarrow x = R \cos \theta$$

$$\begin{aligned} \therefore \text{Resultant (R)} &= \sqrt{x^2 + y^2} \quad \sqrt{\text{Apply } h^2 = p^2 + b^2} \\ &= \sqrt{(R \cos \theta)^2 + (R \sin \theta)^2} \\ &= \sqrt{R^2 (\cos^2 \theta + \sin^2 \theta)} = \sqrt{R^2} = \pm R \end{aligned}$$

Angle base of $\cos$ formula and Another side of $\sin$ formula.



trick Apply.

According to our Syllabus,

Methods of calculation of Resultant forces

(1) Analytical method.

(a) Parallelogram Law of Forces

(b) Triangle Law of Forces

More than two forces are acting.

(c) Polygon Law of Forces

(d) Method of resolution

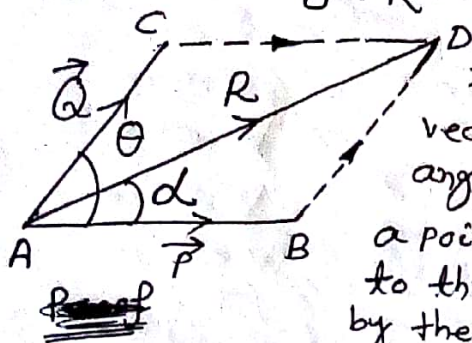
Two forces are acting different line of action.

(2) Graphical Method.

* Parallelogram Law of Forces:-

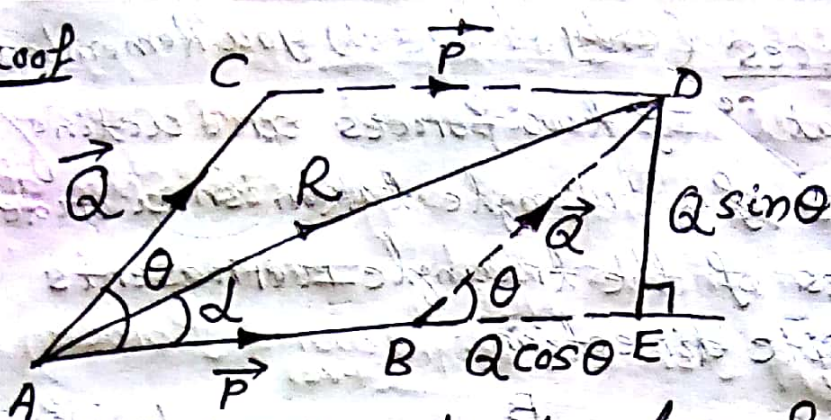
If two forces are acting at a point & are represented in both magnitude & direction by the adjacent sides of Parallelogram, then the diagonal passing through their point of intersection, represents the resultant in both magnitude & direction.

Mathematically, $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$.



If two forces represented by vectors $\vec{AB}$ & $\vec{AC}$ acting under an angle θ are applied to a body at a point A. Their action is equivalent to the action of one force represented by the vector AD, obtained as the diagonal of the Parallelogram constructed on the vectors $\vec{AB}$ & $\vec{AC}$ & directed as shown in figure.

Proof



Draw a perpendicular line from point D which line of action meet at point E.

In ΔADE

$$AD^2 = AE^2 + ED^2 \quad (\because \text{Right Angle } \Delta)$$

$$\Rightarrow (\vec{R})^2 = (AB + BE)^2 + ED^2$$

$$\Rightarrow R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$

$$\Rightarrow R = \sqrt{P^2 + Q^2 \cos^2 \theta + 2PQ \cdot \cos \theta + Q^2 \sin^2 \theta}$$

$$\Rightarrow R = \sqrt{P^2 + Q^2 (\cos^2 \theta + \sin^2 \theta) + 2PQ \cdot \cos \theta}$$

$$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ \cdot \cos \theta}$$

Case-1 If $\theta = 0^\circ$, then $R = \sqrt{(P+Q)^2} \Rightarrow \boxed{R = P+Q}$

Case-2 If $\theta = 90^\circ$, then $R = \sqrt{P^2 + Q^2}$

Case-3 If $\theta = 180^\circ$, then $R = \sqrt{P^2 + Q^2 + 2PQ \cdot \cos 180^\circ}$

$$\Rightarrow R = \sqrt{(P-Q)^2}$$

$$\Rightarrow \boxed{R = P-Q}$$

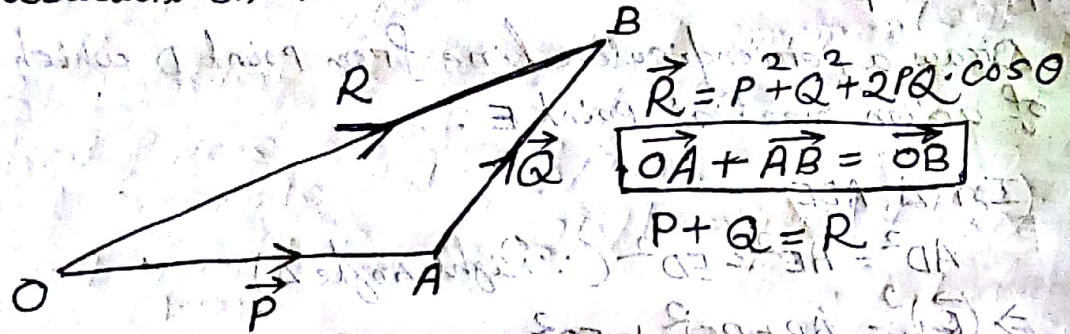
Direction of Resultant Force

In ΔADE , $\tan \alpha = \frac{P}{b} = \frac{Q \sin \theta}{P + Q \cos \theta}$

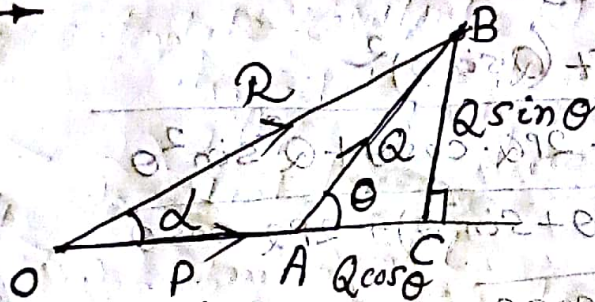
$$\Rightarrow \boxed{\alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)}$$

Triangle Law of Forces (only statement) you Remember.

It states that "If two forces are acting as the adjacent sides of a triangle taken in a order then the closing sides of the triangle represents the resultant in opposite direction."



Proof: →



Draw a $\perp$ line from point B whose line of action meet at point C.

In ΔOBC , $OB^2 = OC^2 + BC^2$

$= (OA + AC)^2 + BC^2$

$= OA^2 + AC^2 + 2 \cdot OA \cdot AC + BC^2$

$\Rightarrow R^2 = P^2 + Q^2 + 2PQ \cdot \cos \theta$

$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ \cdot \cos \theta}$

$\tan \alpha = \frac{BC}{OC} = \frac{Q \sin \theta}{P + Q \cos \theta}$

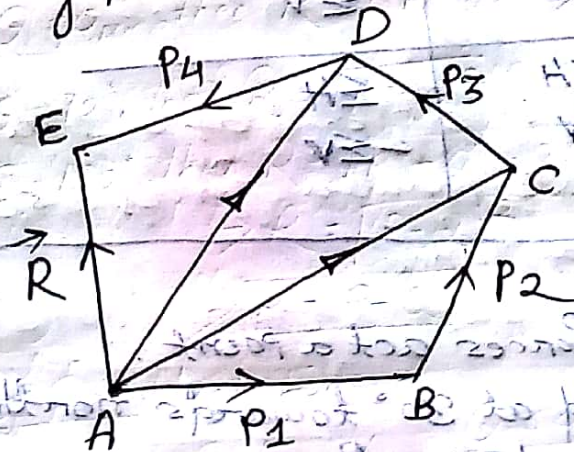
$\Rightarrow \alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)$

Polygon of Law of Forces

It is an extension of triangle law of forces for more than two forces which states that if a no. of forces are acting simultaneously on a particle be represented in both magnitude & direction by the sides of a polygon taken in order, then the resultant of all these forces may be represented in magnitude & direction by the closing side of the polygon taken in order.

Proof :-

Consider the forces P_1, P_2, P_3 & P_4 acting on the body at point 'A' as shown in figure. Line AB represent force P_1 , line BC represent force P_2 , CD for P_3 & DE for P_4 . Polygon is completed by drawing the closing line AE. This closing line AE, represent the resultant of the given system.



$$\vec{AC} = \vec{AB} + \vec{BC}$$

$$\vec{AD} = \vec{AC} + \vec{CD} = \vec{AB} + \vec{BC} + \vec{CD}$$

$$\vec{AE} = \vec{AD} + \vec{DE} = \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE}$$

$$\Rightarrow \boxed{R = P_1 + P_2 + P_3 + P_4}$$

METHODS OF RESOLUTION OF FORCES

Step-1: Resolving all the horizontal components of horizontal forces & find the algebraic sum of all the horizontal components that $\Sigma H = 0$
(Net Leftward Force = Net Rightward Force)
(-ve) (+ve)

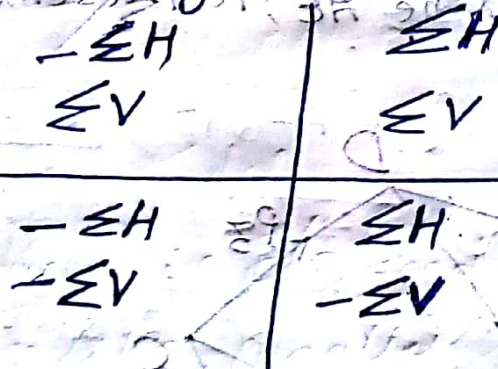
Step-2: Resolving all the vertical components of vertical forces & find the algebraic sum of all the vertical components that $\Sigma V = 0$
(Net upward force = Net downward force)
(+ve) (-ve)

Step-3: Calculate Resultant $(R) = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

Magnitude of the resultant force

Step-4: Resultant force direction $(\theta) = \tan^{-1} \left(\frac{\Sigma V}{\Sigma H} \right)$

Step-5: Remember Symbol



PROBLEMS

- ① Find the angle between two equal forces of magnitude P , when their resultant is P and $P/2$.

BOOK
PROBLEM

2018
(Summer)
Question

Ans Data given

Angle (θ) = ?

Two equal forces of magnitude P which means

one force (P) = P

Second force (Q) = P

Their Resultant is P & $P/2$ i.e. two condition is available here.

Case-1

$$P = P$$

$$Q = P$$

$$\text{Resultant (R)} = P$$

$$\theta = ?$$

Case-2

$$P = P$$

$$Q = P$$

$$\text{Resultant (R)} = P/2$$

$$\theta = ?$$

Solution

For case-1:

$$\text{We know, } R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow P = \sqrt{P^2 + P^2 + 2P \times P \cos \theta}$$

$$\Rightarrow P = \sqrt{2P^2 + 2P^2 \cos \theta}$$

$$\Rightarrow P = \sqrt{2P^2(1 + \cos \theta)}$$

$$\Rightarrow P = \sqrt{2} \times P \sqrt{1 + \cos \theta}$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 = (1 + \cos \theta)^2$$

$$\Rightarrow \frac{1}{\sqrt{2} \times \sqrt{2}} = 1 + \cos \theta$$

$$\Rightarrow \frac{1}{2} = 1 + \cos \theta$$

$$\Rightarrow \frac{1}{2} - 1 = \cos \theta$$

$$\Rightarrow \frac{1-2}{2} = \cos \theta$$

$$\Rightarrow -1/2 = \cos \theta$$

$$\Rightarrow \theta = \cos^{-1}(-1/2)$$

$$\Rightarrow \theta = 120^\circ$$

For case-2:

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow P/2 = \sqrt{P^2 + P^2 + 2P \times P \times \cos \theta}$$

$$\Rightarrow P/2 = \sqrt{2P^2 + 2P^2 \cos \theta}$$

$$\Rightarrow P/2 = \sqrt{2P^2(1 + \cos \theta)}$$

$$\Rightarrow P/2 = \sqrt{2} \times P \sqrt{1 + \cos \theta}$$

$$\Rightarrow \frac{1}{2\sqrt{2}} = \sqrt{1 + \cos \theta}$$

$$\Rightarrow \left(\frac{1}{2\sqrt{2}}\right)^2 = \left(\sqrt{1 + \cos \theta}\right)^2$$

$$\Rightarrow \frac{1}{8} = 1 + \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1}{8} - 1 = \frac{1-8}{8} = -\frac{7}{8}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\frac{7}{8}\right)$$

$$\Rightarrow \theta = 151.04^\circ$$

- ② Find the resultant of two forces 130 N & 110 N respectively, acting at an angle whose tangent is $12/5$.

Book Problem

Ans Data given

Resultant force (R) = ?

Force (P) = 130 N

Force (Q) = 110 N

These two forces acting at an angle of tangent $12/5$ which means $\tan \theta = 12/5$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{12}{5}\right) = 1.176^\circ$$

Solution

We know $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

$$= \sqrt{130^2 + 110^2 + 2 \times 130 \times 110 \times \cos(1.176^\circ)}$$

$$\Rightarrow R = 200 \text{ N}$$

- ③ Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60° , their resultant is $\sqrt{13}$ N.

2018 (winter) old Question

Ans Data given

Force (P) = ?

Force (Q) = ?

Case-1

If two force act at right angle i.e. $\theta = 90^\circ$

Resultant (R) = $\sqrt{10}$ N

Case-2

If two force act an angle 60° , i.e. $\theta = 60^\circ$

Resultant (R) = $\sqrt{13}$ N

Solution

In case-1

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cdot \cos \theta}$$

$$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ \cdot \cos 90^\circ}$$

$$\Rightarrow \sqrt{10} = \sqrt{P^2 + Q^2}$$

$$\Rightarrow 10 = P^2 + Q^2 \dots (1)$$

In case-2

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cdot \cos \theta}$$

$$\Rightarrow \sqrt{13} = \sqrt{P^2 + Q^2 + 2PQ \cdot \cos 60^\circ}$$

$$\Rightarrow \sqrt{13} = \sqrt{P^2 + Q^2 + 2PQ \times \frac{1}{2}}$$

$$\Rightarrow 13 = P^2 + Q^2 + PQ \dots (2)$$

$$\Rightarrow 13 = 10 + PQ \quad (\because P^2 + Q^2 = 10 \text{ From eqn(1)})$$

$$\Rightarrow PQ = 13 - 10 = 3$$

$$\text{We know } (P+Q)^2 = P^2 + Q^2 + 2PQ$$

$$\Rightarrow P+Q = \sqrt{P^2 + Q^2 + 2PQ} = \sqrt{10 + 2 \times 3}$$

$$\Rightarrow P+Q = \sqrt{16}$$

$$\Rightarrow P+Q = 4 \dots (3)$$

$$\text{Also, we know } (P-Q)^2 = P^2 + Q^2 - 2PQ$$

$$\Rightarrow P-Q = \sqrt{P^2 + Q^2 - 2PQ} = \sqrt{10 - 2 \times 3} = \sqrt{4} = 2$$

$$\Rightarrow P-Q = 2 \dots (4)$$

Adding eqn(3) and eqn(4) we get

$$P+Q = 4 \dots (3)$$

$$P-Q = 2 \dots (4)$$

$$2P = 6 \Rightarrow P = 6/2 = 3 \text{ N}$$

put P value in eqn(4)

$$P-Q = 2$$

$$\Rightarrow Q = P-2 = 3-2$$

$$\Rightarrow Q = 1 \text{ N}$$

- ④ Find the magnitude of two forces such that if they act at right angles, their resultant is 5 N, while when they act on an angle of 60° , their resultant is $\sqrt{37}$ N.

2017 (winter)
Question

Ans Data given

Force (P) = ?

Force (Q) = ?

Case-1

If two forces act on right angle $\theta = 90^\circ$

Resultant (R) = 5 N

Case-2

If two forces act on 60°
i.e. $\theta = 60^\circ$

Resultant = R = $\sqrt{37}$ N

Solution

In case-1

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow 5 = \sqrt{P^2 + Q^2 + 2PQ \cos 90^\circ}$$

$$\Rightarrow (5)^2 = (\sqrt{P^2 + Q^2})^2$$

$$\Rightarrow 25 = P^2 + Q^2 \dots \textcircled{1}$$

In case-2

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow \sqrt{37} = \sqrt{P^2 + Q^2 + 2PQ \cos 60^\circ} = \sqrt{P^2 + Q^2 + 2PQ \times \frac{1}{2}}$$

$$\Rightarrow 37 = P^2 + Q^2 + PQ \dots \textcircled{2}$$

$$\Rightarrow 37 = 25 + PQ \quad (\because P^2 + Q^2 = 25 \text{ from eqn (1)})$$

$$\Rightarrow 37 - 25 = PQ$$

$$\Rightarrow 12 = PQ$$

$$\text{Now, } (P+Q)^2 = P^2 + Q^2 + 2PQ = 25 + 2 \times 12 = 49$$

$$\Rightarrow P+Q = \sqrt{49} = 7$$

$$\Rightarrow P+Q = 7 \dots \textcircled{3}$$

$$\text{Similarly } (P-Q)^2 = P^2 + Q^2 - 2PQ = 25 - 2 \times 12 = 1$$

$$\Rightarrow P-Q = \sqrt{1}$$

$$\Rightarrow P-Q = 1 \dots \textcircled{4}$$

Adding eqn (3) & eqn (4), we get

$$P+Q = 7 \dots \textcircled{3}$$

$$P-Q = 1 \dots \textcircled{4}$$

$$2P = 8$$

$$\Rightarrow P = 8/2 = 4 \text{ N}$$

Put P value in eqn (4), we get

$$P-Q = 1$$

$$\Rightarrow Q = P - 1 = 4 - 1 = 3 \text{ N}$$

- ⑤ The greatest and least resultants of two forces are respectively 17 KN & 3 KN. Determine the angle between the two forces when their resultant is $\sqrt{149}$ KN.

Ans Data given

Here, Greatest resultant means whenever we are adding two forces. i.e. $P + Q = 17 \text{ KN}$

Least resultant means whenever we are subtracting two forces i.e. $P - Q = 3 \text{ KN}$

Angle (θ) = ?

Resultant (R) = $\sqrt{149} \text{ KN}$.

Solution

$$P + Q = 17 \text{ --- (1)}$$

$$P - Q = 3 \text{ --- (2)}$$

Adding these two equations, then we get

$$2P = 20$$

$$\Rightarrow P = 10 \text{ KN}$$

Putting value of P in equation 2, we get

$$P - Q = 3 \Rightarrow Q = P - 3 = 10 - 3 = 7 \text{ KN}$$

Now, we know

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow \sqrt{149} = \sqrt{10^2 + 7^2 + 2 \times 10 \times 7 \times \cos \theta}$$

$$\Rightarrow 149 = 149 + 140 \cos \theta$$

$$\Rightarrow 149 - 149 = 140 \cos \theta$$

$$\Rightarrow 0 = 140 \cos \theta$$

$$\Rightarrow \cos \theta = \frac{0}{140} = 0$$

$$\Rightarrow \cos \theta = \cos 90^\circ$$

$$\Rightarrow \boxed{\theta = 90^\circ} \quad (\text{Ans})$$

2018 (Summer)
OLD BACK
Question

- ⑥ Find the greatest and least resultants of two forces whose magnitudes are 12 N and 8 N respectively.

2017 (Summer)
New Question

Ans. Data given

One Force (P) = 12 N

Second force (Q) = 8 N

Greatest Resultant i.e. $P+Q = ?$

Least resultant i.e. $P-Q = ?$

Solution

Greatest Resultant $(P+Q) = 12+8 = 20 \text{ N}$

Least Resultant $(P-Q) = 12-8 = 4 \text{ N}$

- ⑦ The Resultant of two forces P & Q is R.

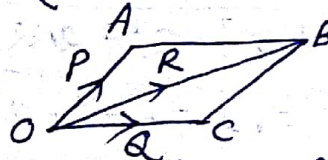
If Q is doubled, then new resultant is perpendicular to P. Prove that $Q=R$.

BOOK PROBLEM

Ans. Data given

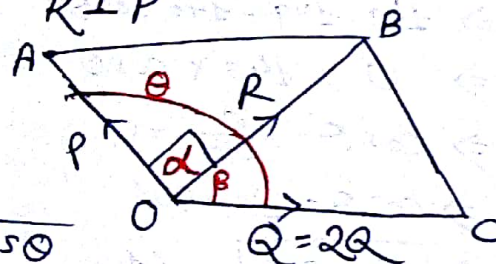
one force is P and other force is Q.

Their resultant is R. (case-1)



Case-2

If Q is double, New resultant is perpendicular to P.
i.e. $Q=2Q$, then $R \perp P$



Solution

We know

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \quad \text{--- (1)}$$

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\Rightarrow \tan 90^\circ = \frac{2Q \sin \theta}{P + 2Q \cos \theta}$$

$$\Rightarrow \frac{1}{0} = \frac{2Q \sin \theta}{P + 2Q \cos \theta}$$

$$\Rightarrow P + 2Q \cos \theta = 0$$

$$\Rightarrow P = -2Q \cos \theta$$

Put this value in eqⁿ (1)

$$R = \sqrt{(-2Q \cos \theta)^2 + Q^2 + 2 \times (-2Q \cos \theta) \times Q \cos \theta}$$

$$\Rightarrow R = \sqrt{4Q^2 \cos^2 \theta + Q^2 - 4Q^2 \cos^2 \theta}$$

$$\Rightarrow R = \sqrt{Q^2}$$

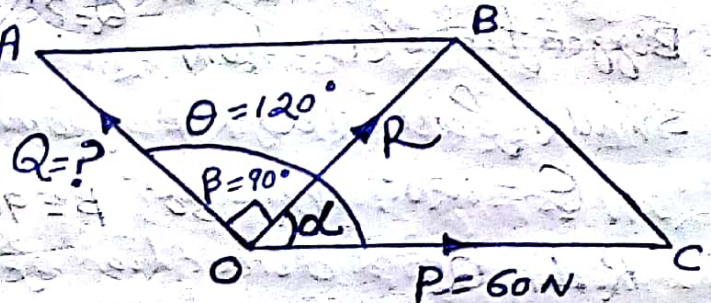
$$\Rightarrow \boxed{R = Q}$$

- ⑧ The Resultant of two concurrent forces is perpendicular to the smaller force and the angle between the forces is 120° . If the bigger force is 60 N , Find the smaller force.

Ans Data given

2017 (WINTER)
Ex-Regular Question

In OABC Parallelogram
P is Bigger force
Q is smaller force.
Resultant (R) is $\perp$ to smaller force.



$$P = 60\text{ N}$$

$$Q = ?$$

$$\theta = 120^\circ$$

$$\alpha = 30^\circ$$

$$\beta = 90^\circ$$

Solution

we know $\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$

$$\Rightarrow \tan 30^\circ = \frac{Q \times \sin 120^\circ}{60 + Q \times \cos 120^\circ}$$

$$\Rightarrow 0.57 = \frac{Q \times 0.86}{60 + Q \times (-0.5)}$$

$$\Rightarrow 0.86 Q = 0.57 \{60 - 0.5 \times Q\}$$

$$\Rightarrow 0.86 Q = 0.57 \times 60 - 0.5 \times Q \times 0.57$$

$$\Rightarrow 0.86 Q = 34.2 - 0.285 Q$$

$$\Rightarrow 0.86 Q + 0.285 Q = 34.2$$

$$\Rightarrow Q(0.86 + 0.285) = 34.2$$

$$\Rightarrow Q \times 1.145 = 34.2$$

$$\Rightarrow Q = \frac{34.2}{1.145} = 29.86\text{ N (Ans)}$$

⑨ Two forces act at angle of 120° . The bigger force is 40N and the resultant is perpendicular to the smaller one. Find the smaller force.

