

Bhubanananda Odisha School of Engineering, Cuttack
Department of Mechanical Engineering



SUBECT- ENGINEERING MECHANICS

PREPARED BY- RAMAN KUMAR SETHI

Engineering Mechanics

Course Code- TH 4(b)

(Common to 1st & 2nd sem)

Period per week:	4	IA:	30 Marks
Total Period:	60	Term End Exam:	70 Marks
Timing of End Exam:	3 Hours	No. Of Credit:	4

Course outcomes:

After completing this course, student will be able to:

1. Analyze the coplanar force system and find out the resultant force of this system by applying basics of mechanics.
2. Determine unknown forces of different engineering systems by applying laws of equilibrium.
3. Apply the principle of friction in various conditions when the object is in static equilibrium.
4. Find the centroid and centre of gravity of various components in engineering system.
5. Analyze different simple machines to find out different influencing parameters viz. Mechanical Advantage, Velocity Ratio and Efficiency.

Course Contents:

Unit – I Basics of mechanics and force system

Significance and relevance of Mechanics, Applied mechanics, Statics, Dynamics.

Space, time, mass, particle, flexible body and rigid body.

Scalar and vector quantity, Units of measurement (SI units) - Fundamental units and derived units.

Force – unit, representation as a vector and by Bow's notation, characteristics and effects of a force, Principle of transmissibility of force, Force system and its classification.

Resolution of a force - Orthogonal components of a force, moment of a force, Varignon's Theorem.

Composition of forces – Resultant, analytical method for determination of resultant for concurrent, non-concurrent and parallel co-planar force systems – Law of triangle, parallelogram and polygon of forces.

Unit- II Equilibrium

Equilibrium and Equilibrant, Free body and Free body diagram, Analytical and graphical methods of analysing equilibrium

Lami's Theorem – statement and explanation, Application for various engineering problems.

Types of beam, supports (simple, hinged, roller and fixed) and loads acting on beam (vertical and inclined point load, uniformly distributed load, couple),

Beam reaction for cantilever, simply supported beam with or without overhang – subjected to combination of Point load and uniformly distributed load.

Beam reaction graphically for simply supported beam subjected to vertical point loads only.

Unit- III Friction

Friction and its relevance in engineering, types and laws of friction, limiting equilibrium, limiting friction, co-efficient of friction, angle of friction, angle of repose, relation between co-efficient of friction and angle of friction.

Equilibrium of bodies on level surface subjected to force parallel and inclined to plane.

Equilibrium of bodies on inclined plane subjected to force parallel to the plane only.

Unit- IV Centroid and centre of gravity

Centroid of geometrical plane figures (square, rectangle, triangle, circle, semi-circle, quarter circle)

Centroid of composite figures composed of not more than three geometrical figures

Centre of Gravity of simple solids (Cube, cuboid, cone, cylinder, sphere, hemisphere)

Centre of Gravity of composite solids composed of not more than two simple solids.

Unit - V Simple lifting machine

Simple lifting machine, load, effort, mechanical advantage, applications and advantages.

Velocity

ratio, efficiency of machines, law of machine.

Ideal machine, friction in machine, maximum Mechanical advantage and efficiency, reversible and non-reversible machines, conditions for reversibility

Velocity ratios of Simple axle and wheel, Differential axle and wheel, Worm and worm wheel, Single purchase and double purchase crab winch, Simple screw jack, Weston's differential pulley block, geared pulley block.

Suggested Learning Resources:

1. Engineering Mechanics by Prof. Bhankhar Bharat Gokaldas (Download from <https://ekumbh.aicte-india.org/dbook.php>)
2. D.S. Bedi, Engineering Mechanics, Khanna Publications, New Delhi (2008)
3. Khurmi, R.S., Applied Mechanics, S. Chand & Co. New Delhi.
4. Bansal R K, A text book of Engineering Mechanics, Laxmi Publications.
5. Ramamrutham, Engineering Mechanics, S. Chand & Co. New Delhi.
6. Dhade, Jamadar & Walawelkar, Fundamental of Applied Mechanics, Pune Vidhyarthi Gruh.
7. Ram, H. D.; Chauhan, A. K., Foundations and Applications of Applied Mechanics, Cambridge University Press.
8. Meriam, J. L., Kraige, L.G., Engineering Mechanics- Statics, Vol. I, Wiley Publications, New Delhi.

Engineering mechanics (CHAPTER-1)

Defn:

→ Mechanics is the branch of physics which deals with the effect of force or multiple forces acting on a body or a system of body, which is at rest or in motion.

→ It deals with the "Macroscopic Approach", i.e. the external effect of the body (velocity, displacement, acceleration etc.).

Types of Body:

(i) Rigid Body:

→ Rigid body is a body in which the distance between all the particles remains constant or in other words the deformation is zero under the action of force.

(ii) Deformable Body:

→ It deforms easily under the action of force.
e.g. rubber.

(iii) Fluid Body:

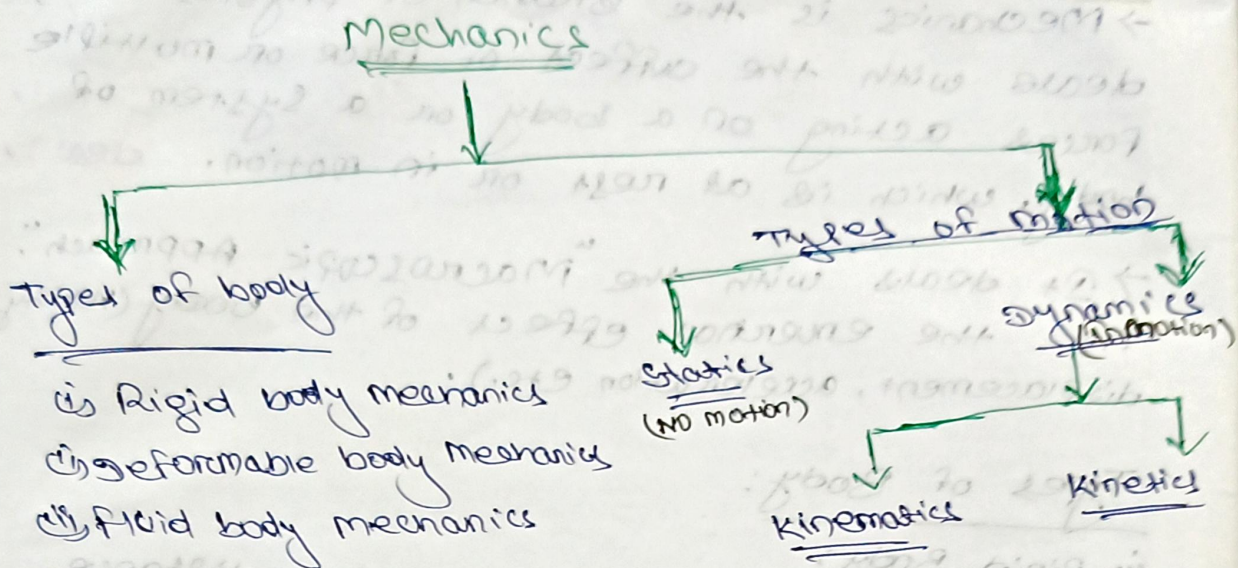
→ Fluid is something that can flow.
e.g. liquid and gas.

* NOTE

Rigid Body Idealization:

In Engineering mechanics we Idealize/Assume that in rigid body there is negligible deformation (no deformation) takes place under the action of external force, in order to simplify our studies.

Classification of Engineering Mechanics:



Statics:

→ Statics is the Branch of mechanics which deals with the body which is at rest.

→ It deals with the forces and their effect while acting upon a body is at rest.

* In Static mechanics the objective is to find out the unknown forces / Reaction forces of a body when it is subjected to any external force. This study helps us to find out the Internal effect acting on a body further.

Dynamics

→ Dynamics is the branch of mechanics which deals with the body which is in motion.

→ Dynamics deals with the forces and their effects while acting upon a body if in motion.

Kinematics: (without considering the forces)

→ It is the branch of dynamics which deals with the body in motion without any reference of the forces which are responsible for the motion.

Kinetics: (considering the forces)

→ It is the branch of dynamics which deals with the body in motion with the reference of the forces which are responsible for the motion.

Assumptions or Idealization:

(i) Particle:

→ In Engineering mechanics we assume that the entire body represented ^{replaced} as a particle & the particle lies in the center of the body, i.e. center of mass.

→ "Particle" located at the center of the mass.

(ii) Rigid Body:

→ No deformation occurs with the effect of forces.

(iii) Point load:

→ we replaced the entire load of a body into a single load, which acts from the center of the mass. Because we assume that the entire body as a particle.

Fundamental concepts

Space: It is a region, which extends in all direction, and contains everything in it.

→ In space position of body is located with respect to a reference system,

Time:

→ It is measure of succession of an event or events.

→ The time is measured in second.

→ An event can be described by position of point

Mass: It is an indication of the quantity of matter present in a system. (S.I unit is Kg)

Weight: It is the measure of the force of gravity acting on a body ($W = mg$.)

$$\Rightarrow W = \text{Kg} \times \frac{\text{m}}{\text{s}^2}$$

$$2) W = \text{N (S.I unit)}$$

Flexible body:

→ A body, which deforms under the action of applied force is called flexible body.

