BHUBANANANDA ORISSA SCHOOL OF ENGINEERING, CUTTACK

DEPARTMENT OF CIVIL ENGINEERING

LECTURE NOTE ON: : STRUCTURAL DESIGN – I (TH-1)

4TH SEMESTER

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Limit State Method

Limit state method

The acceptable limit of safety and serviceability requirements before failure occurs is called a *limit state*. A limit state is a state of impending failure, beyond which a structure ceases to perform its intended function satisfactorily, i.e. it either collapses or becomes unserviceable.

In the concept of limit state, the structure shall be designed to withstand all loads liable to act on it throughout its life. It shall also satisfy the serviceability requirements such as limitations on deflection and cracking, etc. The aim of the design is to achieve acceptable probabilities that the structure will not become unfit for use for which it is intended, i.e. it will not reach its limit state during its design life period. All the relevant limit states shall be considered in the design. In general, the structure shall be designed for most critical limit state and checked for all other limit states.

To ensure the above objective, the design should be based on the characteristic values of the material strengths and applied loads, which take into account the variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data if available, where such data are not available, they should be based on experience. The design values are derived from the characteristic values through the use of partial safety factors, one for material and the other for loads.

Types of limit states:

1. Limit state of collapse
    a. Limit state of collapse in flexure
    b. Limit state of collapse in compression
    c. Limit state of collapse in compression and uniaxial bending
    d. Limit state of collapse in compression and biaxial bending
    e. Limit state of collapse in shear
    f. Limit state of collapse in torsion
    g. Limit state of collapse in tension
2. Limit state of serviceability

a. Limit state of deflection
b. Limit state of cracking
c. Limit state of vibration, fire resistance, durability, etc.

Limit state of collapse

The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from buckling due to elastic or plastic instability (including the effects of sway where appropriate) or overturning. The resistance to bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most unfavourable combination of loads on the structure using the appropriate partial safety factors.

Limit state of serviceability

Limit state of serviceability relates to the performance or behavior of structure at *working loads*. To satisfy the limit state of serviceability the deflection and cracking shall not be excessive. Durability is taken care of by prescribing appropriate grade of concrete, nominal cover for various exposure conditions, cement content etc.

a. Limit state of deflection

The deflection of a structure or parts thereof shall not adversely affect the appearance or efficiency of the structure or finishes or partitions. This has been dealt with in Cl. 23.2 (Page 37) of IS 456: 2000.

b. Limit state of cracking

Cracking of concrete should not adversely affect the appearance or durability of the structure; the acceptable limits of cracking would vary with the type of structure and environment. The surface width of the cracks should not, in general, exceed 0.3 mm in members where cracking is not harmful and does not have any serious adverse effects upon the preservation of reinforcing steel nor upon the durability of the structures.

In members where cracking in the tensile zone is harmful either because they are exposed to the effects of the weather or continuously exposed to moisture or in contact soil or ground water, an upper limit of 0.2 mm is suggested for the maximum width of cracks. For particularly aggressive environment, such as the 'severe' category in Table 3, the assessed surface width of cracks should not in general, exceed 0.1 mm.

Where specific attention is required to limit the designed crack width to a particular value, crack width calculation may be done using formula given in *Annex F*.

Characteristic Strength of Materials (Cl. 36.1, Page 67)

The term 'characteristic strength' means that value of the strength of the material below which not more than 5 percent of the test results are expected to fall. This implies that there is only 5% probability or chance of actual strength being less than the characteristic strength. In other words characteristic strength has 95% reliability.

Characteristic strength of concrete (Table 2, Page 16)

Characteristic strength of concrete is denoted by f_{ck} (N/mm^2). The characteristic strength of various grades of concrete is given in Table 2, IS 456: 2000 (Page 16). The symbol *M*, in the designation of concrete mix, refers to the mix and the number specifies the compressive strength of 150 mm size cube at 28 days expressed in N/mm^2.

Table 2 Grades of Concrete

(*Clause* 6.1, 9.2.2, 15.1.1 *and* 36.1)

Group	Grade Designation	Specified Characteristic Compressive Strength of 150 mm Cube at 28 Days in N/mm^2
(1)	(2)	(3)
Ordinary Concrete	M 10	10
	M 15	15
	M 20	20
Standard Concrete	M 25	25
	M 30	30
	M 35	35
	M 40	40
	M 45	45
	M 50	50
	M 55	55
High Strength Concrete	M 60	60
	M 65	65
	M 70	70
	M 75	75
	M 80	80

NOTES

1 In the designation of concrete mix M refers to the mix and the number to the specified compressive strength of 150 mm size cube at 28 days, expressed in N/mm^2.

2 For concrete of compressive strength greater than M 55, design parameters given in the standard may not be applicable and the values may be obtained from specialized literatures and experimental results.

Characteristic strength of steel

Until the relevant Indian Standard Specifications for reinforcing steel are modified to include the concept of characteristic strength, the characteristic value shall be assumed as the minimum yield stress/0.2 percent proof stress specified in the relevant Indian Standard Specifications. The characteristic strength of steel is designated by symbol f_y (N/mm^2).

Characteristic Loads (Cl. 36.2, Page 67)

The term 'characteristic load' means that value of load which has a 95 percent probability of not being exceeded during the life of the structure. Since data are not available to express loads in statistical terms, for the purpose of this standard, dead loads given in IS: 875 (Part 1).imposed loads given in IS: 875 (Part2), wind loads given in IS: 875 (Part 3), snow load as given in IS: 875 (Part 4) and seismic forces given in IS: 1893 shall be assumed as the characteristic loads.

Design Strength of Materials (Cl. 36.1, Page 67)

The design strength of the materials is given by

$$f_d = \frac{f}{\gamma_m}$$

where

f_d = characteristic strength of the material and

γ_m = partial safety factor appropriate to the material and the limit state being considered.

Design Loads (36.3.2, Page 68)

The design load, F_d is given by

$$F_d = F\gamma_f$$

where

F = characteristic load *(see* 36.2).and

γ_f = partial safety factor appropriate to the nature of loading and the limit state being considered.

Partial Safety Factor γ_m for Material Strength (Cl. 36.4.2, Page 68)

When assessing the strength of a structure or structural member for the limit stale of collapse, the values of partial safety factor, γ_m should be taken as 1.5 for concrete and 1.15 for steel. A higher value of partial factor of safety (γ_m = 1.5) for concrete has been adopted because there is greater chance of variation of strength of concrete due to improper compaction, inadequate curing, improper batching and mixing and variation in properties of ingredients. The chances of variation in the strength of reinforcement are known to be small and hence a lower value (γ_m = 1.15) has been adopted. It should be noted that γ_m values are a1ready incorporated in the equations and tables given in IS 456: 200 for the limit state design.

Partial Safety Factor γ_f for Loads (Cl. 36.4.2, Page 68)

The partial factor of safety (γ_f) for loads are given in Table 18 of IS 456: 200 (Page 68) shall normally be used.

