Bhubanananda Orissa School of Engineering

ENGINEERING PHYSICS

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UNIT 1
UNITS AND DIMENSIONS

Physics explains the law of nature in a special way. This explanation includes a quantitative description, comparison, and measurement of certain physical quantities. To measure or compare a physical quantity we need to fix some standard unit and dimension of the quantity. In this chapter we will discuss the basic concept of Units and Dimensions and its application to various physical problems.

1.1 Physical quantities -

Law of physics can be expressed through certain measurable quantities which are called as Physical quantities.

Physical quantities are divided into two categories.

(1) Fundamental Quantities

(2) Derived Quantities

1.1.1 Fundamental Quantities

Fundamental quantities are those that do not depend on any other physical quantities for their measurements.

There are different systems of units which will be discussed in the coming sections. In each system of units, there are a set of defined fundamental quantities and fundamental units.

Mass (M), length (L) and time(T) are some of the examples of fundamental quantities.

1.1.2 Derived Quantities

The physical quantities which are expressed in terms of other physical quantities are called as Derived Quantities.

Example- Velocity=Length/Time

Acceleration=Velocity/Time=Length/$(\text{Time})^2$

Force = Mass X Acceleration = Mass X Length/$(\text{Time})^2$

1.2 Unit

Unit is a standard which is used to measure a physical quantity.

1.2.1 Fundamental Units

Fundamental units are those units which are independent and not related to each other. The units of fundamental quantities are called as Fundamental Units.

Example – The unit of length is meter. So, meter is an example of fundamental unit. Similarly, second is the fundamental unit of time and kg is the fundamental unit of mass.

1.2.2 Derived Units

The units of the physical quantities which can be expressed in terms of fundamental units are called as Derived Units.

Example- Area = length X breadth = metre X metre = $(\text{metre})^2$

Velocity = displacement/time=metre/second

1.2.3 Systems of Unit

A complete set of units, both fundamental and derived for all physical quantities is called a system of units. The following systems of units are commonly in use.

1.2.3(A) C. G. S. System

This system is based on **C**entimetre, **G**ram, **S**econd as the fundamental units of length, mass and time respectively. This system is also known as French system. In this system, unit of force is taken as dyne, unit of energy as erg, unit of power as erg/sec and unit of heat energy as calorie.

1.2.3(B) F. P. S. System

This system is based on **F**oot, **P**ound, **S**econd as the fundamental units of length, mass and time respectively. This system is also known as British system. In this system unit of force is taken as 'poundal', unit of energy as 'foot poundal', unit of power as 'foot poundal/sec' and unit of heat energy as British thermal unit (BTU).

1.2.3(C) M. K. S. System

This system is based on **M**etre, **K**ilogram, **S**econd as the fundamental units of length, mass and time respectively. This system is also known as Metric system. In this system unit of force is taken as Newton, unit of energy as Joule, unit of power as Joule/sec and unit of heat energy as Joule.

1.2.3(D) SI Units (Systeme International d'Unites or International System of Units)

In C.G.S and M.K.S. system, there are three fundamental quantities e.g. mass, length, and time and accordingly three fundamental units., which are insufficient to measure some physical quantities. Therefore in 1971, the International Bureau of Weights and Measures decided a system of units which is known as International System of Units and abbreviated as SI System. It is based on the seven fundamental units and two supplementary units.

1.2.3(D)(i) Fundamental Units

The fundamental units used to measure in SI System, are given in the following table.

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Fundamental Physical Quantity	Name of the Unit	Symbol of the Unit
(1) Mass	Kilogram	kg
(2) Length	Metre	m
(3) Time	Second	s
(4) Thermodynamic Temperature	Kelvin	K
(5) Electric Current	Ampere	A
(6) Luminosity	Candela	cd
(7) Amount of substance	Mole	mol

1.2.3(D)(ii) Supplementary units

The supplementary units of SI System, are given in the following table.

Supplementary Physical Quantity	Name of the Unit	Symbol of the Unit
(1) Angle	Radian	Rad
(2) Solid angle	Steradian	Sr

Advantage of SI system:

1. SI unit has broader base. It has seven fundamental units and two supplementary units.
2. SI is a rational system of units i.e., it assigns only one unit to one quantity. For example, for energy of any type be it mechanical or heat or electrical, there is only

one unit Joule (J). But, in C.G.S, the unit of heat energy is calorie, but, that for mechanical energy is erg.

3. SI is a metric system, i.e., multiples and submultiples of the unit can be expressed power of 10.
4. SI is internationally accepted system of units.

1.3.1 DIMENSIONS

Dimensions are the power to which the fundamental units/ quantities be raised in order to represent a physical quantity.

Example:- (1) Area = length X breadth = L X L = $[L^2] = [M^0L^2T^0]$

Here 0, 2, and 0 are the dimensions of Area with respect to mass, length and time.

(2) Velocity $= \frac{\text{Displacement}}{Time} = \frac{[L]}{[T]} = [L^1T^{-1}] = [M^0L^1T^{-1}]$

Here 0, 1, and -1 are the dimensions of velocity with respect to mass, length and time.

1.3.2 DIMENSIONAL FORMULA

Dimensional formula is a formula which tells us, how and which fundamental units must be used to express a physical quantity.

Example:- (1) Volume(V) = length X breadth X height = L X L X L = $[L^3] = [M^0L^3T^0]$

$$=> V = [M^0L^3T^0]$$

This is the dimensional formula of volume.

And 0, 3, and 0 are the dimensions of volume with respect to mass, length and time.

(2) $\text{Acceleration(a)} = \frac{\text{Velocity}}{Time} = \frac{[L^1T^{-1}]}{[T]} = [L^1T^{-2}] = [M^0L^1T^{-2}]$

$$=> a = [M^0L^1T^{-2}]$$

This is the dimensional formula of acceleration. Here 0, 1, and -2 are the dimensions of acceleration with respect to mass, length and time.

(3) Momentum = mass X velocity = $[M^1][L^1T^{-1}] = [M^1L^1T^{-1}]$

(4) Force = mass X acceleration = $[M^1][L^1T^{-2}] = [M^1L^1T^{-2}]$

(5) Moment of a force = force X distance = $[M^1L^1T^{-2}][L^1] = [M^1L^2T^{-2}]$

(6) Work = force X distance = $[M^1L^1T^{-2}][L^1] = [M^1L^2T^{-2}]$

(7) Kinetic Energy (E_k) = $\frac{1}{2}mv^2$

$= [M^1][L^1T^{-1}]^2$ [$\frac{1}{2}$ has been ignored because

$= [M^1][L^2T^{-2}]$ it is dimensionless]

$= [M^1L^2T^{-2}]$

(8) Potential Energy (E_p) = mgh

$= [M^1][L^1T^{-2}][L^1]$

$= [M^1L^2T^{-2}]$

(9) Specific heat capacity (s)

$Q = ms\theta,$ where Q = heat energy, m = mass, s = specific heat capacity, θ = thermodynamic temperature.

$$=> s = \frac{Q}{m\theta} = \frac{[M^1L^2T^{-2}]}{[M^1][K^1]} = [M^0L^2T^{-2}K^{-1}]$$

Here 0, 2, -2 and -1 are the dimensions of specific heat capacity with respect to mass, length, time, and temperature.

(10) Specific Latent Heat (l)

$$Q = ml$$

$$=> l = \frac{Q}{m} = \frac{[[M^1L^2T^{-2}]]}{[M^1]} = [M^0L^2T^{-2}]$$

Here, 0,2, -2 are the dimensions of specific latent heat with respect to mass, length, and time.

(11) Charge (q)

$$i = \frac{q}{t}$$

$$=> q = it = [A^1][T^1] = [A^1T^1]$$

Here 1 and 1 are the dimensions of charge with respect to current and time.

(12) Thermal Conductivity (K)

$$Q = K\frac{A(\theta_1-\theta_2)t}{l}$$

$$=> K = \frac{Ql}{A(\theta_1-\theta_2)t}$$

$$= \frac{[M^1L^2T^{-2}][L^1]}{[L^2][K^1][T^1]} = [M^1L^1T^{-3}K^{-1}]$$

[Where Q= heat energy
l=length of the rod
A=cross sectional area
(θ_1-θ_2) =temperature difference

Here, 1,1, -3, -1 are the dimensions of thermal conductivity with respect to mass, length, time and temperature.

Sl. No	Physical Quantity	Formula	Dimensional Formula	S.I Unit
1	Area (A)	Length x Breadth	$[M^0L^2T^0]$	m^2
2	Volume (V)	Length x Breadth x Height	$[M^0L^3T^0]$	m^3
3	Density (d)	Mass / Volume	$[M^1L^{-3}T^0]$	kgm^{-3}
4	Speed (s)	Distance / Time	$[M^0L^1T^{-1}]$	ms^{-1}
5	Velocity (v)	Displacement / Time	$[M^0L^1T^{-1}]$	ms^{-1}
6	Acceleration (a)	Change in velocity / Time	$[M^0L^1T^{-2}]$	ms^{-2}
7	Acceleration due to gravity (g)	Change in velocity / Time	$[M^0L^1T^{-2}]$	ms^{-2}
8	Linear momentum (p)	Mass x Velocity	$[M^1L^1T^{-1}]$	$kgms^{-1}$
9	Force (F)	Mass x Acceleration	$[M^1L^1T^{-2}]$	N (Newton) ($kgms^{-2}$)
10	Work (W)	Force. Displacement	$[M^1L^2T^{-2}]$	J (Joule) (kgm^2s^{-2})
11	Energy (E)	Work	$[M^1L^2T^{-2}]$	J
12	Impulse (I)	Force x Time	$[M^1L^1T^{-1}]$	Ns
13	Pressure (P)	Force / Area	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
14	Power (P)	Work / Time	$[M^1L^2T^{-3}]$	W (Watt)
15	Universal constant of gravitation (G)	$\frac{Force \times (distance)^2}{(mass)^2}$	$[M^{-1}L^3T^{-2}]$	Nm^2kg^{-2}