2016(W)
Question

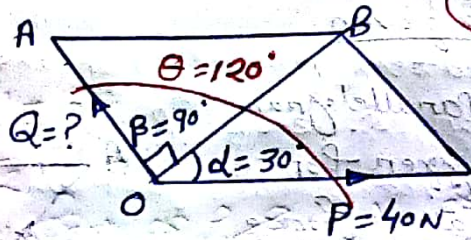
Data given

Bigger force (P) = 40N

Smaller force (Q) = ?

$\theta = 120^\circ$, $\alpha = 30^\circ$, $\beta = 90^\circ$

$R \perp Q$



Solution

We know,

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\Rightarrow \tan 30^\circ = \frac{Q \times \sin 120^\circ}{40 + Q \times \cos 120^\circ}$$

$$\Rightarrow 0.57 = \frac{Q \times 0.86}{40 + Q \times (-0.5)}$$

$$\Rightarrow 0.86Q = 0.57 \{40 - 0.5Q\}$$

$$\Rightarrow 0.86Q = 0.57 \times 40 - 0.57 \times 0.5Q$$

$$\Rightarrow 0.86Q = 22.8 - 0.285Q$$

$$\Rightarrow 0.86Q + 0.285Q = 22.8$$

$$\Rightarrow 1.145Q = 22.8$$

$$\Rightarrow Q = \frac{22.8}{1.145}$$

$$\Rightarrow Q = 19.91 \text{ or } 20\text{N}$$

- ⑩ Two forces equals to $2P$ and P act on a particle. If the 1st is doubled and the second increased by 12 unit. The direction of the resultant is unaltered. Find the value of P .

2017 (summer) Reg. Question

Ans Data given

case-1 Two forces equals to $2P$ & P act on a particle.

i.e. $F_1 = 2P$

$F_2 = P$

case-2 1st is doubled

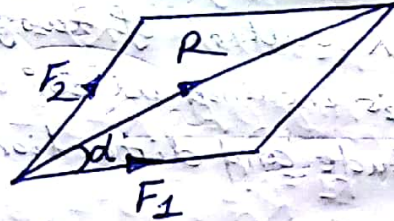
which mean $F_1 = 2 \times 2P = 4P$

2nd is increased 12 unit

which mean $F_2 = P + 12$

As a result, Direction of the resultant is unaltered.

$P = ?$



Solution

In case-1 ; Direction of resultant

$$\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

Formula $\left[\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta} \right]$

$$\Rightarrow \alpha = \tan^{-1} \left[\frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta} \right]$$

$$\Rightarrow \alpha = \tan^{-1} \left[\frac{P \sin \theta}{2P + P \cos \theta} \right] \quad \text{--- (1)}$$

In case-2 : Direction of resultant will be

$$\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\Rightarrow \alpha = \tan^{-1} \left[\frac{(P+12) \sin \theta}{4P + (P+12) \cos \theta} \right] \quad \text{--- (2)}$$

$\therefore$ equation (1) = equation 2

$$\Rightarrow \tan^{-1} \left[\frac{P \sin \theta}{2P + P \cos \theta} \right] = \tan^{-1} \left[\frac{(P+12) \sin \theta}{4P + (P+12) \cos \theta} \right]$$

$$\Rightarrow \frac{P \sin \theta}{2P + P \cos \theta} = \frac{(P+12) \sin \theta}{4P + (P+12) \cos \theta}$$

$$\Rightarrow P [4P + (P+12) \cos \theta] = (P+12) \times (2P + P \cos \theta)$$

$$\Rightarrow 4P^2 + P^2 \cos \theta + 12P \cos \theta = 2P^2 + P^2 \cos \theta + 24P$$

$$\Rightarrow 4P^2 = 2P^2 + 24P$$

$$\Rightarrow 4P^2 - 2P^2 = 24P$$

$$\Rightarrow 2P^2 = 24P$$

$$\Rightarrow \frac{P^2}{P} = \frac{24}{2}$$

$$\Rightarrow \boxed{P = 12\text{ N}}$$

- ⑪ The Resultant of two forces P & 15 N is 20 N .
 20 N is inclined at 60° to the 15 N force. Find the magnitude and direction of P .

DATA GIVEN

IN ODCA Parallelogram

one force = P

Second force

(Q) = 15 N

Resultant (R) = 20 N

$P = ?$

Direction of force P i.e. $\beta = ?$

Solution

In ΔOCA (applying sine Rule)

$$\frac{OA}{\sin \beta} = \frac{AC}{\sin \alpha}$$

$$= \frac{OC}{\sin \{180 - (\alpha + \beta)\}}$$

$$\Rightarrow \frac{15}{\sin \beta} = \frac{P}{\sin \alpha} = \frac{20}{\sin \{180 - (60 + \beta)\}} \quad \text{--- (1)}$$

$$\text{Hence, } \frac{15}{\sin \beta} = \frac{20}{\sin \{180 - (60 + \beta)\}}$$

$$\Rightarrow \frac{\sin (60 + \beta)}{\sin \beta} = \frac{20}{15} = \frac{4}{3}$$

$$\Rightarrow \frac{\sin 60 \times \cos \beta + \cos 60 \times \sin \beta}{\sin \beta} = \frac{4}{3} = 1.33$$

$$\Rightarrow \sin 60 \times \cos \beta + \cos 60 \times \sin \beta = 1.33 \times \sin \beta$$

$$\Rightarrow \frac{\sin 60 \times \cos \beta + \cos 60 \times \sin \beta}{\cancel{\sin \beta} \cos \beta} = \frac{1.33 \times \sin \beta}{\cancel{\sin \beta} \cos \beta}$$

$$\Rightarrow \frac{\sin 60 \times \cos \beta + \cos 60 \times \sin \beta}{\cos \beta} = \frac{1.33 \times \sin \beta}{\cos \beta}$$

(Both side divide $\cos \beta$)

$$\Rightarrow \sin 60 \times \frac{\cos \beta}{\cos \beta} + \cos 60 \times \frac{\sin \beta}{\cos \beta} = 1.33 \times \frac{\sin \beta}{\cos \beta}$$

$$\Rightarrow \sin 60 + \cos 60 \times \tan \beta = 1.33 \tan \beta$$

$$\Rightarrow 0.86 = 1.33 \tan \beta - (-0.95 \tan \beta)$$

$$\Rightarrow 0.86 = 2.28 \tan \beta$$

$$\Rightarrow \tan \beta = \frac{0.86}{2.28} = 0.37$$

$$\Rightarrow \beta = \tan^{-1}(0.37) = 0.35$$

$$\therefore \frac{P}{\sin \alpha} = \frac{20}{\sin(180 - (60 + \beta))}$$

$$\Rightarrow P = \frac{20}{\sin(60 + \beta)} \times \sin \alpha$$

$$\Rightarrow P = \frac{20}{\sin(60 + 0.35)} \times \sin 60^\circ$$

$$\Rightarrow P = 9.94 \text{ N}$$

- 12) Two forces whose magnitude are in ratio 3:5. gives a resultant 28 N. If their angle of inclination is 60° . Find the magnitude of each force.

BOOK PROBLEM

Ans

Data given

Let one force (F_1) = $3x$

second force (F_2) = $5x$

Resultant (R) = 28 N

Angle of inclination (θ) = 60°

$F_1 = ?$

$F_2 = ?$

Solution

We know, $R = \sqrt{P^2 + Q^2 + 2PQ \times \cos \theta}$

$$\Rightarrow 28 = \sqrt{F_1^2 + F_2^2 + 2 \times F_1 \times F_2 \times \cos 60}$$

$$\Rightarrow 28 = \sqrt{(3x)^2 + (5x)^2 + 2 \times 3x \times 5x \times \frac{1}{2}}$$

$$\Rightarrow 28 = \sqrt{9x^2 + 25x^2 + 30x^2 \times \frac{1}{2}}$$

$$\Rightarrow 28 = \sqrt{49x^2}$$

$$\Rightarrow 28 = 7x$$

$$\Rightarrow x = \frac{28}{7} = 4$$

$$\therefore \text{Force } (F_1) = 3x = 3 \times 4 = 12 \text{ N}$$

$$\therefore \text{Force } (F_2) = 5x = 5 \times 4 = 20 \text{ N}$$

- ⑬ Find the magnitude & direction of force which when combine with an 8N vertical force & with 6N Horizontal force.

2015 year
(winter)

Ans Data given

Force (P) = 8N

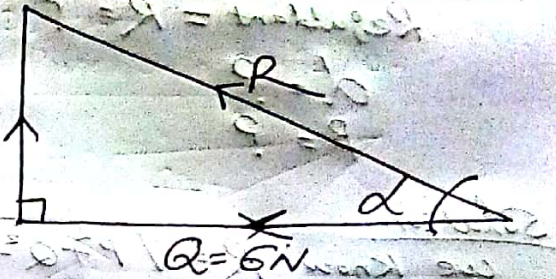
Force (Q) = 6N

Resultant (R) = ?

Direction (α) = ?

P = 8N

Q = 6N



Solution

We know $R = \sqrt{P^2 + Q^2}$

$\Rightarrow R = \sqrt{8^2 + 6^2}$

$\Rightarrow R = \sqrt{64 + 36}$

$\Rightarrow R = \sqrt{100} = 10 \text{ N}$

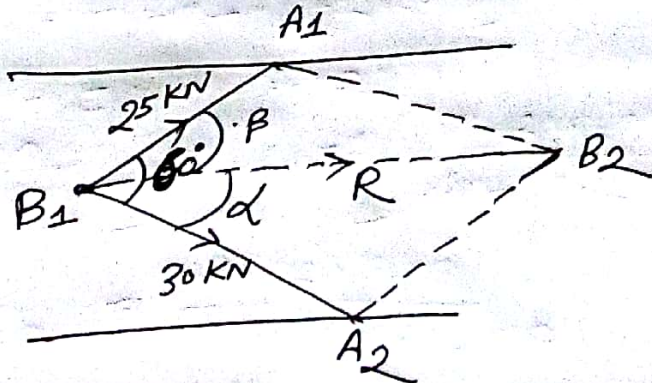
$\therefore \tan \alpha = P/Q$

$\Rightarrow \alpha = \tan^{-1}(P/Q)$

$\Rightarrow \alpha = \tan^{-1}(8/6)$

$\Rightarrow \alpha = 53.13^\circ$

- ⑭ Two person on opposite sides of a canal, pull a boat moving parallel to the sides of a canal, by means of two horizontal ropes. The tension in the ropes are found to be 25 kN & 30 kN, when the angle between them is 60° . Determine the resultant pull on the boat and the angle between each rope & the sides of the canal.



Data given

$$\text{Force (P)} = 30 \text{ kN}$$

$$\text{Force (Q)} = 25 \text{ kN}$$

$$\text{Angle } (\theta) = 60^\circ$$

$$\text{Resultant} = R = ?$$

$$\alpha = ?$$

$$\beta = ?$$

Solution

$$\text{we know } R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow R = \sqrt{30^2 + 25^2 + 2 \times 30 \times 25 \times \cos 60^\circ}$$

$$\Rightarrow R = 9.81 \text{ kN}$$

Direction of resultant

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{25 \sin 60}{30 + 25 \cos 60} \right)$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{21.65}{42.5} \right)$$

$$\Rightarrow \alpha = 0.47^\circ$$

$$\beta = \theta - \alpha = 60 - 0.47^\circ = 59.53^\circ$$

- ⑮ Two forces of 80 N & 70 N act simultaneously at a point. Find the resultant of forces if an angle between is 150° .

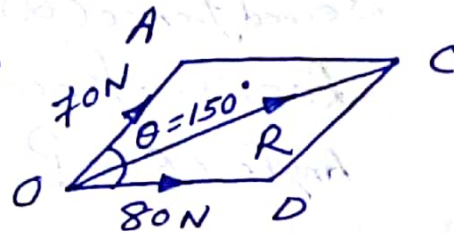
Ans Data given

One force (P) = 80 N

Second force (Q) = 70 N

Resultant (R) = ?

Angle (θ) = 150°



Solution

We know that $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

$$\Rightarrow R = \sqrt{80^2 + 70^2 + 2 \times 80 \times 70 \times \cos 150^\circ}$$

$$\Rightarrow R = 138.31 \text{ N}$$

- ⑯ Find the resultant of two forces equals to 50 N & 30 N acting at angle 60° .

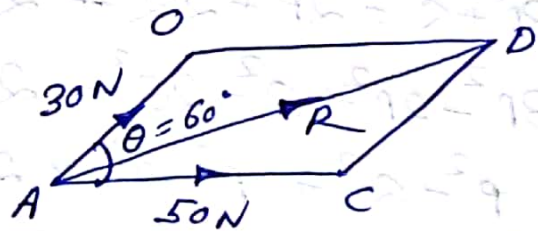
Ans Data given

One force (P) = 50 N

Second force (Q) = 30 N

Angle (θ) = 60°

Resultant force (R) = ?



Solution

We know $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

$$= \sqrt{50^2 + 30^2 + 2 \times 50 \times 30 \times \cos 60^\circ}$$

$$= \sqrt{2500 + 900 + 3000 \cos 60^\circ}$$

$$= 23.29 \text{ N.}$$

(17) The Resultant of two forces i.e. $P+Q$ & $P-Q$ is $\sqrt{3P^2+Q^2}$. Find Angle between of two forces.

Ans Data given

One Force (P) = $P+Q$

Second force (Q) = $P-Q$

Resultant force (R) = $\sqrt{3P^2+Q^2}$

Angle (θ) = ?

Solution

We know

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow \sqrt{3P^2 + Q^2} = \sqrt{(P+Q)^2 + (P-Q)^2 + 2 \times (P+Q) \times (P-Q) \times \cos \theta}$$

$$\Rightarrow 3P^2 + Q^2 = (P+Q)^2 + (P-Q)^2 + 2 \times (P+Q) \times (P-Q) \times \cos \theta$$

$$\Rightarrow 3P^2 + Q^2 = P^2 + Q^2 + 2 \times P \times Q + P^2 + Q^2 - 2 \times P \times Q + 2(P+Q) \times (P-Q) \cos \theta$$

$$\Rightarrow 3P^2 + Q^2 = 2P^2 + 2Q^2 + 2(P+Q)(P-Q) \cos \theta$$

$$\Rightarrow 3P^2 - 2P^2 + Q^2 - 2Q^2 = 2\{P^2 - Q^2\} \times \cos \theta$$

$$\Rightarrow P^2 - Q^2 = P^2 - Q^2 \times 2 \cos \theta$$

$$\Rightarrow \frac{P^2 - Q^2}{P^2 - Q^2} = 2 \cos \theta$$

$$\Rightarrow 1 = 2 \cos \theta$$

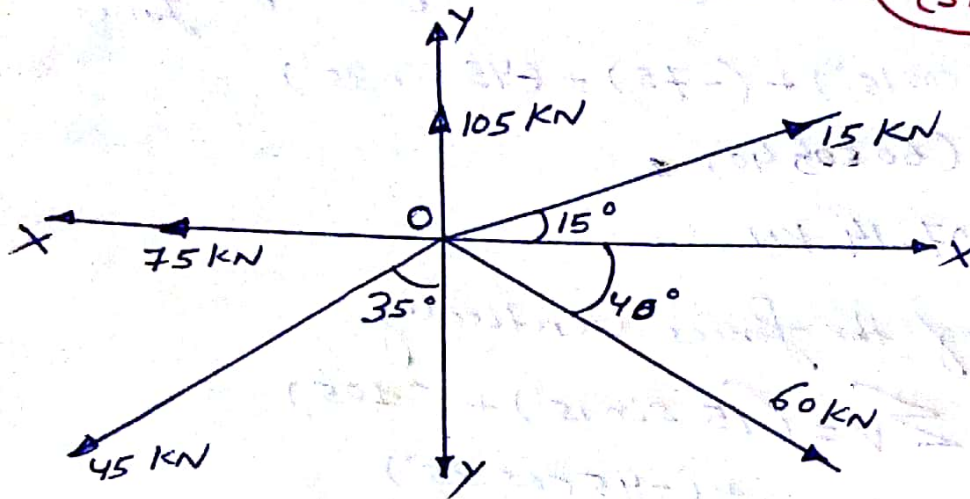
$$\Rightarrow \frac{1}{2} = \cos \theta$$

$$\Rightarrow \cos 60^\circ = \cos \theta$$

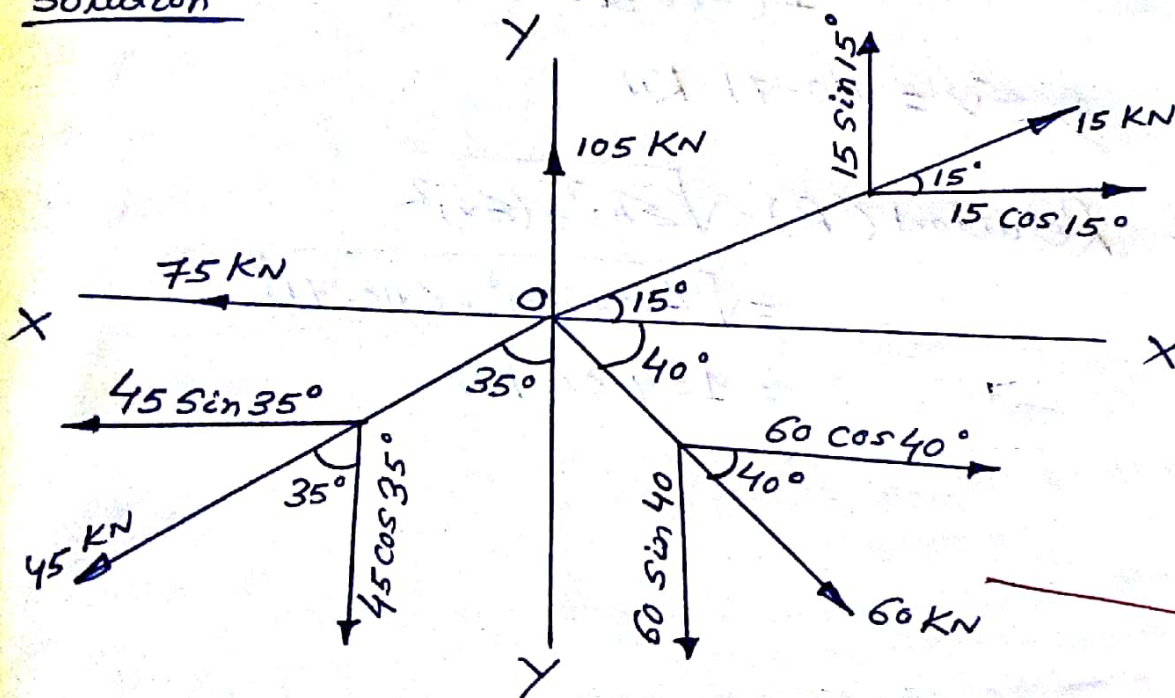
$$\Rightarrow \boxed{\theta = 60^\circ}$$

- ⑮ If Five forces act on a particle as shown in figure, determine the resultant of the forces.

2019 year
(Summer) Question



Solution



Now, see the all horizontal force

$$15 \cos 15^\circ \text{ (+ve sign)}$$

$$75 \text{ kN (-ve sign)}$$

$$45 \sin 35^\circ \text{ (-ve sign)}$$

$$60 \cos 40^\circ \text{ (+ve sign)}$$

See the all vertical force

$$15 \sin 15^\circ \text{ (+ve sign)}$$

$$105 \text{ kN (+ve sign)}$$

$$45 \cos 35^\circ \text{ (-ve sign)}$$

$$60 \sin 40^\circ \text{ (-ve sign)}$$

Resolving all horizontal force. i.e.

ΣH (Here only adding horizontal force)

$$\Rightarrow (15 \cos 15^\circ) + (-75) + (-45 \sin 35^\circ) + (60 \cos 40^\circ)$$

$$= -107.14 \text{ KN}$$

Resolving the forces vertically,

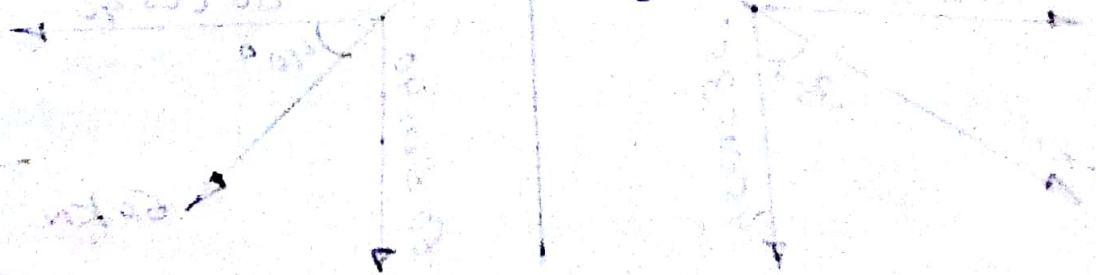
$$\Sigma V = (15 \sin 15^\circ) + (105) + (-45 \cos 35^\circ) + (-60 \sin 40^\circ)$$

$$\Rightarrow \Sigma V = 110.71 \text{ KN}$$

Now, Resultant (R) = $\sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$= \sqrt{(-107.14)^2 + (110.71)^2}$$

$$= 154.06 \text{ KN}$$



①9 The following forces act at a point.

(a) 20 N inclined at 30° towards North of East.

(b) 25 N towards North.

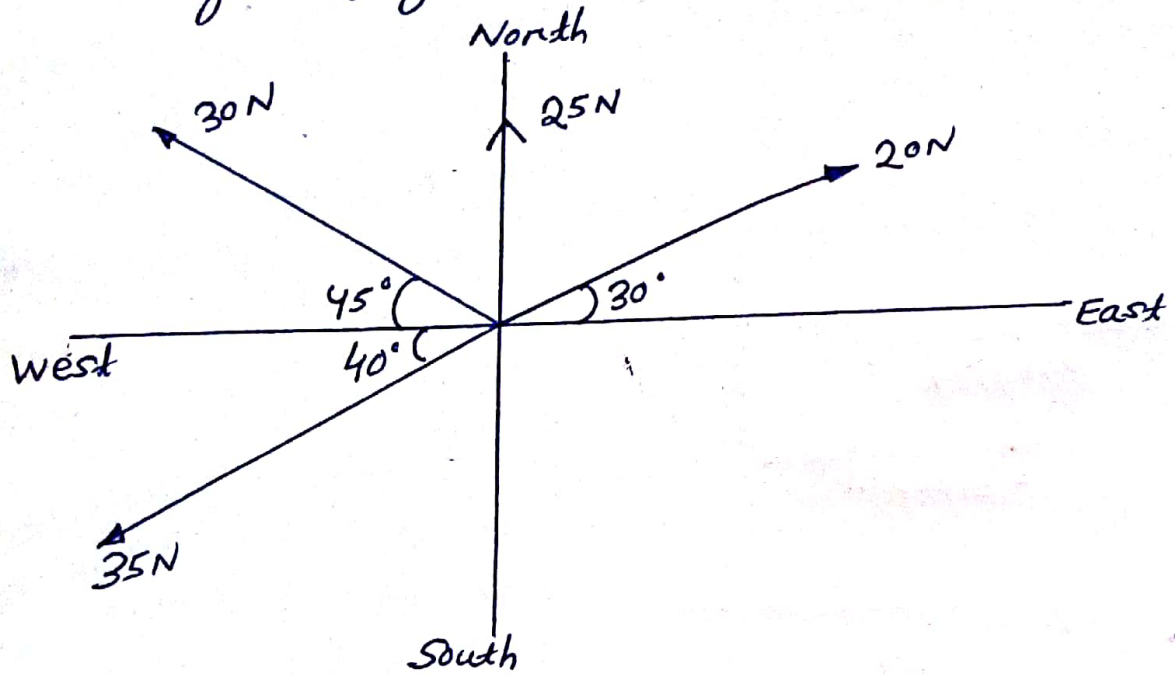
(c) 30 N towards North-West

(d) 35 N inclined at 40° towards South of West.