Table 18 Values of Partial Safety Factor γ_f for Loads

(*Clauses* 18.2.3.1, 36.4.1 *and* B-4.3)

Load Combination	Limit State of Collapse			Limit States of Serviceability		
	DL	*IL*	*WL*	*DL*	*IL*	*WL*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
DL + *IL*	1.5	1.5	1.0	1.0	1.0	–
DL + *WL*	1.5 or 0.9[1)]	–	1.5	1.0	–	1.0
DL + *IL* + *WL*	1.2	1.2	1.2	1.0	0.8	0.8

NOTES

1 While considering earthquake effects, substitute *EL* for *WL*.

2 For the limit states of serviceability, the values of γ_f given in this table are applicable for short term effects. While assessing the long term effects due to creep the dead load and that part of the live load likely to be permanent may only be considered.

[1)] This value is to be considered when stability against overturning or stress reversal is critical.

The factored load, shear, moment and axial load, etc. in limit state design will be 1.5 times the corresponding value for elastic design (working stress method of design). The respective values of dead load and live loads are $w_u = 1.5w,\ W_u = 1.5W,\ M_u = 1.5M,\ V_u = 1.5V,\ P_u = 1.5P$

Idealized stress – strain curve for concrete (Fig. 21, Page 69)

The compressive stress of concrete in the actual structure is assumed to be 0.67 f_{ck}. The factor 0.67 is introduced to account for the size effect based on the assumption that the concrete in the structure develops a strength of only 0.67 times the strength of the cube. The partial factor of safety γ_m = 1.5 is applied in addition to this. Thus, the design curve is obtained as shown in the figure. Thus, the maximum compressive stress in concrete for design purpose is equal to 0.45 f_{ck}. It is to be noted that each curve is parabolic in the initial portion upto a strain of 0.002. At a strain of 0.002 (0.2% strain), the stress remains constant with increasing load, until a strain of 0.35% is reached, when the concrete is said to have failed.

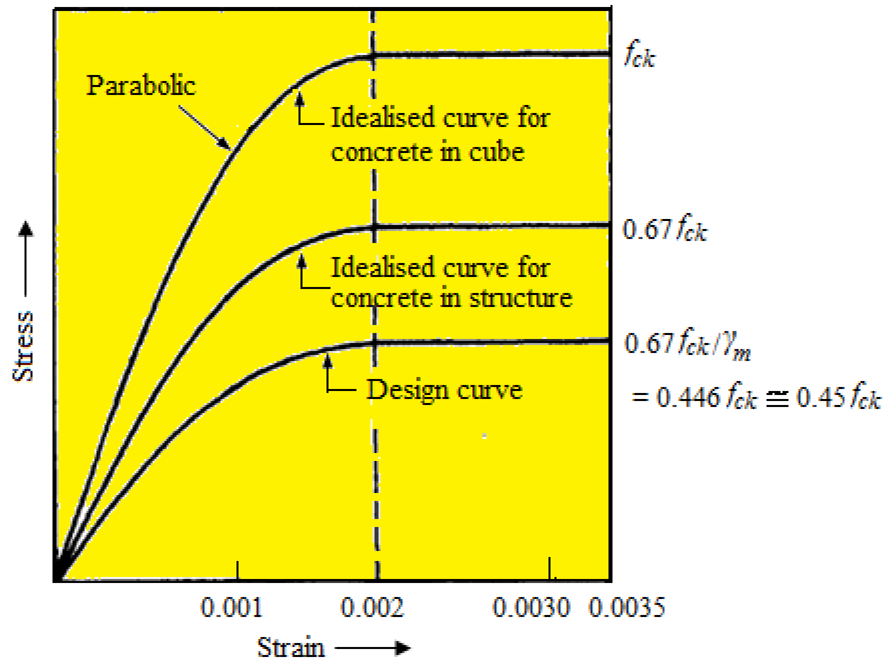


Fig. Stres - strain curve for concrete

Idealized stress – strain curve for steel (Fig. 23A & B, Page 70)

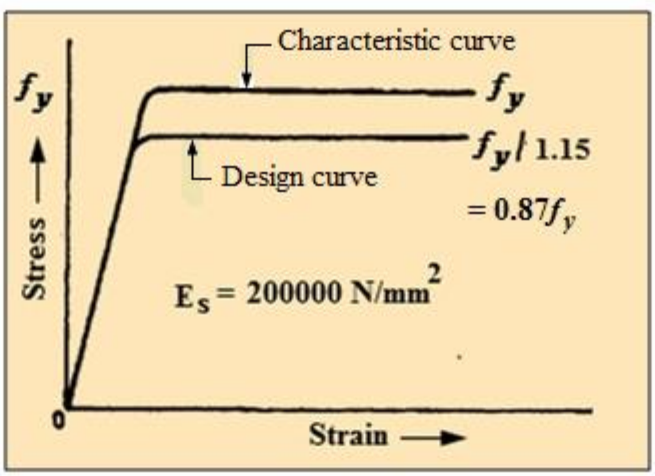


Fig. Steel bar with definite yield point

Idealized stress-strain curve of mild steel and cold-worked deformed bars (Fe 415 and Fe 500) as recommended by IS 456: 2000 is reproduced in Fig. xxxxx. The modulus of elasticity of all types of reinforcing steel, *E* I taken as 2 x 10^5 N/mm^2. The stress-strain relationship for steel in tension and compression is assumed to be the same.

For mild steel the stress is proportional to the strain upto yield point and thereafter the strain increases at a constant stress of f_y. The design yield stress of steel is equal to f_y/γ_m. With $\gamma_m = 1.15$, the design yield stress becomes $0.87 f_y$.

For cold-worked bars (Fe 415 and Fe 500 HYSD bars), there is no definite yield point; hence yield stress is taken as 0.2% proof stress. The stress-strain curve is linear upto a stress of $0.8 f_y$ and the strains are elastic. Thereafter the stress-strain curves are defined as given below.

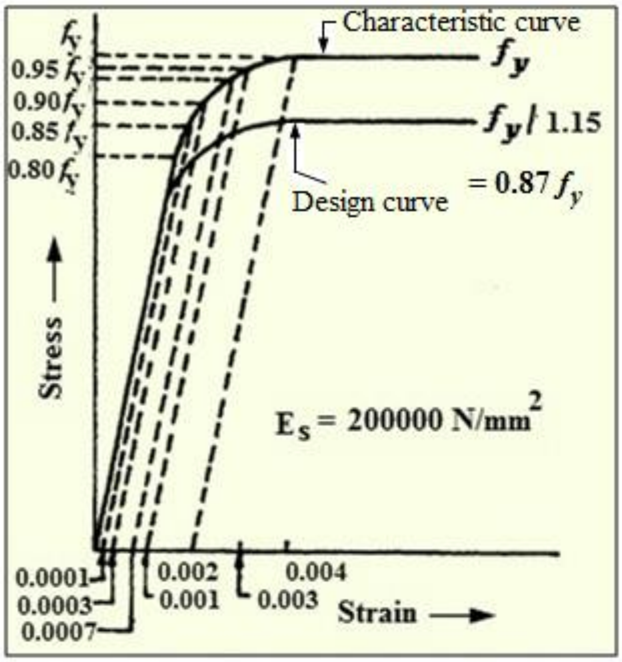


Fig. Cold worked deformed bar

Stress	**Inelastic strain**	**Stress**	**Inelastic strain**
$0.8 f_y$	Nil	$0.95 f_y$	0.0007
$0.85 f_y$	0.0001	$0.975 f_y$	0.001
$0.9 f_y$	0.0003	$1.0 f_y$	0.002

To find out the stress for a given strain value, a stress-strain graph shall be drawn using the values in the table. For plotting the graph, stress-strain relation may be assumed to be linear between above defined points.