16	Moment of inertia (I)	Mass x $(\text{distance})^2$	$[M^1L^2T^0]$	kgm^2
17	Moment of force, moment of couple	Force x distance	$[M^1L^2T^{-2}]$	Nm
18	Surface tension (T)	Force / Length	$[M^1L^0T^{-2}]$	Nm^{-1}
19	Thrust (F)	Force	$[M^1L^1T^{-2}]$	N
20	Tension (T)	Force	$[M^1L^1T^{-2}]$	N
21	Stress	Force / Area	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
22	Strain	Change in dimension / Original dimension	No dimensions $[M^0L^0T^{-0}]$	No unit
23	Modulus of Elasticity (E)	Stress / strain	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
24	Angle (θ), Angular displacement	Arc length / Radius	No dimensions $[M^0L^0T^{-0}]$	Rad
25	Angular velocity (**ω**)	Angle / Time	$[M^0L^0T^{-1}]$	rad s^{-1}
26	Angular acceleration (**α**)	Angular velocity / Time	$[M^0L^0T^{-2}]$	rad s^{-2}
27	Angular momentum (J)	Moment of inertia x Angular velocity	$[M^1L^2T^{-1}]$	kgm^2s^{-1}
28	Velocity gradient	Velocity / Distance	$[M^0L^0T^{-1}]$	s^{-1}
29	Wavelength (**λ**)	Length of a wavelet	$[M^0L^1T^0]$	M
30	Frequency (ν)	Number of vibrations/second or 1/time period	$[M^0L^0T^{-1}]$	Hz or s^{-1}
31	Planck's constant (h)	Energy / Frequency	$[M^1L^2T^{-1}]$	Js
32	Temperature	Temperature	$[M^0L^0T^0K^1]$	K
33	Heat (Q)	Energy	$[M^1L^2T^{-2}]$	J
34	Latent heat (L)	Heat / Mass	$[M^0L^2T^{-2}]$	Jkg^{-1}
35	Specific heat (S)	$\frac{Heat}{mass \times temperature}$	$[M^0L^2T^{-2}K^{-1}]$	$Jkg^{-1}K^{-1}$
36	Thermal expansion coefficient or thermal expansivity	$\frac{Change\ in\ diemnsion}{original\ dimension \times temperature}$	$[M^0L^0T^0K^{-1}]$	K^{-1}
37	Thermal conductivity	$\frac{Heat\ Energy \times thickness}{Area \times temperature}$	$[M^1L^1T^{-3}K^{-1}]$	$Wm^{-1}K^{-1}$
38	Charge (q)	Current x time	$[M^0L^0T^1A^1]$	C (Coulomb)
39	Current density	Current / area	$[M^0L^{-2}T^0A^1]$	A m^{-2}
40	Electric potential (V), voltage, electromotive force	Work / Charge	$[M^1L^2T^{-3}A^{-1}]$	V (Volt)
41	Resistance (R)	Potential difference / Current	$[M^1L^2T^{-3}A^{-2}]$	Ω ohm
42	Capacitance	Charge / potential difference	$[M^{-1}L^{-2}T^4A^2]$	F (Farad)
43	Electrical resistivity or (electrical conductivity)$^{-1}$	$\frac{Resistance \times Area}{length}$	$[M^1L^3T^{-3}A^{-2}]$	Ωm
44	Electric field (E)	Force / Charge	$[M^1L^1T^{-3}A^{-1}]$	NC^{-1}

45	Electric flux	Electric field X area	$[M^1L^3T^{-3}A^{-1}]$	Nm^2C^{-1}
46	Electric dipole moment	Torque / electric field	$[M^0L^1T^1A^1]$	C m
47	Electric field strength or electric intensity	Potential difference / distance	$[M^1L^1T^{-3}A^{-1}]$	Volt/meter or NC^{-1}
48	Magnetic field (B), magnetic flux density, magnetic induction	$\frac{Force}{current \times length}$	$[M^1L^0T^{-2}A^{-1}]$	T (Tesla)
49	Magnetic flux (Φ)	Magnetic field X area	$[M^1L^2T^{-2}A^{-1}]$	Wb (Weber)
50	Inductance	Magnetic flux / current	$[M^1L^2T^{-2}A^{-2}]$	H (Henry)
51	Magnetic field strength (H), magnetic intensity or magnetic moment density	Magnetic moment / volume	$[M^0L^{-1}T^0A^1]$	Am^{-1}
52	Heat energy, internal energy	Force X (distance)	$[M^1L^2T^{-2}]$	J
53	Kinetic energy	$\frac{1}{2} \times mass \times (velocity)^2$	$[M^1L^2T^{-2}]$	J
54	Potential energy	Mass X acceleration due to gravity X height	$[M^1L^2T^{-2}]$	J
55	Efficiency	$\frac{Output\ work\ or\ energy}{Input\ work\ or\ energy}$	No dimensions $[M^0L^0T^0]$	No unit
56	Permittivity of free space	$\frac{Charge \times charge}{4\pi \times electric\ force \times (distance)^2}$	$[M^{-1}L^{-3}T^4A^2]$	$F\ m^{-1}$
57	Permeability of free space	$\frac{2\pi \times force \times distance}{current \times current \times length}$	$[M^1L^1T^{-2}A^{-2}]$	NA^{-2}
58	Refractive index	$\frac{Speed\ of\ light\ in\ vacuum}{Speed\ of\ light\ in\ medium}$	No dimensions $[M^0L^0T^0]$	No unit

1.4.1 Dimensional Equation

When a physical quantity is written on the left-hand side and its dimensional formula is written on the right hand side of a equation then it is called as dimensional equation.

Example:- Work(W) = Force X displacement

$= [M^1L^1T^{-2}][L^1]$

$= [M^1L^2T^{-2}]$

$=> W = [M^1L^2T^{-2}]$ ----------------------(i)

This equation is known as dimensional equation.

1.4.2 Use of dimensional analysis

Dimensional analysis has following three uses.

(i) To convert the value of a physical quantity from one system to another.

(ii) Derive a relation between various physical quantities.

(iii) To check the correctness of a given relation.

1.4.3 Principle of Homogeneity

It states that the dimensional formula of every term on both sides of a correct relation must be same.

1.4.4 To convert the value of a physical quantity from one system to another

Using dimensional analysis, we can find the numerical value of a physical quantity in any system if the numerical value is known in another system.

Example:- convert a work of 1 Joule in to erg.

Solution:-

(M.K.S.)	(WORK)	(C.G.S.)
System-1	$[M^1L^2T^{-2}]$	System-2
(Value known)	a=1	(Value unknown)
M_1=1 kg	b=2	M_2=1 g
L_1=1 m	c=-2	L_2=1 cm
T_1=1 s		T_2=1 s
n_1=1		n_2=?

Physical quantity as represented in system 1 = $n_1[M_1^aL_1^bT_1^c]$

Physical quantity as represented in system 2 = $n_2[M_2^aL_2^bT_2^c]$

Since the quantity are same in both the system, then

$$\mathbf{n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]}$$

$$=> \mathbf{n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c}$$

Substituting the value as given in the problem we get,

$$\mathbf{n_2 = 1\left[\frac{1kg}{1g}\right]^1\left[\frac{1m}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2} = 1\left[\frac{1000g}{1g}\right]^1\left[\frac{100cm}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2} = 10^7}\text{ erg}$$

∴ 1 Joule = 10^7 erg

1.4.5 To derive a relation between various physical quantities by the method of dimensional analysis:

Relation of one physical quantity with others can be derived when the factors on which this quantity depends are known to us.

Example:- Obtain an expression for centripetal force required to move a body of mass 'm', with velocity 'v' in a circle of radius 'r'.

Solution:- Let the centripetal force depend upon m, v, r as follows:

$F \propto m^a$, $F \propto v^b$, $F \propto r^c$

Here a, b, c are dimensionless constants.

$F \propto m^av^br^c$

$=> F = k\, m^av^br^c$ ------------------------------------- (i)

Here k is a dimensionless constant.

Writing the dimensional formulae of physical quantities in above equation, we get

$$[M^1L^1T^{-2}] = [M^1]^a[L^1T^{-1}]^b[L^1]^c$$

$$=> [M^1L^1T^{-2}] = [M^aL^{b+c}T^{-b}]$$

Now comparing the dimensions on both sides, we get

a = 1 ----------------------------(ii)
b+c = 1 -------------------------(iii)
-b = -2 ---------------------------(iv)

From equation (iv), b=2,

Now substituting the value of b in equation (iii), we get c=-1.

Putting the values of a=1, b=2, c=-1 in equation (i), we get

$$\mathbf{F = k\frac{mv^2}{r}}$$

This is the required expression.

1.5 Checking the dimensional correctness of Physical relations.

To check the correctness of a relation, we find the dimensional formula of every term on both sides of the relation. If the dimensions are same then the relation is said to be dimensionally correct.

Example(1):- To check the correctness of given relation.

$$s = ut + \frac{1}{2}at^2$$

Solution:- Given relation is $s = ut + \frac{1}{2}at^2$

Dimensional formula of s = Displacement = $[L^1]$=$[M^0L^1T^0]$----------------- (i)

Dimensional formula of ut = $[M^0L^1T^{-1}][T^1] = [M^0L^1T^0]$----------------------(ii)

Dimensional formula of $\frac{1}{2}\mathbf{at^2} = [M^0L^1T^{-2}][T^2] = [M^0L^1T^0]$------------------ (iii)

From the above equations we get dimensional formula of every term are same. Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

Example(2):- To check the correctness of given relation.

$$v = u + at$$

Solution:- Given relation is $v = u + at$

Dimensional formula of v = final velocity = $[M^0L^1T^{-1}]$ ----------------------(i)

Dimensional formula of u = initial velocity = $[M^0L^1T^{-1}]$ ---------------------(ii)

Dimensional formula of at = $[M^0L^1T^{-2}][T^1] = [M^0L^1T^{-1}]$------------------ (iii)

From the above equations we get dimensional formula of every term are same. Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

Example(3):- To check the correctness of given relation.

$$v^2 - u^2 = 4as$$

Solution:- Given relation is $v^2 - u^2 = 4as$

Dimensional formula of $v^2 = [M^0L^1T^{-1}]^2 = [M^0L^2T^{-2}]$ ------------------------(i)

Dimensional formula of $u^2 = [M^0L^1T^{-1}]^2 = [M^0L^2T^{-2}]$ -------------------------(ii)

Dimensional formula of 2as= $[M^0L^1T^{-2}][L^1] = [M^0L^2T^{-2}]$ ------------------(iii)

From the above equations we get dimensional formula of every term are same. Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

Question:- To check the correctness of following relation.

(1) $t = 2\pi\sqrt{\frac{g}{l}}$ where g=acceleration due to gravity and l=length of thread,

t = time period

(2) $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ where n=frequency, l = length, T=tension and m= mass/length

(3) $h = \frac{r\rho g}{2T\cos\theta}$ where ρ=density, g=acceleration ant T=surface tension, h = height

(4) $v = \sqrt{\frac{E}{\rho}}$ where v = velocity, E= coefficient of elasticity and ρ=density

(5) $v = \sqrt{\frac{T}{m}}$ where v = velocity, T=tension and m= mass/length

(6) $v = rn$ where v = velocity, r = radius n=frequency

Solution:-

(1) Given relation is $t = 2\pi\sqrt{\frac{g}{l}}$

Dimensional formula of $t = [M^0L^0T^1]$ -- (i)

Dimensional formula of $2\pi\sqrt{\frac{g}{l}} = \sqrt{\frac{[M^0L^1T^{-2}]}{[L^1]}} = \sqrt{[M^0L^0T^{-2}]}$

$= [M^0L^0T^{-2}]^{1/2}$

$= [M^0L^0T^{-1}]$ --------------------------(ii)

From the above equations we get that the dimensional formula of the terms on both the sides are not same. Therefore according to Principle of Homogeneity the given relation is dimensionally incorrect.