Rigid body:

→ A body which does not deform under the action of applied force

Scalar & vector quantity:

Scalar quantity:

→ quantities which are described by their magnitude known as scalar quantities/quantity

e.g mass (m), Length (L), time (t), volume (V), temp (T) etc

vector quantity:

→ quantities which are described by their magnitude and direction both known as vector quantity

e.g velocity (v), force (F), acceleration (a), momentum etc

1.4 UNITS OF MEASUREMENT [SI UNITS]

Fundamental units : Length, Mass and Time are the basic fundamental quantities and unit of these quantities are known as fundamental units.

Derived units : Units of other than fundamental quantities may be derive from the basic units referred as derived units. Examples: (1) Area is result of multiplication of two lengths quantity as L^2 . (2) Velocity is length divided by time as $\frac{L}{T}$. (3) Force is product of mass & acceleration as $kg \cdot m/sec^2$ [N].

SI units : By international agreement in in 1960, the international system of units known as S.I. Unit is accepted and used all over the worldwide. The symbols and notation of SI units and their derivatives are standardize to avoid any possibility of confusion.

Table 1.1: Fundamental SI units

Sr. No.	Fundamental Quantity	Name of SI unit	Symbol
1	Length	Meter	m
2	Mass	Kilogram	kg
3	Time	Second	s
4	Electrical current	Ampere	A
5	Temperature	Kelvin	K
6	Luminous intensity	Candela	cd

FORCE

→ Force is defined as an agent which produces or tends to produce, destroys or tends to destroy the motion of the body whether the body is in rest condition or in motion.

→ Force is a vector quantity i.e. it has, some magnitude as well as direction. It is represented as $\vec{F}$. Its unit is Newton (N).

Effect of a Force:

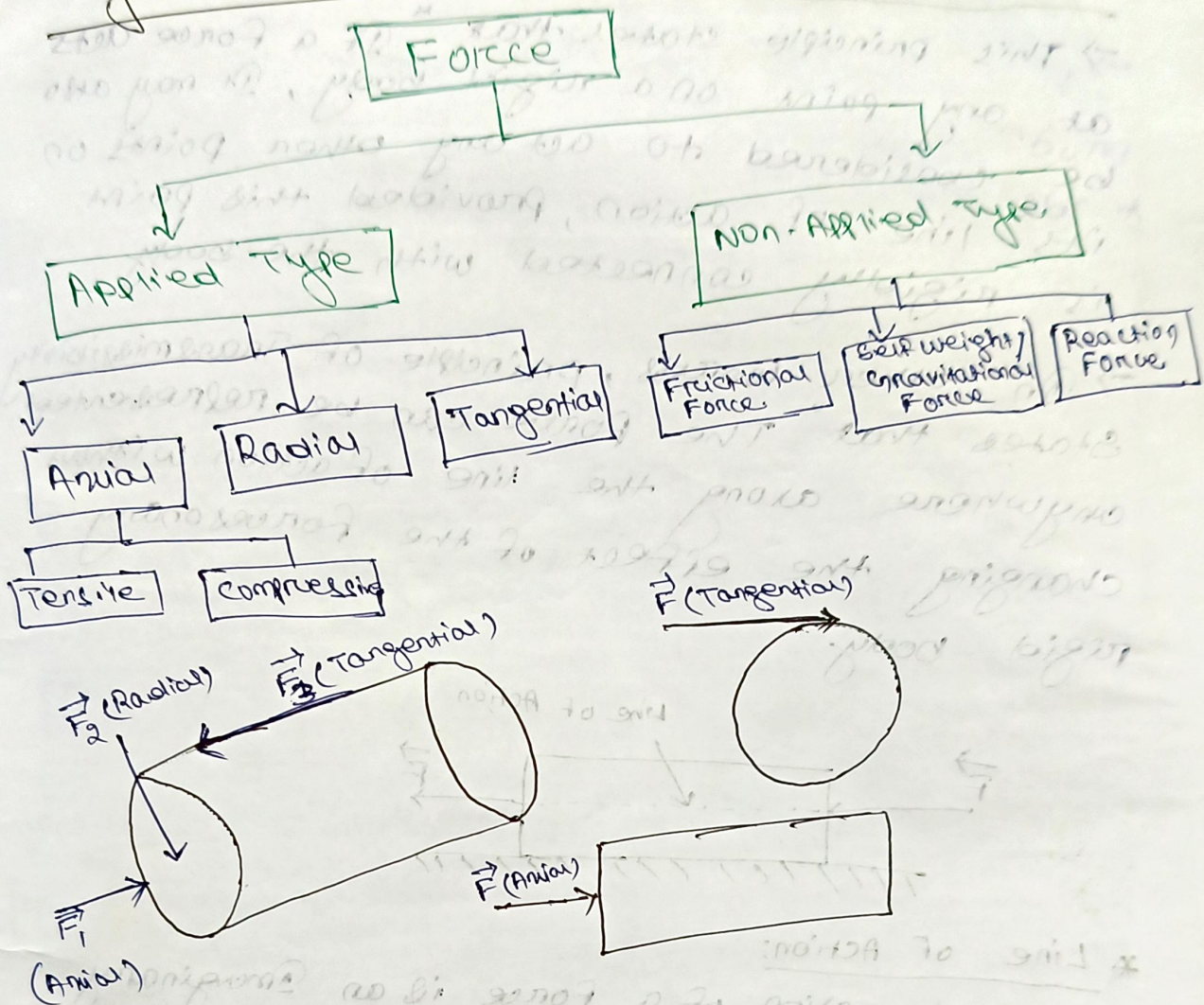
A Force may produce following effect in a body:

- i) It may change the motion of a body.
- ii) It may retard the motion of a body.
- iii) It may retard the forces already acting on a body, thus bringing it to rest or in equilibrium.
- iv) It may increase the effect of the forces.
- v) It may give rise to the internal stresses in a body, on which it acts.

Characteristics of a Force:

- i) Magnitude (10 N, 15 N, 20 N etc.)
- ii) Direction (vertical, horizontal, Angle, sense)
↗ right ↖ left
↑ up ↓ down
- iii) Nature of the Force (Push or Pull)
- iv) Point of action / Application (The point at which or through which the force acts on a body)

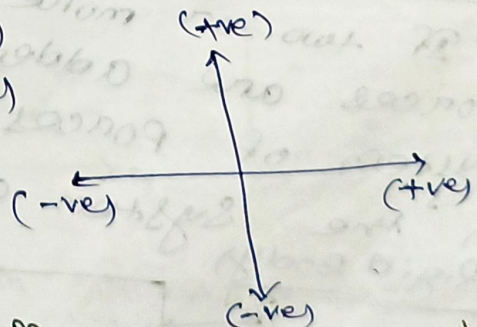
Types of Force:



* Except this categorisation Force may be also classified as
 a) contact type &
 b) non-contact type

Sign convention of Force:

$\rightarrow$ (+ve) $\uparrow$ (+ve)
 $\leftarrow$ (-ve) $\downarrow$ (-ve)

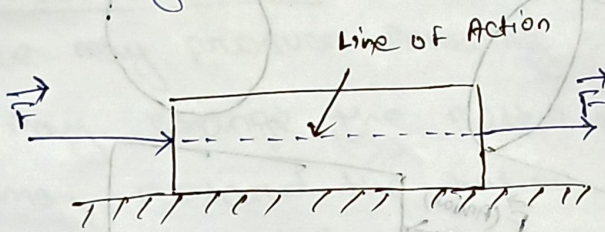


* Some other forces are Mechanical force, spring force, Equilibrium force, resultant force etc.

Principle of Transmissibility: 70 299T

→ This principle states that "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body."

→ In other words, principle of Transmissibility states that "The force can be represented anywhere along the line of action without changing the effect of the force on any rigid body."

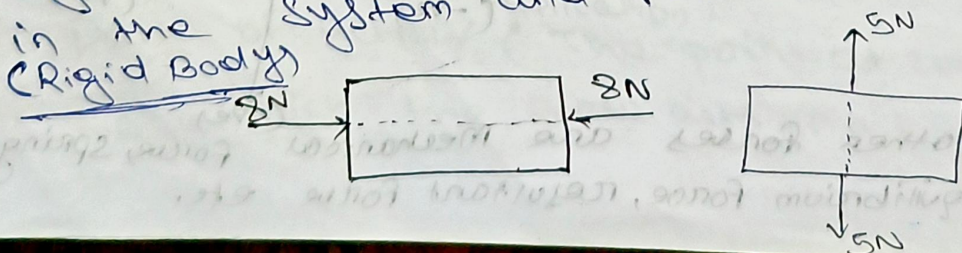


* Line of Action:

→ Line of action of a force is an imaginary straight line drawn through the point of application of the force in the direction of the force.

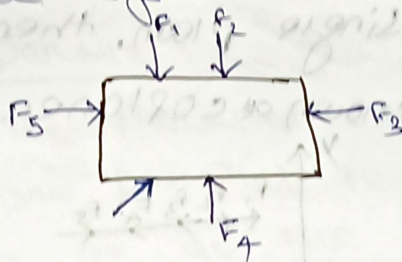
Principle of Superposition of Forces:

→ If two or more equal and opposite collinear forces are added or removed from the system of forces, there will be no change in the system and position of the body (Rigid Body).