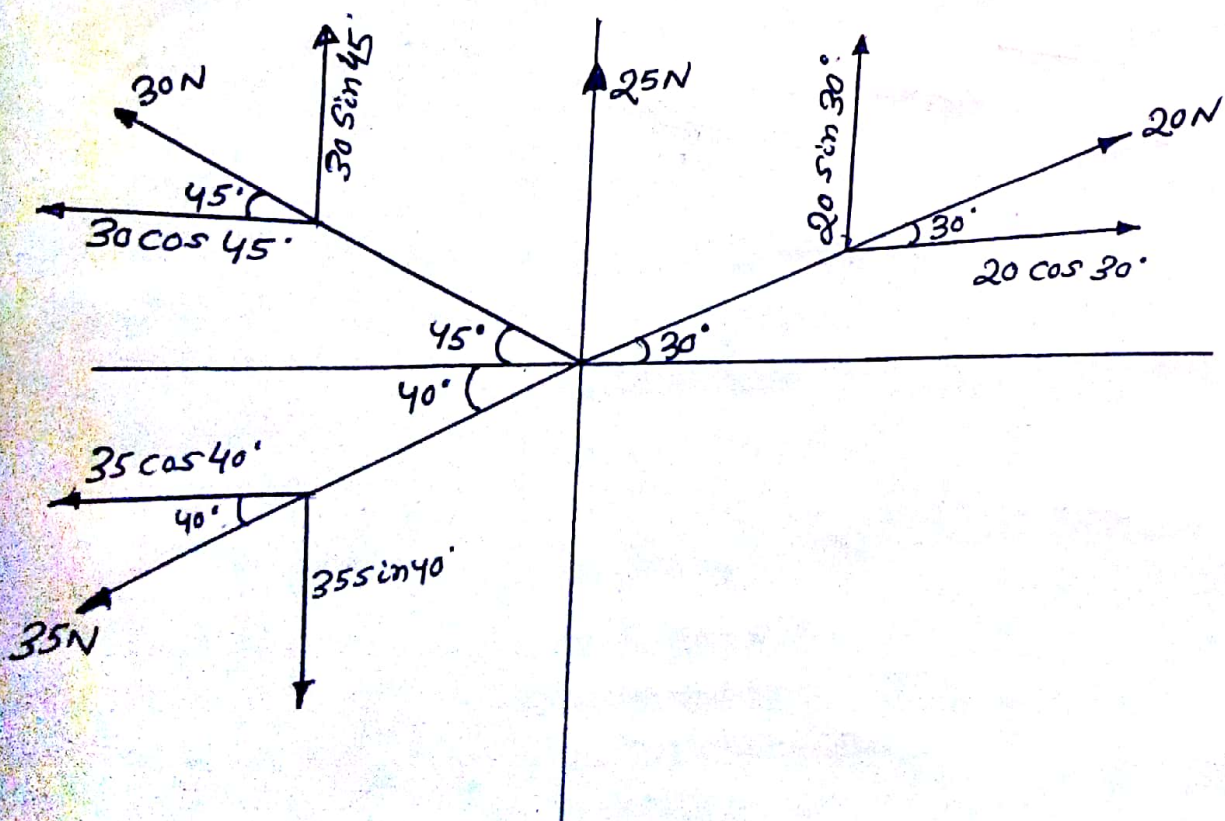
Find the magnitude & direction of resultant force.

Solution

The system of given forces is shown in figure.



Now, solving this figure.



Now, See the all horizontal force.

$$20 \cos 30 \text{ (+ve)}$$

$$30 \cos 45 \text{ (-ve)}$$

$$35 \cos 40 \text{ (-ve)}$$

Now, see all vertical force.

$$20 \sin 30 \text{ (+ve)}$$

$$25 \text{ N (+ve)}$$

$$30 \sin 45 \text{ (+ve)}$$

$$35 \sin 40 \text{ (-ve)}$$

Resolving all horizontal force

$$\Sigma H = (20 \cos 30) + (-30 \cos 45) + (-35 \cos 40)$$

$$\Rightarrow \Sigma H = -38.75 \text{ N}$$

Resolving all vertical force

$$\Sigma V = (20 \sin 30) + (25) + (30 \sin 45) + (-35 \sin 40)$$
$$= 33.72 \text{ N}$$

$$\therefore \text{Resultant (R)} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(-38.75)^2 + (33.72)^2}$$

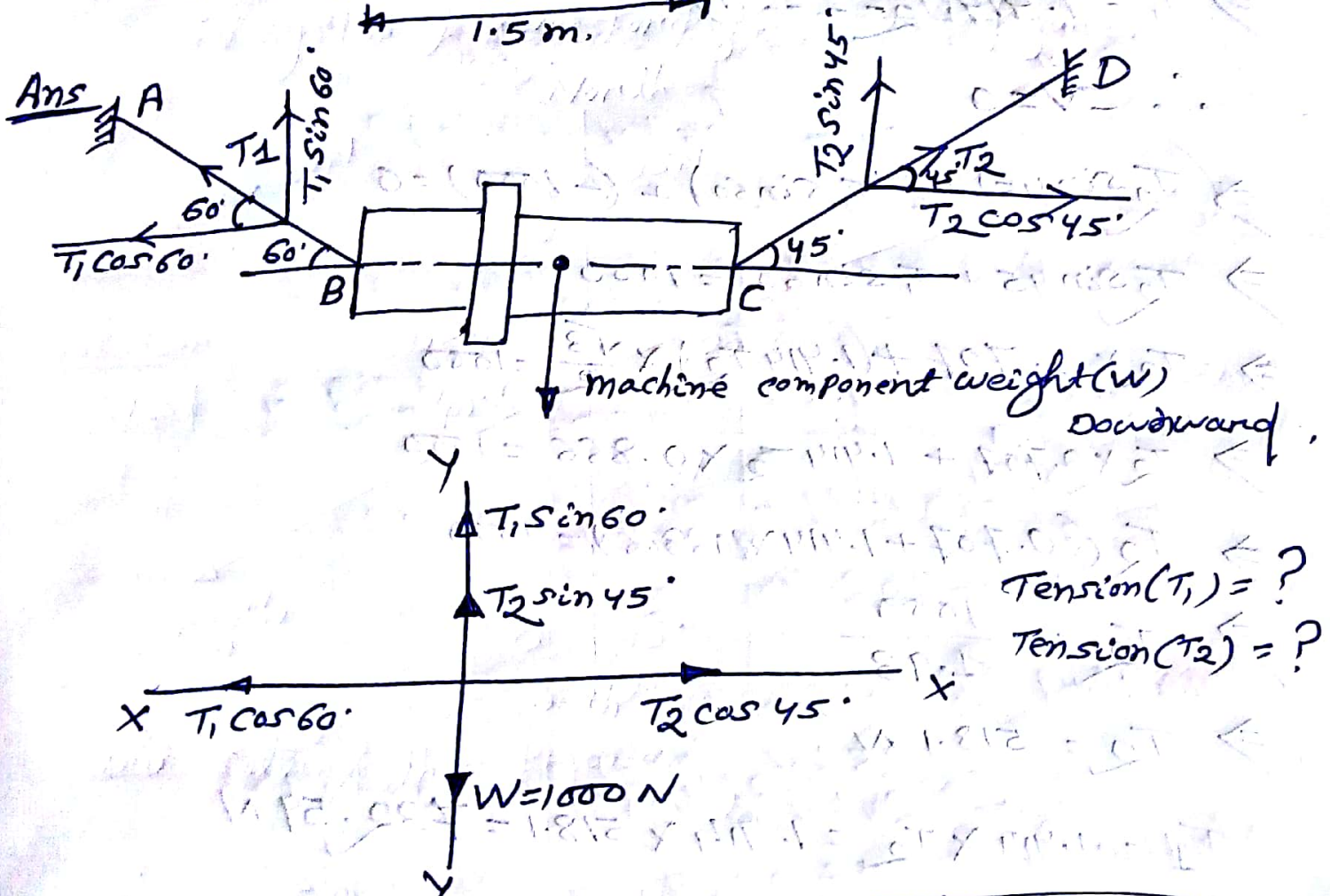
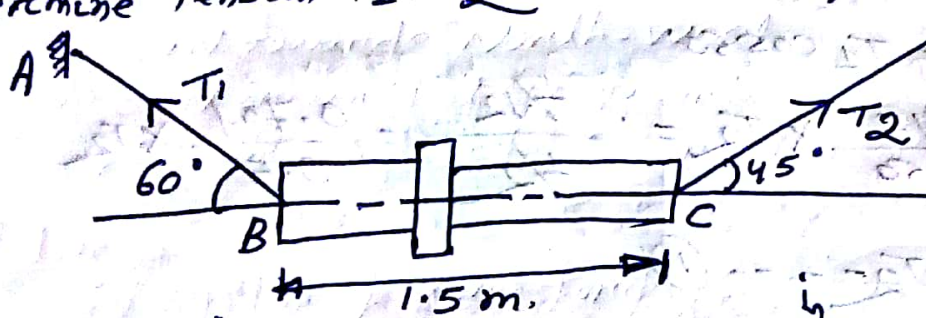
$$= 51.36 \text{ N}$$

$$\text{Direction } (\theta) = \tan^{-1} \left(\frac{\Sigma V}{\Sigma H} \right)$$

$$= \tan^{-1} \left(\frac{33.72}{-38.75} \right)$$

$$=$$

②0 A machine component 1.5 m long and weight 1000 N is supported by two ropes AB & CD as shown in figure. Determine Tension T_1 & T_2 in the rope AB & CD.



$$\therefore \sum H = 0$$

$$\Rightarrow (T_2 \cos 45) + (-T_1 \cos 60) = 0$$

$$\Rightarrow T_2 \cos 45 = T_1 \cos 60$$

$$\Rightarrow T_1 = \frac{\cos 45}{\cos 60} \times T_2 = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{0.707}{0.5} \times T_2$$

$$\Rightarrow T_1 = 1.414 T_2 \text{ --- (1)}$$

$$\therefore \sum V = 0$$

$$\Rightarrow (T_2 \sin 45) + (T_1 \sin 60) + (-1000) = 0$$

$$\Rightarrow T_2 \sin 45 + T_1 \sin 60 = 1000$$

$$\Rightarrow T_2 \times 0.707 + (1.414 T_2) \times \frac{\sqrt{3}}{2} = 1000$$

$$\Rightarrow T_2 \times 0.707 + 1.414 T_2 \times 0.866 = 1000$$

$$\Rightarrow T_2 (0.707 + 1.414 \times 0.866) = 1000$$

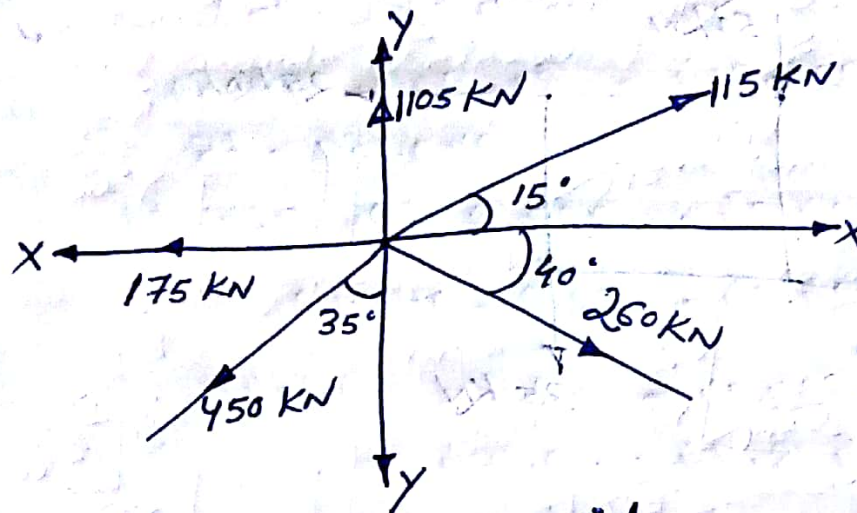
$$\Rightarrow T_2 = \frac{1000}{1.93}$$

$$\Rightarrow T_2 = 518.1 \text{ N}$$

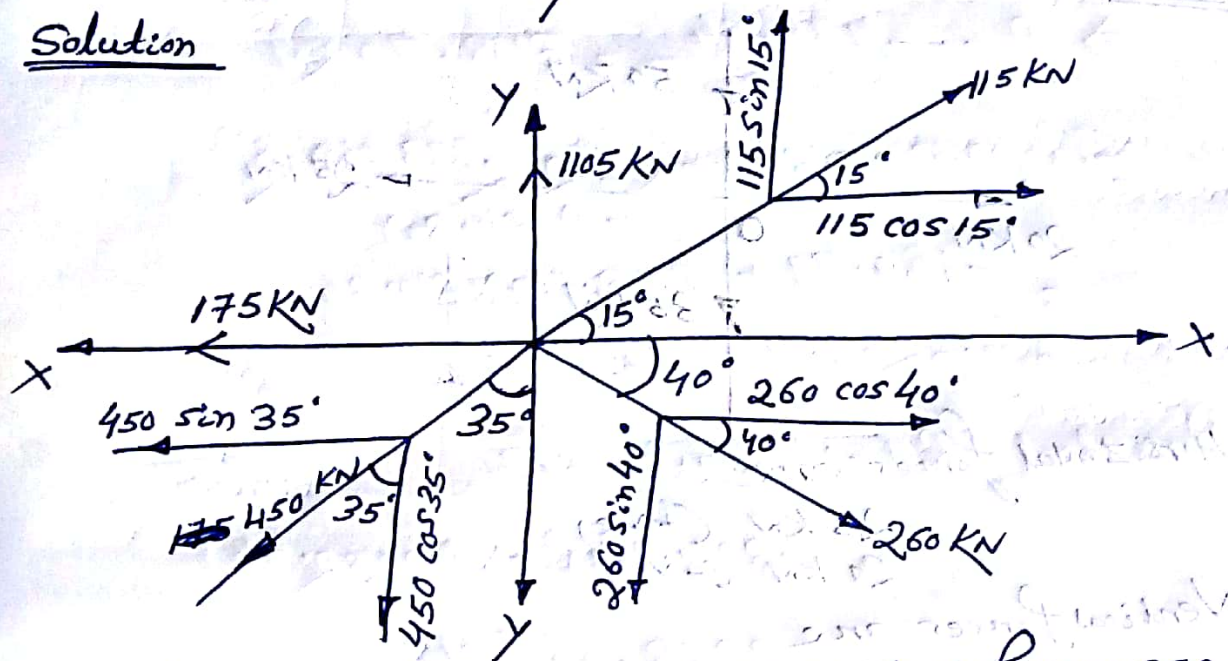
$$T_1 = 1.414 \times T_2 = 1.414 \times 518.1 = 732.59 \text{ N}$$

(Ans)

21) If Five Forces act on a particle as shown in figure. Determine the resultant force & it's direction.



Solution



The horizontal forces are:

$$\begin{aligned} &115 \cos 15^\circ (+ve) \\ &175 \text{ kN } (-ve) \\ &450 \sin 35^\circ (-ve) \\ &260 \cos 40^\circ (+ve) \end{aligned}$$

The Vertical forces are

$$\begin{aligned} &115 \sin 15^\circ (+ve) \\ &1105 \text{ kN } (+ve) \\ &450 \cos 35^\circ (-ve) \\ &260 \sin 40^\circ (-ve) \end{aligned}$$

$$\text{Now, } \Sigma H = (115 \cos 15^\circ) + (-175) + (-450 \sin 35^\circ) + (260 \cos 40^\circ)$$

$$= -243.08 \text{ kN}$$

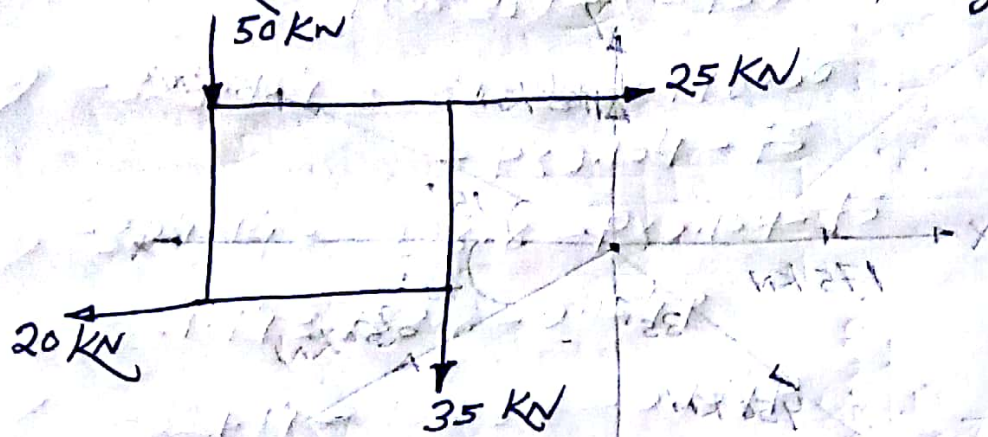
$$\Sigma V = (115 \sin 15^\circ) + (1105) + (-450 \cos 35^\circ) + (-260 \sin 40^\circ)$$

$$= 1392.71 \text{ kN}$$

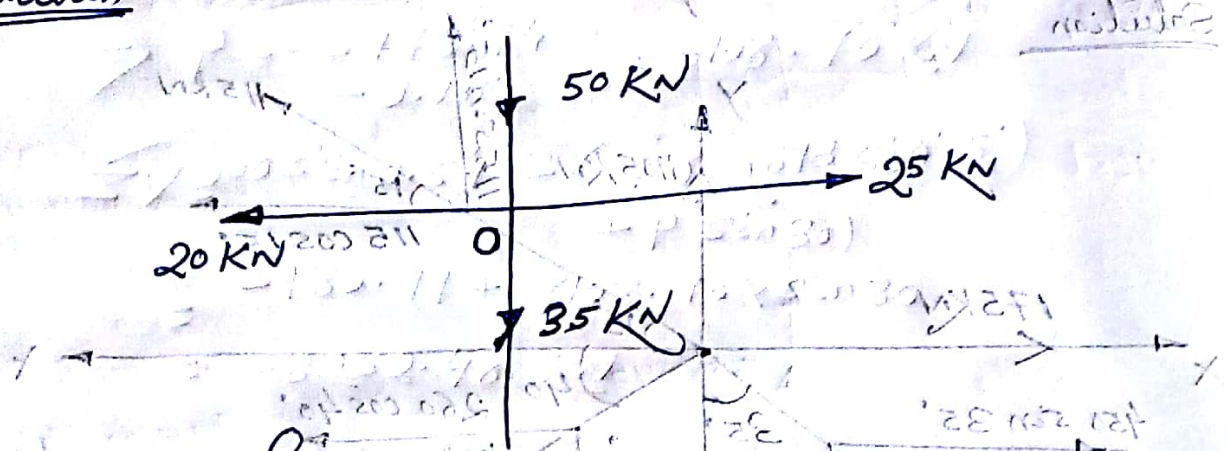
$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-243.08)^2 + (1392.71)^2} = 1413.76 \text{ kN}$$

$$\text{Direction } (\theta) = \tan^{-1} \left(\frac{\Sigma V}{\Sigma H} \right) = \tan^{-1} \left(\frac{1392.71}{-243.08} \right) = -1.39^\circ$$

② A system of forces are acting the corners of a rectangular block as shown in diagram. Find Resultant force.



Solution



Horizontal forces are

$$25 \text{ kN (+ve)}$$

$$20 \text{ kN (-ve)}$$

Vertical forces are

$$50 \text{ kN (-ve)}$$

$$35 \text{ kN (-ve)}$$

Now Resolving the forces horizontally

$$\Sigma H = (25)^2 + (-20)^2 = 25 - 20 = 5 \text{ kN}$$

Resolving the forces vertically

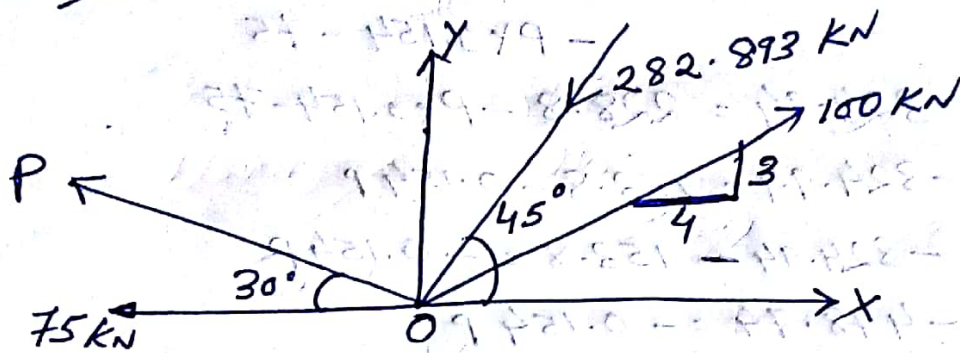
$$\Sigma V = (-50) + (-35) = -85 \text{ kN}$$

$$\text{Resultant (R)} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(5)^2 + (-85)^2}$$

$$= 85.15 \text{ kN}$$

- 23) If five forces act on a particle as shown in figure. & the algebraic sum of horizontal component of all these forces is -324.904 kN . Calculate the value of P & the resultant of all forces.

2018(W)
Question

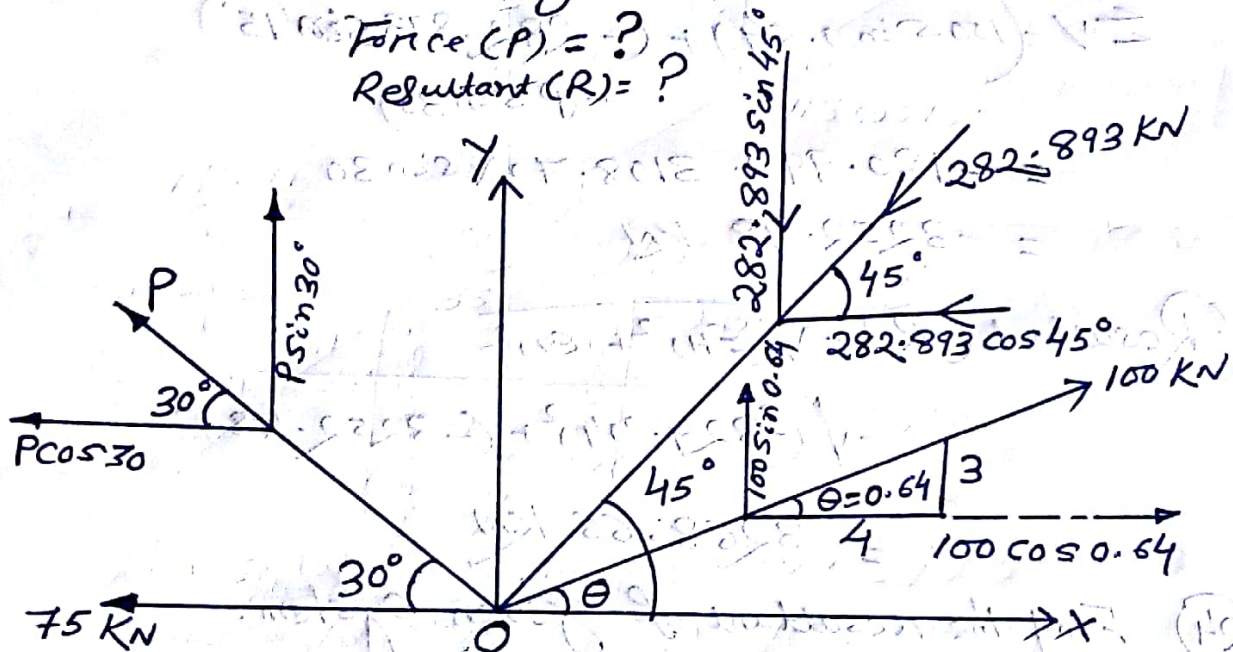


Solution

ΣH Value is given -324.904 kN

Force (P) = ?