(2) Given relation is $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Dimensional formula of $n = [M^0L^0T^{-1}]$-- (i)

Dimensional formula of $\frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{[L^1]}\sqrt{\frac{[M^1L^1T^{-2}]}{[M^1L^{-1}]}} = \frac{1}{[L^1]}\sqrt{[M^0L^2T^{-2}]}$

$= \frac{1}{[L^1]}[M^0L^2T^{-2}]^{1/2} = \frac{1}{[L^1]}[M^0L^1T^{-1}]$

$= [M^0L^0T^{-1}]$ ------------------(ii)

From the above equations we get dimensional formula of every term are same. Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

(3) Given relation is $h = \frac{r\rho g}{2T\cos\theta}$

Dimensional formula of $h = [M^0L^1T^0]$--(i)

Dimensional formula of $\frac{r\rho g}{2T\cos\theta} = \frac{[L^1][M^1L^{-3}][M^0L^1T^{-2}]}{[M^1L^0T^{-2}]}$

$= [M^0L^{-1}T^0]$ ---------------(ii)

From the above equations we get dimensional formula of every term are not same. Therefore according to Principle of Homogeneity the given relation is incorrect.

(4) Given relation is $v = \sqrt{\frac{E}{\rho}}$

Dimensional formula of $v = [M^0L^1T^{-1}]$ -- (i)

Dimensional formula of $\sqrt{\frac{E}{\rho}} = \sqrt{\frac{[M^1L^{-1}T^{-2}]}{[M^1L^{-3}]}} = \sqrt{[M^0L^2T^{-2}]}$

$= [M^0L^2T^{-2}]^{1/2}$

$= [M^0L^1T^{-1}]$ ----------------(ii)

From the above equations we get dimensional formula of every term are same. Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

(5) Given relation is $v = \sqrt{\frac{T}{m}}$

Dimensional formula of $v = [M^0L^1T^{-1}]$ -- (i)

Dimensional formula of $\sqrt{\frac{T}{m}} = \sqrt{\frac{[M^1L^1T^{-2}]}{[M^1L^{-1}]}} = \sqrt{[M^0L^2T^{-2}]}$

$= [M^0L^2T^{-2}]^{1/2}$

$= [M^0L^1T^{-1}]$ ----------------(ii)

From the above equations we get dimensional formula of every term are same. Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

(6) Given relation is v = r*n*

Dimensional formula of $v = [M^0L^1T^{-1}]$ -- (i)

Dimensional formula of $rn = [M^0L^1T^0][M^0L^0T^{-1}]$

$= [M^0L^1T^{-1}]$ ----------------(ii)

From the above equations we get dimensional formula of every term are same. Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

QUESTIONS FOR SEMESTER EXAMINATION

VERY SHORT ANSWER QUESTIONS(2 Marks each)

1. Name two quantities which are dimensionless in nature.
   [Ans. Angle and strain]
2. Name two quantities which have dimensional formula [$M^1L^{-1}T^{-2}$]
   [Ans. Pressure and Stress
3. Obtain the dimension of (i) pressure (ii) Kinetic Energy
   [Ans.(i) [$M^1L^{-1}T^{-2}$] and (ii) [$M^1L^2T^{-2}$]
4. What is meant by a unit?
   [Ans. Unit is a standard which is used to measure a physical quantity.
5. How are Fermi and angstrom related with each other?
   [Ans. 1 angstrom = 10^5 Fermi
6. Write the dimensional formula of force and work.
   [Ans. Force = [$M^1L^1T^{-2}$] and Work = [$M^1L^2T^{-2}$]
7. Write the SI unit and dimensional formula of electric Potential.
   [Ans. Volt and [$M^1L^2T^{-3}A^{-1}$]
8. Write down the dimensional formula for Gravitational constant.
   [Ans. [$M^{-1}L^3T^{-2}$]

SHORT ANSWER QUESTIONS (5 Marks each)

1. Write the fundamental units in S.I. system.
2. Prove that the dimensions of kinetic energy and potential energy are same.
3. Check the correctness of the following (i) v^2-u^2=2as, (ii) $s = ut + \frac{1}{2}at^2$
4. Obtain the dimensional formula of Thermal Conductivity.
5. Obtain the dimensional formula of Specific heat capacity.

LONG ANSWER QUESTIONS (10 Marks each)

1. States the principle of Homogeneity and check the correctness of following relation. (i) $t = 2\pi\sqrt{\frac{g}{l}}$ and (ii) $v = \sqrt{\frac{E}{\rho}}$, where ....
2. Prove that (i) 1 Newton = 10^5 dyne and (ii) 1Joule = 10^7 erg by the method of dimensional analysis.
3. Time period of a simple pendulum (T) depends upon length of the pendulum and acceleration due to gravity; find the expression for time period by the method of dimensional analysis.

UNIT 2
SCALARS AND VECTORS

We have come across with different types of physical quantities, in one dimensional motion, only two directions are possible. So directional aspect of the quantities like position, displacement, velocity and acceleration can be taken care by using positive and negative signs. But in the case of motion in two dimensions (plane) or three dimensions (space), an object can have large number of directions. In order to deal with such situation effectively, we need to introduce the concept of scalar and vector quantities.

In this chapter we shall discuss the definition of scalar and vector quantities, its applications to solve different physical problems and how they can be multiplied.

2.1 Scalar Quantities and Vector Quantities

The physical quantities are classified into two categories (i) Scalar Quantities and (ii) Vector quantities.

2.1.1 Scalar quantities

Scalar quantities are those quantities which require only the magnitude for their complete specification.

Example:- mass, length, volume, time, distance, speed, density, energy, temperature, electric charge, electric potential etc.

2.1.2 Vector quantities

Vector quantities are those quantities which require magnitude as well as direction for their complete specification and satisfies the law of vector addition.

Example:- Displacement, velocity, acceleration, force, momentum, electric field, magnetic field, magnetic moment etc.

2.1.3 Vector

A directed line segment is called as vector. When it is written over the head of a physical quantity, then it represents a vector quantity.

2.1.4 Representation of a vector

A vector '$\vec{A}$' (Fig. 2.1) can be represented by an arrow 'OP' of finite length directed from initial point O to the terminal point P. The length of arrow represents the magnitude of vector and the arrow head denotes the direction of the vector. A vector is written with an arrow head over its symbol like '$\vec{A}$'. The magnitude of given vector is represented by modulus of vector ($|\vec{A}|$) or simply 'A'.

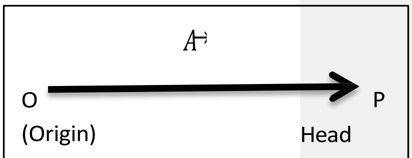

Fig. 2.1 Representation of a vector

2.1.4 Types of vector

There are different types of vectors that are generally used in maths and science, are discussed as follows.

(i) **Null vector:-**It is a vector having zero magnitude and an arbitrary direction. It is represented by a point or dot (•). When a null vector is added or subtracted from a given vector, the resultant vector is same as the given vector. Dot product of a null vector with any other vector is always zero. Cross product of a null vector with any other vector is also a null vector. (We will discuss dot product and cross product in a coming section)

(ii) **Unit vector :-** Any vector whose magnitude is one unit is called as a unit vector. A unit vector only specifies the direction of given vector. A unit vector in the direction of vector '$\vec{A}$' is written as $\hat{A}$ and is read as 'A cap'.

$$\vec{A} = \hat{A}A \qquad \text{or} \ \hat{A} = \frac{\vec{A}}{A}$$

In three dimensional coordinate system, unit vectors along positive X, Y and Z-axes are usually represented by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively. These unit vectors are mutually perpendicular to each other (Fig.2.2)

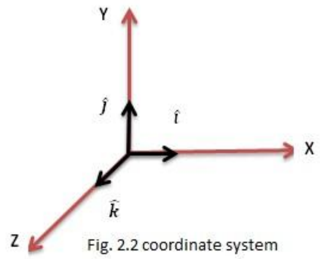

Fig. 2.2 coordinate system

(iii) **Collinear vectors**

Vector having a common line of action are called as collinear vectors. There are of two types of collinear vectors.

(a) Parallel vectors (θ=0^0):- Two vectors acting along same direction irrespective of their magnitude are called as parallel vectors. Vectors '$\vec{A}$' and '$\vec{B}$' shown in fig. 2.3 are parallel vectors. Angle between them is zero.

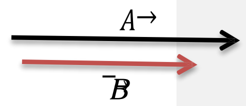

Fig. 2.3 Parallel vectors

(b) Anti-parallel vectors (θ=180^0):- Two vectors acting along opposite direction irrespective of their magnitude are called as anti-parallel vectors. Vectors '$\vec{A}$' and '$\vec{B}$' shown in fig. 2.4 are anti-parallel vectors. Angle between them is 180^0.

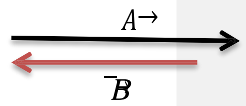

Fig. 2.4 Anti-Parallel vectors

(iv) **Perpendicular vectors (θ=90^0)**

Two vectors are said to be perpendicular when they are normal to each other (irrespective of their magnitude). Vectors '$\vec{A}$' and '$\vec{B}$' shown in fig. 2.5 are perpendicular vectors. Angle between them is 90^0.

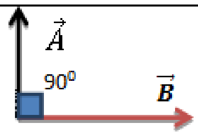

Fig. 2.5 Perpendicular vectors

(v) **Equal vectors**

Two vectors are said to be equal if they possess same magnitude and direction. Vectors '$\vec{A}$' and '$\vec{B}$' shown in fig. 2.6 are equal vectors. All equal vectors are parallel vectors.

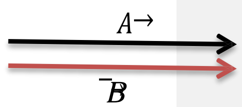

Fig. 2.6 Equal vectors

(vi) **Negative vector**

A vector is said to be negative vector of another one if they possess same magnitude and opposite direction. Vectors '$\vec{A}$' and '$\vec{B}$' shown in fig. 2.7 are negative vector to each other. All negative vectors are anti-parallel vectors.

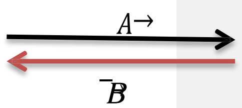

Fig. 2.7 Negative vectors

(vii) Co-initial vectors

A number of vectors are said to be co-initial when they have common initial point. Vectors ‘$\vec{A}$’, ‘$\vec{B}$’, ‘$\vec{C}$’,‘$\vec{D}$’ and ‘$\vec{E}$’ shown in fig. 2.8 are co-initial vectors, started from a common point P.

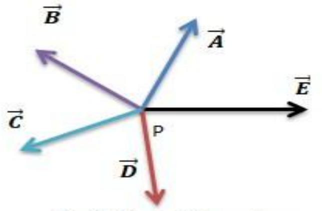

Fig. 2.8 Co-initial vectors

(viii) Co-planar vectors

A number of vectors are said to be co-planar when they are lying in the same plane. Vectors ‘$\vec{A}$’, ‘$\vec{B}$’, ‘$\vec{C}$’,‘$\vec{D}$’ and ‘$\vec{E}$’ shown in fig. 2.9 are co-planar vectors, present in the same plane.