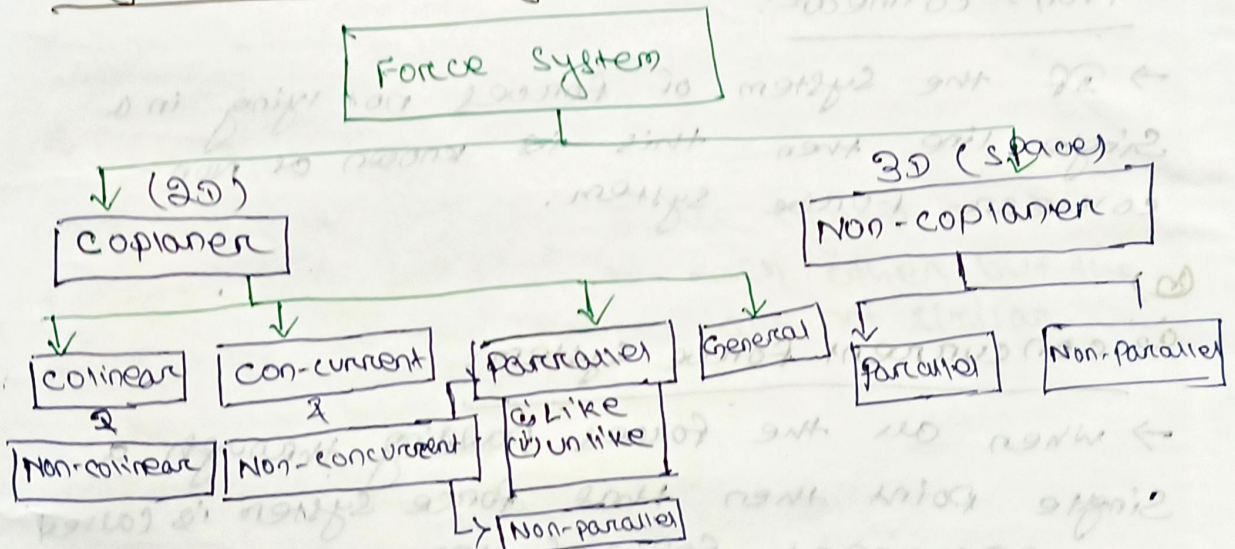


FORCE SYSTEM

→ When two or more forces act on a body, they are called to form a system of forces or Force system.

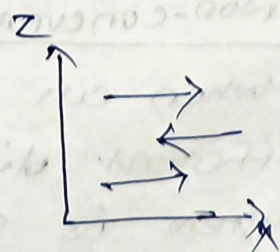
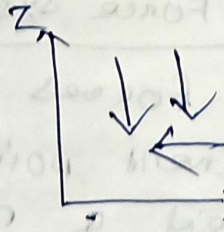
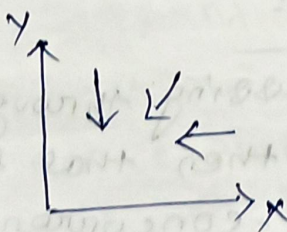


Types of Force system:



① Coplaner Force System:

→ The forces, when acting in a single plane i.e. (X-Y)/(Y-Z) or in (X-Z) plane are known as coplaner force system.



* Force direction may be in outward or in-inwardly towards the point

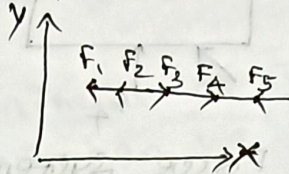
→ Outward from the point

← Towards the point

(a)

is coplanar collinear Force system:

→ If the forces are acting in a single straight line as well as all the forces are acting on a single plain, then this force system ~~is~~ ^{is} known as coplanar collinear force system.



(ii)

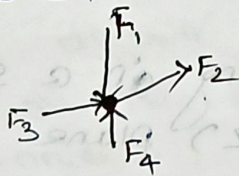
Non - collinear:

→ If the system of forces not lying in a single line then this is known as non-collinear force system.

(b)

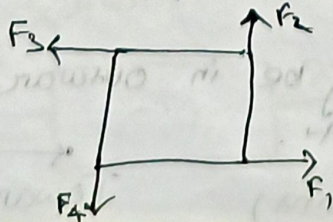
is concurrent force system:

→ when all the forces passing through a single point then that force system is called a concurrent force system.



is non-concurrent Force system:

→ when all the forces passing through a different different point then that force system is called a non-concurrent force system.



②

Parallel Force System:

→ In Parallel Force System all the forces are parallel to each other.

Like Parallel:

→ In Like Parallel Force System all the forces are parallel to each other and the direction of all the forces are same.

$\longrightarrow F_1$

$\longrightarrow F_2$

$\longrightarrow F_3$

Unlike Parallel:

→ In Unlike Parallel Force System all the forces are parallel to each other but the direction of the forces are not similar.

$\longleftarrow F_1$

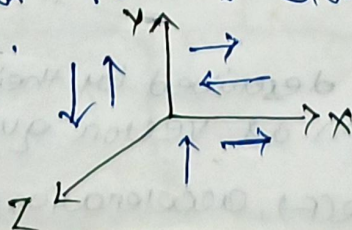
$\longrightarrow F_2$

$\longleftarrow F_3$

② Non-coplanar Force System:

→ When the system of forces are not acting on a single plane (i.e. ~~are~~ acting on different planes) it is called noncoplanar force system.

→ In non-coplanar force system the line of action of forces do not lie in the same plane.



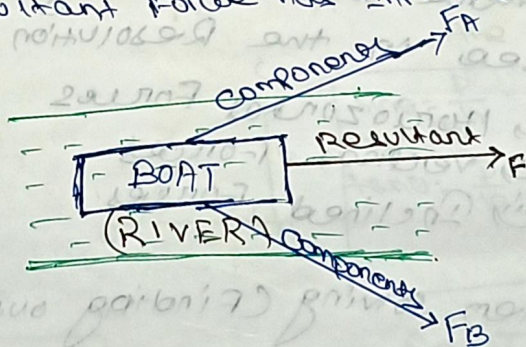
Resolution of a Force:

→ The process of splitting up the given force into two or a number of components / direction without changing the effect on the body is called resolution of a force.

→ In other words, when a single force acting on a body is replaced by two forces or multiple forces acting in such a way that the effect produced by this replaced forces is a same effect as that of single force. Then it is called resolution of a force.

* The resolution of a force is the reverse action of the addition of the component vectors.

* The resultant force has infinite no. of components.



* Here the resultant force $\vec{F}$ is separated by two components $\vec{F}_A$ & $\vec{F}_B$.

* These two forces ($\vec{F}_A$ & $\vec{F}_B$) are called components of a force, and the force which is broken into two parts ($\vec{F}$) is called Resultant of the resolved part.

Resultant Force:

→ If a number of forces are acting simultaneously on a particle, then it is possible to find out a single force which could replace them i.e. which would produce the same effect as produced by the given forces. This single force is called "Resultant Force" and the given forces are called its components.

Types of component Forces:

- i) Perpendicular components/Orthogonal components:
- ii) Non-perpendicular components/Non-orthogonal components:

* Method of Resolution of a Force:

→ Method of Resolution of a Force, simply means to find out the values of its various components of a single force (Resultant force).

→ Generally there are two types of components as stated above i.e. perpendicular and non-perpendicular components.

Resolution of a Force into two-Perpendicular components/Orthogonal components:

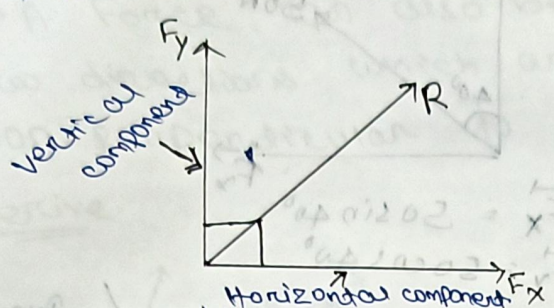
→ Generally there are three types of forces we have to see in the Resolution of forces, these are

- i) Horizontal Forces
- ii) Vertical Forces
- iii) Inclined Forces.

→ While problem solving (Finding out the Resultant force) it is easy to observe or calculate the effect of the Horizontal forces as well as the Vertical forces, but it is very difficult to find out the effect of the Inclined forces because they are not acting on a certain angle and they act in different angles. e.g. ($30^\circ, 40^\circ, 50^\circ, 70^\circ, 120^\circ, 150^\circ$ etc.)

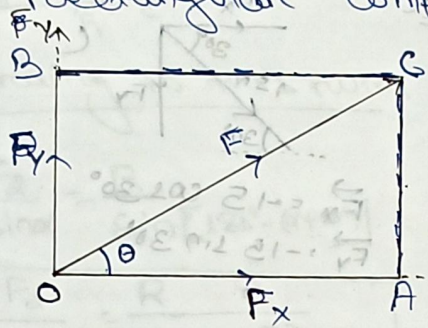
→ In case of perpendicular component forces we have to split the resultant force into

Two components in such a way that they are (two components) (Horizontal component & vertical component) are perpendicular to each other.



* Here F_x and F_y are two perpendicular components of Resultant 'R'.

→ Perpendicular components are also considered as the rectangular components.



Notes:

* In the Incline force calculation we have followed up by some Inclination angle (Angle is known), either it is given directly or we have to find out this incline angle by the help of other given angle.