Example: For Fe 415 grade steel,

At 0.8f_y, the strain will be $\frac{0.8 f_y}{E} = \frac{0.8 \times 415}{2 \times 10^5} = 0.00166 (Elastic\ strain)$.

At 0.85f_y, the strain will be $= \frac{0.85 \times 415}{2 \times 10^5} (elastic) + 0.0001 (inelastic) = 0.00186$

For the given strain, now the stress can be found from the graph.

The design stress-strain curve can be similarly obtained by substituting f_{yd} for f_y.

For Fe 415 grade of steel,

Stress level 0.85f_{yd}, Design stress $= \frac{0.85 f_y}{\gamma_m} = \frac{0.85 \times 415}{1.15} = 306.7\ N/mm^2$ and

$$\text{Strain} = \frac{306.7}{2 \times 10^5} (elastic) + 0.0001 (inelastic) = 0.00163$$

Stress level 0.9f_{yd}, Design stress $= \frac{0.9 f_y}{\gamma_m} = \frac{0.9 \times 415}{1.15} = 324.8\ N/mm^2$ and

$$\text{Strain} = \frac{324.8}{2 \times 10^5} (elastic) + 0.0003 (inelastic) = 0.00192$$

The salient points of design stress-strain curve for two types of cold-worked steel (Fe 415 and Fe 500) are given in table.

Stress level	**Fe 415 steel:** $f_y = 415\ N/mm^2$		**Fe 500 steel:** $f_y = 500\ N/mm^2$	
	Strain	Stress (N/mm^2)	Strain	Stress (N/mm^2)
$0.8 f_{yd}$	0.00144	288.7	0.00174	347.8
$0.85 f_{yd}$	0.00163	306.7	0.00195	369.6
$0.9 f_{yd}$	0.00192	324.8	0.00226	391.3
$0.95 f_{yd}$	0.00241	342.8	0.00277	413.0
$0.975 f_{yd}$	0.00276	351.8	0.00312	423.9
$1.0 f_{yd}$	0.00380	360.9	0.00417	434.8

Limit State of Collapse: Flexure

Assumptions:

Design for the limit state of collapse in flexure shall be based on the assumptions given *below:*

a) Plane sections normal to the axis remain plane after bending.

b) The maximum strain in concrete at the outermost compression fibr is taken as 0.0035 in bending.

c) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given in Fig. 21 **(Cl. 38.1, Page 69)**. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67times the characteristic strength. The partial safety factor γ = 1.5 shall be applied in addition to this.

d) The tensile strength of the concrete is ignored.

e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typica1 curves are given in Fig. 23 **(Cl. 38.1, Page 70)**. For design purposes the partial safety factor γ_m equal to 1.15 shall be applied.

f) The maximum strain in the tension reinforcement in the section at failure shall not be less than:

$$\frac{f_y}{1.15E_s}+0.002$$

where $f_y =$ Characteristic strength of steel, and

$E_s =$ Modulus of elasticity of steel

This assumption is intended to ensure ductile failure, that is, the tensile reinforcement has to undergo a certain degree of inelastic deformation before the concrete fails in compression.

Stress block parameters and depth of neutral axis:

IS 456: 2000 recommends the flexural compressive strength of concrete equal to 0.67f_{ck} with a partial factor of safety γ_m = 1.5 applied over it. The corresponding stress-strain diagram is shown in the Fig. xx.

Fig. yyy shows the single reinforced beam section, strain variation and stress variation across the depth of the section.

Strain in the concrete is proportional to the distance from neutral axis.

$$\frac{OC}{OD}=\frac{x_1}{x_u}=\frac{0.002}{0.0035}$$

Depth of parabolic portion, $OC = x_1 = \frac{4}{7} x_u$

$$CD = x_2 = OC - OD = x_u - \frac{4}{7} x_u$$

Depth of rectangular portion, $x_1 = \frac{3}{7} x_u$

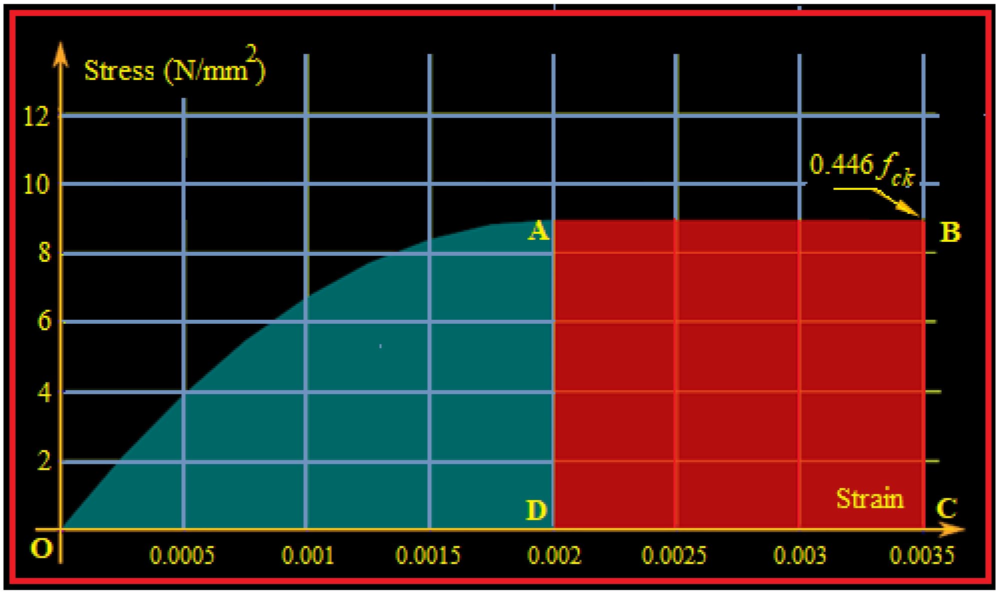


Fig. Design stress-strain curve for concrete

Area of rectangular portion, ABCD of the stress block,

$$A_1 = \frac{3}{7} x_u \times 0.446 f_{ck} = 0.19 f_{ck} x_u$$

Area of parabolic portion, OAD of the stress block,

$$A_2 = \frac{2}{3} \times \frac{4}{7} x_u \times 0.446 f_{ck} = 0.17 f_{ck} x_u$$