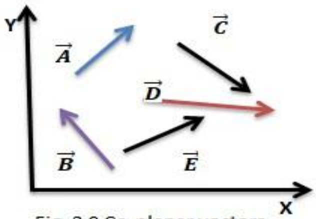

Fig. 2.9 Co-planar vectors

(ix) Position Vector

Vectors that indicate the position of a point in a coordinate system is called as position vector. Let point P(x,y,z) present in three coordinate system then‘$\vec{R}$’ is the position vector of point P from the origin O(0,0,0) as shown in the fig 2.10.

Position vector can be written as

$$\vec{R} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

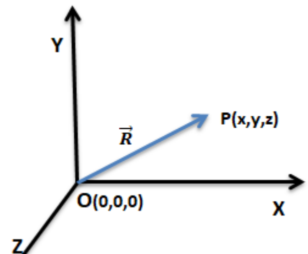

Fig. 2.10 Position Vector

2.2 Addition of vectors

Vectors cannot be added according to the simple algebra, because vectors have magnitude along with direction. So vector can be added as follows.

2.2.1 Triangle law of vector addition

It is a law for the addition of two vectors. It can be stated as follows:

“If two vectors acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle taken in same order, then the resultant vector is represented in magnitude and direction by third side of that triangle taken in opposite order”.

Let two vectors $\vec{A}$ and $\vec{B}$ acting at a point be represented by two sides $\overrightarrow{OP}$ and $\overrightarrow{PQ}$ of triangle OPQ, taken in same order (Fig. 2.11). According to triangle law of vector addition, the third side $\overrightarrow{OQ}$ represent the resultant vector $\vec{R}$, taken in opposite order.

$$\vec{R} = \vec{A} + \vec{B}$$

It can be mathematically proved that:

$$\mathbf{R} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

And $$\beta = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

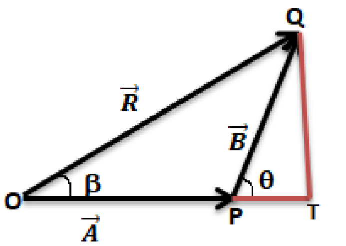

Fig. 2.11 Vector addition by Triangle law

2.2.2 Parallelogram's law of vector addition

Two vectors can also be added by using parallelogram's law of vector addition. It can be stated as follows:

> "If two vectors acting simultaneously on a body are represented in magnitude and direction by two sides of a parallelogram drawn from a point, then the resultant vector is represented in magnitude and direction by the diagonal of the parallelogram passing through that point".

Let two vectors $\vec{A}$ and $\vec{B}$ acting at a point, be represented by two sides $\vec{OP}$ and $\vec{OQ}$ of Parallelogram OPTQ, drawn from a point (Fig. 2.12). According to parallelogram law of vector addition, the diagonal $\vec{OT}$ of parallelogram represent the resultant vector $\vec{R}$, passing through that point.

$$\vec{R} = \vec{A} + \vec{B}$$

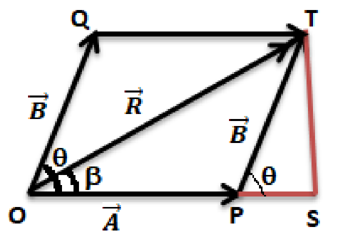

Fig. 2.12 Parallelogram law

It can be mathematically proved that:

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

And $\quad \beta = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$

2.3 Resolution of Vectors in a plane

Resolution of a vector is the process of obtaining the component vectors which when combined according to the law of vector addition, produce the given vector.

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Let $\vec{OP}\ (= \vec{R})$ be the position vector of point P(x,y) in XY-plane(Fig. 2.13).From P draw the perpendiculars PQ and PB on X-axis and Y-axis respectively. It makes an angle θ with X-axis.

Let $\hat{\imath}$ and $\hat{\jmath}$ are the unit vectors along x-axis and Y-axis respectively.

Consider $\vec{R}$ resolves into two components horizontal component $\vec{R_x}$ along X-axis and vertical component $\vec{R_y}$ along Y-axis.

According to triangle law of vector addition in triangle OPQ we can write $\vec{R} = \vec{R_x} + \vec{R_y}$

$$\Rightarrow \vec{R} = \hat{\imath}R_x + \hat{\jmath}R_y \text{ ------------------------ (i)}$$

In the ΔOPQ, $\cos\theta = \frac{OQ}{OP}$

$$\Rightarrow \cos\theta = \frac{R_x}{R}$$

$$\Rightarrow R_x = R\cos\theta \text{ ------------------ (ii)}$$

Again in ΔOPQ, $\sin\theta = \frac{QP}{OP} = \frac{OB}{OP}$

$$\Rightarrow \sin\theta = \frac{R_y}{R}$$

$$\Rightarrow R_y = R\sin\theta \text{ ---------------------- (iii)}$$

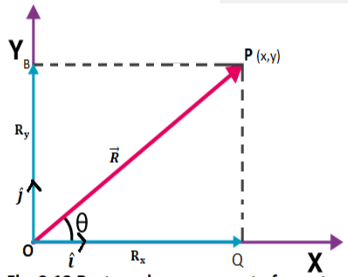

Fig. 2.13 Rectangular component of a vector

Now putting the values of equation (ii) and (iii) in equation (i), we get:

$$\vec{R} = \hat{\imath}R\cos\theta + \hat{\jmath}R\sin\theta$$

2.4 Vector multiplication

There are two ways in which two vectors can be multiplied together.

(i) Scalar Product or Dot product
(ii) Vector Product or Cross product

2.4.1 Scalar product or Dot product

Dot product between two vectors is defined as the product of their magnitude and the cosine of the smaller angle between them.

It is written by putting a dot (•) between two vectors. The result of this product does not possess any direction. Hence it is also called as Scalar product.

Consider two vectors $\vec{A}$ and $\vec{B}$ drawn from a point and inclined to each other at angle θ as shown in the Fig.2.14.

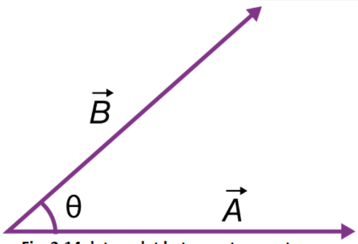

Fig. 2.14 dot produt between two vectors

Dot product of $\vec{A}$ and $\vec{B}$ is given by

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

2.4.1(a) Characteristics of Dot product

Dot product obeys the following characteristics:

(i) **Commutative**

Dot product of two vectors is commutative in nature.

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

(ii) **Distributive**

Dot product of a vector with the sum of a number of vectors is equal to the sum of the dot product of the vector taken with other vectors separately. Mathematically, it can written as

$$\vec{A} \cdot (\vec{B} + \vec{C} + \vec{D}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} + \vec{A} \cdot \vec{D}$$

(iii) **Perpendicular vectors**

For two perpendicular vectors $\vec{A}$ and $\vec{B}$, the angle between them $\theta = 90^0$

$$\vec{A} \cdot \vec{B} = AB\cos\boldsymbol{\theta} = AB\cos 90^0 = (AB)(0) = 0 \qquad ; \cos 90^0 = 0$$

Thus, the dot product of two non-zero vectors, which are perpendicular to each other is always zero. This statement is known as condition of perpendicularity.

Since $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are mutually perpendicular to each other (Fig.2.2).

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0$$

(iv) **Parallel vectors**

For two parallel vectors $\vec{A}$ and $\vec{B}$, the angle between them $\theta = 0^0$

$$\vec{A} \cdot \vec{B} = AB\cos\boldsymbol{\theta} = AB\cos 0^0 = (AB)(1) = AB \qquad [\cos 0^0 = 1$$

(v) **Antiparallel vectors**

For two antiparallel vectors $\vec{A}$ and $\vec{B}$, the angle between them θ=180^0

$\vec{A} \cdot \vec{B} = AB \cos \boldsymbol{\theta} = AB \cos 180^0 = (AB)(-1) = -AB \qquad [cos 180^0 = -1$

(vi) **Equal vectors**

Vectors are equal if they possess same magnitude and direction i.e., θ=0^0

∴ Dot product between two equal vectors is given by

$\vec{A} \cdot \vec{A} = AA \cos \boldsymbol{\theta} = AA \cos 0^0 = (AA)(1) = A^2 \qquad [cos 0^0 = 1$

Dot product between two equal vectors is equal to the square of the magnitude of the either one.

$\therefore \hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$

(vii) **Dot product in terms of rectangular component**

Let A_x, A_y, A_z and B_x, B_y, B_z are the rectangular components of two vectors $\vec{A}$ and $\vec{B}$ respectively.

Then $\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$

And $\vec{B} = B_x \hat{\imath} + B_y \hat{\jmath} + B_z \hat{k}$

$$\therefore \vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_xB_x(\hat{\imath} \cdot \hat{\imath}) + A_xB_y(\hat{\imath} \cdot \hat{\jmath}) + A_xB_z(\hat{\imath} \cdot \hat{k})$$

$$+ A_yB_x(\hat{\jmath} \cdot \hat{\imath}) + A_yB_y(\hat{\jmath} \cdot \hat{\jmath}) + A_yB_z(\hat{\jmath} \cdot \hat{k})$$

$$+ A_zB_x(\hat{k} \cdot \hat{\imath}) + A_zB_y(\hat{k} \cdot \hat{\jmath}) + A_zB_z(\hat{k} \cdot \hat{k})$$

Since $\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{\imath} = 0$,
$\hat{\imath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0$,
$\hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\jmath} = 0$,
$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$

$$= A_xB_x(1) + A_xB_y(0) + A_xB_z(0)$$

$$+ A_yB_x(0) + A_yB_y(1) + A_yB_z(0)$$

$$+ A_zB_x(0) + A_zB_y(0) + A_zB_z(1)$$

$$\Rightarrow \vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Therefore, dot product of two vectors is defined as the sum of the product of their rectangular components along the three coordinate axes.

Problem(1):- Find the dot product between $\vec{A} = 3\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ and $\vec{B} = 4\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$

Solution:- Here given **A_x=3, A_y=2, A_z=5** and **B_x=4, B_y=3, B_z=7**

We know that $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

= 3 X 4 + 2 X 3 + 5 X 7

= 12+6+35=53

Problem(2):- Find the dot product between $\vec{A} = 2\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$ and $\vec{B} = 3\hat{\imath} - 6\hat{\jmath} + \hat{k}$

Solution:- Here given **A_x=2, A_y=5, A_z=6** and **B_x=3, B_y=-6, B_z=1**

We know that $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

= 2 X 3 + 5 X (-6) + 6 X 1

= 6-30+6=-18

Problem(3):- Find the dot product between $\vec{A} = 5\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{B} = 2\hat{\imath} - 3\hat{\jmath}$

Solution:- Here given **A_x=5, A_y=2, A_z=3** and **B_x=2, B_y=-3, B_z=0**

We know that $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

= 5 X 2 + 2 X (-3) + 3 X 0

= 10-6+0=4

Problem(4):- Find the dot product between $\vec{A} = 6\hat{\imath} + 2\hat{k}$ and $\vec{B} = 3\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

Solution:- Here given **A_x=6, A_y=0, A_z=2** and **B_x=3, B_y=4, B_z=6**

We know that $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

= 6 X 3 + 0 X 4 + 2 X 6

= 18+0+12=30

Problem(5):- Find the dot product between $\vec{A} = 3\hat{\imath} + 2\hat{\jmath}$ and $\vec{B} = 4\hat{\imath} + 3\hat{\jmath}$

Solution:- Here given **A_x=3, A_y=2, A_z=0** and **B_x=4, B_y=3, B_z=0**

We know that $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

= 3 X 4 + 2 X 3 + 0 X 0

= 12+6+0=18

2.4.2 Cross product or Vector product

Cross product of two vectors $\vec{A}$ and $\vec{B}$ defined as a single vector $\vec{C}$ whose magnitude is equal to the product of their individual magnitude and sine of the smaller angle between them and is directed along the normal to the plane containing $\vec{A}$ and $\vec{B}$.