Derive

In $\triangle OAC$

$$\frac{a}{c} = \cos \theta$$

$$\Rightarrow \frac{F_x}{F} = \cos \theta$$

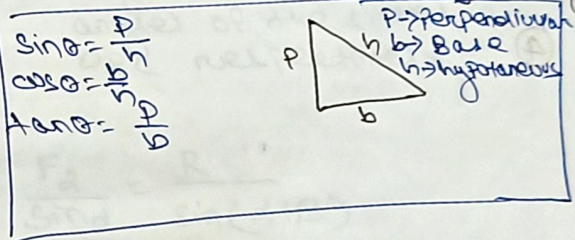
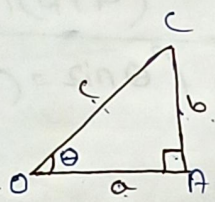
$$\Rightarrow \boxed{F_x = F \cos \theta}$$

again

$$\frac{b}{c} = \sin \theta$$

$$\Rightarrow \frac{F_y}{F} = \sin \theta \quad (\because AC = OB = F_y)$$

$$\Rightarrow \boxed{F_y = F \sin \theta}$$



$$\begin{aligned} \sin \theta &= \frac{p}{c} \\ \cos \theta &= \frac{b}{c} \\ \tan \theta &= \frac{p}{b} \end{aligned}$$

or In $\triangle OBC$

$$\frac{b}{c} = \sin (90^\circ - \theta)$$

$$\Rightarrow \frac{F_y}{F} = \cos \theta$$

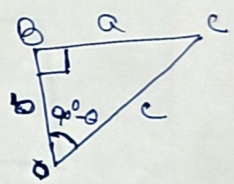
$$\Rightarrow \boxed{F_y = F \cos \theta}$$

again

$$\frac{a}{c} = \cos (90^\circ - \theta)$$

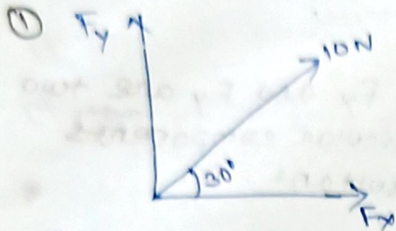
$$\Rightarrow \frac{F_x}{F} = \sin \theta$$

$$\Rightarrow \boxed{F_x = F \sin \theta}$$



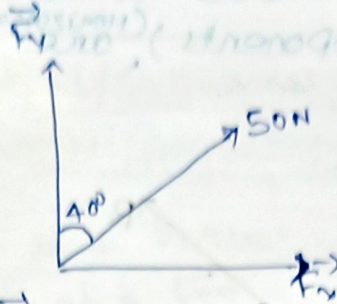
Notes: → The "cos" component of force is always in the direction along axis from angle (θ) on given force.

problems:



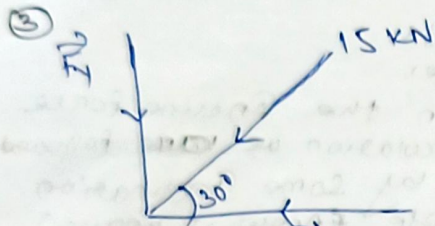
$$\vec{F}_x = 10 \cos 30^\circ$$

$$\vec{F}_y = 10 \sin 30^\circ$$



$$\vec{F}_x = 50 \sin 40^\circ$$

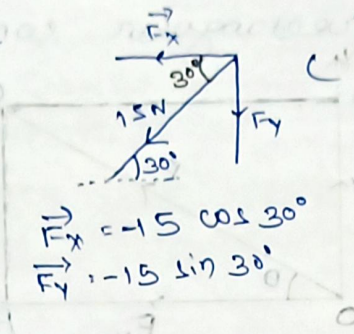
$$\vec{F}_y = 50 \cos 40^\circ$$



$$\vec{F}_x = -15 \cos 30^\circ$$

$$\vec{F}_y = -15 \sin 30^\circ$$

or

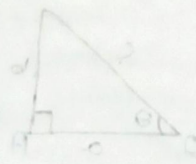


$$\vec{F}_x = -15 \cos 30^\circ$$

$$\vec{F}_y = -15 \sin 30^\circ$$

Reference
→ Alternate
Angle

④



① 37°

② 53°

③ 37°

④ 53°

⑤ 37°

⑥ 53°

⑦ 37°

⑧ 53°

⑨ 37°

⑩ 53°

Moment of Force

Def:

- It is the turning effect produced by a force on the body, on which it acts, about a point.
- The ability of a force to produce rotational motion, or the rotational effect of a force.
- It is also known as Torque.
- moment depend upon two factors:

Torque = Force $\times$ moment arm

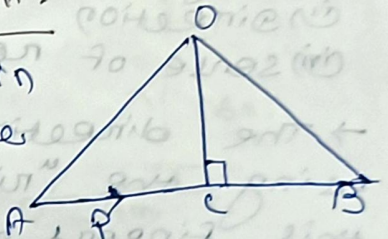
$$\tau = F \times l$$

moment arm = perpendicular distance between the line of action of force and the axis of rotation.

$$\text{moment} = \text{Force} \times \text{perpendicular distance}$$

Graphical representation of moment:

→ consider a force P represented, in magnitude and direction, by the line AB .



→ Let O be a point, about which the moment of this force is required to be found out.

→ So in order to find out this moment we have to draw a perpendicular line to AB from the point O .

Now moment of force P about O point

$$= P \times OC$$

or

$$AB \times OC$$

$$= 2 \times \frac{1}{2} \times AB \times OC$$

$$= 2 \times \text{Area of } \triangle OAB$$

→ Hence moment of a force about a point is equal to twice of the Area of the triangle.

units of moment:

Since the moment of a force is the product of force and distance, so the unit of moment will be, $\boxed{\text{N-m}}$ $\rightarrow$ S.I. unit

(i) KN-m

(ii) Nm etc

$$1 \text{ KN-m} = 10^6 \text{ N-mm}$$

Types of moment:

Two types

(i) Clockwise moment (C.W) ~~(+ve)~~

(ii) Anticlockwise moment (C.C.W) (+ve)

Note:

It is not necessary that you have to choose C.W moment $\rightarrow$ -ve or C.C.W moment $\rightarrow$ +ve, you can choose C.W moment $\rightarrow$ +ve & C.C.W moment $\rightarrow$ -ve

Note:

$\rightarrow$ moment of force is a vector quantity, so it has

(i) magnitude (Force $\times$ $\perp$ distance)

(ii) direction

(iii) sense of rotation.

$\rightarrow$ The direction of moment will be obtained by using the "right hand thumb" rule. To do this fingers of the right hand are folded such that they follow the sense of rotation. The thumb then points along the moment axis which gives the direction and sense of the moment.

Note: During problem solving you have to choose either C.W moment +ve & C.C.W moment -ve or C.C.W moment +ve & C.W moment -ve.

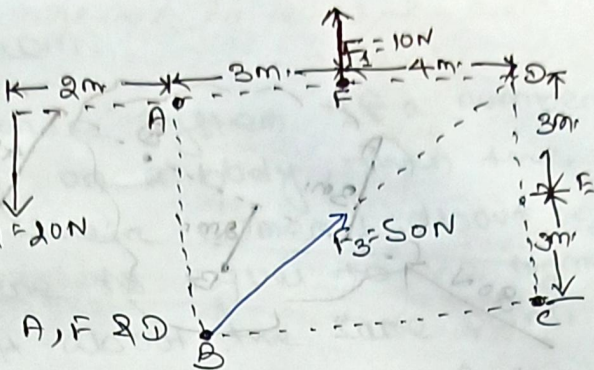
Principle of moment:
If a body is in equilibrium under the action of number of forces acting simultaneously on the body, then the algebraic sum of the clockwise moments of the forces about a point is equal to the sum of anticlockwise moments of the forces about the same point.

Mathematically,

$$\sum M = 0$$

The algebraic sum of moments of all the forces about a point, in their plane is zero.

- Clk moment of F_3 about A, F & D



5015

$\therefore F_2 = 10 \times 3 = 30 \text{ N-m}$ (Clockwise)

Note:

Taking c.w moment $\rightarrow -ve$
c.c.w moment $\rightarrow +ve$

$$M_{F_2} = 10 \times 4 = -40 \text{ N-m (CCW)}$$

$M_A^{F_2} = 20 \times 2 = 40 \text{ N}\cdot\text{m}$ (C.C.W)

$$M_c = 20 \times 9 = 180 \text{ N}\cdot\text{m} \quad (\text{clockwise})$$

(iii) $\vec{F}_2$ is a inclined force.

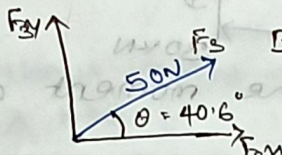
In $\triangle BCD$,

$$\tan \theta = \frac{p}{b}$$

$$2) \tan \theta = \frac{2}{1}$$

$$2) \text{ } \Theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$\Rightarrow \Theta = 40.6^\circ$$



$$F_{BX} = 50 \cos 40.6^\circ = 37.96$$

$$F_{3y} = 50 \sin 40.6^\circ = 32.53$$

$$M_{F_3} = (F_{3x, \text{component}} \times r_x) + (F_{3y, \text{component}} \times r_y)$$

$$= \frac{(50 \cos 40.6^\circ \times 6)}{\text{c.c.w (Horizontal component)}} + \frac{0}{\text{(vertical component)}}$$

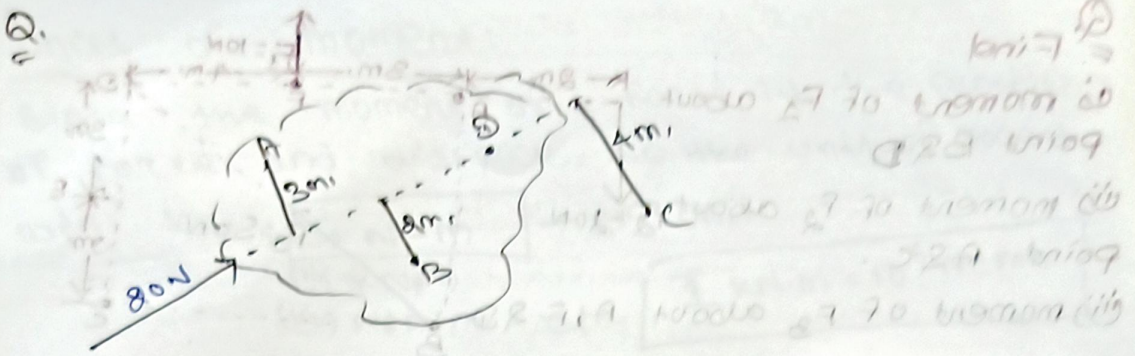
∴ If a force passes through the center of rotation, then the force does not produce any moment

$$= 227.76 \text{ N-m}$$

$$M_F = (50 \cos 40.6 \times 6) - (50 \sin 40.6 \times 3)$$

$$\begin{aligned} M^F_3 &= (50 \cos 40.6 \times 6) - (50 \sin 40.6 \times 7) \\ &= 227.78 - 227.78 \\ &= 0 \end{aligned}$$

Note $\vec{F}_3$ Force passes through point D, so we can directly take $M_D = 0$.