Total area of the stress block, $A = 0.17 f_{ck} x_u + 0.19 f_{ck} x_u$

$$= 0.36 f_{ck} x_u$$

Centroid of the stress block from extreme compression fibre,

$$\bar{y} = \frac{A_1 x_1 + A_2 x_2}{A}$$

$$= \frac{0.19 f_{ck} x_u \times \frac{1}{2}\left(\frac{3}{7}x_u\right) + 0.17 x_u\left(\frac{3}{7}x_u + \frac{3}{8}\times\frac{4}{7}x_u\right)}{0.36 f_{ck} x_u}$$

$$= \frac{2.1x_u}{0.36} = 0.42x_u$$

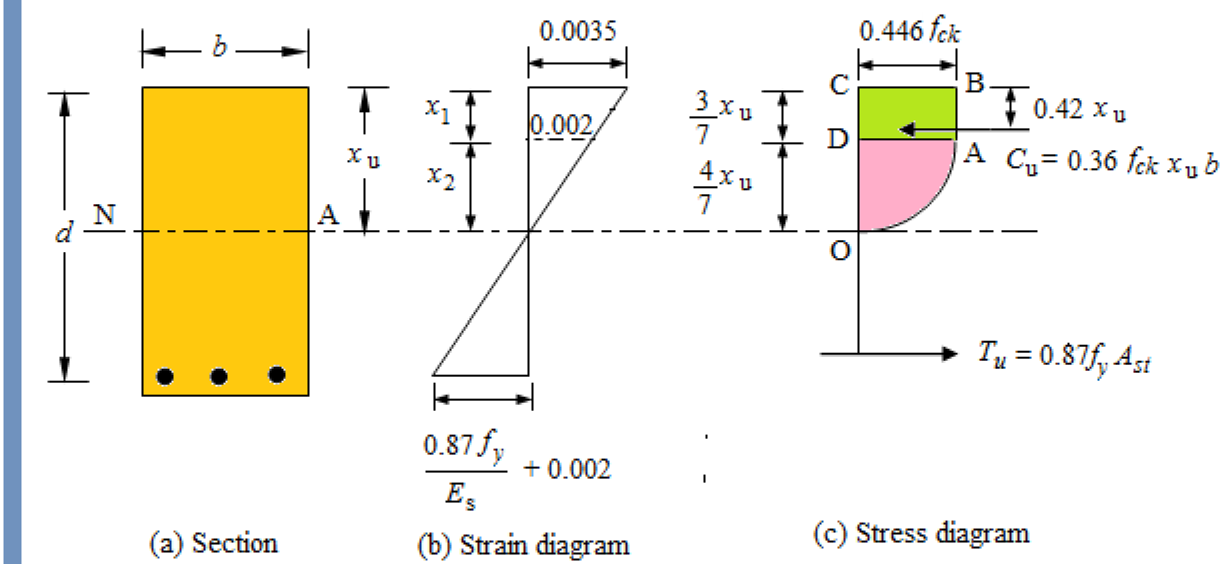


Fig. stres-strain variation across section

Provision of IS 56: 2000 requires that maximum strain in steel should not be less than $\frac{0.87}{E_s} + 0.002$. *This limits depth of neutral axis.*

From the strain diagram,

$$\frac{x_{u,\max}}{d - x_{u,\max}} = \frac{\varepsilon_{cu}}{\varepsilon_{su}}$$

$$\frac{x_{u,\max}}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{su}} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002}$$

$$\frac{x_{u,\max}}{d} = \frac{0.0035E_s}{0.0055E_s + 0.87 f_y}$$

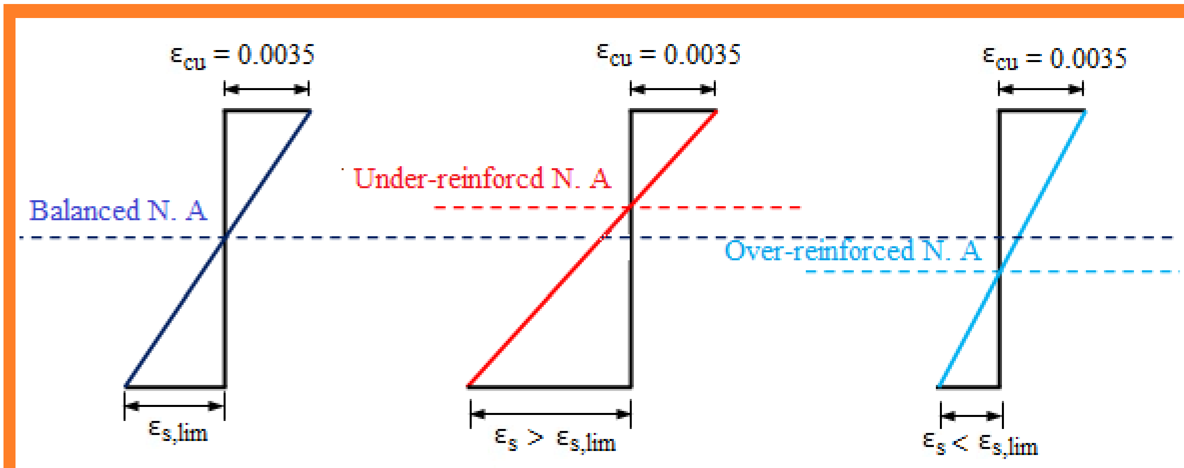
Substituting the value of $E_s = 2\times10^5\ N/mm^2$, we get $$\frac{x_{u,\max}}{d} = \frac{700}{1100 + 0.87 f_y}$$

The values of limiting depth of neutral axis for various grades of steel are as given in the table.

Grade of steel	Fe 250	Fe 415	Fe 500	Fe 550
Strain at failure (ε_{su})	0.0031	0.0038	0.042	0.0044
$\frac{x_{u,\max}}{d}$	0.53	0.48	0.46	0.44

Modes of failure and types of section:

Based on extensive test results, maximum strain criteria have been accepted as the failure criteria for a reinforced concrete member in flexure. A reinforced concrete member is assumed to have failed if the strain, ε_c in extreme compression fibre reaches ultimate value of $\varepsilon_{cu} (= 0.0035)$. At this stage, the actual strain (ε_s) in steel can have three possible values which decides the nature of the section.