It is written by putting a cross (×) between two vectors. The resultant of this product possesses a direction. Hence it is also called as vector product.

Consider two vectors $\vec{A}$ and $\vec{B}$ drawn from a point and inclined to each other at angle θ as shown in the Fig.2.15.

Cross product of $\vec{A}$ and $\vec{B}$ is given by

$$\vec{A} \times \vec{B} = \vec{C} = AB \sin\theta \hat{n}$$

Where $\hat{n}$ is the unit vector of $\vec{C}$ directed perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

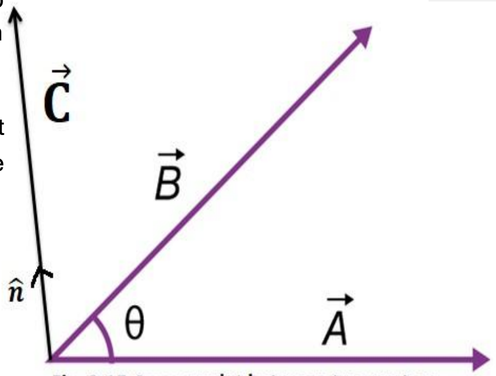

Fig. 2.15 Cross produt between two vectors

Right hand thumb rule:-

Imagine the normal to the plane (PQRS) containing $\vec{A}$ and $\vec{B}$ to be held by your right hand with the thumb erect. If the fingers curl directed from $\vec{A}$ to $\vec{B}$ then the direction of the thumb gives the direction of $\vec{A} \times \vec{B}$.

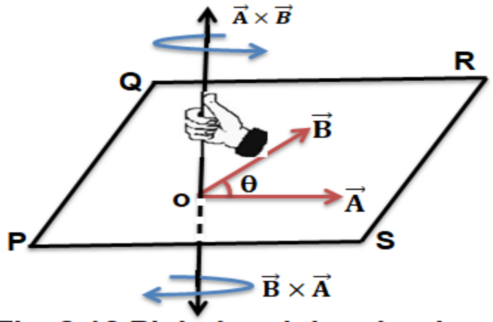

Fig. 2.16 Right hand thumb rule for cross product

2.4.2(a) Characteristics of Cross product

Cross product obeys the following characteristics:

(i) **Not Commutative**

Cross product of two vectors is not commutative in nature.

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

(ii) **Distributive**

Cross product of a vector with the sum of a number of vectors is equal to the sum of the cross product of the vector taken with other vectors separately. Mathematically, it can be written as

$$\vec{A} \times (\vec{B} + \vec{C} + \vec{D}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} + \vec{A} \times \vec{D}$$

(iii) **Perpendicular vectors**

For two perpendicular vectors $\vec{A}$ and $\vec{B}$, the angle between them θ=90^0

$\vec{A} \times \vec{B} = AB \sin\boldsymbol{\theta}\, \hat{n} = AB \sin 90^0 \hat{n} = (AB)(1)\hat{n} = AB\hat{n}$ $\quad [sin 90^0 = 1$

=> $|\vec{A} \times \vec{B}| = AB$

Thus, the magnitude of the cross product of two perpendicular vectors is equal to the product of their individual magnitude.

Since $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors mutually perpendicular to each other (Fig.2.2).

$\hat{i} \times \hat{j} = \hat{k}$ $\qquad$ $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ $\qquad$ $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\qquad$ $\hat{i} \times \hat{k} = -\hat{j}$

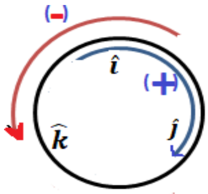

Fig. 2.17 Cyclic Rule

(iv) **Parallel vectors**

For two parallel vectors $\vec{A}$ and $\vec{B}$, the angle between them θ=0⁰

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n} = AB \sin 0^0 \hat{n} = (AB)\,(0)\,\hat{n} = (0)\hat{n} = null\ vector$$

$$[\sin 0^0 = 0]$$

(v) **Antiparallel vectors**

For two antiparallel vectors $\vec{A}$ and $\vec{B}$, the angle between them θ=180⁰

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n} = AB \sin 180^0 \hat{n} = (AB)(0)\,\hat{n} = (0)\hat{n} = null\ vector$$

$$[\sin 180^0 = 0]$$

(vi) **Equal vectors**

Vectors are equal if they possess same magnitude and direction i.e., θ=0⁰

∴ Cross product between two equal vectors is given by

$$\vec{A} \times \vec{A} = AA \sin\theta\, \hat{n} = AA \sin 0^0 \hat{n} = (AA)(0)\,\hat{n} = (0)\hat{n} = null\ vector$$

Cross product of two equal vectors is always a null vector.

$\therefore |\hat{i} \times \hat{i}| = |\hat{j} \times \hat{j}| = |\hat{k} \times \hat{k}| = 0$

(vii) **Cross product in terms of rectangular component**

Let A_x, A_y, A_z and B_x, B_y, B_z are the rectangular components of two vectors $\vec{A}$ and $\vec{B}$ respectively.

Then $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

And $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\therefore \vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x(\hat{i} \times \hat{i}) + A_xB_y(\hat{i} \times \hat{j}) + A_xB_z(\hat{i} \times \hat{k})$$

$$+ A_yB_x(\hat{j} \times \hat{i}) + A_yB_y(\hat{j} \times \hat{j}) + A_yB_z(\hat{j} \times \hat{k})$$

$$+ A_zB_x(\hat{k} \times \hat{i}) + A_zB_y(\hat{k} \times \hat{j}) + A_zB_z(\hat{k} \times \hat{k})$$

$$= A_xB_x(0) + A_xB_y(\hat{k}) + A_xB_z(-\hat{j}) +$$
$$A_yB_x(-\hat{k}) + A_yB_y(0) + A_yB_z(\hat{i}) +$$
$$A_zB_x(\hat{j}) + A_zB_y(-\hat{i}) + A_zB_z(0)$$

Since $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{i} = -\hat{k}$
$\hat{i}\times\hat{k} = -\hat{j},\ \hat{k}\times\hat{i} = \hat{j}$
$\hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{j} = -\hat{i}$
$|\hat{i}\times\hat{i}| = |\hat{j}\times\hat{j}| = |\hat{k}\times\hat{k}| = 0$

$$=> \vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

The cross product in terms of rectangular component of two can also be written in the form of determinant, as follows

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Problem(1):- Find the Cross product between $\vec{A} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} + 7\hat{k}$

Solution:- Here given **A_x=3, A_y=2, A_z=5** and **B_x=4, B_y=3, B_z=7**

We know that $\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

$$= (2\times7 - 5\times3)\hat{i} + (5\times4 - 3\times7)\hat{j} + (3\times3 - 2\times4)\hat{k}$$

$$= (14 - 15)\hat{i} + (20 - 21)\hat{j} + (9 - 8)\hat{k}$$

$$= -\hat{i} - \hat{j} + \hat{k}$$

Problem(2):- Find the cross product between $\vec{A} = 2\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{B} = 3\hat{i} - 6\hat{j} + \hat{k}$

Solution:- Here given **A_x=2, A_y=5, A_z=6** and **B_x=3, B_y=-6, B_z=1**

We know that $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & 6 \\ 3 & -6 & 1 \end{vmatrix} = \hat{i}\begin{vmatrix} 5 & 6 \\ -6 & 1 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 5 \\ 3 & -6 \end{vmatrix}$

$$=\hat{i}(5\times1 - 6\times(-6)) - \hat{j}(2\times1 - 6\times3) + \hat{k}(2\times(-6) - 5\times3)$$

$$=\hat{i}(5 + 36) - \hat{j}(2 - 18) + \hat{k}(-12 - 15)$$

$$=\hat{i}(41) - \hat{j}(-16) + \hat{k}(-27)$$

$$=41\hat{i} + 16\hat{j} - 27\hat{k}$$

LIST OF QUESTIONS FOR SEMESTER EXAMINATION

VERY SHORT ANSWER QUESTIONS(2 Marks each)

1. Define Scalar and Vector Quantity? Give one example of each of them.
2. State the characteristics of null vector.
3. State the triangle law of vector addition.
4. Define coplanar vector.
5. Find the dot product between $\vec{A} = 3\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 5\hat{i} - 3\hat{j}$
   [Ans. Here given **A_x=3, A_y=2, A_z=3** and **B_x=5, B_y= -3, B_z=0**
   We know that $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$
   $= 3 \times 5 + 2 \times (-3) + 3 \times 0$
   $= 15-6+0=9$
6. Find the dot product between $\vec{A} = 5\hat{i} + 6\hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + 3\hat{j}$
   [Ans.Here given **A_x=5, A_y=6, A_z=1** and **B_x=2, B_y=3, B_z=0**
   We know that $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$
   $= 5 \times 2 + 6 \times 3 + 1 \times 0$
   $= 10+18+0=28$

SHORT ANSWER QUESTIONS (5 Marks each)

1. State and explain triangle law of vector addition.
2. State and explain Parallelogram law of vector addition.
3. Find the Cross product between $\vec{A} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{B} = 4\hat{i} + 6\hat{j} + 7\hat{k}$

[Ans. Here given **A_x=2, A_y=4, A_z=3** and **B_x=4, B_y=6, B_z=7**

We know that $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 3 \\ 4 & 6 & 7 \end{vmatrix} = \hat{i}\begin{vmatrix} 4 & 3 \\ 6 & 7 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 4 \\ 4 & 6 \end{vmatrix}$

$=\hat{i}(4 \times 7 - 3 \times 6) - \hat{j}(2 \times 7 - 3 \times 4) + \hat{k}(2 \times 6 - 4 \times 4)$

$=\hat{i}(28 - 18) - \hat{j}(14 - 12) + \hat{k}(12 - 16)$

$=\hat{i}(10) - \hat{j}(2) + \hat{k}(-4)$

$=10\hat{i} + 2\hat{j} - 4\hat{k}$

4. Drive the resolution of a vector on horizontal and vertical components.
5. Find the Cross product between $\vec{A} = 3\hat{i} + 2\hat{j}$ and $\vec{B} = 4\hat{i} + 7\hat{j}$
   [Ans. Here given **A_x=3, A_y=2, A_z=0** and **B_x=4, B_y=7, B_z=0**

We know that $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 4 & 7 & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} 2 & 0 \\ 7 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 0 \\ 4 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 2 \\ 4 & 7 \end{vmatrix}$

$= \hat{i}(2 \times 0 - 0 \times 7) - \hat{j}(3 \times 0 - 0 \times 4) + \hat{k}(3 \times 7 - 2 \times 4)$

$= \hat{i}(0 - 0) - \hat{j}(0 - 0) + \hat{k}(21 - 8)$

$= \hat{i}(0) - \hat{j}(0) + \hat{k}(13)$

$= 13\hat{k}$

LONG ANSWER QUESTIONS (10 Marks each)

1. Define dot product and discuss characteristics of dot product.
2. Define cross product and discuss characteristics of cross product.