Find

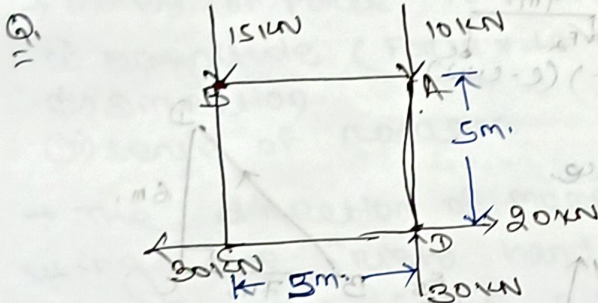
$$\textcircled{i} M_A = 80 \times 3 = +240 \text{ N}\cdot\text{m} \quad [\text{↺ (c.c.w)}]$$

$$\textcircled{ii} M_B = 80 \times 2 = -160 \text{ N}\cdot\text{m} \quad [\text{↻ (c.w)}]$$

$$\textcircled{iii} M_C = 20 \times 4 = +80 \text{ N}\cdot\text{m} \quad [\text{↺ (c.w)}]$$

$$\textcircled{iv} M_D = 20 \times 0 = 0$$

Sense of rotation



Calculate the moment at point B of various forces.

$$\sum M_B = ?$$

Taking c.w moment +ve
c.c.w moment -ve

$$\sum M_B = (15 \times 0) + (10 \times 5) - (20 \times 5) - (30 \times 5) + (30 \times 5)$$

$$= 0 + 50 - 100 - 150 + 150$$

$$= -50 \text{ N}\cdot\text{m}$$

$\therefore \sum M_B$ is 50 N·m (c.c.w) direction.

Varignon's Theorem:

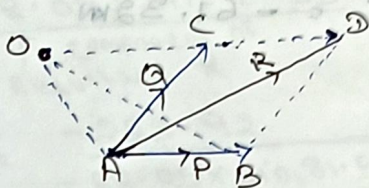
→ In a coplanar force system if a number of forces are acting on a body, then the algebraic sum of their moments about a point in their plane is equal to the moment of their resultant about the same point.

→ It is used to find the location of the point of application of resultant force w.r.t. any point or axis.

Sum of moment of Individual Forces = moment of resultant force alone

Proof

$$\sum M_O = R \times x$$



Take a point O along the line of action of the force $\vec{CD}$, such that $CD = OC$

moment of force $\vec{P}$, about point 'O'

$$= 2 \times \text{Area of } \triangle AOB$$

moment of force $\vec{Q}$, about point 'O'

$$= 2 \times \text{Area of } \triangle AOC$$

moment of resultant $\vec{R}$, about point 'O'

$$= 2 \times \text{Area of } \triangle AOD$$

$$\triangle AOD = \triangle AOC + \triangle ACD$$

$$\Rightarrow \triangle AOD = \triangle AOC + \triangle AOB$$

moment of Resultant force $\vec{R}$

$$\vec{R} = 2 \times \text{Area of } \triangle AOD$$

$$\Rightarrow \vec{R} = 2 \times \text{Area of } (\triangle AOC + \triangle AOB)$$

We know,

$$\triangle ACD = \triangle AOB$$

$$\triangle ABD = \triangle AOB$$

So,

$$\triangle ACD = \triangle AOB$$

proved

COMPOSITION OF FORCES

Resultant Force:

As we discussed earlier, Resultant Force is a single force which produces the same effect as a number of forces acting together.

Composition of Forces:

→ The process of finding out the resultant force, of a number of given force is called composition, compounding of forces.

Method of composition of forces / method of finding Resultant force:

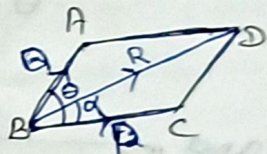
- ① Analytical method
- ② Graphical method

Analytical method For finding Resultant Force:

→ The resultant force, of a given system of forces, may be found out analytically by the following methods:

- ① parallelogram law of force
- ② Method of Resolution

Parallelogram Law of Forces

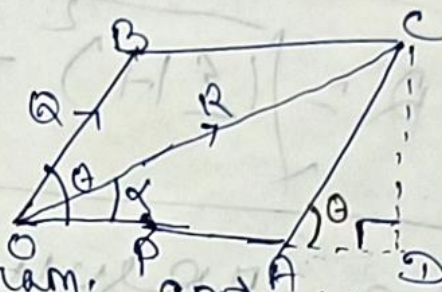


→ In a parallelogram opposite sides are parallel to each other.

→ Parallelogram Law of Forces states that "If, two, forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram; then resultant may be represented in magnitude and direction, by the diagonal of the parallelogram, which passes through their point of intersection."

Derive parallelogram law of forces:

consider two forces $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the two adjacent sides of the parallelogram, and the resultant force $\vec{R}$ is represented in magnitude and direction by the diagonal of the parallelogram.



In $\triangle OCD$,

$$OC^2 = OD^2 + CD^2$$

$$\Rightarrow OC^2 = (OA + AD)^2 + CD^2$$

$$\Rightarrow R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$

$$\Rightarrow R^2 = P^2 + Q^2 \cos^2 \theta + 2 \times P \times Q \cos \theta + Q^2 \sin^2 \theta$$

$$\Rightarrow R^2 = P^2 + Q^2 \cos^2 \theta + Q^2 \sin^2 \theta + 2PQ \cos \theta$$

$$\Rightarrow R^2 = P^2 + Q^2 (\cos^2 \theta + \sin^2 \theta) + 2PQ \cos \theta$$

$$\Rightarrow \boxed{R^2 = P^2 + Q^2 + 2PQ \cos \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$\text{So, } \boxed{R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}} \quad (\text{magnitude of } \vec{R})$$

Again $\tan \alpha = \frac{CD}{OD} \quad (\because \tan \theta = \frac{P}{Q})$

$$\Rightarrow \tan \alpha = \frac{CD}{OA + AD}$$

$$\Rightarrow \tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\text{So } \boxed{\alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)}$$

(direction of $\vec{R}$)

In $\triangle ACD$

$$\angle CAD = \theta \quad (\because \angle BOA = \angle CAD \text{ corresponding angle})$$

$$\sin \theta = \frac{P}{h}$$

$$\Rightarrow \sin \theta = \frac{CD}{AD}$$

$$\Rightarrow \sin \theta = \frac{CD}{Q} \quad (\because \angle BOA = \angle CAD = \theta)$$

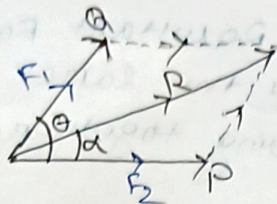
$$\Rightarrow CD = Q \sin \theta$$

Similarly, $\cos \theta = \frac{b}{h}$

$$\Rightarrow \cos \theta = \frac{AD}{AC}$$

$$\Rightarrow \cos \theta = \frac{AD}{Q}$$

$$\Rightarrow AD = Q \cos \theta$$



Note:

Whenever there are two forces acting on a plane intersecting each other, we use "parallelogram law of forces" for finding out of "Resultant".

* This principle applies where,

- i) There are two concurrent forces,
- ii) These forces have same ~~size~~ (direction)

Mathematically,
Resultant force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\text{or } R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

where,

P and Q = Two concurrent forces acting on a single point

θ = $\angle$ is the angle between these two forces P & Q.

and

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$\text{or } \tan\alpha = \frac{Q \sin\theta}{P + Q \cos\theta}$$

F_1 and $F_2 \rightarrow$ Forces whose resultant is required to be found out.