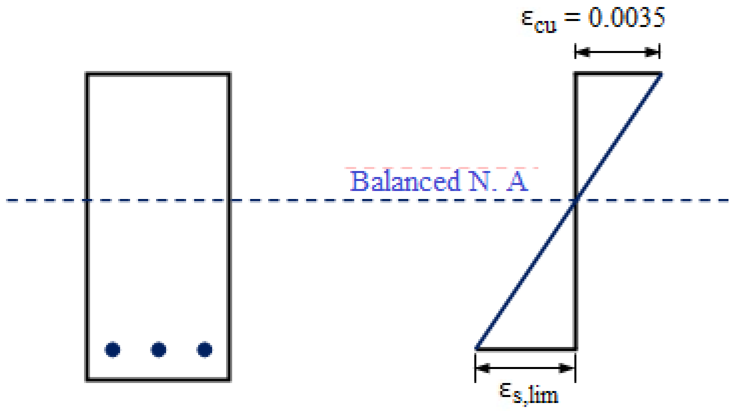
(a) Balaanced section (b) Under-reinforced section (c) Over-reinforced section

Fig. Strain diagrams

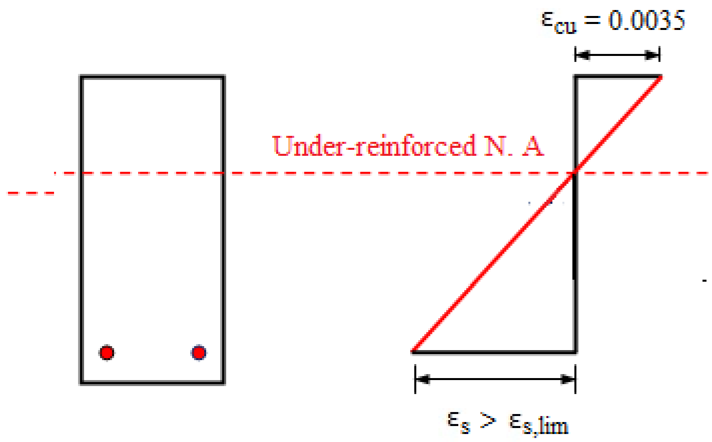
a) *Balanced section*: Actual strain ε_s in steel is equal to failure strain or ultimate strain ε_{su} of steel, i.e., $\varepsilon_s = \varepsilon_{s,\lim}$.

b) *Under-reinforced section*: Actual strain ε_s in steel is more than failure strain or ultimate strain ε_{su} of steel, i.e., $\varepsilon_s > \varepsilon_{s,\lim}$.

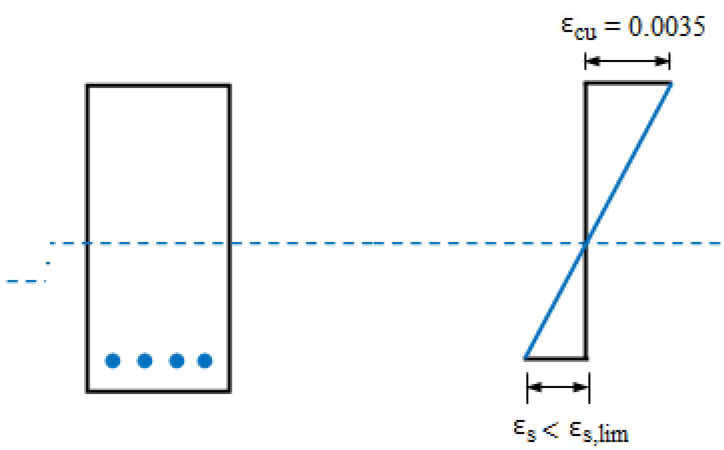
c) Over-reinforced section: Actual strain ε_s in steel is less than failure strain or ultimate strain ε_{su} of steel, i.e., $\varepsilon_s < \varepsilon_{s,\lim}$.



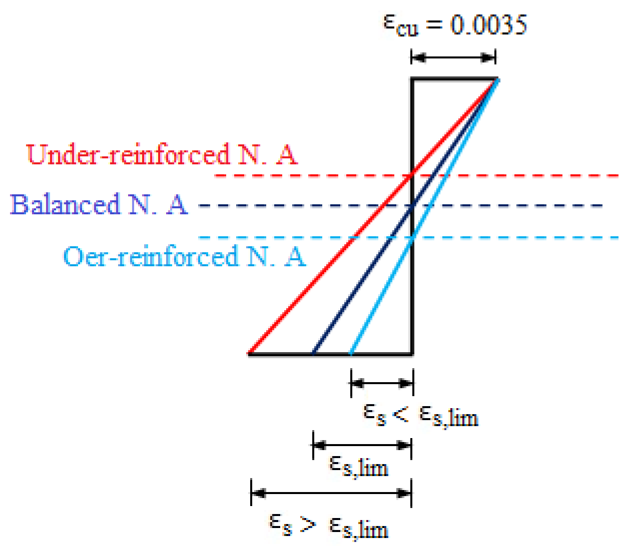
(a) Balaanced section



(b) Under-reinforced section



(c) Over-reinforced section



Singly reinforced rectangular section

A singly reinforced rectangular beam section with strain diagram and stress diagram are shown in the Figure . The formulae are derived by the assumptions as in limit state method.

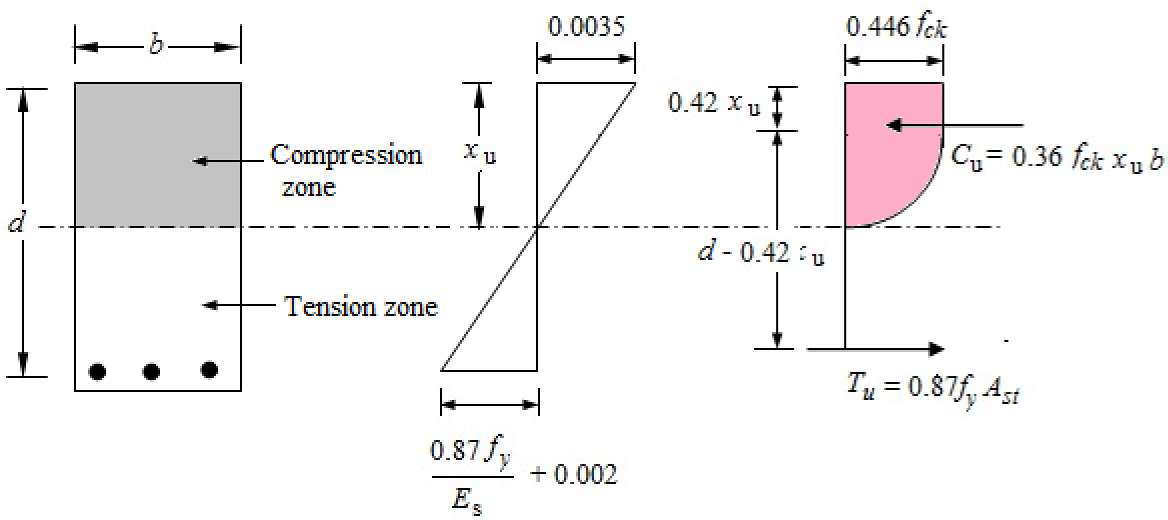


Fig. Singly reinforced beam

Neutral axis:

Total compressive force on the section, $C_u = 0.36 f_{ck} x_u b$

Total tensile force on the section, $T_u = 0.87 f_y A_{st}$

For equilibrium of the section, $C_u = T_u$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \qquad (1)$$

The value of the depth of neutral axis, x_u as obtained from the above equation should not exceed $x_{u,\max}$ for a given section. If $x_u > x_{u,\max}$, the depth of neutral axis shall be taken as $x_{u,\max}$.