UNIT 3
KINEMATICS

Motion is common to everything in the universe. We walk, run, and ride a bicycle. Even when we are sleeping, air moves into and out of our lungs and blood flows in arteries and veins. We see leaves falling from trees and water flowing down a dam. Automobiles and planes carry people from one place to the other. Motion is change in position of an object with time. In this chapter we are going to discuss basic concept of kinematics (i.e., study of motion of bodies without the consideration of the force involved).

3.1.1 Rest

A body is said to be at rest when it does not change its position with respect to the surrounding or a specified reference frame.

The position of a body can be specified by its projections on the three axes of a rectangular co-ordinate system having its origin O at a fixed point. Distances of these projections from O are called as Position co-ordinates. If the co-ordinates do not change with time, the body is said to be at rest.

3.1.2 Motion

A body is said to be in motion if it changes its position with respect to the surroundings or a specified reference frame.

As the body moves along any path in space, its projections move in straight line along the three axes. The actual motion can be reconstructed from the motion of these three projections.

3.2.1 Displacement

Displacement is a vector connecting between the initial and final position of the body and directed away from the initial position towards the final position.

The shortest distance between initial and final position of the body is called as displacement.

Displacement can be represented as "s" and dimensional formula is $s = [M^0L^1T^0]$ and its SI unit is Metre (m).

3.2.2 Speed

Speed of a body is defined as the distance covered by the body in one second.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{s}{t}$$

Dimensional formula of speed is $[M^0L^1T^{-1}]$ and SI unit is Metre/ Second (m/s).

3.2.3 Velocity $(\bar{v}\rightarrow)$

Velocity of a body is defined as the rate of change of displacement of the body.

$$\text{Average velocity } (\bar{v}_{av}) = \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\Delta \vec{s}}{\Delta t}$$

The velocity at a particular instant of time is called as instantaneous velocity.

$$\text{Instantaneous velocity}(\vec{v}) = \frac{d\vec{s}}{dt}$$

Dimensional formula of velocity is $[M^0 L^1 T^{-1}]$ and SI unit is Metre/ Second (m/s).

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3.2.4 Acceleration ($\vec{a}$)

Acceleration is defined as the rate of change of velocity of a body.

$$\text{Acceleration } (\vec{a}) = \frac{\text{Change in Velocity}}{\text{Change in time}}$$

$$\Rightarrow \vec{a} = \frac{d\vec{v}}{dt} = \frac{d\left(\frac{d\vec{s}}{dt}\right)}{dt} \qquad ; \vec{v} = \frac{d\vec{s}}{dt}$$

$$\Rightarrow \vec{a} = \frac{d^2\vec{s}}{dt^2}$$

Dimensional formula of acceleration is $[M^0 L^1 T^{-2}]$ and SI unit is m/s^2 or ms^{-2}

3.2.5 Force ($\vec{F}$)

Force is an external agent which is used as pull or pushes to change or try to change the state of rest or uniform motion or the direction of motion of a body.

$$\textbf{Force } (\vec{F}) = \text{mass} \times \text{acceleration}$$

$$\Rightarrow \vec{F} = m\vec{a}$$

$$\Rightarrow \vec{F} = m\frac{d\vec{v}}{dt}$$

Dimensional formula of Force, $\mathbf{F} = ma = [M^1][L^1T^{-2}] = [M^1L^1T^{-2}]$

And SI unit is kgm/s^2 or Newton

3.3.1 Equations of Motion under Gravity (downward motion)

Consider a body is falling freely from a point „O" with initial velocity u = 0. It reaches a point „P" after „t" second and acquires a velocity „v" due to the uniform acceleration due to the gravity „g". Here „v" is called as final velocity just before hitting the ground. It covers a distance „h" while moving from „O" to „P".

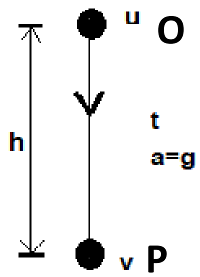

Fig. 3.1 Body fall from a height

Therefore, the equations of motion under gravity in downward motion can be written as follows:

1. Velocity time relation $v = u + at = gt$
2. Displacement time relation $h = ut + \frac{1}{2}at^2 = \frac{1}{2}gt^2$
3. Velocity displacement relation $v^2 = u^2 + 2ah = 2gh$

   Displacement in „nth" second $S_{nth} = u + \frac{a}{2}(2n - 1) = \frac{g}{2}(2n - 1)$

As $h = \frac{1}{2}gt^2$, **t =** $\sqrt{\frac{2h}{g}}$

This is the time of descent. It can be seen that this does not depend on the mass of falling object provided air resistance is neglected.

3.3.2 Equations of Motion under Gravity (upward):

Consider a body thrown upward from a point „O" with initial velocity „u" at time t=0 (Fig. 3.2). It reaches a point „P" after „t" second and acquires a velocity „v" due to the uniform acceleration due to the gravity „g". Here „v" is called as final velocity after travelling a height h.

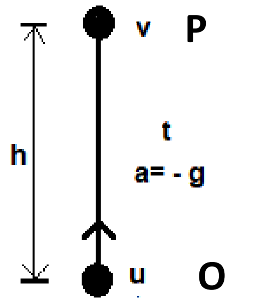

Fig. 3.2 a body is thrown in to a height

The equations of motion under gravity in upward motion can be written as follows:

1. Velocity time relation $v = u + at = u - gt$
2. Displacement time relation $h = ut + \frac{1}{2}at^2 = ut - \frac{1}{2}gt^2$
3. Velocity displacement relation $v^2 = u^2 + 2ah = u^2 - 2gh$
4. Displacement in „nth" second $s_{nth} = u + \frac{a}{2}(2n - 1) = u - \frac{g}{2}(2n - 1)$

At maximum height, v = 0, so, t = $\frac{u}{g}$. This is the time of ascent.

Hence, $h = ut - \frac{1}{2}gt^2 = \frac{u^2}{g} - \frac{u^2}{2g} = \frac{u^2}{2g}$

This is the maximum height reached by the object when thrown with an initial velocity of u.

*Sign convention used:- Physical quantities upward direction are taken to be positive and downward direction are taken to be negative or vice-versa.

3.4 Circular motion

A body is said to move in circular motion, if it moves in such a way that its distance from a fixed point always remains constant.

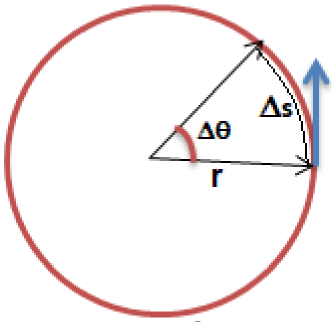

Fig. 3.3 Circular motion

3.4.1 Angular Displacement (Δθ)

Angular Displacement of a body undergoing circular motion is defined as the angle turned by its radius vector.

It is a vector quantity. It acts along the axis of rotation as per the right hand thumb rule.

$$\Delta\theta = \frac{\Delta s}{r}$$

Here angular displacement is dimensionless quantity$[M^0L^0T^0]$ and its SI unit is radian (rad).

3.4.2 Angular Velocity (ω)

Angular velocity of a body undergoing circular motion is defined as the rate of change of angular displacement with time.

It is a vector quantity and act along the axis of rotation i.e. along the same direction as displacement vector.

$$\boldsymbol{\omega} = \frac{d\theta}{dt}$$

Here dimensional formula of angular velocity is $[M^0L^0T^{-1}]$ and its SI unit is radian per second (rad/s).

3.4.3 Angular acceleration (α)

Angular acceleration of a body undergoing circular motion is defined as the rate of change of angular velocity with time.

It is a vector quantity and acts along the axis of rotation.

$$\alpha = \frac{d\boldsymbol{\omega}}{dt}$$

Here dimensional formula of angular acceleration is $[M^0L^0T^{-2}]$ and its SI unit is radian per second ^{2} (rad/s^2).

3.5.1 Relation between Linear displacement(Δs) and Angular Displacement (Δθ):-

We know that $\Delta\theta = \frac{\Delta s}{r}$

$$=> \Delta s = r\,\Delta\theta, \text{ this is the scalar form.}$$

In vector form, Linear Displacement = $d\vec{s} = d\vec{\theta} \times \vec{r}$

3.5.2 Relation between Linear velocity (v) and Angular velocity (ω):-

We know that the angular velocity,

$$\boldsymbol{\omega} = \frac{d\theta}{dt}$$

$$=> \boldsymbol{\omega} = \frac{d\left(\frac{s}{r}\right)}{dt} = \frac{1}{r}\frac{ds}{dt} \qquad ;\Delta\theta=\Delta s/r$$

$$=> \boldsymbol{\omega} = \frac{1}{r}v \qquad ;v = \frac{ds}{dt}$$

$=> v = r\omega$, this is linear speed (scalar form).

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

3.5.3 Relation between Linear acceleration (a) and Angular acceleration (α):-

We know that the angular acceleration,

$$\alpha = \frac{d\boldsymbol{\omega}}{dt}$$

$$=> \alpha = \frac{d\left(\frac{v}{r}\right)}{dt} = \frac{1}{r}\frac{dv}{dt} \qquad ;\omega=v/r$$

$$=> \alpha = \frac{1}{r}a \qquad ;a = \frac{dv}{dt}$$

$=> a = r\alpha$, This is linear acceleration or tangential acceleration.

In vector form $\vec{a} = \vec{\alpha} \times \vec{r}$

Example (1):- A car goes round a curve path, of radius 8 m, with a velocity 54 km h^{-1}. What is its angular velocity?

Answer:- Given velocity, $\mathrm{v} = 54\ \mathrm{km\ h^{-1}} = 54 \times \frac{5}{18}\ \mathrm{m\ s^{-1}} = 15\ \mathrm{m\ s^{-1}}$

and r = 8 m

We know that, $\mathrm{v} = \mathrm{r}\omega$

$$=> 15 = 8\,\omega$$

$$=> \omega = \frac{15}{8}\, rad\ s^{-1} = 1.875\, rad\ s^{-1}$$

Example (2):- A particle moving in a circle of radius 10 m has its angular velocity increased, in one minute, by 90 rad min^{-1}. Calculate
(i) Angular acceleration
(ii) Tangential or linear acceleration.