Resultant (R) = ?



Horizontal forces are

$$100 \cos 0.64 \text{ (+ve)}$$

$$282.893 \cos 45^\circ \text{ (+ve)}$$

$$P \cos 30^\circ \text{ (-ve)}$$

$$75 \text{ kN (-ve)}$$

$$\begin{cases} \tan \theta = P/b = 3/4 \\ \Rightarrow \theta = \tan^{-1}(3/4) \\ \Rightarrow \theta = 0.64 \end{cases}$$

Vertical forces are

$$100 \sin 0.64 \text{ (+ve)}$$

$$282.893 \sin 45^\circ \text{ (-ve)}$$

$$P \sin 30^\circ \text{ (+ve)}$$

Now, $\sum H = (100 \cos 0.64) + (282.893 \cos 45)$
 $(-P \cos 30) + (-75)$

$\Rightarrow -324.904 = 80.20 + 148.60$
 $- P \times 0.154 - 75$

$\Rightarrow -324.94 = 228.8 - P \times 0.154 - 75$

$\Rightarrow -324.94 = 153.8 - 0.154 P$

$\Rightarrow -324.94 - 153.8 = -0.154 P$

$\Rightarrow -478.74 = -0.154 P$

$\Rightarrow P = \frac{-478.74}{-0.154} = 3108.70 \text{ KN}$

$\sum V = (100 \sin 0.64) + (-282.893 \sin 45)$
 $+ (P \sin 30)$

$= -180.99 + 3108.70 \times \sin 30$

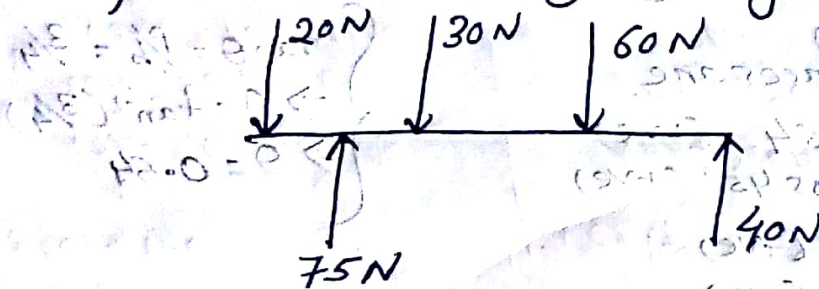
$= -3252.48 \text{ KN}$

Resultant (R) = $\sqrt{(\sum H)^2 + (\sum V)^2}$

$= \sqrt{(-324.904)^2 + (-3252.48)^2}$

$= 3268.66 \text{ KN}$

(24) Find the resultant of given system.



Solution here no horizontal force present.

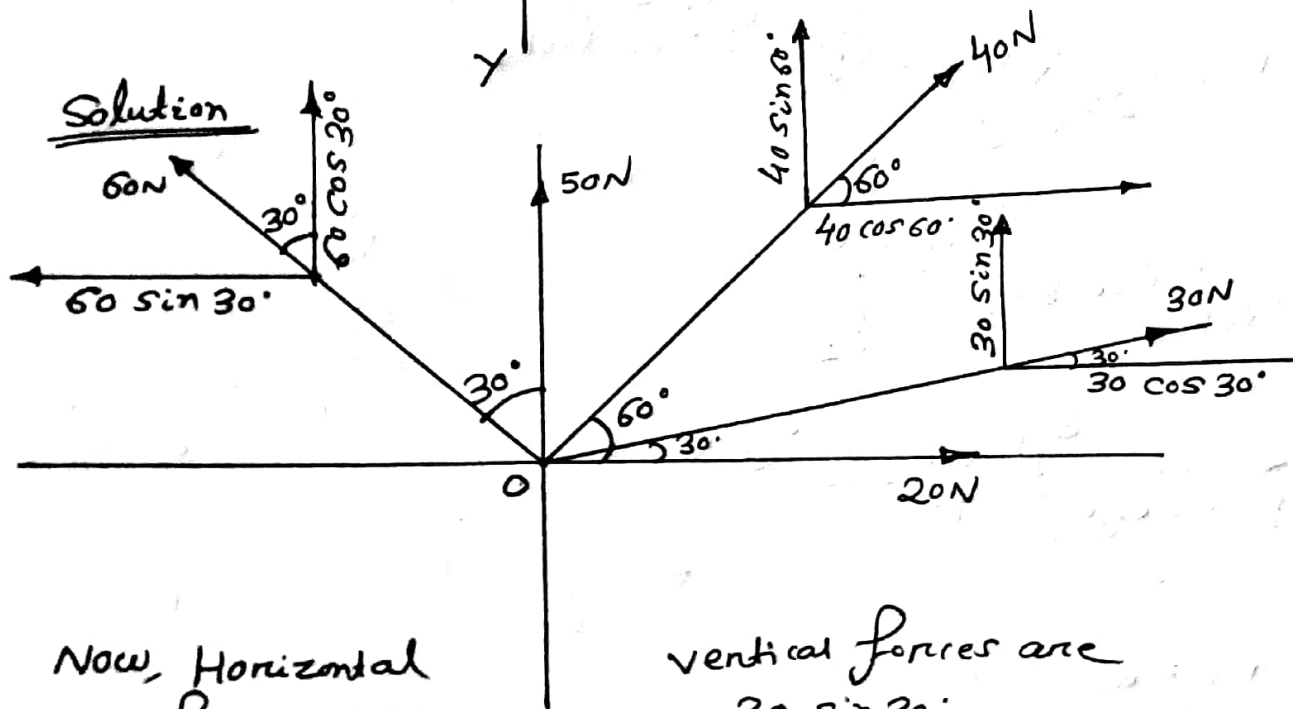
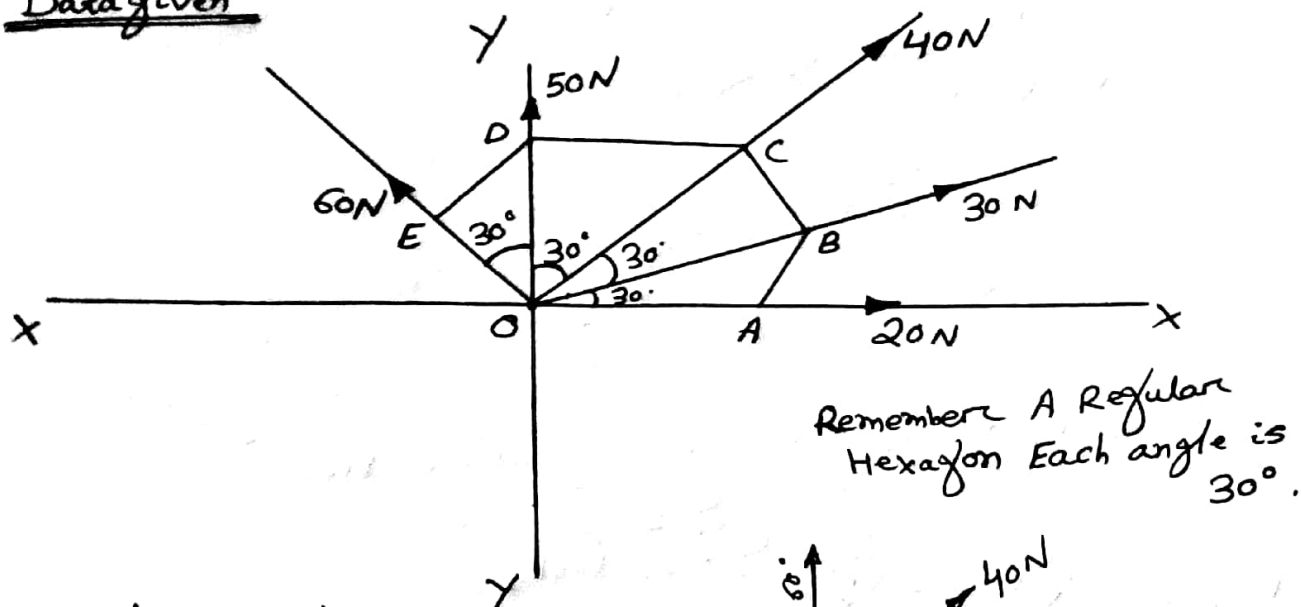
So, $R = \sqrt{(\sum H)^2 + (\sum V)^2}$

$\Rightarrow R = \sum V$

$\Rightarrow R = (40) + (75) + (-20)$
 $+ (-30) + (-60) = 5 \text{ N}$

(25) The forces 20 N, 30 N, 40 N, 50 N, 60 N are acting at one point of a regular hexagon towards the other five angular points taken in order. Find the magnitude and direction of the resultant force.

Data given



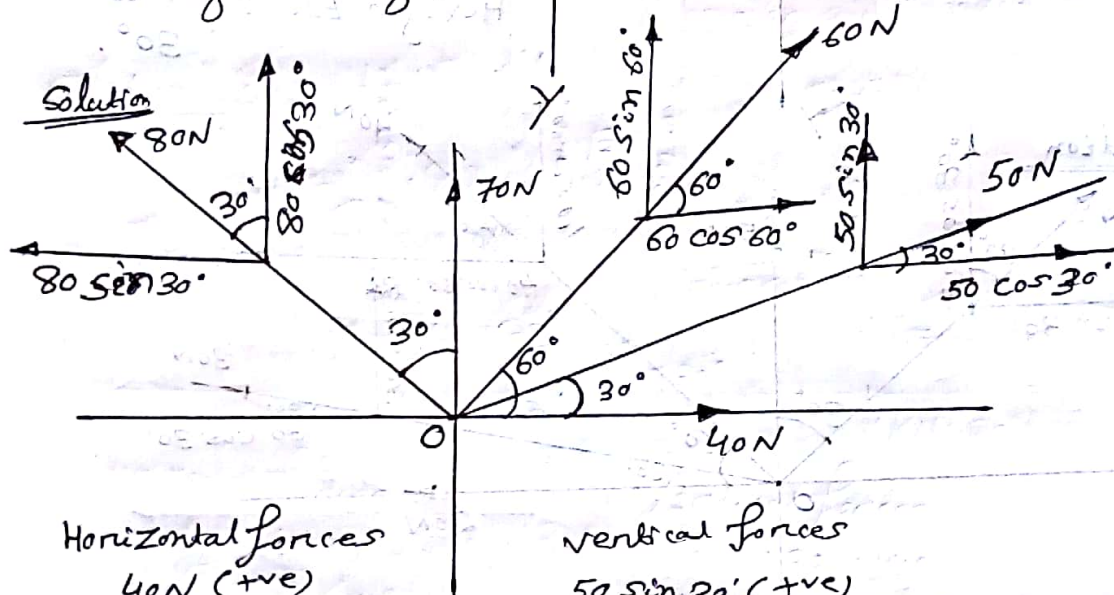
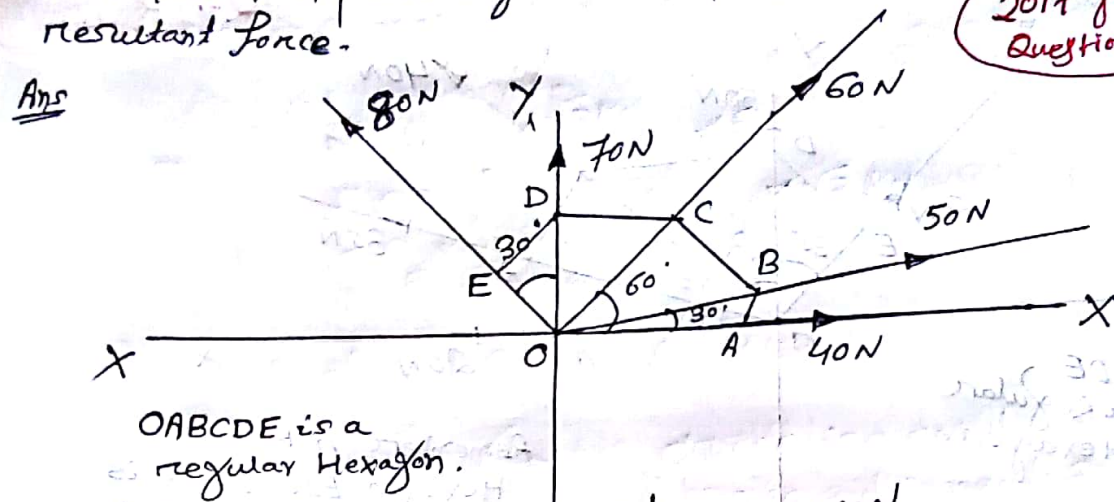
Now, Horizontal forces are
 20 N
 $30 \cos 30^\circ$
 $40 \cos 60^\circ$
 $60 \sin 30^\circ$

vertical forces are
 $30 \sin 30^\circ$
 $40 \sin 60^\circ$
 50 N
 $60 \cos 30^\circ$

- 26) The forces 40 N, 50 N, 60 N, 70 N and 80 N are acting at one of the angular points of a regular hexagon towards the other five angular points taken in order. Find the magnitude and direction of the resultant force.

2017 year Question

Ans



Horizontal forces
 40 N (+ve)
 $50 \cos 30^\circ$ (+ve)
 $60 \cos 60^\circ$ (+ve)
 $80 \cos 30^\circ$ (-ve)

Vertical forces
 $50 \sin 30^\circ$ (+ve)
 $60 \sin 60^\circ$ (+ve)
 $80 \sin 30^\circ$ (+ve)
 70 N (+ve)

$$\Sigma H = (40) + (50 \cos 30^\circ) + (60 \cos 60^\circ) + (80 \cos 30^\circ)$$

$$\Rightarrow \Sigma H = \cancel{21.77} \text{ N } 69.61 \text{ N}$$

$$\Sigma V = (50 \sin 30^\circ) + (60 \sin 60^\circ) + (70) + (80 \sin 30^\circ)$$

$$\Rightarrow \Sigma V = \cancel{76.73} \text{ N } 14.64$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(\cancel{21.77})^2 + (\cancel{76.73})^2}$$

$$= \cancel{79.75} \text{ N } 71.13 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{\Sigma V}{\Sigma H} \right) = \tan^{-1} \left(\frac{\cancel{76.73}}{\cancel{21.77}} \right) = 11.87^\circ$$

- (27) Two forces one of which is double the other, has resultant of 260 N. If the direction of the large force is reversed and the other remain unaltered, the resultant reduces to 180 N. Determine the magnitude of the forces and the angle between them.

2014 year

Ans Data given

Case-1

One force (F_1) = P

Second force (F_2) = 2P

Resultant (R) = 260 N

Case-2

$F_1 = P$

$F_2 = -2P$ (As it is reversed sign is changed)

R = 180 N (Reduce)

∴ Magnitude of two forces i.e. $F_1 = ?$ & $F_2 = ?$
Angle (θ) = ?

Solution

Solving case-1:

We know $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$\Rightarrow (260)^2 = \sqrt{P^2 + (2P)^2 + 2 \times P \times 2P \times \cos\theta}$$

$$\Rightarrow (260)^2 = P^2 + 4P^2 + 4P^2 \times \cos\theta$$

$$\Rightarrow 260^2 = 5P^2 + 4P^2 \times \cos\theta \quad \dots (1)$$

For solving case-2

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\Rightarrow 180 = \sqrt{P^2 + (-2P)^2 + 2 \times P \times (-2P) \times \cos\theta}$$

$$\Rightarrow (180)^2 = P^2 + 4P^2 - 4P^2 \times \cos\theta$$

$$\Rightarrow 180^2 = 5P^2 - 4P^2 \cos\theta \quad \dots (2)$$

Adding both equation.

$$260^2 = 5P^2 + 4P^2 \times \cos\theta$$

$$180^2 = 5P^2 - 4P^2 \cos\theta$$

$$260^2 + 180^2 = 10P^2$$

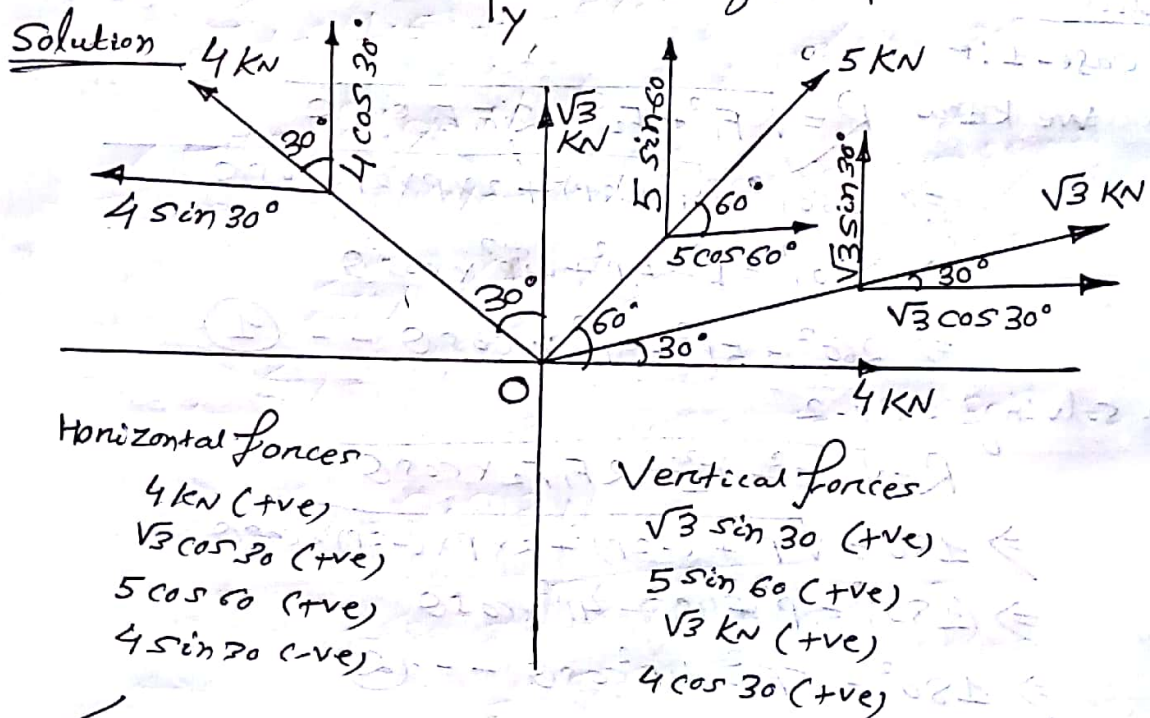
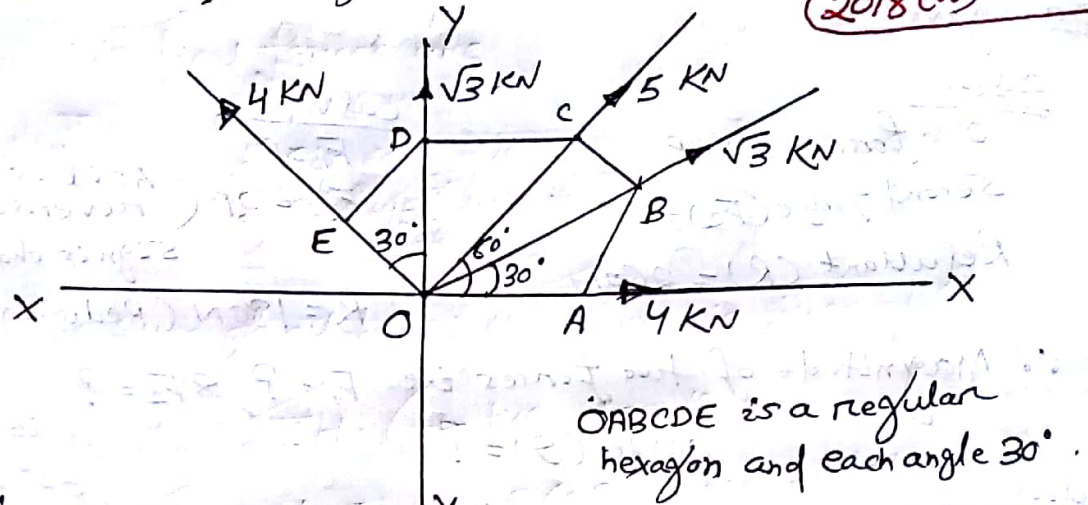
$$\Rightarrow P^2 = \frac{260^2 + 180^2}{10}$$

$$\Rightarrow P = \sqrt{\frac{260^2 + 180^2}{10}} = 100 \text{ N}$$

$F_1 = P = 100 \text{ N}$ & $F_2 = 2P = 2 \times 100 = 200 \text{ N}$
Putting the value of P in eqn(1), we get Angle θ

- (28) Five forces of $4, \sqrt{3}, 5, \sqrt{3}$ and 4 kN respectively act at one of the angular points of a regular hexagon towards five other angular points. Find Analytically the direction & magnitude of resultant force.