Lever arm, $z = d - 0.42 x_u$

Moment of resistance:

1) For balanced section,

Moment of resistance = total compression x lever arm

= total tension x lever arm

Considering compressive force (under-reinforced and balanced section),

$$M_u = 0.36 f_{ck} x_u b (d - 0.42 x_u)$$

$$M_u = 0.36 f_{ck} \frac{x_u}{d}\left(1 - 0.42 \frac{x_u}{d}\right) b d^2 \qquad (2)$$

For limiting value of neutral axis, substituting $x_{u,\max}$ for x_u and $M_{u,\lim}$ for $M_{u,\lim}$,

we have
$$M_{u,\lim} = 0.36 f_{ck} \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) b d^2$$

$$= Q_{\lim} b d^2$$

where

$$Q_{\lim} = \frac{M_{u,\lim}}{b d^2}$$

$$= 0.36 f_{ck} \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) \qquad (3)$$

$Q_{\lim}$ is known as moment of resistance factor for balanced rectangular section.

Limiting moment of resistance factor for various grades of material

$f_{ck}, N/mm^2$	$f_y, N/mm^2$			
	250	415	500	550
20	**2.96**	**2.76**	2.66	2.58
25	3.70	3.45	3.33	3.23
30	4.44	4.14	3.99	3.87

2) For under-reinforced section

Moment of resistance, $M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$

Substituting the value of $x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

$$M_u = 0.87 f_y A_{st} \left(d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b} \right)$$

$$= 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} b d} \right) \qquad (4)$$

Dividing both sides by $0.87 f_y b d^2$

$$\frac{M_u}{0.87 f_y b d^2} = \frac{A_{st}}{bd} \left(1 - \frac{f_y A_{st}}{f_{ck} b d} \right)$$

Substituting $A_{st} = \dfrac{p_t b d}{100}$, $\dfrac{M_u}{0.87 f_y b d^2} = \left(\dfrac{p_t}{100} \right) - \dfrac{f_y}{f_{ck}} \left(\dfrac{p_t}{100} \right)^2$

$$\frac{f_y}{f_{ck}} \left(\frac{p_t}{100} \right)^2 - \left(\frac{p_t}{100} \right) - \frac{M_y}{0.87 f_y b d^2} = 0$$

which on solving gives

$$\frac{p_t}{100} = \frac{1 - \sqrt{1 - \dfrac{4.6}{f_{ck}} \times \dfrac{M_u}{bd^2}}}{2 \dfrac{f_y}{f_{ck}}}$$

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}\times\frac{M_u}{bd^2}}}{f_y/f_{ck}}\right] \qquad (5)$$

The positive value of the root of the equation will give larger area of steel than $p_{t,\lim}$ and hence not considered.

From equation (5), for given values of f_y and f_{ck} and for different values of $\frac{M_u}{bd^2}$, the reinforcement percentage p_t can be plotted and tabulated. SP:16 contains chart 1 to 18 and tables 1 to 4 based on the above equation.

Limiting reinforcement

Substituting $\frac{M_{u,\lim}}{bd^2}$ for $\frac{M_u}{bd^2}$ in equation (5)

For M20 grade of concrete and Fe 415 grade steel

$$Q_{\lim} = \frac{M_{u,\lim}}{bd^2} = 2.76$$

Hence
$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 2.76}}{415/20}\right] = 0.96$$

For different grades of concrete and steel, the values of $p_{t,\lim}$ are tabulated as

Limiting Percentage Of Reinforcement $p_{t,lim}$ For Singly Reinforced Rectangular Section

Grade of concrete $f_{ck}, N/mm^2$	**Grade of steel** $f_y, N/mm^2$			
	Fe 250	Fe 415	Fe 500	Fe 550
M15	1.32	0.72	0.57	0.50
M20	**1.75**	**0.96**	0.76	0.66
M25	2.19	1.20	0.95	0.83
M30	2.63	1.44	1.14	0.99

Types of problems:

There are three types of problems in singly reinforced beams.

Type I: Determination of moment of resistance of a section.

The moment of resistance of a singly reinforced rectangular section is determined as follows.

1) Determine the depth of neutral axis from the following equation.

$$\frac{x_u}{d} = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b\, d^2}$$

2) Compare this with the limiting depth of neutral axis, $x_{u,\max}$ determined from the following equation.

$$\frac{x_{u,\max}}{d} = \frac{700}{1100 + 0.87\, f_y}$$

If $\frac{x_u}{d} < \frac{x_{u,\max}}{d}$, the section is under-reinforced. The moment of resistance is determined from any of the following formulae.

$$M_u = 0.36\, f_{ck}\, \frac{x_u}{d}\left(1 - 0.42\frac{x_u}{d}\right) b\, d^2$$

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1 - \frac{f_y\, A_{st}}{f_{ck}\, b\, d}\right)$$

$$M_u = 0.36\, f_{ck}\, x_u\, b\left(d - 0.42 x_u\right)$$

If $\frac{x_u}{d} = \frac{x_{u,\max}}{d}$, the section is balanced. The moment of resistance is determined from the following formula.

$$M_{u,\lim} = 0.36\; f_{ck}\, \frac{x_{u,\max}}{d}\left(1 - 0.42\frac{x_{u,\max}}{d}\right) b\, d^2$$

If $\frac{x_u}{d} > \frac{x_{u,\max}}{d}$, the section is over-reinforced. The moment of resistance of such section is limited to $M_{u,\lim}$ since code has prohibited the use of over-reinforced section.

Type II: Determination of actual stresses in a given section subjected to a given B. M.

This problem will be dealt with a numerical later with example for better clarity of understanding.

Type III: Design a section to resist a given ultimate design moment.

Design of section requires determination of

1. dimension of beam, i.e., width b and effective depth d, and
2. area of steel so as to develop a given moment of resistance.

Ideally, the objective of the designer is to design a balanced section so that the ultimate stresses in both the materials are developed simultaneously. However such a design may not be economical. Practical considerations may demand to fix some uniform cross-sectional dimensions. In such situation, the design may result in a singly reinforced balanced section, under-reinforced section or doubly reinforced section. However, if the section has to be singly reinforced, an under-reinforced section is always desirable in limit state method.

The following steps may be followed for the design of beam section.

1. The width is generally decided by the functional architectural requirement of the building.
2. The depth is then determined by considering the beam as balanced section.

$$d_{bal} = \sqrt{\frac{M_u}{Q_{\lim} b}}$$

Design of a balanced beam with this depth is hardly possible since it requires a fixed percentage of reinforcement to be provided for perfectly balanced section. If more area of steel is provided, the section becomes over-reinforced. Therefore under-reinforced section with larger depth remains the only option.