Answer:- Here given

$$\text{Change in angular velocity} = 90 \text{ rad min}^{-1} = \frac{90}{60} \text{ rad s}^{-1} = 1.5 \text{ rad s}^{-1}$$

and time = 60 s, radius r =10 m

(i) Angular acceleration, $\propto = \frac{\text{Change in angular velocity}}{\text{time}} = \frac{1.5}{60} \text{ rad s}^{-2} = \frac{1}{40} \text{ rad s}^{-2}$

(ii) Linear acceleration, $a = \text{r} \propto = 10 \times \frac{1}{40} \text{ m s}^{-2} = 0.25 \text{ m s}^{-2}$

3.6 Projectile motion

Projectile is a body thrown with an initial velocity in the vertical plane and then it moves in two dimensions under the action of gravity alone without being propelled by any engine or fuel.

The path of projectile is called its trajectory.

Example:-

(i) A cricket ball thrown into the space.
(ii) A fruit falling from a tree.
(iii) A bullet fired from a rifle.
(iv) A bag dropping from an aeroplane.
(v) A football kicked into the space.

3.7 Projectile fired at an angle θ with the horizontal.

Consider a projectile fired from a point O with a velocity „u" at an angle θ with horizontal. The projectile rises to the maximum height „H" at point P and fall back at Q, lying on the same level of projection (Fig. 3.4).

Since the projectile moves in X-Y plane, so velocity „u" can be resolved in to two components as follows.

(i) u cosθ along horizontal direction. This component is uniform since it does not depend upon acceleration due to gravity „g".
(ii) u sinθ along vertical direction. This component is non-uniform since the acceleration due to gravity (g) acts exactly opposite to it. It is zero at the maximum height.

The above mentioned two components are independent to each other, since they are mutually perpendicular.

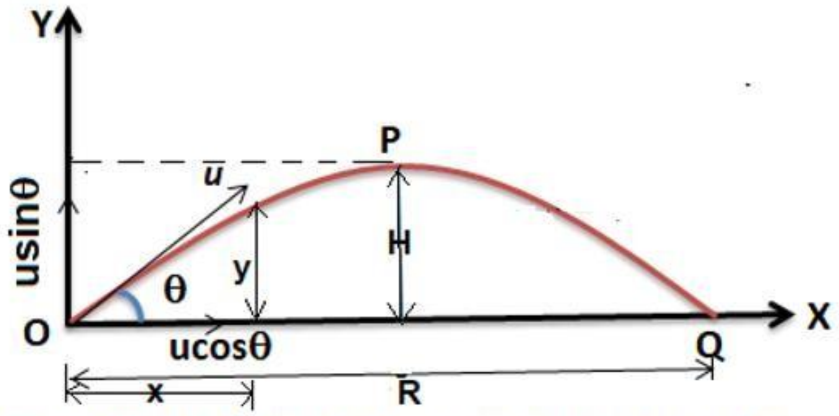

Fig. 3.4 Projectile fired at an angle θ with horizontal

The following important results can be derived in this case.

3.7.1 Equation of trajectory

It is an equation which connecting between horizontal displacement and vertical displacement of the projectile motion.

Applying equation of motion, $\mathrm{s} = \mathrm{ut} + \frac{1}{2}\mathrm{at}^2$along horizontal direction we get,

$\mathrm{x} = \mathrm{ucos}\boldsymbol{\theta}\ \mathrm{t} + \frac{1}{2} \times 0 \times \mathrm{t}^2$;Since $a = g\cos 90^0 = 0$, along

Here, horizontal direction, s = horizontal displacement = x,

u = horizontal displacement = $\mathrm{ucos}\theta$

Hence, $\mathrm{x} = \mathrm{ucos}\boldsymbol{\theta}\ \mathrm{t}$

$=> t = \frac{x}{u\cos\theta}$-------------------------------------(1)

Applying equation of motion, $\mathrm{s} = \mathrm{ut} + \frac{1}{2}\mathrm{at}^2$---(2)

along the vertical direction. So,
s = vertical displacement = y
u = Initial vertical velocity = u sinθ
a = acceleration due to gravity = -g

Therefore, from the above equation (2), we can write

$$y = u\sin\theta\, t + \frac{1}{2}(-g)t^2$$

$$y = u\sin\theta\, t - \frac{1}{2}gt^2 \text{---} (3)$$

Now putting the value of equation (1) in equation (3), we get

$$y = u\sin\boldsymbol{\theta}\ \frac{x}{u\cos\boldsymbol{\theta}} - \frac{1}{2}g\left(\frac{x}{u\cos\boldsymbol{\theta}}\right)^2$$

$$=> y = x\tan\boldsymbol{\theta} - \frac{gx^2}{2u^2\cos^2\theta} \text{-------------(4)}$$

This equation is known as equation of trajectory. Since equation (4) satisfies the equation of parabola, so the path of the projectile is parabolic.

3.7.2 Maximum height (H)

It is the maximum distance travelled by the projectile in vertical direction. It is travelled due to vertical component of velocity.

Applying equation of motion,

$$v^2 - u^2 = 2as \text{-----------------------------------(5)}$$

Here we have considered the motion of projectile in the vertical direction, so we can write:

s = maximum height = H
u = Initial vertical velocity = u sinθ
v = final vertical velocity at the highest point= 0
a = acceleration due to gravity = -g

Therefore, from the above equation (5), we get:

$$0^2 - u^2 sin^2\boldsymbol{\theta} = 2(-g)H$$

$$=> u^2\sin^2\boldsymbol{\theta} = 2gH$$

$$=> H = \frac{u^2\sin^2\boldsymbol{\theta}}{2g} \text{------------------------------------(6)}$$

3.7.3 Time of ascent (t)

It is time taken by the projectile to reach the maximum height from the point of projection.

Applying equation of motion,

$$v = u + at \text{-----------------------------------(7)}$$

Here we have considered the motion of projectile form „O" to „P" in the vertical direction only, so we can write:

u = Initial vertical velocity = u sinθ
v = final vertical velocity at the highest point = 0
a = acceleration due to gravity = -g
t = time of ascent

Therefore, from the above equation (7), we get:

$$0 = \text{u} \ \sin \boldsymbol{\theta} + (-\text{g})\text{t}$$

$$=> \text{gt} = \text{u} \ \sin \boldsymbol{\theta}$$

$$=> \text{t} = \frac{\text{u} \ \sin \boldsymbol{\theta}}{\text{g}} \text{-----------------------------(8)}$$

The above equation represents the time of ascent of the projectile.

3.7.4 Time of descent (t)

It is time taken by the projectile to reach the level of projection from the maximum height.

3.7.5 Total time of flight (T)

It is total time taken by the projectile to come back to the ground from which it was protected.

If the air resistance is neglected, then time of descent is equal to the time of ascent=t.

Therefore,

$$\text{Total time of flight(T)} = \text{time of ascent(t)} + \text{time of descent(t)}$$

$$=> T = \frac{\text{u} \ \sin\boldsymbol{\theta}}{\text{g}} + \frac{\text{u} \ \sin\boldsymbol{\theta}}{\text{g}}$$

$$=> T = \frac{2\text{u} \ \sin\boldsymbol{\theta}}{\text{g}} \text{-----------------------------(9)}$$

3.7.6 Horizontal range (R)

It is the distance travelled by the projectile in the horizontal direction during its time of flight. The horizontal range is travelled due to the horizontal component of velocity which is uniform.

$$\text{R} = \text{horizontal velocity} \ \times \text{Total time of flight}$$

$$=> \text{R} = \text{u} \cos \theta \ \times \frac{2\text{u} \ \sin\boldsymbol{\theta}}{\text{g}}$$

$$=> \text{R} = \frac{\text{u}^2 2 \sin \boldsymbol{\theta} \ \cos\theta}{\text{g}}$$

$$=> \text{R} = \frac{\text{u}^2 \sin 2\boldsymbol{\theta}}{\text{g}} \text{-----------------------(10)} \qquad [\sin 2\boldsymbol{\theta} = 2 \sin \boldsymbol{\theta} \ \cos \theta$$

3.7.7 Condition for maximum Horizontal range (R_{max})

We know that the horizontal range of projectile motion is

$$R = \frac{u^2 \sin 2\boldsymbol{\theta}}{g}$$

It is clear that horizontal range depends upon the value of velocity of projection „u" and angle of projection „θ". For fixed value of „u", horizontal range depends upon the angle of projection „θ". Horizontal range will be maximum if „sin2θ" is maximum. *i.e.,*

$$\sin 2\theta = 1$$

$$=> sin\,2\theta = \sin 90^0$$

$$=> 2\theta = 90^0$$

$$=> \theta = 45^0$$

This is the condition for maximum horizontal range. Maximum horizontal range can be written as

$$R_{max} = \frac{u^2 \times 1}{g}$$

$$=> R_{max} = \frac{u^2}{g} \text{--------------------------} \quad (11)$$

This is the maximum horizontal range travelled by the projectile fired at an angle ϴ=45^0.

LIST OF QUESTIONS FOR SEMSTER

VERY SHORT ANSWER QUESTIONS(2 Marks each)

1. Define rest and motion.
2. Distinguish between distance and displacement.
3. How do you define acceleration? State the unit in which it is measured.
4. What do you mean by horizontal range? Under what condition it is maximum?
5. A body starts from rest and covers a distance by acquiring a velocity of 20m/s in 5 sec. Find the acceleration. [Ans. $4m/s^2$
6. A body covers a distance of 50m along a straight line in 5 sec. Calculate the speed at the end of 5 sec? [Ans. 10m/s
7. Explain the circumstances in which a body has zero average velocity.

   [Answer

   Average velocity = Net displacement/time taken

   Suppose a car covers a distance of 50 m in 5 seconds and comes back to initial position in 5 seconds. The average speed of car is 10 m/s, but average velocity is zero because displacement is zero.

   A body is moving in a circular path and complete one rotation. So, the total displacement is zero. Therefore, average velocity is zero.

SHORT ANSWER QUESTIONS (5 Marks each)

1. Find the relationship between liner velocity and angular velocity.
2. Write short notes on (i) Velocity, (ii) Acceleration, (iii) Displacement
3. Find the relation between linear acceleration and angular acceleration.
4. A body starting from rest moves with an acceleration of 5 m/s^2. Calculate its velocity when it covered a distance of 20m. [Ans. 14.14m/s
5. A body possessing an initial velocity of 10m/s moves an acceleration 2m/s^2. Calculate its velocity at the end of 4 sec. [Ans. 18m/s
6. Show that the path of a projectile fired at an angle „θ" with horizontal is parabolic in nature.

LONG ANSWER QUESTIONS (10 Marks each)

1. A projectile is fired with a velocity „u" with making an angle „θ" with horizontal. Find the following expressions
   (i) Equation of trajectory
   (ii) Maximum height attained
   (iii) Time of ascent
   (iv) Total time of flight
   (v) Horizontal range
   (vi) Condition for maximum horizontal range.
2. A body travels along a circular path of radius 10m. It completes half of circumference in three sec. Calculate it"s (i) Distance covered, (ii) displacement, (iii) speed and (iv) velocity.