2018 (W) OLD Back



$$\sum H = (4) + (\sqrt{3} \cos 30^\circ) + (5 \cos 60^\circ) + (4 \sin 30^\circ)$$

$$= 3.45 \text{ kN}$$

$$\sum V = \sqrt{3} \sin 30^\circ + 5 \sin 60^\circ + \sqrt{3} + 4 \cos 30^\circ$$

$$= -0.88 \text{ kN}$$

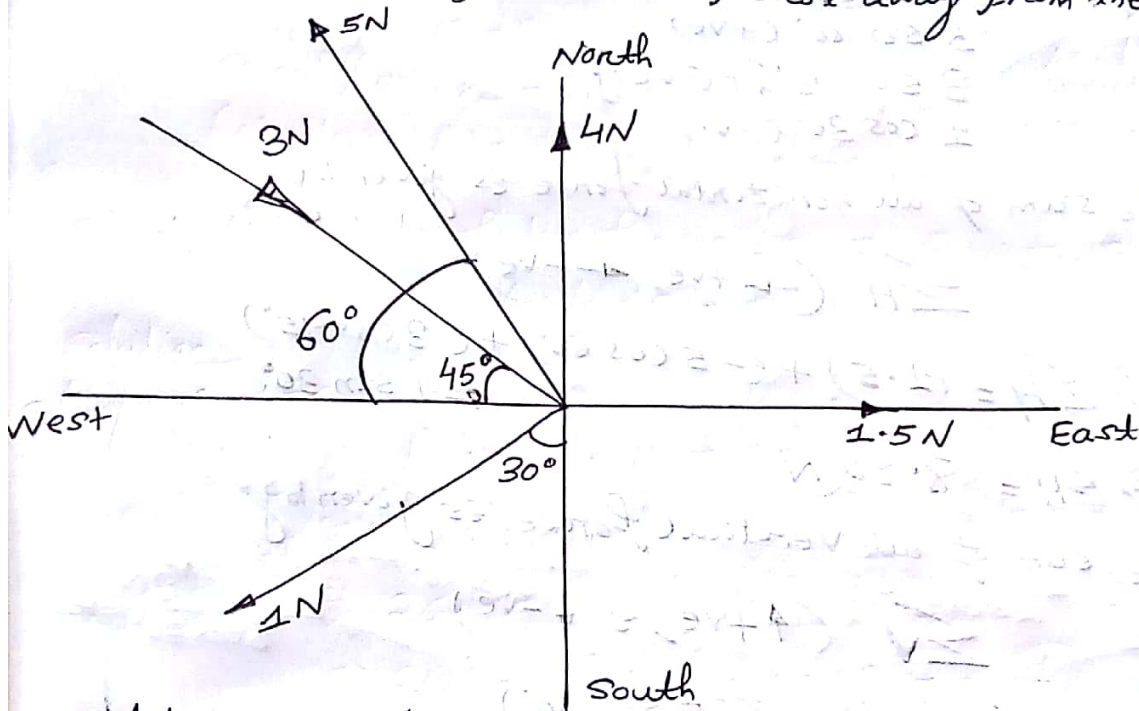
$$R = \sqrt{EH^2 + EV^2} = 3.56 \text{ kN}$$

$$\theta = \tan^{-1} \left(\frac{EV}{EH} \right) = -14.30^\circ$$

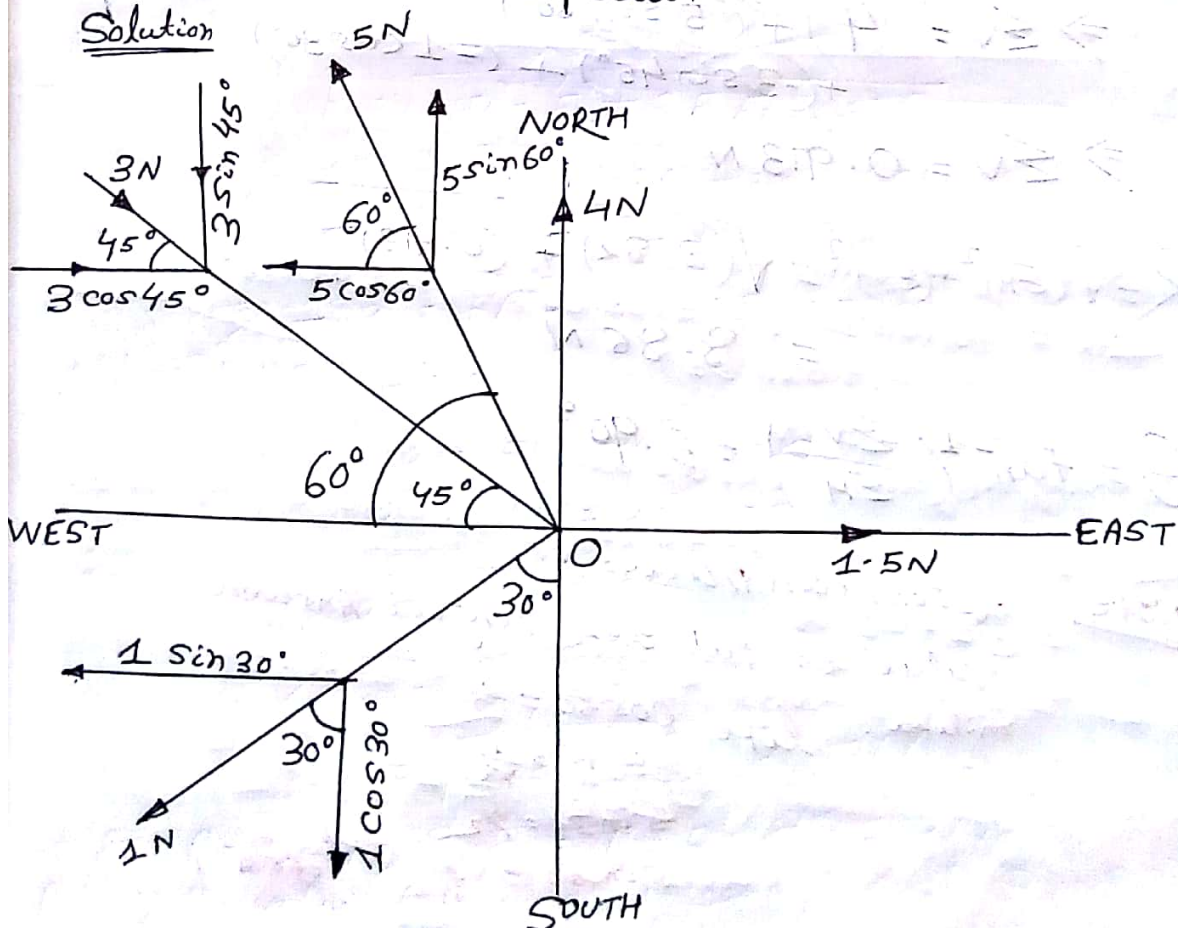
(shift $\rightarrow$ mode $\rightarrow$ 3 (Deg) option $\rightarrow$ A = Symbol)
calculator, convert degree)

29) Find the magnitude & direction of resultant of following System of force.

- (i) 4N due north
- (ii) 3N Acting North^{of} west towards the point at 45° Angle.
- (iii) 1.5 acting due east
- (iv) 1N 30° west of south away from the point.
- (v) 5N acting 60° North of west away from the point.



Solution



Now, Horizontal forces are

$$1.5 \text{ N (+ve)}$$

$$5 \cos 60^\circ \text{ (-ve)}$$

$$3 \cos 45^\circ \text{ (-ve)}$$

$$1 \sin 30^\circ \text{ (-ve)}$$

Vertical forces are

$$4 \text{ N (+ve)}$$

$$5 \sin 60^\circ \text{ (+ve)}$$

$$3 \sin 45^\circ \text{ (+ve)}$$

$$1 \cos 30^\circ \text{ (-ve)}$$

The sum of all horizontal force is given by

$$\sum H \text{ (} \rightarrow \text{ +ve, } \leftarrow \text{ -ve)}$$

$$\Rightarrow \sum H = (1.5) + (-5 \cos 60^\circ) + (-3 \cos 45^\circ) + (-1 \sin 30^\circ)$$

$$\Rightarrow \sum H = -8.82 \text{ N}$$

The sum of all vertical force is given by

$$\sum V \text{ (} \uparrow \text{ +ve, } \downarrow \text{ -ve)}$$

$$\Rightarrow \sum V = (4) + (5 \sin 60^\circ)$$

$$+ (-3 \sin 45^\circ) + (-1 \cos 30^\circ)$$

$$\Rightarrow \sum V = 0.913 \text{ N}$$

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-8.82)^2 + (0.913)^2}$$
$$= 8.86 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{\sum V}{\sum H} \right) = 5.90^\circ$$

Note Resolution process for
value of put as a perfect Answer
milika. like $\cos 60^\circ = \frac{1}{2}$

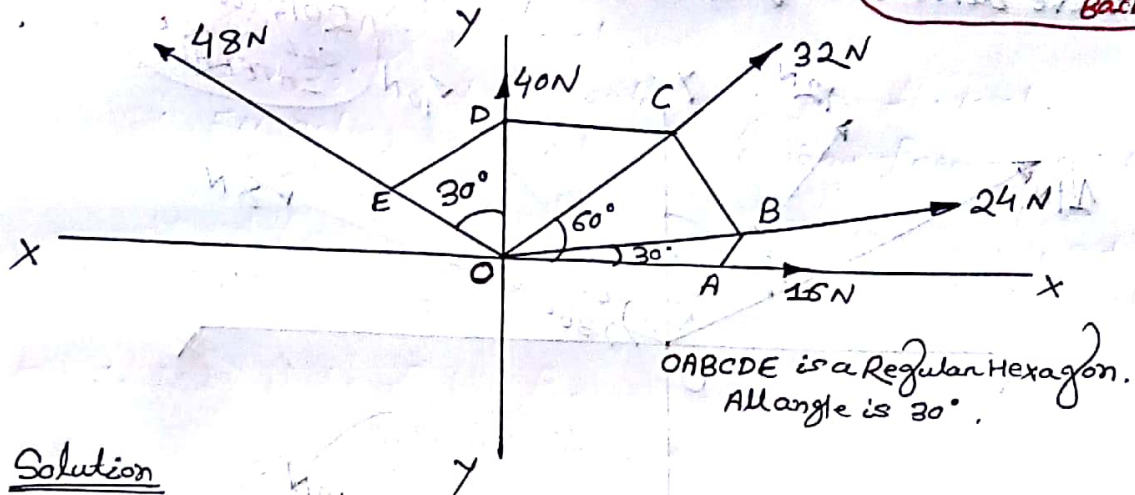
$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 30^\circ = \frac{1}{2}$$

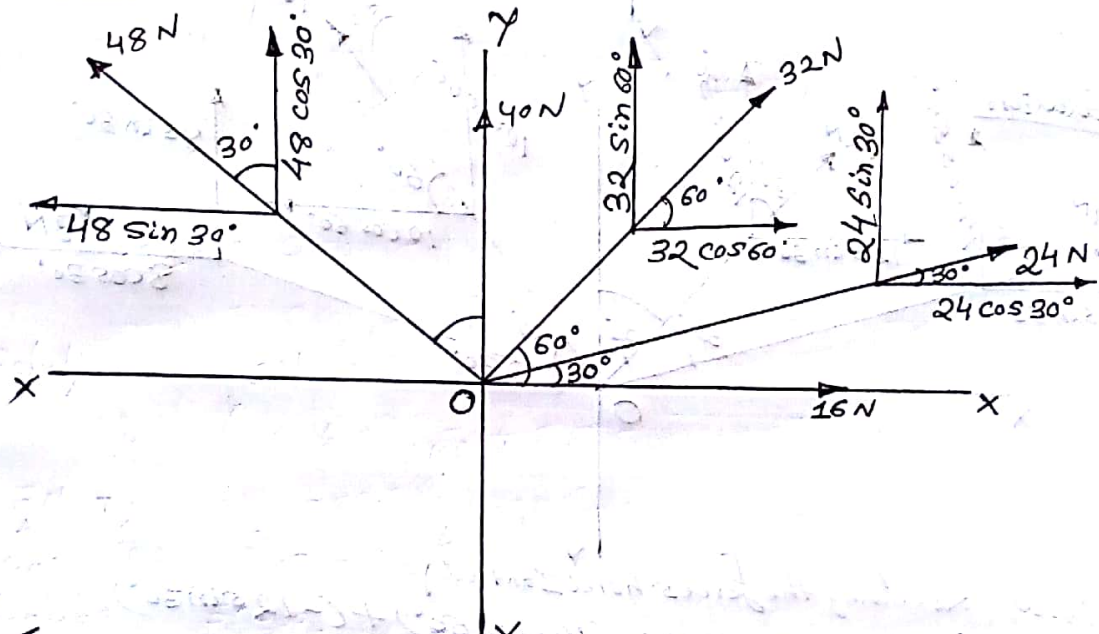
$$\sin 60^\circ = \frac{\sqrt{3}}{2} \text{ put as a perfect Answer}$$

- 30) The forces 16 N, 24 N, 32 N, 40 N & 48 N are acting on one of the angular points of a regular Hexagon towards the other five angular points taken in order. Find the magnitude & direction of resultant force.

2018 (Winter) OLD BACK



Solution



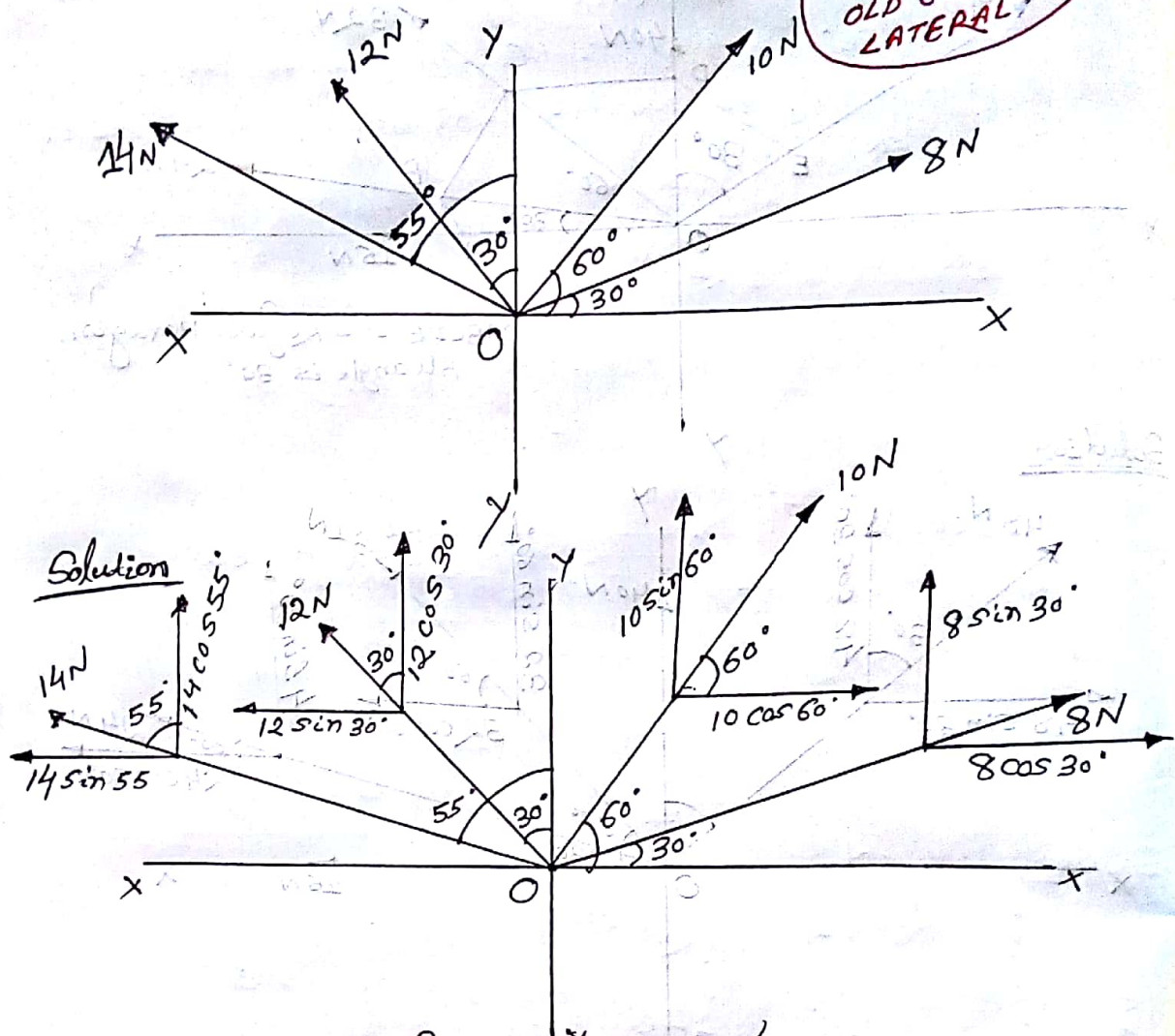
$$\begin{aligned}\Sigma H &= (16) + (24 \cos 30^\circ) + (32 \cos 60^\circ) + (-48 \sin 30^\circ) \\ &= 16 + 24 \times \frac{\sqrt{3}}{2} + 32 \times \frac{1}{2} - 48 \times \frac{1}{2} \\ &= 28.78 \text{ N}\end{aligned}$$

$$\begin{aligned}\Sigma V &= 24 \sin 30^\circ + 32 \sin 60^\circ + 40 + 48 \cos 30^\circ \\ &= 24 \times \frac{1}{2} + 32 \times \frac{\sqrt{3}}{2} + 40 + 48 \times \frac{\sqrt{3}}{2} \\ &= 121.28 \text{ N}\end{aligned}$$

$$\theta = \tan^{-1} \left(\frac{\Sigma V}{\Sigma H} \right) = \tan^{-1} \left(\frac{121.28}{28.78} \right) = 76.65^\circ$$

- 31) Find the magnitude & direction of the resultant of the concurrent forces of 8N, 10N, 12N & 14N making angles of 30° , 60° , 120° & 145° respectively with positive direction of X-axis.

2017 (WINTER)
OLD COURSE
LATERAL



Now, Resolving the forces horizontally

$$\begin{aligned}\sum H &= (8 \cos 30^\circ) + (10 \cos 60^\circ) + (-12 \sin 30^\circ) \\ &\quad + (-14 \sin 55^\circ) \\ &= -5.53 \text{ N}\end{aligned}$$

Resolving the forces vertically

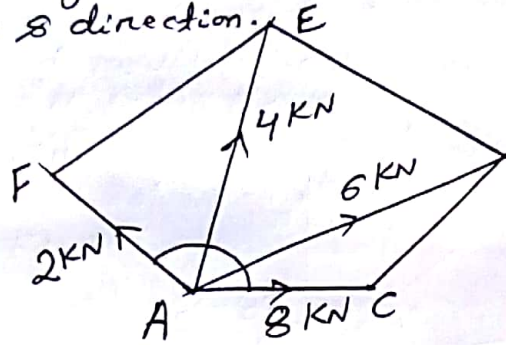
$$\begin{aligned}\sum V &= (8 \sin 30^\circ) + (10 \sin 60^\circ) + (12 \cos 30^\circ) + (14 \cos 55^\circ) \\ &= 31.08 \text{ N}\end{aligned}$$

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-5.53)^2 + (31.08)^2} = 31.56 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\sum V}{\sum H}\right) = \tan^{-1}\left(\frac{31.08}{-5.53}\right) = -79.91^\circ$$

(Convert Degree by using calculator)

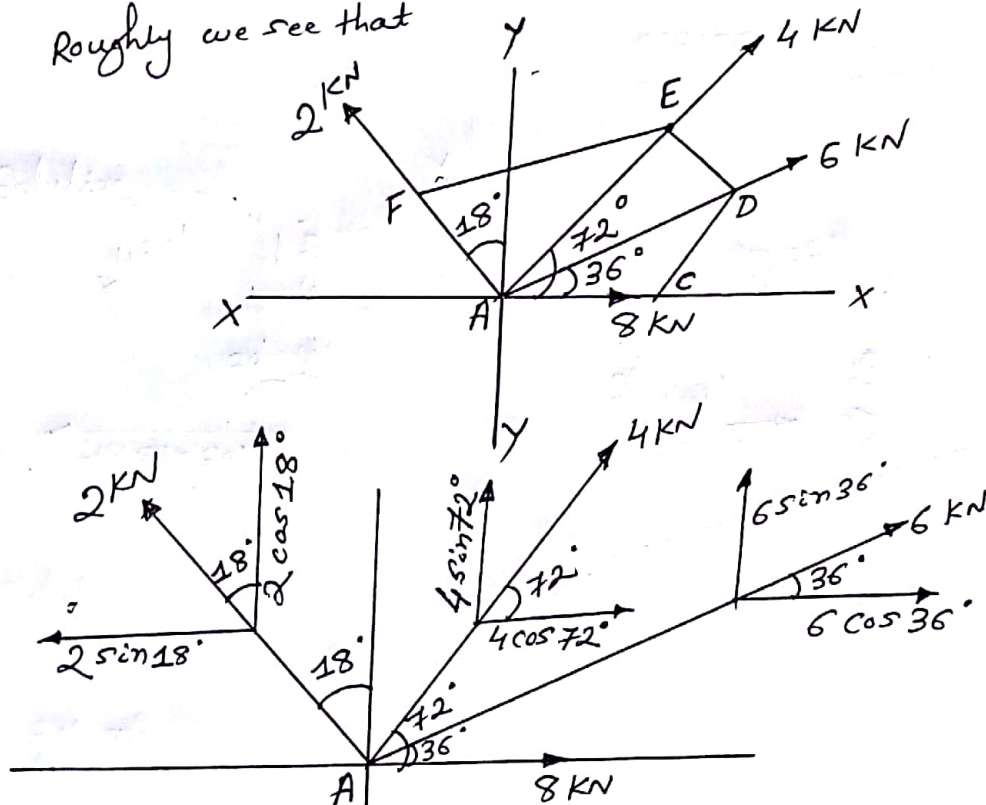
- 32) Forces of 2 kN, 4 kN, 6 kN, 8 kN respectively act on a regular pentagon as shown in Figure. Find resultant in magnitude & direction.



(Remember
Regular Pentagon
Angle total = 108°)
Each angle = $\frac{108}{3}$
 $= 36^\circ$

Solution

Roughly we see that



Now

$$\sum H = (8) + (6 \cos 36^\circ) + (4 \cos 72^\circ) + (-2 \sin 18^\circ)$$

$$= 4.86 \text{ kN}$$

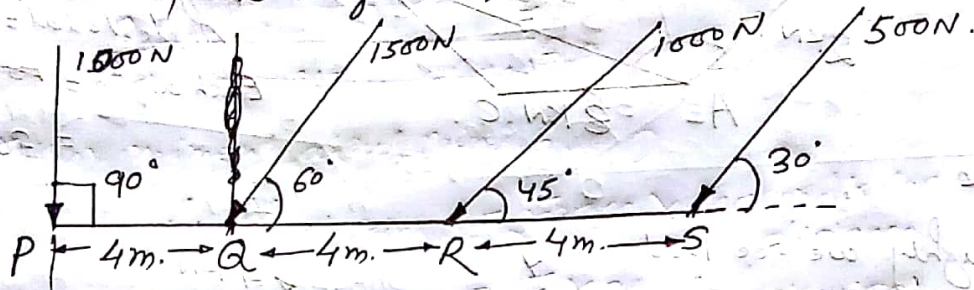
$$\sum V = (6 \sin 36^\circ) + (4 \sin 72^\circ) + (2 \cos 18^\circ)$$

$$= -3.61 \text{ kN}$$

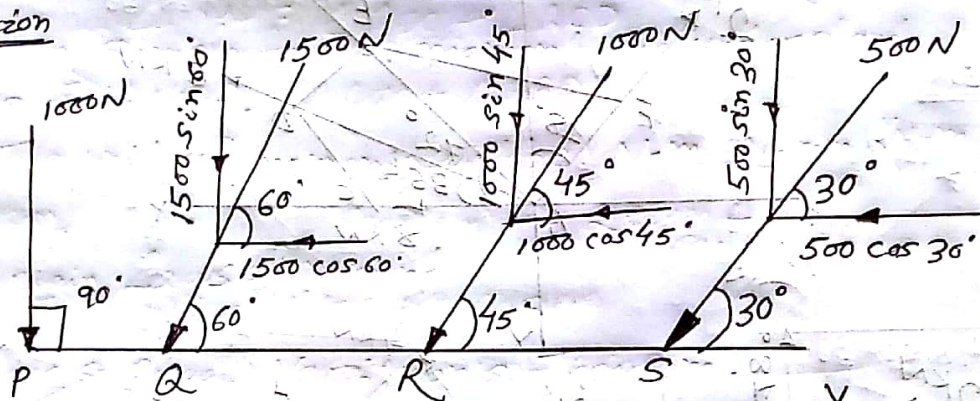
$$\text{Resultant (R)} = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(4.86)^2 + (-3.61)^2} = 6.05 \text{ kN}$$

$$\theta = \tan^{-1} \left(\frac{\sum V}{\sum H} \right) = -36.60^\circ$$

- 33) A horizontal line PQRS is 12m. long, where $PQ = QR = RS = 4m$. Forces of 1000 N, 1500 N, 1000 N, 500 N act at P, Q, R, S respectively with downward direction. The line of action of these forces make angles of 90° , 60° , 45° & 30° respectively with PS. Find the magnitude & direction of resultant force.