3. Assume 5% increase in effective depth for $d \le 500\,mm$ (Case 1) and 10% increase in depth for $500 \le d \le 1000\,mm$ (Case 2).
4. Assume one layer of 20 mm diameter bars for Case 1 and two layers of 20 mm diameter bars for Case 2.
5. If the clear cover to the reinforcement is 30 mm,

   effective cover = 30 + 10 = 40 mm (Case1)

   = 30 + 20 + 10 = 60 mm (Case2)
6. The overall depth can be obtained as

$$D = d + 40\,mm \;\; (Case1)$$
$$= d + 60\,mm \;\; (Case2)$$

7. The value so obtained shall be rounded upto nearest 25 mm.
8. The revised effective is determined as

$$d = D - 40\,mm \;\; (Case1)$$
$$= D - 60\,mm \;\; (Case2)$$

9. Determine $\frac{M_u}{bd^2}$, p_t and A_{st} using the relevant equations for under reinforced section.

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}\times\frac{M_u}{bd^2}}}{f_y/f_{ck}}\right] \quad \text{or}$$

$$M_u = 0.87 f_y A_{st}(d - 0.42\,x_u)$$

Also determine the $A_{st,\lim}$.

10. Select the suitable diameter of bar and numbers such that $A_{st} > A_{st,reqd}$ and also $A_{st} < A_{st,\lim}$.

Type IV: Given factored moment, M_u and dimension of section (b x d), determine the area of reinforcement

The following steps may be followed for the design of beam section.

1. Determine the limiting moment of resistance

$$M_{u,\lim} = 0.36\, f_{ck}\frac{x_{u,\max}}{d}\left(1-0.42\frac{x_{u,\max}}{d}\right)bd^2$$

There are three possibilities.

a) If $M_u = M_{u,\lim}$, it will result a balanced section. The area of steel is found by the following ways.

$$M_{u,\lim} = 0.87 f_y A_{st}\, d\left(1-0.42\frac{x_{u,\lim}}{d}\right) \quad \text{or}$$

$$p_{t,\lim} = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}\times\frac{M_{u,\lim}}{bd^2}}}{f_y/f_{ck}}\right]$$

b) If $M_u > M_{u,\lim}$, it will result in a over-reinforced section. Since, design of over-reinforced section is prohibited by code; a double reinforced section shall be designed.

c) If $M_u < M_{u,\lim}$, it will result in an under-reinforced section.

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} b d}\right) \quad \text{Or}$$

$$p_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{bd^2}}}{f_y / f_{ck}} \right] \quad \text{Or}$$

$$A_{st} = 0.5 \left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{bd^2}}}{f_y / f_{ck}} \right] bd$$

Numerical

Problem 1.

A rectangular beam of 230 mm wide and 520 mm effective depth is reinforced with 4 -16 mm diameter bars. Find out the depth of neutral axis and specify the type of beam. The materials are M20 grade of concrete and HYSD reinforcement of grade Fe 415. Also find out the depth of neutral axis if the reinforcement is increased to 4-20 mm diameter bars.

Solution:

Case I: $A_{st} = 4 \times 201 = 804\, mm^2$

Total compression $= 0.36 f_{ck}\, b\, x_u = 0.36 \times 20 \times 230\, x_u = 1656\, x_u$

Total tension $= 0.87 f_{ck}\, A_{st} = 0.87 \times 415 \times 804 = 29284\, N$

Equating $1656\, x_u = 290284$

$x_u = 175.3\, mm$

Limiting value of neutral axis, $x_{u,\max} = 0.48\, d = 0.48 \times 520 = 250\, mm$

$x_u < x_{u,\max}$

Section is under-reinforced.

Case II: $A_{st} = 4 \times 314 = 1256\, mm^2$

Total compression $= 0.36 f_{ck}\, b\, x_u = 0.36 \times 20 \times 230\, x_u = 1656\, x_u$

Total tension $= 0.87 f_{ck}\, A_{st} = 0.87 \times 415 \times 1256 = 453479\, N$

Equating $1656\, x_u = 453479$

$x_u = 273.8\, mm$

Limiting value of neutral axis, $x_{u,\max} = 0.48\, d = 0.48 \times 520 = 250\, mm$

$x_u > x_{u,\max}$

Section is over-reinforced.

Depth of neutral axis, $x_u = x_{u,\max} = 250\, mm$

Problem 2.

A singly reinforced rectangular beam of width 230 mm wide and 460 mm effective depth is reinforced with 3-20 mm diameter bars. Find out the factored moment of resistance of the section. The materials are M20 grade of concrete and HYSD reinforcement of grade Fe 415. Also find out the factored moment of resistance of the section if the reinforcement is increased to 5-20 mm diameter bars.

Solution:

Case I: $A_{st} = 3 \times 314 = 942\, mm^2$

Total compression $= 0.36\, f_{ck}\, b\, x_u = 0.36 \times 20 \times 230\, x_u = 1656\, x_u$

Total tension $= 0.87\, f_{ck}\, A_{st} = 0.87 \times 415 \times 942 = 340109\, N$

Equating $1656\, x_u = 340109$

$x_u = 205.4\, mm$

Limiting value of neutral axis, $x_{u,\max} = 0.48\, d = 0.48 \times 460 = 220.8\, mm$

$x_u < x_{u,\max}$

The section is under-reinforced and $x_u = 205.4\, mm$

Moment of resistance, $M_u = 0.87\, f_y\, A_{st}\left(d - 0.42\, x_u\right)$

$= 0.87 \times 415 \times 942\left(460 - 0.42 \times 205.4\right) \times 10^{-6}$

$= 127.12\ kN$

Alternatively, $M_u = 0.36\, f_{ck}\, b\, x_u\left(d - 0.42\, x_u\right)$

$= 0.36 \times 20 \times 230 \times 205.4\left(460 - 0.42 \times 205.4\right) \times 10^{-6}$

$= 127.12\ kN$

Case II: $A_{st} = 4 \times 314 = 1256\, mm^2$

Total compression $= 0.36\, f_{ck}\, b\, x_u = 0.36 \times 20 \times 230\, x_u = 1656\, x_u$

Total tension $= 0.87\, f_{ck}\, A_{st} = 0.87 \times 415 \times 1256 = 453479\, N$

Equating $1656\, x_u = 453479$

$x_u = 273.8\, mm$

Limiting value of neutral axis, $x_{u,\max} = 0.48\,d = 0.48 \times 460 = 220.8\,mm$

$$x_u > x_{u,\max}$$

The section is over-reinforced and $x_u = x_{u,\max} = 220.8\,mm$

Moment of resistance, $M_u = 0.36\,f_{ck}\,b\,x_u\,(d - 0.42\,x_u)$

$$= 0.36 \times 20 \times 230 \times 220.8(460 - 0.42 \times 220.8) \times 10^{-6}$$

$$= 134.29\ kN.m$$

The actual stress in the steel, f_{st}, when the section is subjected to $M_{u,\lim}$ is determined by equating the total tension and total compression.

$$A_{st}\ f_{st} = 1656\,x_u$$

$$1256\ f_{st} = 1656 \times 220.8$$

$$f_{st} = \frac{1656 \times 220.8}{1256} = 291.1\,N/mm^2$$

as against $0.87\ f_y = 0.87 \times 415 = 361\,N/mm^2$

Problem 3.

A singly reinforced rectangular beam is subjected to a bending moment of 36 kN-m at working loads. The width of the beam is 200 mm. Find the depth and steel area for balanced section. The materials are M20 grade of concrete and mild steel reinforcement.