[Ans. (i)31.4m, (ii)20m (iii)10.466m/s, and (iv)6.666m/s

UNIT 4 – WORK AND FRICTION

4.1 WORK:

Work is said to be done if the force applied on a body displaces the body and the force has a component along the direction of displacement.

$W = \vec{F} \cdot \vec{s} = Fs \cos\theta$

Where, W = work done

F = magnitude of the force

s = magnitude of the displacement

θ = angle between the force and displacement

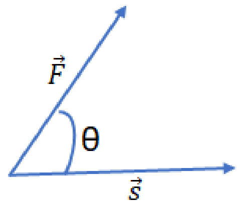

Figure 4.1

- If θ = 0° and W = $Fs \cos 0° = +Fs$. Here, work done is *positive* which means work is said to be done *upon* the body.

  Example: when an external horizontal force is applied to a body and the body moves in a horizontal direction, the work done by this external horizontal force is positive.

- If θ = 90° and W = $Fs \cos 90° = 0$,

  Example: A person carrying a box over his head and walking in the horizontal direction. In this case, *work done by the gravity* is zero.

- If θ = 180° and W = $\cos 180° = -Fs$. *Negative* work done means work is done *by* the body.

  Example: Work done by the force of friction is negative.

- When the force is applied, if there is *no* displacement, then also work done is zero.

Unit:

The SI unit of work is Joule (J) and the CGS unit is Erg.

The SI unit and dimensions of work and energy are same.

Dimension:

$$[\mathbf{W}] = [\mathbf{F}][\mathbf{s}][\mathbf{cos\theta}] = [\mathbf{MLT^{-2}}] \times [\mathbf{L}] = [\mathbf{ML^2T^{-2}}]$$

4.2 FRICTION:

Let us say there is an *almirah* placed on the *floor*. One person tries to push it. He exerts force, the almirah does not move in the beginning. Then, the person increases force little by little and at one point the almirah starts to move.

Let us analyse this situation. When the person is applying force, the force must have some effect (the force must create acceleration). But *apparently* there is no effect. Why is it happening? It is because when the person is applying force, the *floor* is exerting an equal amount of force on the almirah. Hence, the effect of force is getting cancelled. When the person is increasing the force, the force on the almirah by the floor is increasing too. However, there is a *limit* to the force by the floor. Once, it is reached, the almirah starts to move. However, when the almirah is moving, the floor is still applying force on the almirah. The force tries to oppose the motion of the almirah. In this example, the force on the almirah by the floor arising because of the contact between them is *frictional force*.

In this chapter, we will formally discuss the concept of friction, the types of friction and the laws regarding friction.

Definition:

The force which opposes or tend to oppose the relative motion between two surfaces in contact is called as force of friction.

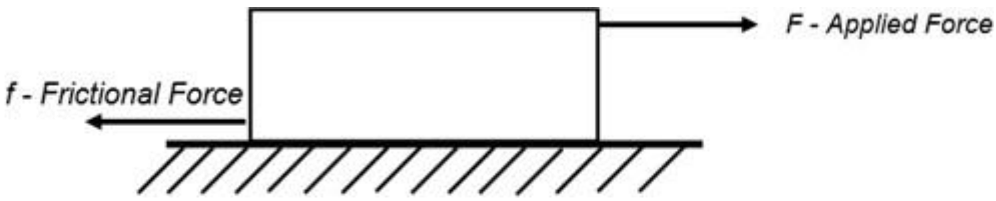

Figure 4.2

Force of friction is created because of the inter-locking of two surfaces in contact.

4.2 TYPES OF FRICTION:

Friction can be classified into four types.

1. Static Friction
2. Kinetic (dynamic) friction
3. Rolling friction
4. Fluid friction (viscosity)

In this chapter we will focus on static and kinetic friction and the laws regarding them.

1. **Static Friction:**

- The force of friction which comes into play when there is no relative motion between two surfaces in contact is called as force of static friction. Force of static friction is equal and opposite to the applied force till the body is at rest.
- For example, a person or a group of persons are trying to push a heavy object. Initially, a small force is applied, and the magnitude of force is increased gradually. The magnitude of static friction increases gradually too. As long as the object is in static condition, the floor exerts an equal and opposite force on the object. As in the below figure, the applied force is towards the right, hence the frictional force is towards the left.

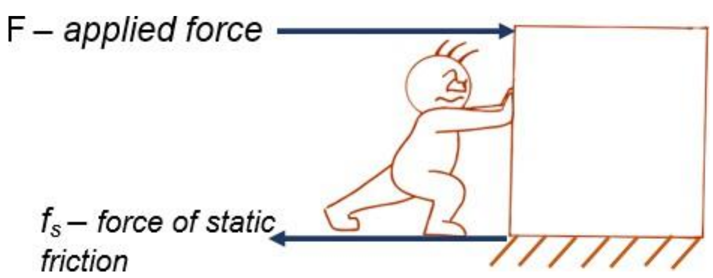

Figure 4.3

- Static friction is a self-adjusting force.
- The maximum value of static friction is called the **limiting friction**.

$$f_L = \mu_s R \text{ ----------------------- (1)}$$

Where, f_L – Force of limiting friction
μ_s – Coefficient of static friction
R – Normal reaction

Once the limiting friction is reached, the body starts to move, and kinetic friction comes to picture.

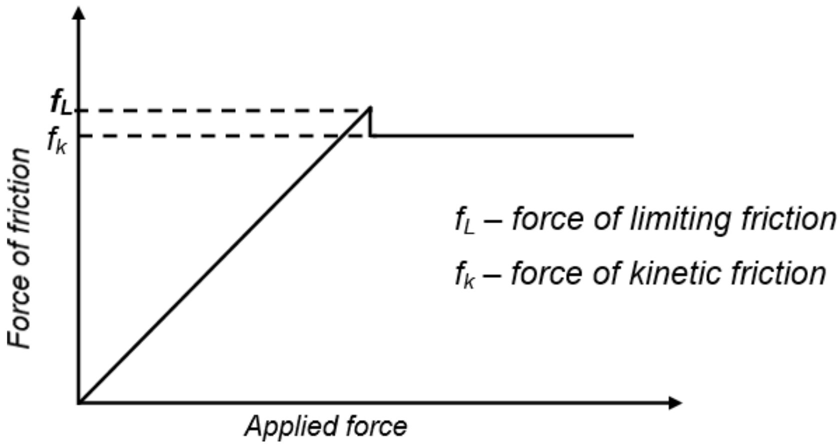

Figure 4.4

2. Kinetic (dynamic) Friction:

The force of kinetic friction comes into play when there is relative motion between two surfaces in contact is called as force of kinetic friction or dynamic friction or sliding friction. The direction of the frictional force is such that the relative slipping is opposed by the friction.

$$\text{Hence, } f_k = \mu_k R \text{----------------------- (2)}$$

Where, f_k – Force of kinetic friction
μ_k – Coefficient of kinetic friction
R – Normal reaction

4.4. LAWS OF LIMITING FRICTION:

Statements about factors upon which the force of limiting friction between two surfaces depends, are termed as laws of limiting friction. They are stated as below.

i. The direction of force of friction is always opposite to the direction of *intended* motion (the body is at rest).
ii. The force of limiting friction depends on the nature and state of polish of the surfaces in contact and act tangentially to the interface between the two surfaces.
iii. The magnitude of limiting friction f_L is directly proportional to the magnitude of the normal reaction R between the two surfaces in contact.

$$f_L \propto R$$

iv. The magnitude of the limiting friction between two surfaces is independent of the area and shape of the surfaces in contact as long as the normal reaction remains same.

4.5. COEFFICIENT OF FRICTION

- The frictional force (f) is directly proportional to the normal reaction force (R) and the proportionality constant μ is called the coefficient of friction.

$$\mu = \frac{f}{R}$$

- Hence, the coefficient of friction is defined as the ratio of the frictional force to the normal force.
- The coefficient of friction is determined experimentally.
- As the unit and dimension of frictional force and normal force are same, μ is unit and dimensionless.
- The coefficient of friction depends on the nature of the bodies in contact, their material and the surface roughness.

Example 1:

A box of mass 30 kg is pulled on a horizontal surface by applying a horizontal force. If the coefficient of kinetic friction between the box and the horizontal surface is 0.25, find the force of friction exerted by the horizontal surface on the box during its motion.

Answer:

Mass m = 30 kg, $\mu_k = 0.25$

$$f_k = \mu_k R = \mu_k mg = 0.25 \times 30 \times 9.8 = 73.5 \text{ Newton}$$

Example 2:

A body of mass 10 kg is placed on a rough horizontal surface at rest. The coefficient of friction between the body and & surface is μ = 0.1. Find the force of friction acting on the body.

Answer:

Since, the body is at rest, the force of static friction will come into play which is equal to applied force.

Since, applied force is zero; the force of static friction is zero.

Example 3:

Find the force of friction in situation as shown in the below figure. Take g = 10 m/s^2

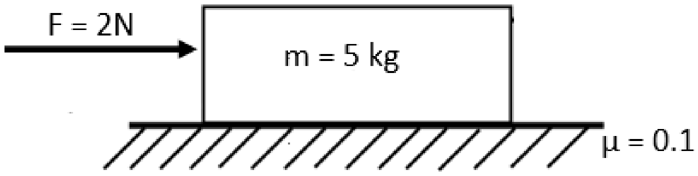

Figure 4.5

Answer:

The magnitude of limiting friction $f_L = \mu R = 0.1 \times 5g = 5N$

We see, the applied force is smaller than the force of limiting friction i.e., $F < f_L$

So, the force of static friction = magnitude of applied force = 2 N.

4.6 METHOD TO REDUCE FRICTION:

The following methods can be used to reduce friction when friction creates hurdle in the performance of machines or for similar necessary reasons

i. **By polishing or rubbing:**
The roughness of a surface can be reduced by rubbing or polishing it. The polishing makes a surface smooth and reduces friction.

ii. **Lubrication or use of talcum powder:**
Friction can be reduced by using lubricants like oil and grease or talcum powder as they form a thin film between different parts of a machine. This film covers up the pores & the lumps present on the surfaces of different parts and hence improves the smoothness.

iii. **By converting sliding friction to rolling friction:**
Rolling friction is lesser than sliding friction. Hence, ball bearings can be placed between the moving parts a machine to avoid direct contact between them. This reduces friction.

iv. **Streamlining:**
The objects that move in fluid, for example, ship, boat or plane, the shape of the body can be streamlined to reduce the friction between the body and the fluid.

LIST OF QUESTIONS FOR SEMESTER EXAMINATION

I. Very short type questions (2 marks each)

1. Define work. Write its formula and unit.
2. Define friction.
3. Define limiting friction. Write its formula.
4. Define coefficient of friction. State its unit and dimension.
5. Define kinetic friction and state its formula.

II. Short type questions (5 marks each)

1. Define limiting friction. State the laws of limiting friction.
2. Write five methods to reduce friction.

III. Long type questions (10 marks each)

1. Compare static and kinetic friction. State the laws of limiting friction.
2. Define friction and coefficient of friction. What are the methods to reduce friction?