Solution



Now, Horizontal forces are

$$500 \cos 30^\circ \text{ (-ve)}$$

$$1000 \cos 45^\circ \text{ (-ve)}$$

$$1500 \cos 60^\circ \text{ (-ve)}$$

Vertical forces are

$$1000 \text{ N (-ve)}$$

$$1500 \sin 60^\circ \text{ (-ve)}$$

$$1000 \sin 45^\circ \text{ (-ve)}$$

$$500 \sin 30^\circ \text{ (-ve)}$$

$$\Sigma H = (-500 \cos 30^\circ) + (-1000 \cos 45^\circ) + (-1500 \cos 60^\circ)$$

$$= -500 \times \frac{\sqrt{3}}{2} - 1000 \times \frac{1}{\sqrt{2}} - 1500 \times \frac{1}{2}$$

$$= -899.67 \text{ N}$$

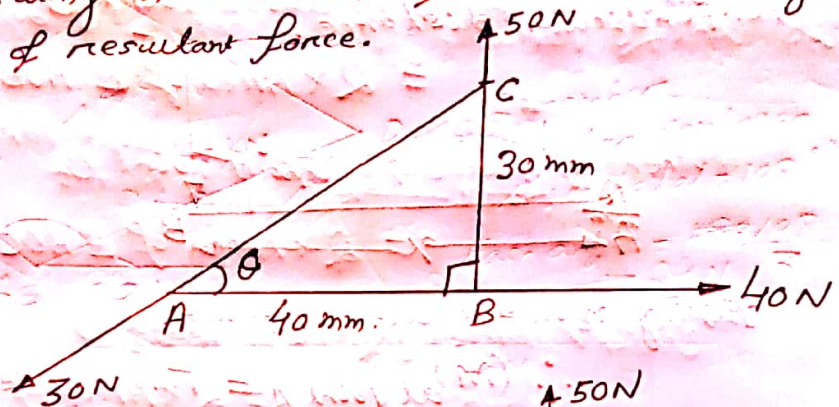
$$\Sigma V = (-1000) + (-1500 \sin 60^\circ) + (-1000 \sin 45^\circ)$$

$$+ (-500 \sin 30^\circ)$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 1221.45 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = -47.43^\circ \text{ (convert degree by using calculator)}$$

(34) A Triangle ABC has its side AB = 40 mm along positive X-axis & side BC = 30 mm along Y-axis. Three forces of 40 N, 50 N & 30 N act along the sides AB, BC & CA respectively. Determine magnitude of resultant force.



Solution

$$\Delta ABC, \sin \theta = \frac{BC}{AC}$$

$$\Rightarrow \sin \theta = \frac{30}{\sqrt{AB^2 + BC^2}} = \frac{30}{\sqrt{40^2 + 30^2}}$$

$$\Rightarrow \sin \theta = \frac{30}{50} = 0.6$$

~~$\cos \theta = \frac{BC}{AC}$~~

$$\cos \theta = \frac{AB}{AC}$$

~~$\Rightarrow \cos \theta = \frac{30}{50}$~~

$$\Rightarrow \cos \theta = \frac{40}{50} = 0.8$$

Horizontal forces are

40 N (+ve)

30 cos θ (-ve)

Vertical forces are

50 N (+ve)

30 sin θ (-ve)

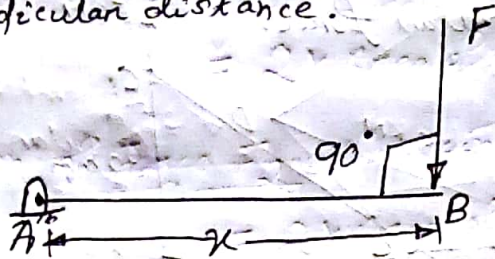
$$\begin{aligned} \therefore \sum H &= (40) + (-30 \cos \theta) = 40 - 30 \cos \theta \\ &= 40 - 30 \times 0.8 \\ &= 16 \text{ N} \end{aligned}$$

$$\begin{aligned} \sum V &= (50) + (-30 \sin \theta) = 50 - 30 \sin \theta \\ &= 50 - 30 \times 0.6 \\ &= 32 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Resultant (R)} &= \sqrt{(\sum H)^2 + (\sum V)^2} \\ &= \sqrt{(16)^2 + (32)^2} \\ &= 35.8 \text{ N} \end{aligned}$$

Moment

It can be defined as the product of Force & Perpendicular distance.



Taking moment about point A

$$\therefore (M)_{\text{at point A}} = \text{Force} \times \text{Perpendicular distance} \\ = F \times x \text{ (clockwise)}$$

→ In simple terms, moment can be understood as rotation.

→ The unit of moment will be, N-m, N-mm, ~~KN-m~~.

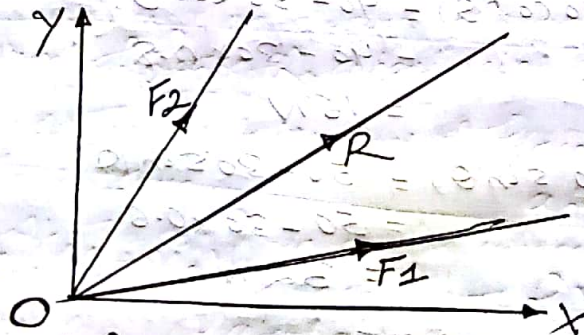
Question

Explain Laws of moment
or

State & Proof Varignon's Theorem

STATEMENT

If a no. of coplanar forces are acting simultaneously on a particle then the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about their same point.



Moment of Force F_1 about at point O $= (M)_1 = F_1 \times x_1$

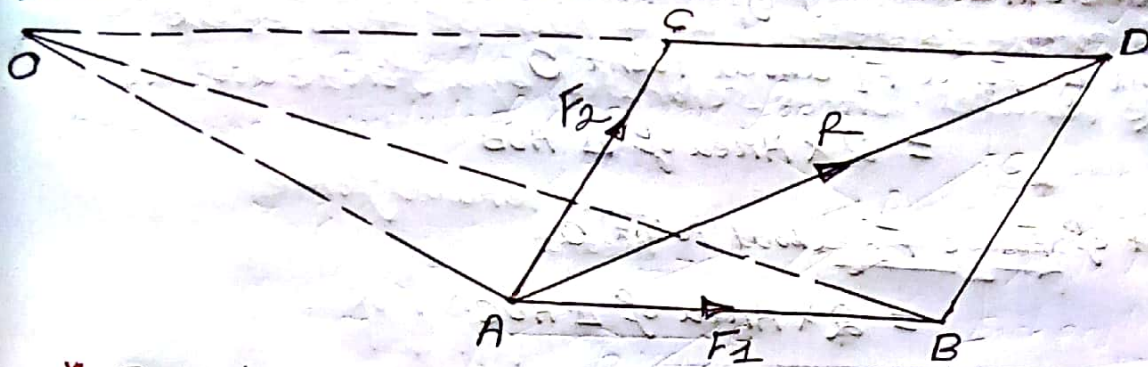
Moment of Force F_2 about at point O $= (M)_2 = F_2 \times x_2$

Proof

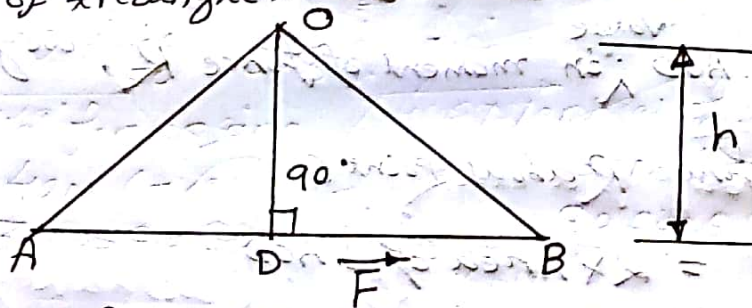
Moment of Force F_1 & F_2 about point O
= Moment of Resultant force R about point O

$$\Rightarrow \sum (M_1 + M_2) = R \times x$$

PROOF



- * Considering point O about which the moment is to be taken. After that from point O , draw a line which is parallel to the line of Action Force F_1 on base AB and intersecting the line AC at point C .
- * Here AB & AC adjacent sides of Parallelogram. Now complete the $ACDB$ Parallelogram. The diagonal of Parallelogram representing magnitude & direction by the resultant of Force F_1 & F_2 . Here, R is the resultant force. Now, Join the point O from point A & B . (Join OA & OB).
- * According to graphical representation of the moment, i.e. The moment of force about a point is equal to twice of the Area of triangle.



Moment of force $\vec{F}$ about point O

$$(M)_{\text{At Point } O} = \vec{F} \times h \quad \text{--- (1)}$$

$$\begin{aligned} \text{Area of the Triangle} &= \frac{1}{2} \times AB \times OD \\ &= \frac{1}{2} \times \vec{F} \times h \end{aligned}$$

$$\Rightarrow 2 \times \text{Area of Triangle} = \vec{F} \times h \quad \text{--- (2)}$$

Equating the eqⁿ (1) & eqⁿ (2), we get

$$\boxed{\text{Moment of Force} = 2 \times \text{Area of Triangle}}$$

From Geometrical Configuration;

Moment of Force F_1 about Point O

$$= 2 \times \text{Area of } \triangle AOB$$

Moment of Force F_2 about Point O

$$= 2 \times \text{Area of } \triangle AOC$$

Moment of Force R about point O

$$= 2 \times \text{Area of } \triangle AOD$$

Now,

$$\triangle AOD = \triangle AOC + \triangle ACD$$

$$\Rightarrow \triangle AOD = \triangle AOC + \triangle ABD$$

$$(\because \triangle ACD = \triangle ABD)$$

Parallelogram, 2 equal Δ same

$$\Rightarrow \triangle AOD = \triangle AOC + \triangle AOB$$

$$(\because \triangle ABD = \triangle AOB$$

same Base line)

Putting this $\triangle AOD$ ^{value} in moment of force R , we get

$\therefore$ Moment of Force R about Point O

$$= 2 \times \text{Area of } \triangle AOD$$

$$= 2 \times \text{Area of } (\triangle AOC + \triangle AOB)$$

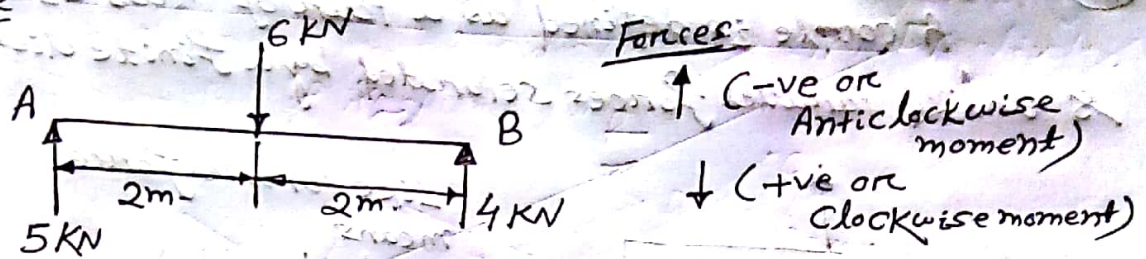
$\Rightarrow$ Moment of Force R about Point O

$$= \text{Moment of Force } F_1 \text{ \& } F_2 \text{ about Point O}$$

$$\Rightarrow \left\{ \sum (M_1 + M_2) \right\}_{\text{about O point}} = R \times \perp$$

Moment PROBLEMS

Example



We know Resultant force $(R) = \sqrt{(\sum H)^2 + (\sum V)^2}$ $(\because \sum H = 0)$

$$\Rightarrow R = \sqrt{\sum V^2}$$

$$\Rightarrow R = \sum V$$

$$\Rightarrow R = (5) + (-6) + (-4) = -3 \text{ kN}$$

According to Varignon's theorem

$$R \times x = (\sum M) \text{ at any point} \quad \text{--- (1)}$$

$$\begin{aligned} \Rightarrow (\sum M)_{\text{At Point A}} &= (5 \times 0) + (6 \times 2) + (-4 \times 4) \\ &= 12 - 16 \\ &= -4 \text{ kN (clockwise)} \end{aligned}$$

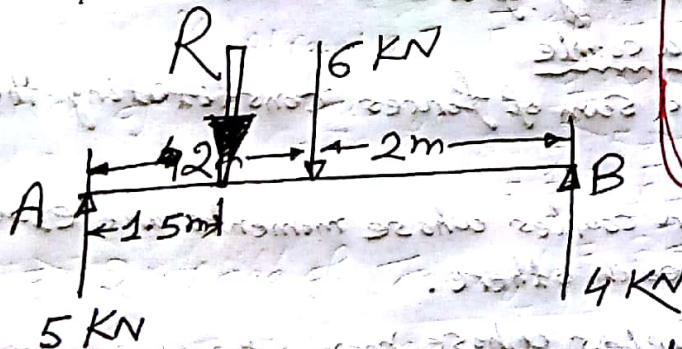
Now, putting this value in equation (1), we get the x distance, where resultant force act.

$$R \times x = (\sum M)_{\text{Point A}}$$

$$\Rightarrow -3 \times x = -4$$

$$\Rightarrow x = \frac{-4}{-3} = \frac{4}{3}$$

$$\Rightarrow x = 1.3 \text{ metre}$$

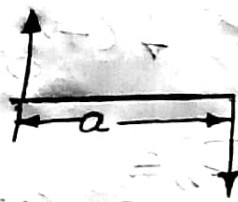


Here Resultant force R act in Downward direction (clockwise), maintain distance 1.5m. about the Point A.

Remember,
In semester
Question, if ASK
Magnitude, direction
& position of
the resultant,
then you can understand
Moment law is
applied
For solution point view

Couple

A couple is defined as a pair of two equal & unlike parallel forces separated by a finite distance.



Like parallel force means

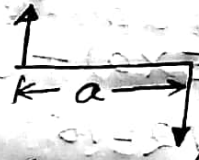


Unlike parallel force means

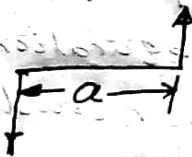


Classification of couple

① Clockwise couple



② Anticlockwise couple



Example of couple

Winding of a clock

unscrewing the cap of a bottle

Locking or unlocking of a lock with a key

Opening or closing of a water tap.

Characteristics of a couple

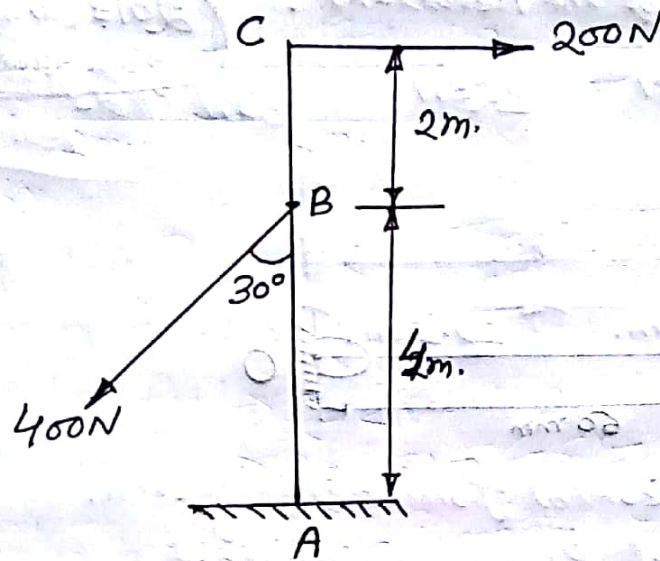
(1) The algebraic sum of forces constituting the couple is zero.

(2) Two coplanar couples whose moments are equal & opposite, balance each other.

(3) Any two couples whose moments are equal & of same sign are equivalent both in magnitude and direction.

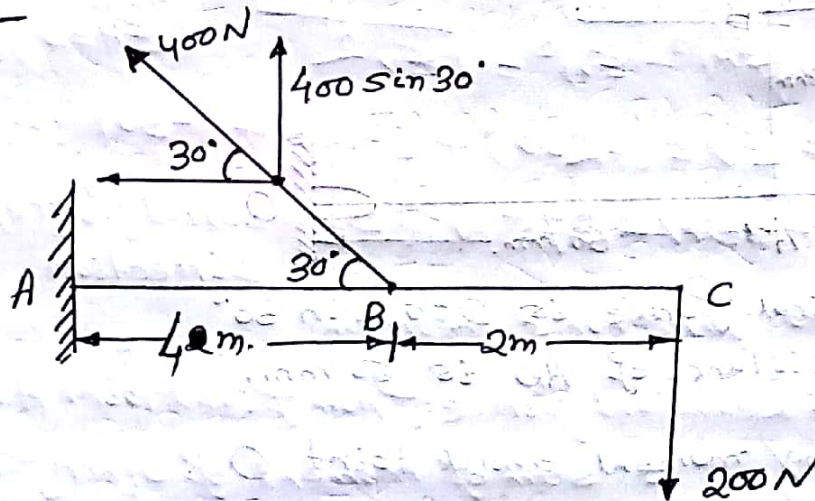
PROBLEM-35

Find the sum of moments of the forces shown in figure, about point A.



2017 (winter)
year Question

Solution

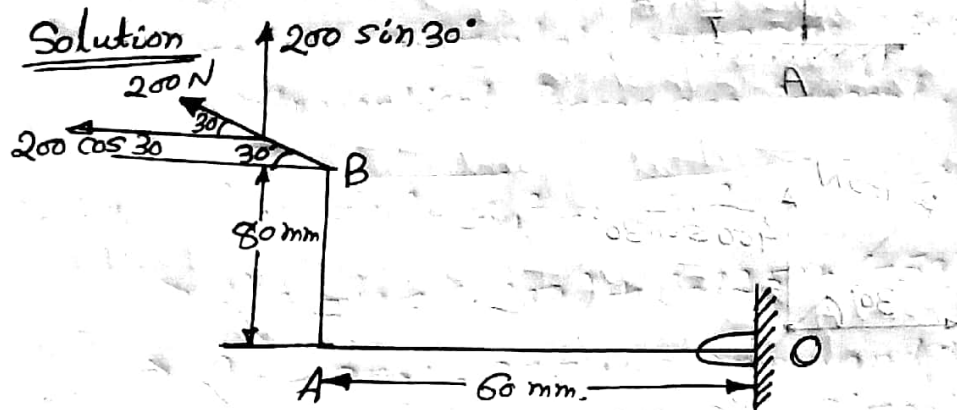
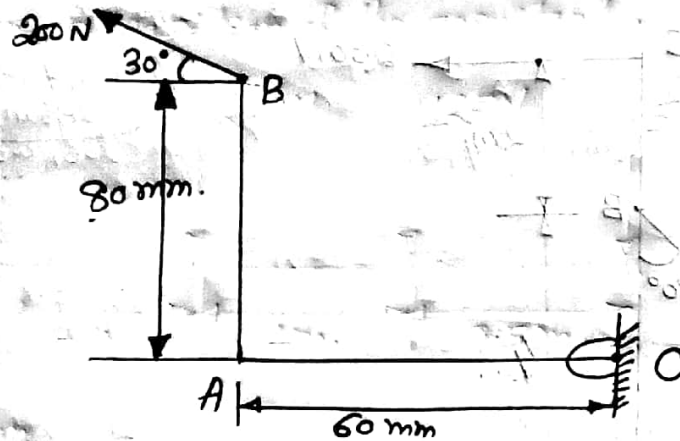


Sum of moment about Point A .

$$\begin{aligned} (\sum M)_{A \text{ Point A}} &= \{ (400 \sin 30^\circ \times 4) + (200 \times 6) \} \\ &= -800 + 1200 \\ &= 400 \text{ N}\cdot\text{m (clockwise)} \end{aligned}$$

- 36) A force of 200 N is acting on a bracket OAB hinged at 'O' shown in figure. Find the moment of force about O.

2013 (winter)
Year Question



Here Vertical force is $200 \sin 30^\circ$
Distance of AO is 60 mm.

Now, Taking moment about Point O.

$$(\sum M)_{\text{Point O}} = (200 \sin 30^\circ \times 60)$$

$$= 6000 \text{ N-mm.}$$

Or

$$= \frac{6000}{1000} \text{ N.m}$$

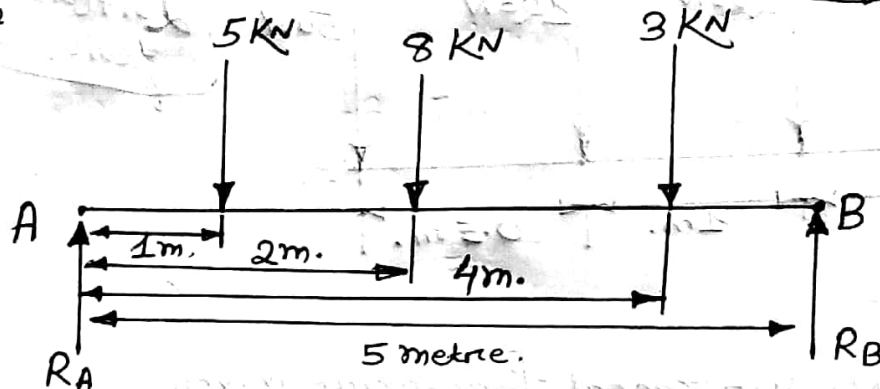
$$= 6 \text{ N.m.}$$

we know
 $1 \text{ m} = 1000 \text{ mm}$

- ③7 A Beam AB of Length 5m. supported horizontally at A & B carries point loads of 5 kN, 8 kN and 3 kN at a distance of 1m, 2m and 4m. from the support A respectively. Find the reaction At A and B.

2014 (winter)
year Question

Solution



Here no Horizontal forces are present. (only vertical forces are present).

$$\text{So, } R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

$$\Rightarrow \boxed{R = \sum V}$$

Now we can write $\sum V = 0$

$$\Rightarrow (R_A) + (-5) + (-8) + (-3) + (R_B) = 0$$

$$\Rightarrow R_A + R_B = 16 \quad \text{--- (1)}$$

$\therefore$ Taking moment about point A

$$(\sum M)_{\text{About point A}} = 0$$

$$\Rightarrow (R_A \times 0) + (5 \times 1) + (8 \times 2) + (3 \times 4) + (-R_B \times 5) = 0$$

$$\Rightarrow 5 + 16 + 12 = 5 \times R_B$$

$$\Rightarrow 5 R_B = 33$$

$$\Rightarrow R_B = \frac{33}{5} = 6.6 \text{ kN}$$

Put this R_B value in eqⁿ(1), we will get R_A .

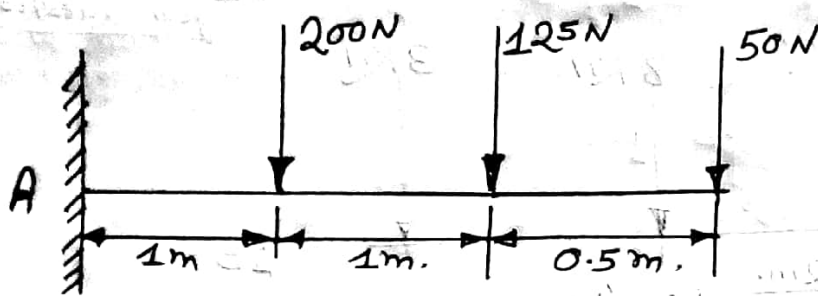
$$R_A + R_B = 16$$

$$\Rightarrow R_A = 16 - R_B$$

$$\Rightarrow R_A = 16 - 6.6$$

$$\Rightarrow R_A = 9.4 \text{ kN} \quad (\text{Ans})$$

- 38) A system of parallel forces acting on a lever is shown in figure. Determine the magnitude, direction & position of the resultant.



2017 (winter)
year Question

Solution

Here no horizontal forces are given.
only vertical forces are present.

$$\text{Now, } R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{\Sigma V^2} \quad (\Sigma H = 0)$$

$$\Rightarrow R = \Sigma V = (-200) + (-125) + (-50)$$

$$\Rightarrow R = -375 \text{ N} \quad \text{(Downward force)}$$

Direction

According to Varignon's theorem,

Moment of Resultant force from fixed point A.

= sum of moment of other forces.