$\theta \rightarrow$ Angle between the forces $\vec{F}_1$ & $\vec{F}_2$

$\alpha \rightarrow$ Angle which the resultant force makes with one of the forces ($\vec{F}_2$)

$$\alpha = \tan^{-1} \left| \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} \right|$$

$$\text{or } \alpha = \tan^{-1} \left(\frac{Q \sin\theta}{P + Q \cos\theta} \right)$$

problem:

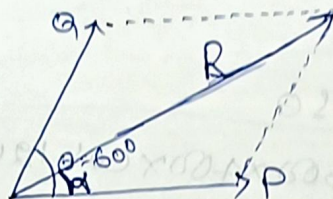
Q. calculate the resultant of two forces having magnitude 600 N and 400 N, and acting at 60° to one another, when they have

- ① Same sense
- ② opposite sense

Solve

① Same sense:

$$P = 600 \text{ N}, Q = 400 \text{ N}, \theta = 60^\circ$$



$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$
$$= \sqrt{(600)^2 + (400)^2 + 2 \times 600 \times 400 \cos 60^\circ}$$

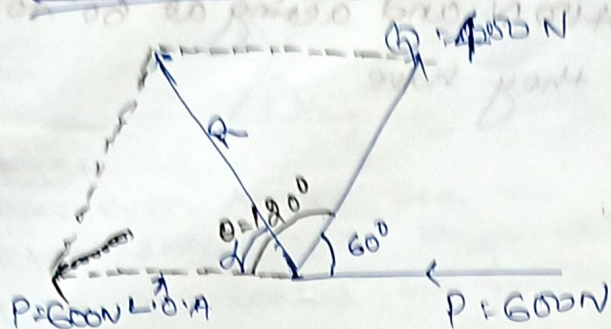
$$R = 871.773 \text{ N}$$

$$\alpha = \tan^{-1} \left| \frac{Q \sin \theta}{P + Q \cos \theta} \right|$$

$$= \tan^{-1} \left| \frac{400 \sin 60^\circ}{600 + 400 \cos 60^\circ} \right|$$

$$\alpha = 23.413^\circ$$

② Opposite Sense



$$P = 600\text{ N}$$

$$Q = 400\text{ N}$$

$$\theta = 120^\circ$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$= \sqrt{(600)^2 + (400)^2 + 2 \times 600 \times 400 \times \cos 120^\circ}$$

$$= 529.15\text{ N}$$

$$\alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)$$

$$= \tan^{-1} \left(\frac{400 \sin 120}{600 + 400 \cos 120} \right)$$

$$= 40.89^\circ$$

Note: Case-I

① If $\theta = 0$ (When the forces act along the same line).

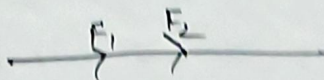
$$R = F_1 + F_2$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$= \sqrt{P^2 + Q^2 + 2PQ} \quad (\because \cos 0^\circ = 1)$$

$$= \sqrt{(P + Q)^2}$$

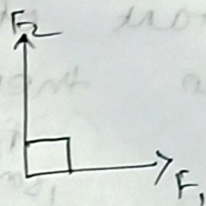
$$\text{So, } R = P + Q$$



② Case-2

→ If $\theta = 90^\circ$, when the forces act at right angle.

$$R = \sqrt{F_1^2 + F_2^2}$$



$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$= \sqrt{P^2 + Q^2} \quad (\because \cos 90^\circ = 0)$$

③ If $\theta = 180^\circ$, when the forces act along the same straight line but in opposite direction.

$$R = F_1 - F_2$$



$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$= \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P - Q)^2}$$

$$R = P - Q$$

④ If the two forces (i.e. when $F_1 = F_2 = F$) are equal
 $P = Q = F$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$= \sqrt{F^2 + F^2 + 2F^2 \cos \theta}$$

$$= \sqrt{2F^2 + 2F^2 \cos \theta}$$

$$= \sqrt{2F^2 (1 + \cos \theta)}$$

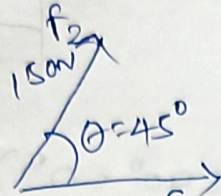
$$= \sqrt{2F^2 \cdot 2 \cos^2 \left(\frac{\theta}{2} \right)}$$

$$= \sqrt{4F^2 \cos^2 \left(\frac{\theta}{2} \right)}$$

$$R = 2F \cos \left(\frac{\theta}{2} \right)$$

Q. Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these forces, if the angle between them is 45°

Solⁿ



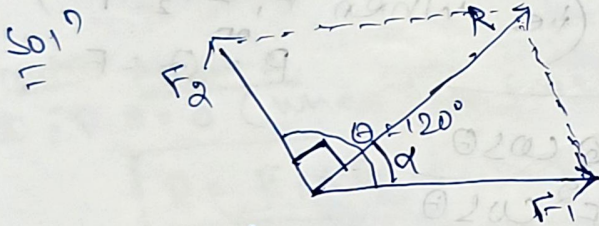
$$F_1 = 100 \text{ N}, F_2 = 150 \text{ N}, \theta = 45^\circ$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(100)^2 + (150)^2 + 2 \times 100 \times 150 \times \cos 45^\circ}$$

$$= 232 \text{ N.}$$

Q. Two forces act at an angle of 120° . The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.



$$F_1 = 40 \text{ N}$$

$$\theta = 120^\circ$$

$$\alpha = 120^\circ - 90^\circ = 30^\circ$$

We know that,

$$\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\Rightarrow \tan 30^\circ = \frac{F_2 \sin 120^\circ}{40 + F_2 \cos 120^\circ}$$

$$\Rightarrow \tan 30^\circ$$

$$\Rightarrow 0.577 = \frac{F_2 \times 0.866}{40 + F_2 (-0.5)}$$

$$\Rightarrow 23.08 - 0.2885 F_2 = 0.866 F_2$$

$$\Rightarrow 1.1545 F_2 = 23.08$$

$$\Rightarrow F_2 = 20.61 \approx 20 \text{ N.}$$

Any

Method of Resolution

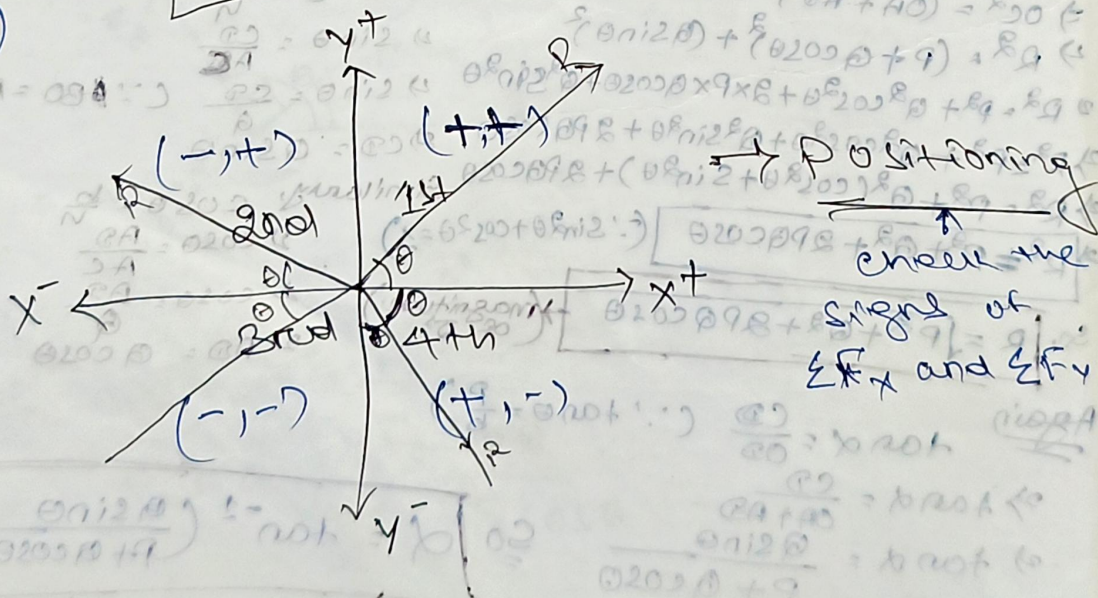
- ① Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (ΣH)
- ② Resolve all the forces vertically and find the algebraic sum of all the vertical components (ΣV)
- ③ The Resultant R of the given forces will be given by the equation.

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \rightarrow \text{Magnitude}$$

- ④ The resultant force will be inclined at an angle θ , with the horizontal, such that,

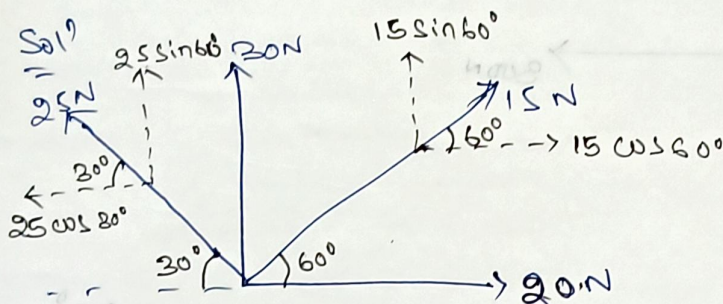
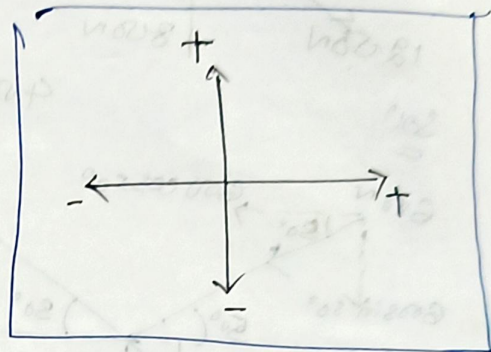
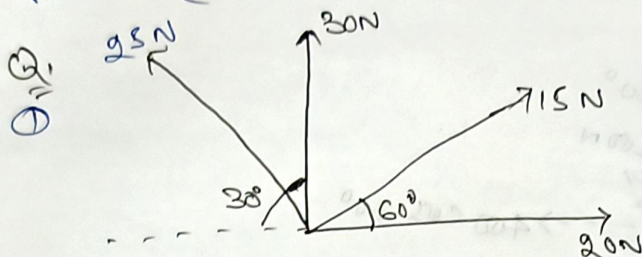
$$\tan \theta = \frac{|\Sigma V|}{|\Sigma H|} \rightarrow \text{Direction}$$

⑤



Note:

→ When there are more than two forces acting on a single point as well as all the forces are in a single plane, we use "method of resolution" for finding the resultant of these forces (coplanar concurrent forces).



$$\sum F_x = 20\text{N} + 15 \cos 60^\circ - 25 \cos 30^\circ$$

$$= 5.849\text{N}$$

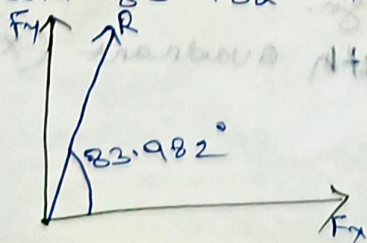
$$\sum F_y = 30\text{N} + 15 \sin 60^\circ + 25 \sin 30^\circ$$

$$= 55.49\text{N}$$

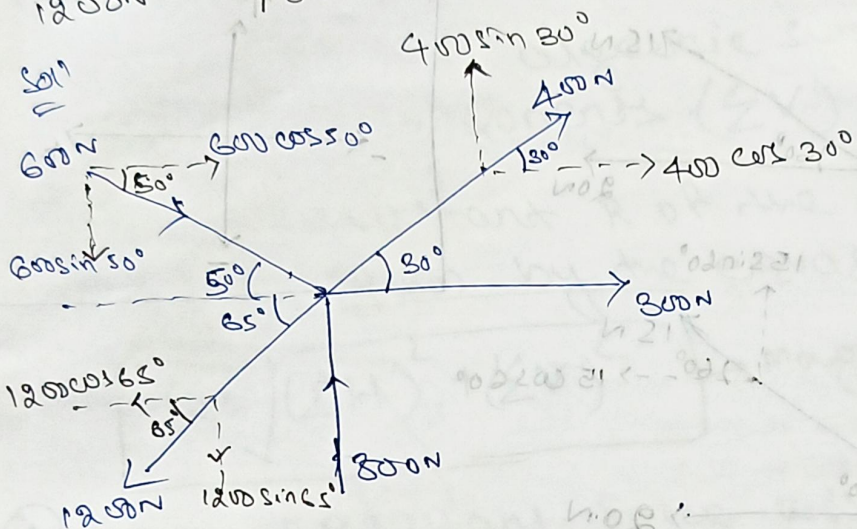
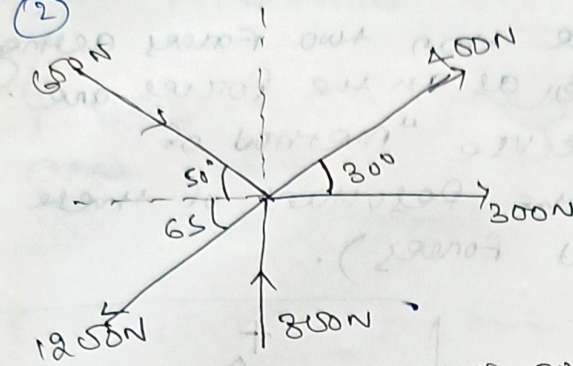
$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{(5.89)^2 + (55.49)^2} = 55.79\text{N}$$

$$\theta = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right) = \tan^{-1} \left(\frac{55.49}{5.89} \right) = 83.982^\circ$$

Note: The sign of $\sum F_x$ and $\sum F_y$ both are positive, so the resultant lies in 1st quadrant and 83.982° inclined to x -axis.



Q1
Q2



$$\Sigma F_x = 300 \text{ N} + 400 \cos 30^\circ + 600 \cos 50^\circ - 1200 \cos 65^\circ$$

$$= 524.94 \text{ N}$$

$$\Sigma F_y = 800 + 400 \sin 30^\circ - 600 \sin 50^\circ - 1200 \sin 65^\circ$$

$$= -547.196 \text{ N}$$

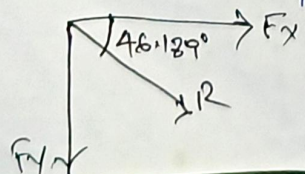
$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(524.94)^2 + (-547.196)^2}$$

$$R = 758.277 \text{ N}$$

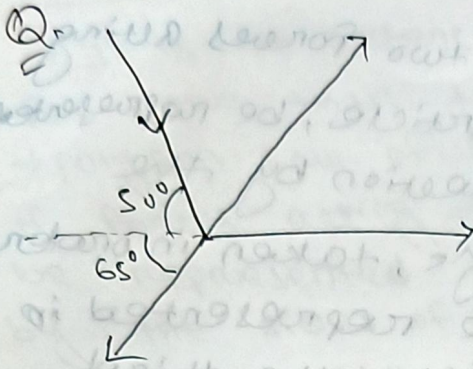
$$\theta = \tan^{-1} \left(\frac{\Sigma F_y}{\Sigma F_x} \right) = \tan^{-1} \left(\frac{-547.196}{524.94} \right) =$$

$$\theta = 46.189^\circ$$

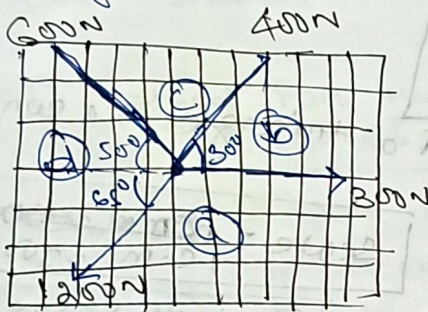
NOTE: θ will always kept in +ve sign. no need to write it in -ve sign.
 → The resultant lies 4th quadrant ($x \rightarrow +ve, y \rightarrow -ve$)



Graphical method



→ First we have to draw space diagram. Exact angle and exact / Actual direction, in the graph page.



→ Sectioning it in C.W or in C.C.W direction, by using bow's notation,

→ In the vector diagram starting from a convenient point and then go on adding all the forces vectorially one by one (keeping in view the direction of the forces) to some suitable scale.

→ The graphical method for the resultant force is briefly discussed below. There is two method (graphically)

- Triangle law of forces (Two forces)
- Polygon law of forces (more than two forces)

GRAPHICAL METHOD

Triangle Law of Forces:

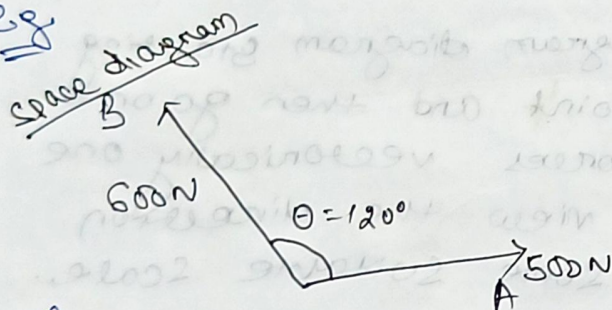
→ It states that "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order, their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."

$$\vec{A} + \vec{B} = \vec{R}$$

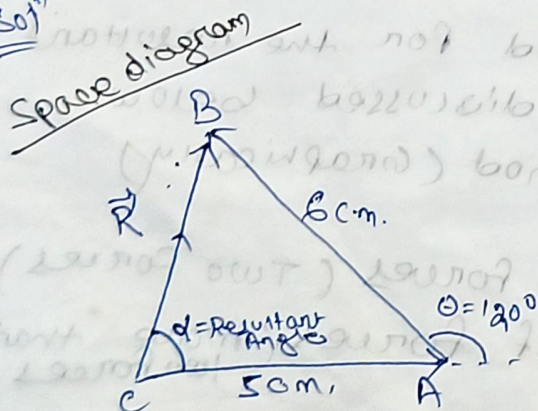
* Forming two sides of a triangle in a continuous line form.

Scale: ~~1 cm = 100 N~~
1 cm = 100 N

Eg

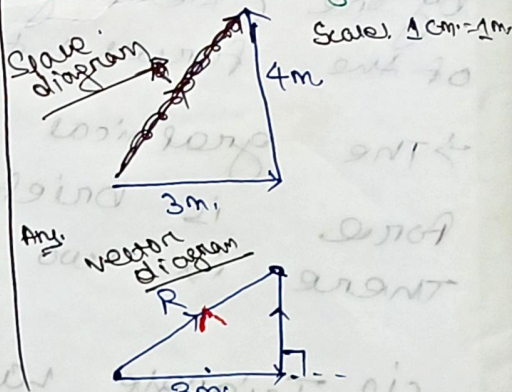


Soln



Try to Solve

Q. A girl moves 3m towards east, then she takes 90° left turn and moves 4m north. Find displacement of girl.



d → Resultant Angle (Angle inclined to horizontal side)
 $\vec{CB}$ → magnitude of Resultant (Third side of the triangle taken in opposite order)

Polygon Law of Forces:

- It is an extension of "Triangle Law of Forces".
for more than two forces.
→ Polygon Law of Forces states that "If a number of forces acting simultaneously on a particle be represented in magnitude and direction, by the sides of a polygon taken in order, then the resultant of all these forces may be represented in magnitude and direction by the closing side of the polygon, taken in opposite order."

Graphical method for the Resultant Force by Polygon law of forces:

① Construction of space diagram (position diagram):

→ It means the construction of a diagram showing the various along with their magnitude and lines of action.