From fixed point A.

$$\Rightarrow R \times x = (200 \times 1) + (125 \times 2)$$

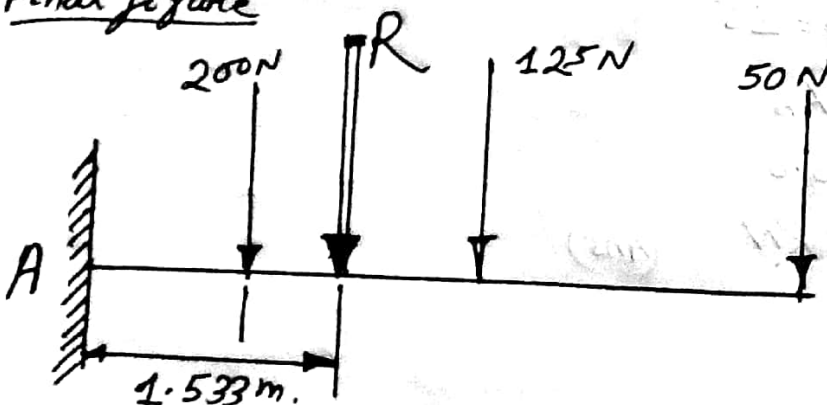
$$+ (50 \times 2.5)$$

$$\Rightarrow -375 \times x = 200 + 250 + 125$$

$$\Rightarrow x = \frac{575}{-375} = -1.533 \text{ metre}$$

(clockwise)

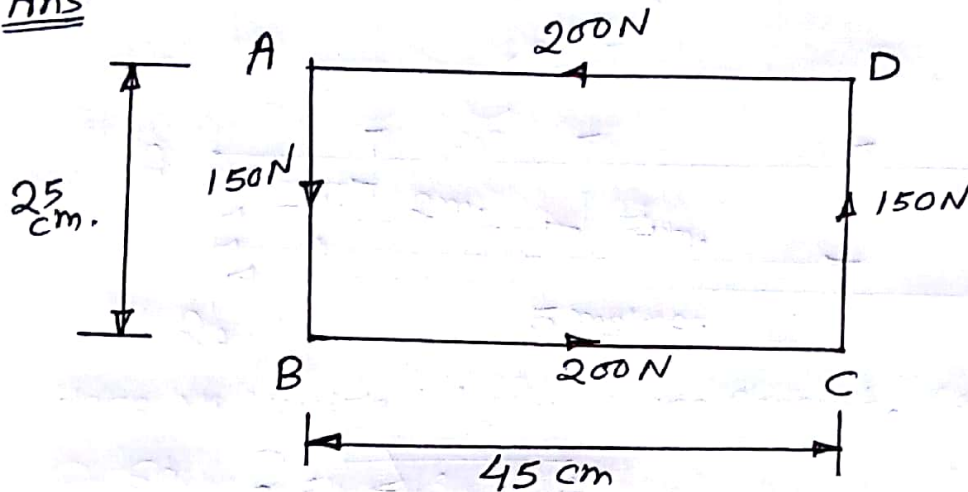
Final figure



- 39) ABCD is a rectangle, in which $AB = CD = 25 \text{ cm}$. and $BC = DA = 45 \text{ cm}$. Forces of 150 N each acting along AB and CD and forces of 200 N each act along BC & DA. Find the resultant moment of two couple.

2014 (winter)
year Question

Ans



Taking resultant moment of two couple at point B.

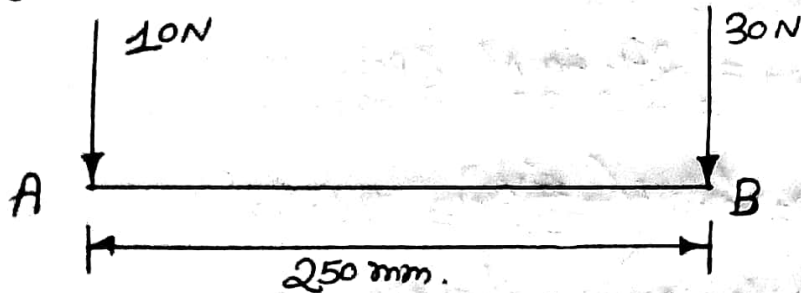
$$\begin{aligned} (\sum M)_{\text{point A}} &= (-150 \times 45) + (-200 \times 25) \\ &= -11750 \text{ Ncm} \end{aligned}$$

$$= -11750 \text{ Nmm}$$

- 40) Two like parallel forces of 10 N and 30 N act at the ends of a rod 250 mm long. Find magnitude of the resultant force and the point where it acts.

2017 (summer)
year Question

Ans



$$\text{Resultant force } (R) = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(\Sigma V)^2} = \Sigma V$$

$$\Rightarrow R = (-10) + (-30) = -40 \text{ N}$$

Now; According to Varignon's theorem

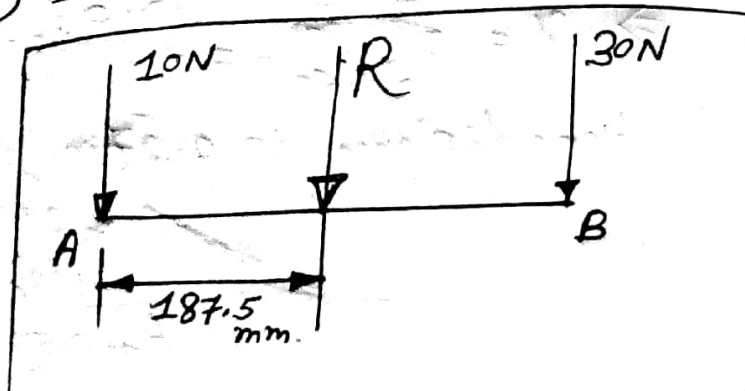
$$(\Sigma M)_{\text{At point A}} = R \times x$$

$$\Rightarrow (-10 \times 0) + (30 \times 250) = -40 \times x$$

$$\Rightarrow x = \frac{7500}{-40}$$

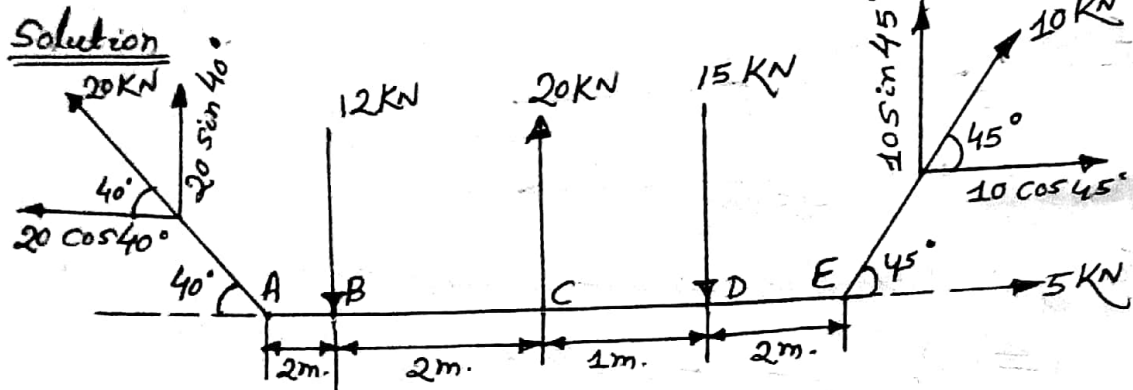
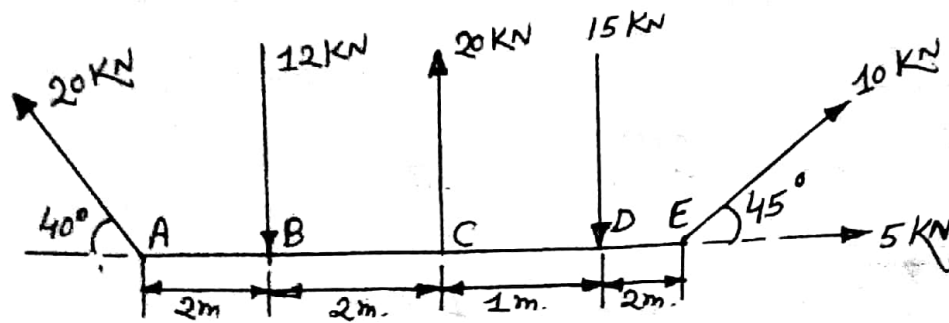
$$\Rightarrow x = -187.5 \text{ mm}$$

(clock wise
~~distance~~)



- 41) A beam is subjected to forces as shown in figure. Find the magnitude, direction and the position of the resultant force.

2017 Question (Summer)



$$\begin{aligned} \text{Resultant force } (R) &= \sqrt{(\sum H)^2 + (\sum V)^2} \\ &= \sqrt{\{(5) + (-20 \cos 40^\circ)\}^2 + \{(20 \sin 40^\circ) + (-12) + (-15) + 10 \sin 45^\circ\}^2} \\ &= \sqrt{(18.33)^2 + (16.41)^2} \\ &= 24.60 \text{ N (Positive value)} \end{aligned}$$

going upward direction

$$\text{Direction } (\theta) = \tan^{-1} \left(\frac{\sum V}{\sum H} \right) = 41.83^\circ$$

Now, According to Varignon's theorem

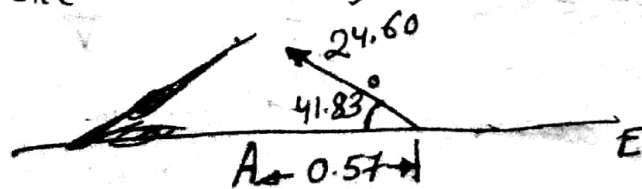
$$R \times x = (\sum M)_{\text{point A}}$$

$$\Rightarrow R \times x = (20 \sin 40^\circ \times 0) + (12 \times 2) + (-20 \times 4) + (15 \times 5) + (-10 \sin 45^\circ \times 7)$$

$$\Rightarrow 24.60 \times x = 14.05$$

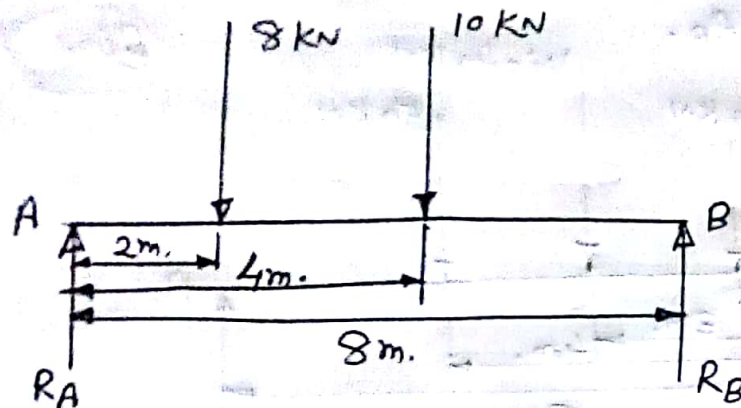
$$\Rightarrow x = \frac{14.05}{24.60} = 0.57 \text{ metre}$$

Here distance is 0.57 m.



- 42) A simply supported beam of 8 m. span has two concentrated loads of 8 kN & 10 kN at 2 m. & 4 m from left hand support. Find the reaction at two supports.

Ans



$$R = \sqrt{(\sum H)^2 + (\sum V)^2} \quad \sum H = 0$$

$$\Rightarrow R = \sum V$$

$$\Rightarrow \sum V = 0 \quad (\uparrow = +)$$

$$\Rightarrow (R_A) + (-8) + (-10) + (R_B) = 0$$

$$\Rightarrow R_A + R_B = 18 \text{ kN} \quad \text{--- (1)}$$

Taking moments about point A

$$(\sum M)_{\text{Point A}} = 0$$

$$\Rightarrow (R_A \times 0) + (8 \times 2) + (10 \times 4) + (-R_B \times 8) = 0$$

$$\Rightarrow 16 + 40 = 8R_B$$

$$\Rightarrow R_B = \frac{56}{8} = 7 \text{ kN}$$

Putting this R_B value in equation (1), we get

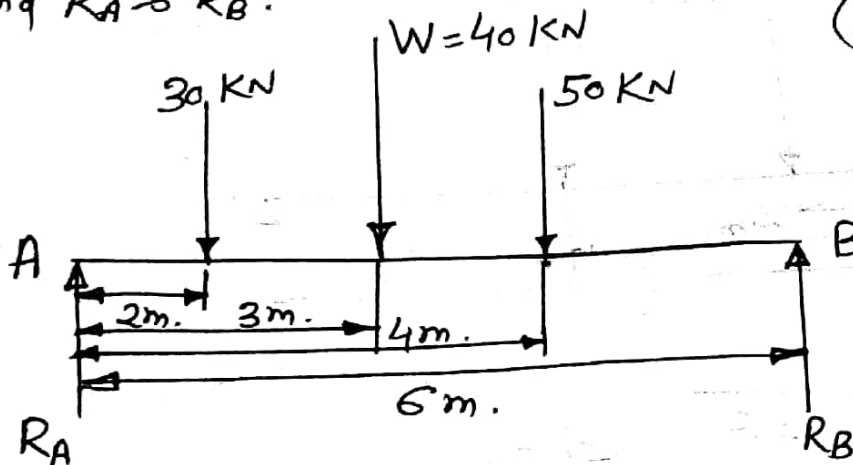
$$R_A + R_B = 18$$

$$\Rightarrow R_A + 7 = 18$$

$$\Rightarrow R_A = 18 - 7 = 11 \text{ kN}$$

43) A Simply Supported Beam weight 40 kN is supported at A & B, AB = 6 meters. Two point loads of 30 kN and 50 kN are acting at distance of 2m, 8m. from left end. Find R_A & R_B . (Book) Problem.

Ans



$$\sum V = 0$$

$$\Rightarrow (R_A) + (-30) + (-40) + (-50) + (R_B) = 0$$

$$\Rightarrow R_A + R_B = 120 \text{ kN} \quad \text{--- (1)}$$

$$(\sum M)_{\text{About 'A' point}} = 0$$

$$\Rightarrow (-R_A \times 0) + (30 \times 2) + (40 \times 3) + (50 \times 4) + (-R_B \times 6) = 0$$

$$\Rightarrow 60 + 120 + 200 = 6R_B$$

$$\Rightarrow R_B = \frac{380}{6} = 63.33 \text{ kN}$$

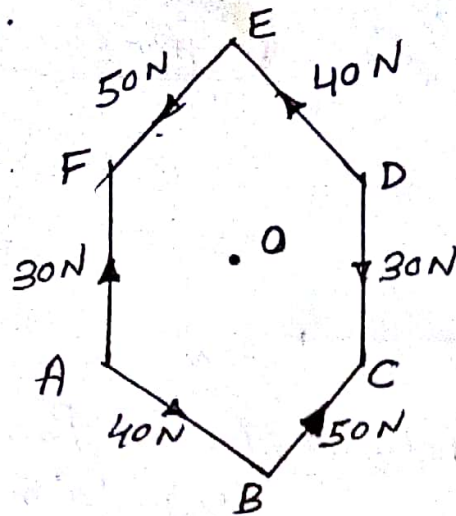
Putting this R_B value in equation (1), we get

$$R_A + R_B = 120$$

$$\Rightarrow R_A + 63.33 = 120$$

$$\Rightarrow R_A = 120 - 63.33 = 56.67 \text{ kN}$$

- (44) A Regular Hexagon has sides equal to 20 cm. Forces of 30 N each act along AF & DC, 40 N each along AB & DE, 50 N each along BC & EF as shown in figure. Find the moment of the ~~force~~ forces about point 'O' which keep the system in the equilibrium.



2015 (Winter)
Year Question

Every base 20 cm.

Solution

Draw a perpendicular line from Point O about line AB which meet at G.

Join OA & OB.

Now, OAB is an equilateral Δ .

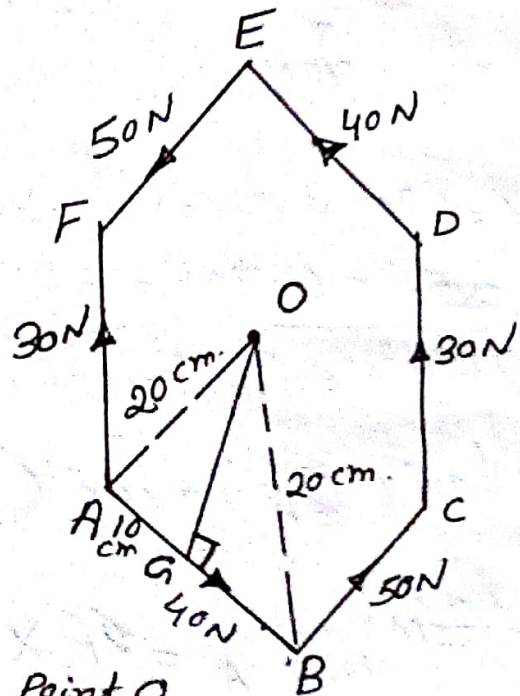
$$\text{Now, } OG = \sqrt{(OA)^2 - (AG)^2}$$

$$\Rightarrow OG = \sqrt{(20)^2 - (10)^2}$$

$$\Rightarrow OG = 17.32 \text{ cm.}$$

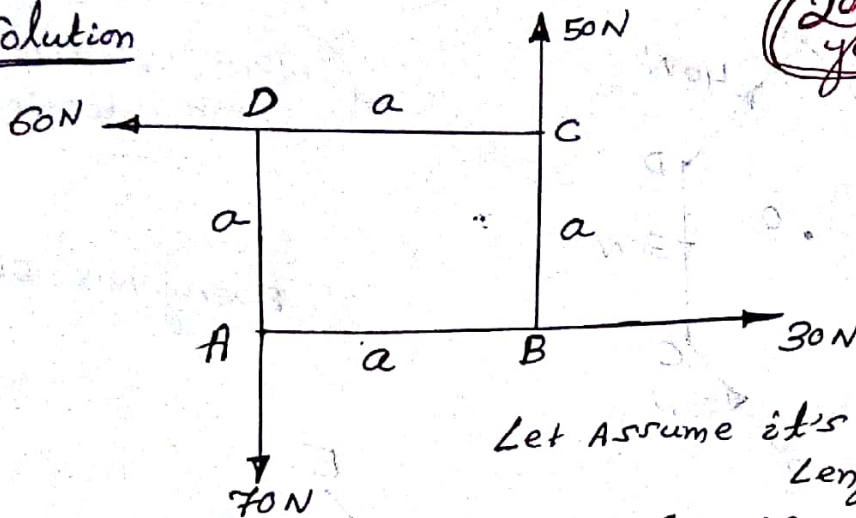
Sum of moment force about Point O

$$\begin{aligned} (\sum M)_{\text{Point O}} &= (-OG \times 40) + (-OG \times 50) + (-OG \times 30) \\ &\quad + (-OG \times DE) + (-OG \times 50) + (OG \times 30) \\ &= (-17.32 \times 40) + (-17.32 \times 50) + (-17.32 \times 30) \\ &\quad + (-17.32 \times 40) + (-17.32 \times 50) + (17.32 \times 30) \\ &= \underline{\hspace{2cm}} \text{ N. cm.} \end{aligned}$$



- 45) Forces of 30N, 50N, 60N & 70N act along the sides of a square ABCD taken in order. Find the magnitude, direction & position of the resultant.

Solution



(2017 (Summer) year Question)

Let Assume it's all face Length 'a' m.

($\because$ ABCD is a square)

$$\Sigma H = (30) + (-60) = -30 \text{ N}$$

$$\Sigma V = (50) + (-70) = -20 \text{ N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-30)^2 + (-20)^2} = 36.05 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{-20}{-30}\right) = 33.69^\circ$$

Now,

$$(\Sigma M)_{\text{Point A}} = R \times r$$

$$\Rightarrow a \cdot (-50 \times a) + (-60 \times a) = R \times r$$

$$\Rightarrow a \times \{(-50) + (-60)\} = R \times r$$

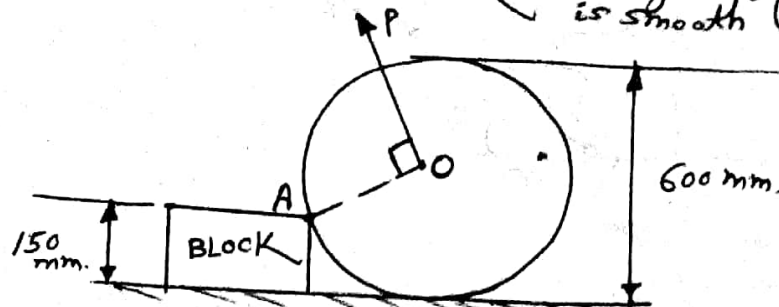
$$\Rightarrow a \times (-110) = R \times r$$

$$\Rightarrow -110 \times a = R \times r$$

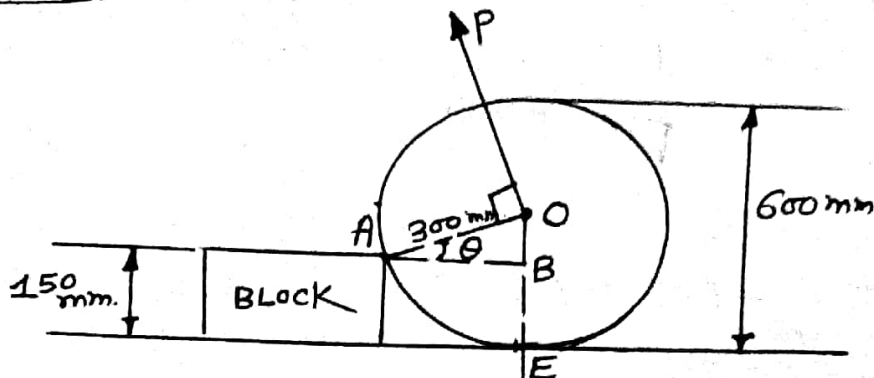
$$\Rightarrow r = \frac{-110 \times a}{R}$$

$$\Rightarrow r = \frac{-110 \times a}{36.05} \text{ meter.}$$

- 46) A uniform wheel of 600 mm diameter, weight 5 kN rest against a rigid rectangular block of 150 mm height as shown in figure. Find the Least pull, through the centre of the wheel required just to turn the wheel over corner A of the block. Also find the reaction of the block. Taking surface is smooth. 2016 (Winter) Question



Solution



Extend centre 'O' point through downward where line meet point E. Join A & B point.

Diameter of wheel (D) = 600 mm, so OA = radius = 300 mm.

Height of block or BE = 150 mm.

Weight of wheel (W) = 5 kN

Least pull (P) = ?

Reaction at point A = R_A = ?

Now, OAB is a right angle Triangle.

$$OB = OE - BE = 300 - 150 = 150 \text{ mm.}$$

$$AB = \sqrt{(OA)^2 - (OB)^2} = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm.}$$

$$\sin \theta = \frac{OB}{OA} = \frac{150}{300}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{150}{300}\right) = 30^\circ$$

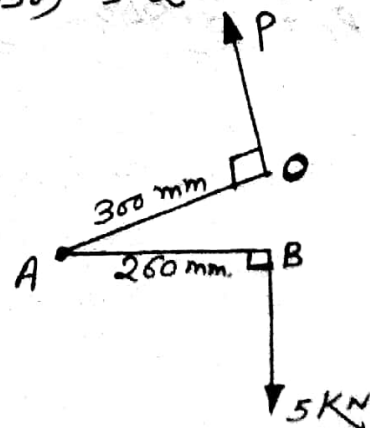
Taking moment about Point A.

$$(\sum M)_{A \text{ point}} = 0$$

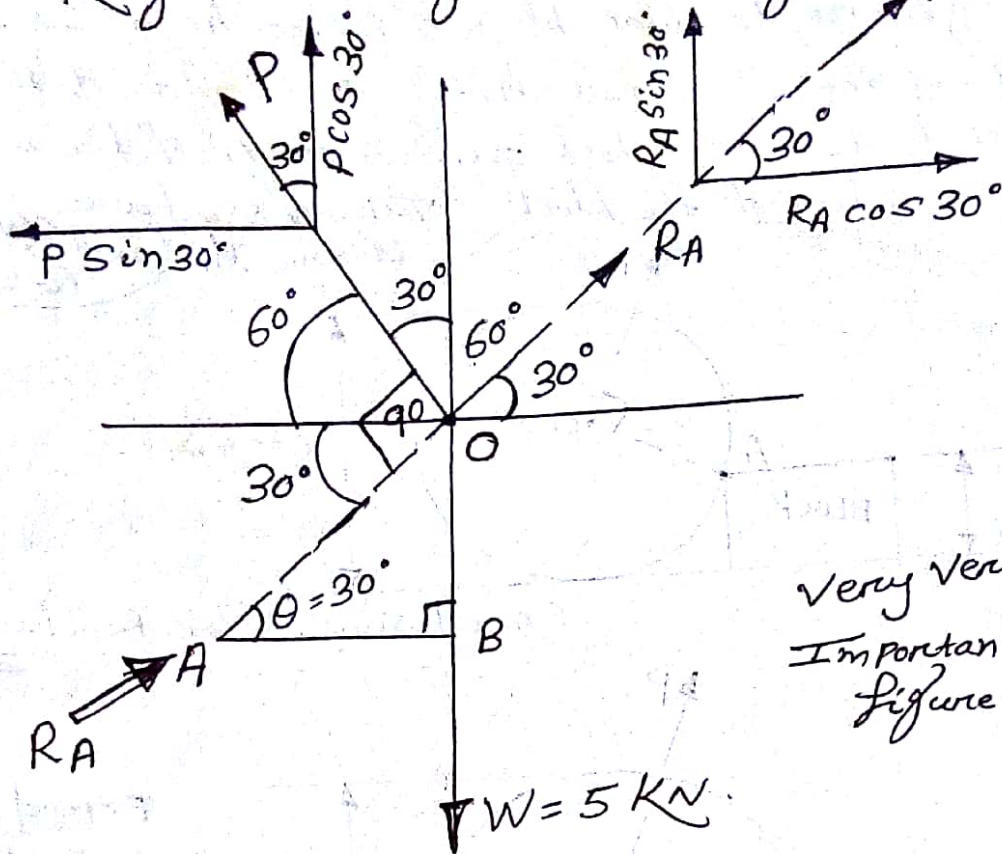
$$\Rightarrow (260 \times 5) + (-P \times 300) = 0$$

$$\Rightarrow 260 \times 5 = P \times 300$$

$$\Rightarrow P = \frac{1300}{300} = 4.33 \text{ kN}$$



Marking this figure carefully.



Very Very
Important
figure.

$$\therefore \sum H = 0$$

$$\Rightarrow (R_A \cos 30^\circ) + (-P \sin 30^\circ) = 0$$

$$\Rightarrow R_A \cos 30^\circ = P \sin 30^\circ$$

$$\Rightarrow R_A = \frac{4.33 \times \sin 30^\circ}{\cos 30^\circ}$$

$$\Rightarrow R_A = \text{--- kN}$$

OR

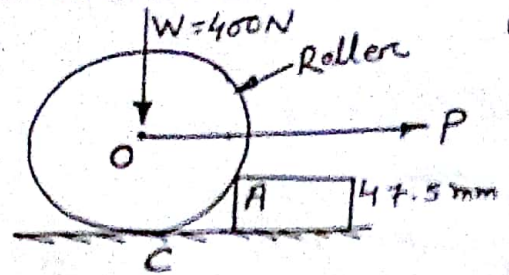
$$\therefore \sum V = 0$$

$$\Rightarrow (R_A \sin 30^\circ) + (P \cos 30^\circ) + (-5) = 0$$

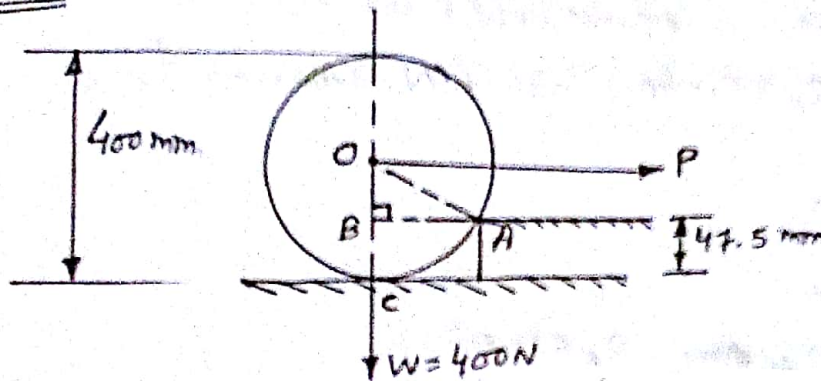
$$\Rightarrow R_A \sin 30^\circ = 5 - 4.33 \times \cos 30^\circ$$

$$\Rightarrow R_A = \frac{3.848}{\sin 30^\circ} = \text{--- kN.}$$

- 47) A roller 400 mm in diameter & weighing 400 N is to be subjected to pivoting motion about an edge A, by applying a force P as shown in the figure. Determine the magnitude of horizontal force P passing through the centre of the roller & corresponding reaction R_A exerted by the edge A. Assume surface will be smooth. 2017 (Winter)



Solution

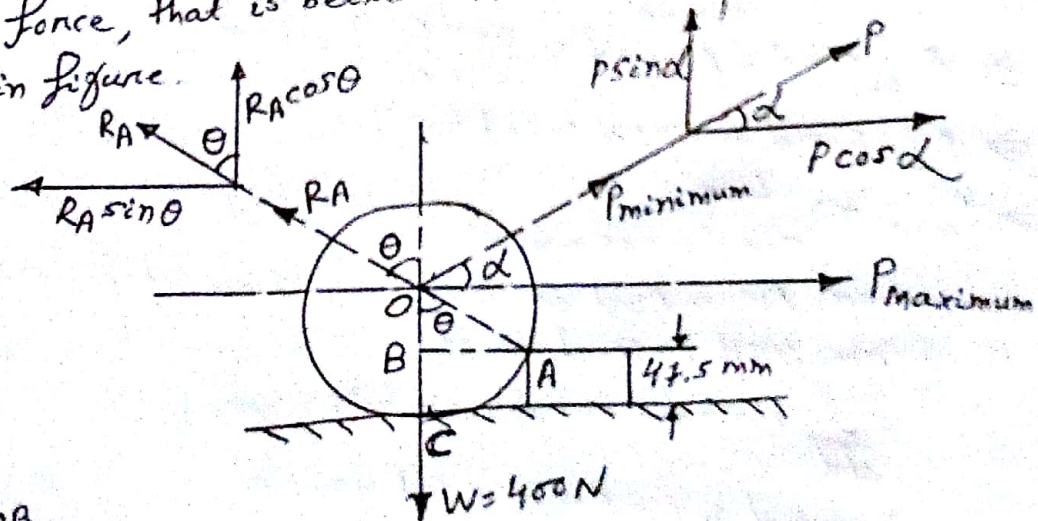


Now, OBA is a Right angle triangle.

$$OB = OC - BC = 200 \text{ mm} - 47.5 \text{ mm} = 152.5 \text{ mm}$$

$$AB = \sqrt{(OA)^2 - (OB)^2} = \sqrt{(200)^2 - (152.5)^2} = 129.39 \text{ mm}$$

One thing is that, centrally, the force will be maximum to rolling the roller very much difficult. So, we can calculate ($P_{\min}$) force, that is better. Now inclined force P will be draw in figure.



ΔAOB

$$\sin \theta = \frac{AB}{OA}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{AB}{OA} \right) = \sin^{-1} \left(\frac{129.39}{200} \right) = 40.31^\circ \text{ or } 40^\circ$$

Now, $\sum H = 0$

case-1 ~~Consider P~~ (Consider $P_{\max}$)

$$\Rightarrow \cancel{P \cos \theta} + \cancel{(-R_A \sin \theta)} = 0$$

$$\Rightarrow (P) + (-R_A \sin \theta) = 0$$

$$\Rightarrow P = R_A \sin \theta$$

$$\Rightarrow P = R_A \times \sin 40^\circ \quad \text{--- (1)}$$

$$\sum V = 0$$

$$\Rightarrow (R_A \cos \theta) + (-W) = 0$$

$$\Rightarrow R_A \cos 40^\circ = 400$$

$$\Rightarrow R_A = \frac{400}{\cos 40^\circ} = 512.16 \text{ N}$$

Put R_A Value in equation (1), we get

$$P_{\max} = R_A \times \sin 40^\circ = 335.6 \text{ N. (Maximum force)}$$

Case-2

$$P_{\min} = ?$$

$$\text{so, } \sum H = 0$$

$$\Rightarrow (P \cos \alpha) + \cancel{(-R_A \sin \theta)} = 0$$

$$\Rightarrow P \cos \alpha = R_A \sin \theta$$

$$\Rightarrow R_A = \frac{P \cos \alpha}{\sin 40^\circ}$$

$$\sum V = 0$$

$$\Rightarrow P \sin \alpha + R_A \cos \theta + (-400) = 0$$

$$\Rightarrow P \sin \alpha + \frac{\cos \alpha}{\sin 40^\circ} \times \cos \theta = 400$$

$$\Rightarrow P \left[\sin \alpha + \cos \alpha \times \frac{\cos \theta}{\sin \theta} \right] = 400$$

$$\Rightarrow P \left[\sin \alpha + \cos \alpha \times \cot 40^\circ \right] = 400$$

$$\Rightarrow P_{\min} = \frac{400}{\sin \alpha + \cos \alpha \times \cot 40^\circ} \quad \text{--- (2)}$$

Whenever Down Part is maximum. Now, derivative the term. with respect to α .

$$\frac{d}{d\alpha} (\sin \alpha + \cos \alpha \times \cot 40^\circ) = 0$$

$$\Rightarrow \cos \alpha + (-\sin \alpha \times \cot 40^\circ) = 0$$

$$\Rightarrow \cos \alpha = \sin \alpha \times \cot 40^\circ$$

$$\Rightarrow \frac{\cos \alpha}{\sin \alpha} = \cot 40^\circ \Rightarrow \cot \alpha = \cot 40^\circ$$

$$\Rightarrow \alpha = 40^\circ$$

Now, putting α value in equation 2, we get

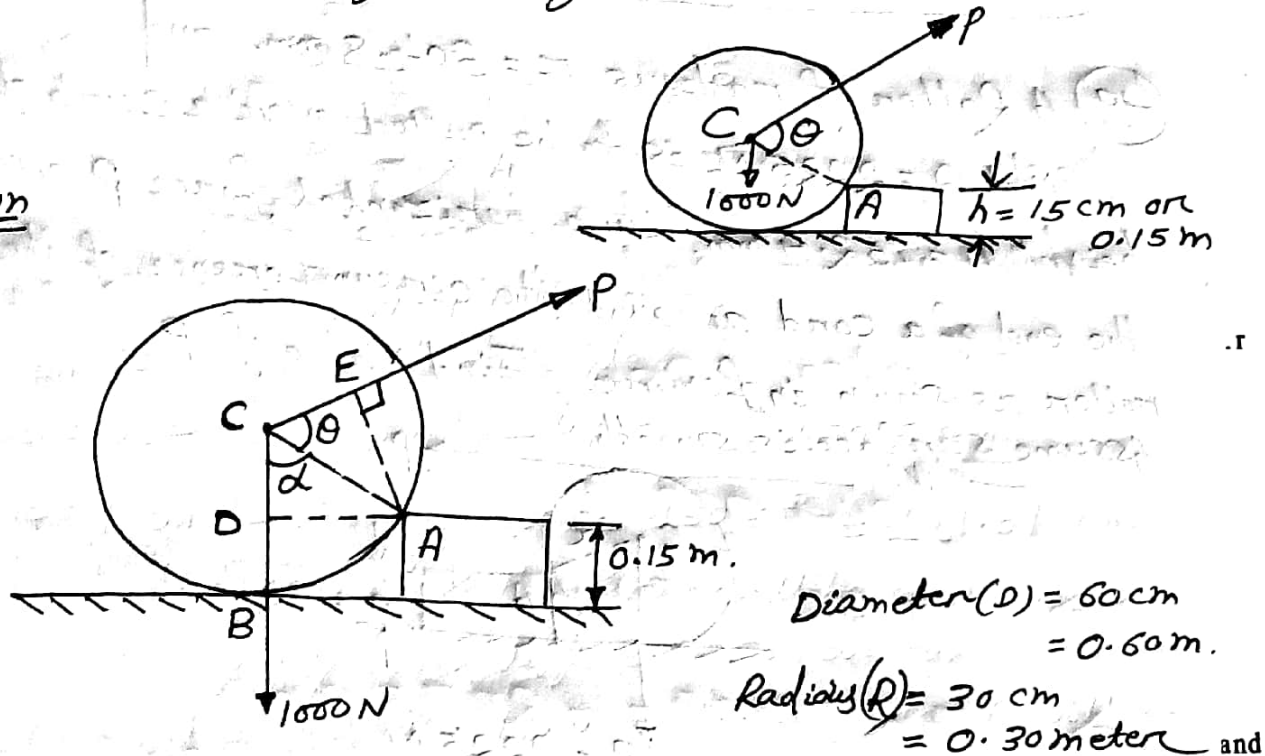
$$(P_{\min})_{\text{force}} = \frac{400}{\sin \alpha + \cos \alpha \times \cot 40^\circ}$$

$$= \frac{400}{\sin 40 + \cos 40 \times \cot 40^\circ}$$

$$= 298.04 \text{ N}$$

- 48) A Uniform wheel of 60 cm. diameter weighing 1000 N rest against a rectangular block 15 cm. High. Find the least force required which when acting through the centre of the wheel will just turn the wheel over corner of the block. Also find Angle θ which this least force shall be make with AC. Figure is given below. Surface is smooth.

Solution



$$(\sum M)_{\text{Point A}} = 0$$

$$\Rightarrow (-1000 \times AD) + (P \times AE) = 0 \quad \text{--- (1)}$$

$$AD = \sqrt{AC^2 - DC^2} = \sqrt{r^2 - (BC - BD)^2} = \sqrt{r^2 - (r - h)^2}$$

$$\Rightarrow AD = \sqrt{(0.30)^2 - (0.30 - 0.15)^2}$$

$$\Rightarrow AD = 0.25 \text{ m.}$$

$$\Delta ACE, \sin \theta = \frac{AE}{AC} \Rightarrow AE = AC \sin \theta = r \sin \theta$$

$$\text{From equation (1): } (1000 \times AD) + (P \times AE) = 0$$

$$\Rightarrow P \times AE = 1000 \times AD$$

$$\Rightarrow P \times r \sin \theta = 1000 \times 0.25$$

$$\Rightarrow P = \frac{1000 \times 0.25}{r \sin \theta} = \frac{1000 \times 0.25}{(0.30) \times \sin \theta}$$

$$\Rightarrow P = \frac{833.33}{\sin \theta}$$

Here, when we calculate $P_{\min}$ force, denominator will be maximum.

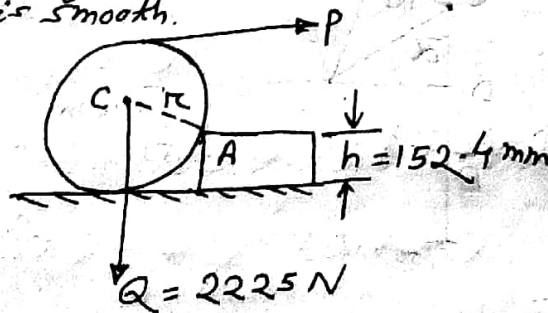
So, $\sin \theta$ is maximum when value is 1

$$\sin \theta = 1 = \sin 90^\circ$$

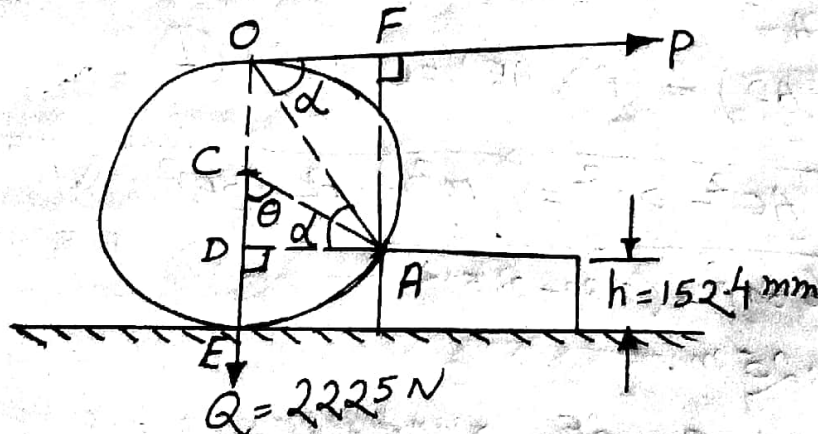
$$\Rightarrow \theta = 90^\circ$$

$$P_{\min} = \frac{833.33}{\sin 90^\circ} = 833.33 \text{ N}$$

(49) A Roller of radius $r = 304.8 \text{ mm}$. and weight $Q = 2225 \text{ N}$ is to be pulled over a curb of height $h = 152.4 \text{ mm}$. by a horizontal force P applied to the end of a cord around the circumference of the roller as shown in figure. Find Value of P . Assume surface is smooth.



Solution



Mark Triangle ACD & ODA

$$\Delta ACD, AC = r = 304.8 \text{ mm.}$$

$$CD = CE - DE = 304.8 - 152.4 = 152.4 \text{ mm.}$$

$$AD = \sqrt{(AC)^2 - (CD)^2} = \sqrt{(304.8)^2 - (152.4)^2}$$

$$\Rightarrow AD = 263.94 \text{ mm.}$$

ΔODA ,

$$OA = \sqrt{OD^2 + AD^2}$$

$$\tan \alpha = \frac{OD}{AD}$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{OC + CD}{AD} \right)$$

$$\Rightarrow \alpha = 60^\circ$$

$$= \sqrt{(OC + CD)^2 + (263.94)^2}$$

$$= \sqrt{(304.8 + 152.4)^2 + (263.94)^2}$$

$$= 527.91 \text{ mm.}$$

Now, ΔOFA ,

$$\sin \alpha = \frac{AF}{OA}$$

$$\Rightarrow AF = OA \times \sin \alpha = 527.91 \times \sin 60^\circ$$

$$\Rightarrow AF = 527.91 \times 0.866 = 457.18 \text{ mm}$$

$$(\sum M)_{\text{Point A}} = 0$$

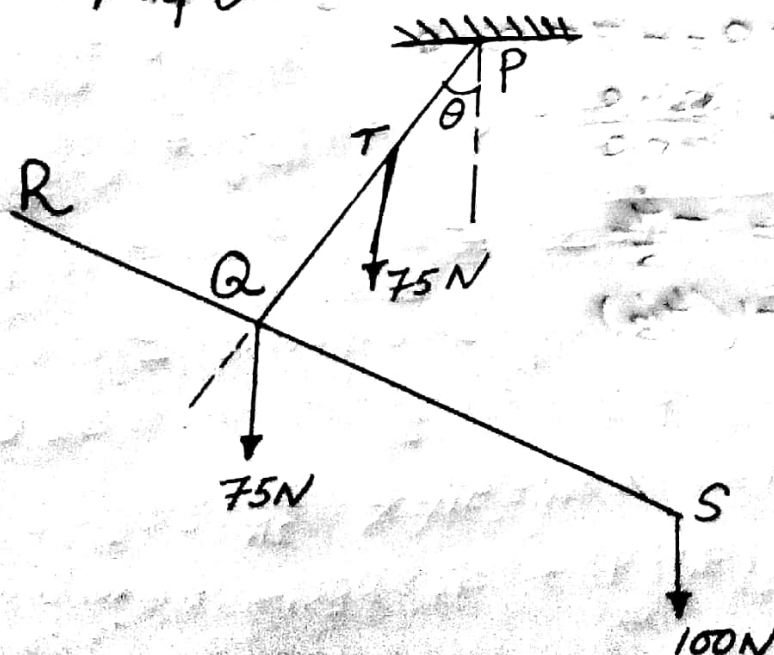
$$\Rightarrow (-Q \times AD) + (AF \times P) = 0$$

$$\Rightarrow AF \times P = Q \times AD$$

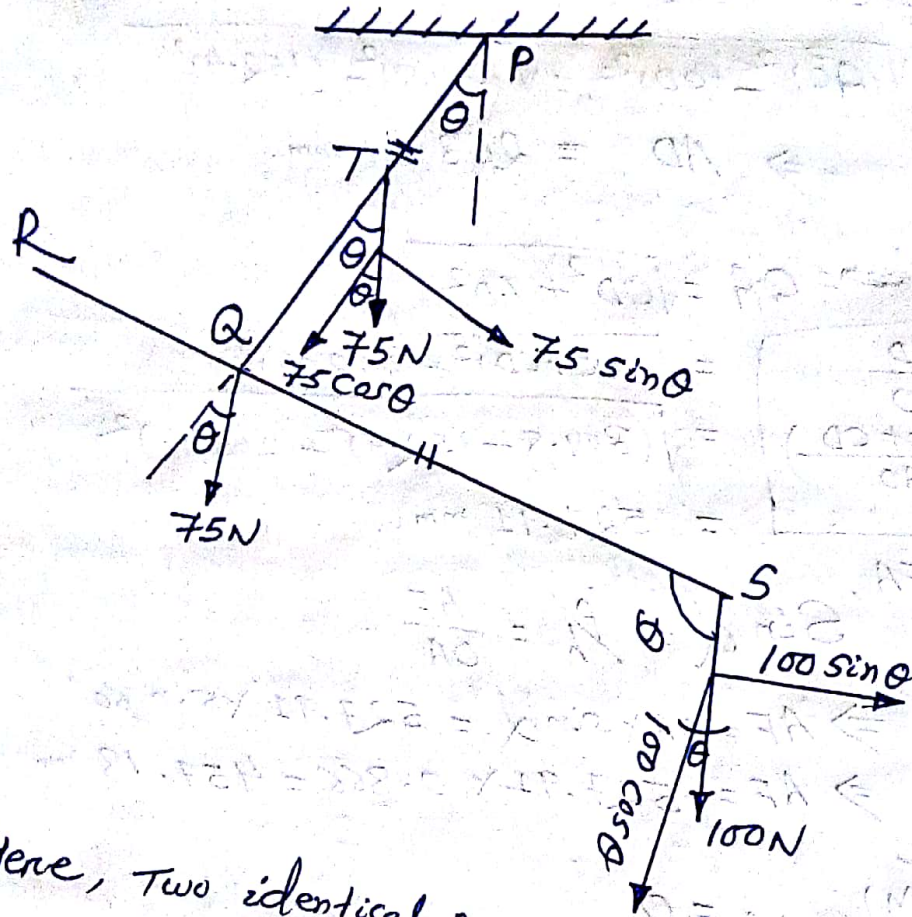
$$\Rightarrow P = \frac{Q \times AD}{AF} = \frac{2225 \times 263.94}{457.18} = 1284.54 \text{ N}$$

- (50) Two identical Prismatic bars PQ & RS each weighing 75 N are welded together to form a Tee and are suspended in a vertical plane as shown in Figure-1. Find θ .

2018 (Summer)
Year Question



Solution



Here, Two identical prismatic bar that means $(PQ = RS)$ is always same. Here also $QS = QT$ is same

($\sum M$) point Q = 0

$$\Rightarrow (100 QS \cos \theta) + (QT \times 75 \sin \theta) = 0$$

$$\Rightarrow 100 QS \cos \theta = -75 \sin \theta$$

$$\Rightarrow 100 \cos \theta = -75 \sin \theta \quad (\because QS = QT)$$

$$\Rightarrow \frac{100}{-75} = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \tan \theta = -\frac{4}{3}$$

$$\Rightarrow \theta = \tan^{-1}(4/3)$$