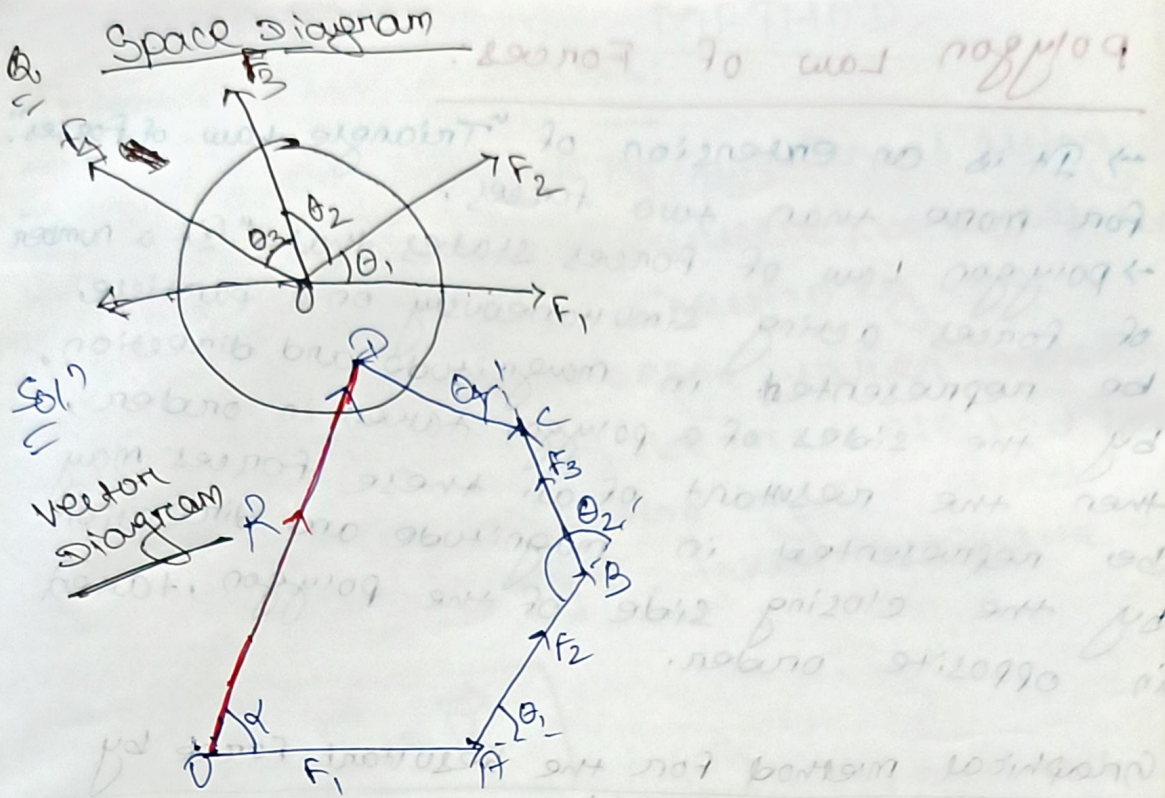
② Use of Bow's notation:

→ All the ~~method~~ forces in the space diagram are named by using the Bow's notation. It is a convenient method in which every force is named by two capital letters, placed on its either sides in the space diagram.

③ Construction of vector diagram (Force diagram):

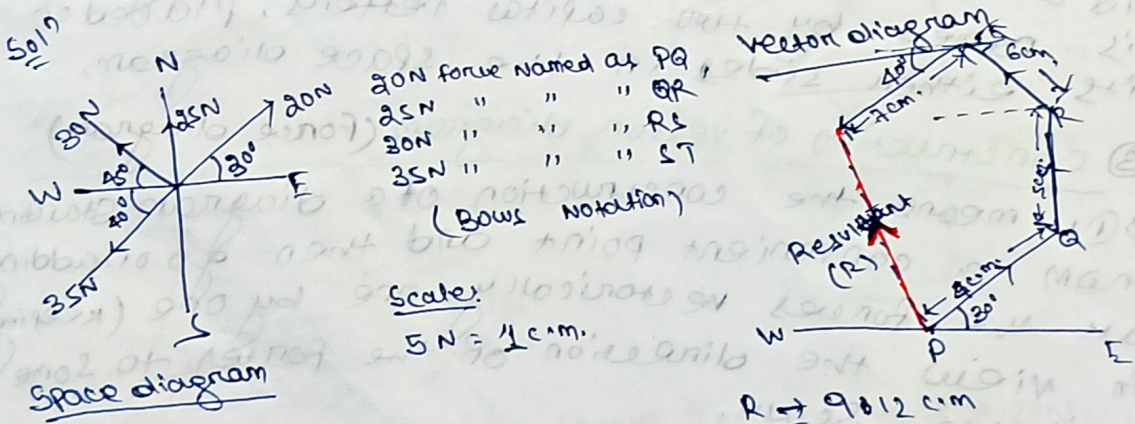
→ It means the construction of a diagram starting from a convenient point and then go on adding all the forces vectorially one by one (keeping in view the direction of the force) to some suitable scale.

* Now the closing side of the polygon, taken in opposite order will give the magnitude of the resultant force (to the scale) and its direction.



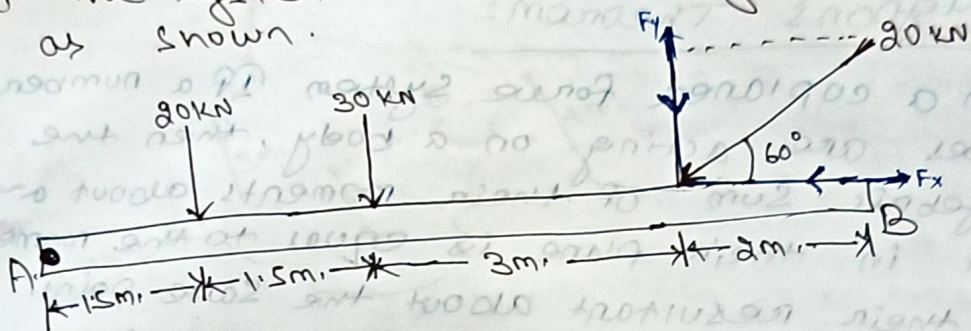
- Q. Find the magnitude and direction of resultant using polygon law of forces.
- Soln
- (i) 20N, at 30° North of East
 - (ii) 25N towards North
 - (iii) 30N towards North of West at 45°
 - (iv) 35N ~~at 135°~~ Inclined at 40° towards South of West.

Find the magnitude and direction of resultant using polygon law of forces.



$R = 9.12 \text{ cm}$
 $= 9.12 \times 5$
 $= 45.60 \text{ N}$
 $\theta = 132^\circ \text{ with east}$
 $\theta = 48^\circ \text{ with west}$

Q. The system of forces acting on a beam AB is as shown.



Determine the magnitude, direction and location of point of application of resultant, in the given force system with point A.

Solⁿ

We can solve the resultant force by "method of resolution".

$$\sum H = -20 \cos 60^\circ = -10 \text{ kN}$$

$$\sum V = -20 - 30 - 20 \sin 60^\circ = -67.32 \text{ kN}$$

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

$$= \sqrt{(-10)^2 + (-67.32)^2}$$

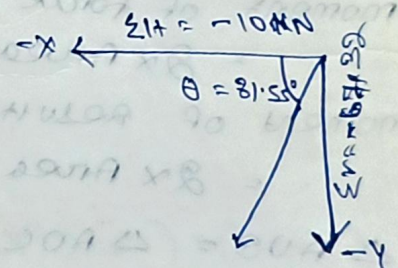
$$= 68.05 \text{ kN}$$

Both $\sum H$ and $\sum V$ in -ve sign, so the resultant lies in 3rd quadrant.

$$\theta = \tan^{-1} \left| \frac{\sum V}{\sum H} \right|$$

$$= \tan^{-1} \left| \frac{-67.32}{-10} \right|$$

$$= 81.55^\circ$$



According to Varignon's Theorem,

Sum of moment of individual force = moment of Resultant force alone.

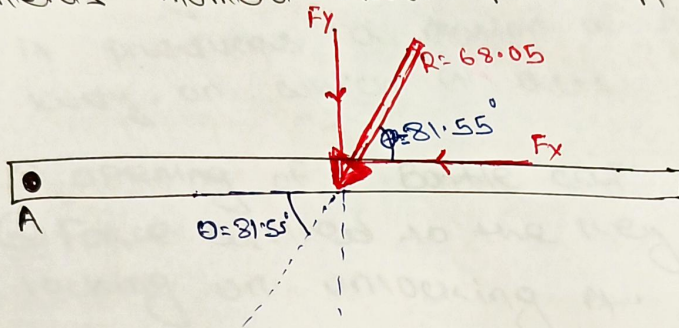
here

$$\sum M_A = R \times x$$

Note: The resultant force lies in an inclined angle so that in calculation, we have to resolve this force into two components (the resultant force lies somewhere in the above beam).

$$\begin{aligned}\sum M_A &= -20 \times 1.5 - 30 \times 3 - 20 \sin 60^\circ \times 6 + 0 \text{ (F}_x \text{ component)} \\ &= -223.92 \text{ kN-m}\end{aligned}$$

Let's assume that the resultant force is acting on ~~any~~ point at a distance of x , due to the inclined angle of resultant force it also have two components and we have to calculate these components moment about point A.



$$R \times x = \sum M_A$$

$$\Rightarrow \frac{-68.05 \times \sin 81.55^\circ \times x}{\text{F}_y \text{ component of Resultant R}} = -223.92$$

$\Rightarrow$ Fx component line of action lies in the point A, so the moment at point A is zero for Fx component.

$$\Rightarrow x = \frac{-223.92}{-68.05 \times \sin 81.55^\circ}$$

$$\Rightarrow x = 3.32 \text{ m.}$$

So the resultant lies at 3.32 m. distance from point A.