Solution:

Factored moment M_u = 1.5 x working moment = 15 x 36 = 54 kN.m

For M20 grade of concrete and Fe 250, $Q_{\lim} = 2.96$

$$d = \sqrt{\frac{M_{u,\lim}}{Q_{\lim}\,b}} = \sqrt{\frac{54 \times 10^6}{2.96 \times 200}} = 302\,mm$$

For area of steel,

$$M_{u,\lim} = 0.87\ f_y\ A_{st}\ d\left(1 - 0.42\frac{x_{u,\lim}}{d}\right)$$

$$54 \times 10^6 = 0.87 \times 250\,A_{st} \times 302(1 - 0.42 \times 0.53)$$

$$54 \times 10^6 = 57995\,A_{st}$$

$$A_{st} = 1057\,mm^2$$

Problem 4.

Design a singly reinforced rectangular beam subjected to an applied factored moment of 120 kN.m. Assume the width of section as 230 mm. The materials are M20 grade of concrete and HYSD reinforcement of grade Fe 415.

Solution:

$M_u = 120\,kN.m$	$f_y = 415\,N/mm^2$
$b = 230\,mm$	$Q_{\lim} = 2.76$
$f_{ck} = 20\,N/mm^2$	$p_{\lim} = 0.96$

Balanced depth of neutral axis, $$d = \sqrt{\frac{M_{u,\lim}}{Q_{\lim} b}} = \sqrt{\frac{120\times 10^6}{2.76\times 230}} = 434.8\,mm$$

Increase the depth by 5% and add 40 mm effective cover.

$$D = 1.05\times 434.8 + 40 = 496.5\,mm$$

Taking D as nearest multiple of 25 mm, D = 500 mm

$$d = 500 - 30\,(\text{clear cover}) - 10\,(\text{upto centroid of reinforcement})$$

$$= 460\,mm$$

$$\frac{M_u}{bd^2} = \frac{120\times 10^6}{230\times 460^2} = 2.47$$

$$A_{st} = 0.5\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}\times\frac{M_u}{bd^2}}}{f_y/f_{ck}}\right]bd$$

$$A_{st} = 0.5\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 2.47}}{415/20}\right]230\times 460 = 873.96\,mm^2$$

Alternatively, $$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}\times\frac{M_u}{bd^2}}}{f_y/f_{ck}}\right] = 0.826$$

$$A_{st} = \left(\frac{p_t}{100}\right)b\,d = \frac{0.826}{100}\times 230\times 460 = 873.91\,mm^2$$

$$A_{st,\lim} = \left(\frac{p_{t,\lim}}{100}\right)b\,d = \frac{0.96}{100}\times 230\times 460 = 1015.68\,mm^2$$

Provide 3-20 mm # = 3 x 314 = 942 mm^2.

Thus, $$A_{st,reqd} < A_{st,provided} < A_{st,\lim}$$

Dimension of designed beam $b\times D = 230\,mm\times 500\,mm$

$$A_{st} = 3-20\,mm\# = 942\,mm^2$$

Problem 5.

A rectangular cantilever beam of size 230 mm width and 500 mm effective depth is subjected to a bending moment of 80 kNm at working loads. Find the steel area required. The materials are M20 grade of concrete and HYSD reinforcement of grade Fe 415.

Solution:

$M_w = 120\,kN.m$	$f_y = 415\,N/mm^2$
$b = 230\,mm$	$Q_{\lim} = 2.76$
$f_{ck} = 20\,N/mm^2$	$p_{t,\lim} = 0.96$

Factored moment, $M_u = 1.5\times 80 = 120\,kN.m$

$$M_{u,\lim} = Q_{\lim}\,b\,d^2 = 2.76\times 230\times 500^2\times 10^{-6}$$

$$= 158.7\,kN.m$$

$$M_u < M_{u,\lim}$$

The beam will be designed as under-reinforced.

Area of steel

$$\frac{M_u}{bd^2} = \frac{120 \times 10^6}{230 \times 500^2} = 2.09$$

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}} \times \frac{M_u}{bd^2}}}{f_y / f_{ck}}\right]$$

$$= 50\left[\frac{1-\sqrt{1-\frac{4.6}{20} \times 2.09}}{415/20}\right] = 0.673$$

$$A_{st} = \left(\frac{p_t}{100}\right) b\,d = \frac{0.673}{100} \times 230 \times 500 = 773.95\, mm^2$$

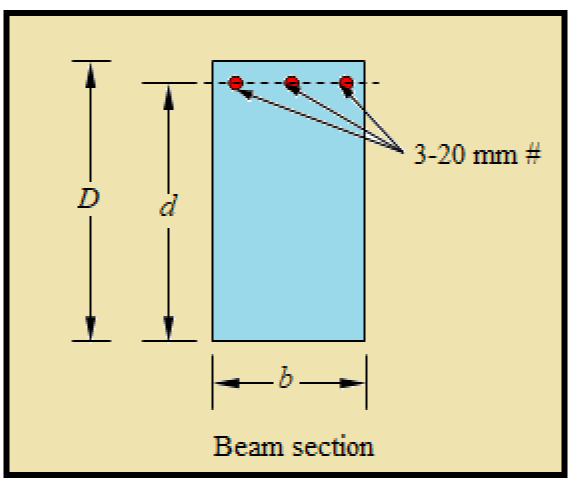
$$A_{st,\lim} = \left(\frac{p_{t,\lim}}{100}\right) b\,d = \frac{0.96}{100} \times 230 \times 500 = 1104\, mm^2$$

Provide 3-20 mm # = 3 x 314 = 942 mm^2

Thus, $A_{st,reqd} < A_{st,provided} < A_{st,\lim}$

Dimension of designed beam $b \times d = 230\, mm \times 500\, mm$

$$A_{st} = 3 - 20\, mm\# = 942\, mm^2$$



Problem 6.

A beam of 230 mm width and 400 mm effective depth is reinforced with 3-16 mm ϕ mild steel bars in tension. Compare both flexural capacities M_u and the strain in steel when (a) M20 mix is used and (b) M25 mix is used. Comment on the calculations.

Solution:

$3-16\,mm\ \varphi$	$f_{ck} = 20\,N/mm^2$
$b = 230\,mm$	$f_{ck} = 25\,N/mm^2$
$d = 400\,mm$	$f_y = 250\,N/mm^2$

Case I:

Area of steel, $A_{st} = 3 \times 201 = 603\,mm^2$

Depth of neutral axis $0.36\,f_{ck}\,x_u\,b = 0.87\,f_y\,A_{st}$

$$0.36 \times 20 \times 230\,x_u = 0.87 \times 250 \times 603$$

$$1656 x_u = 131152.5$$

$$x_u = 79.2\,mm$$

Limiting depth of neutral axis, $x_{u,\max} = 0.53 \times d = 0.53 \times 400 = 212\,mm$

Since, $x_u < x_{u,\max}$, the section is under-reinforced.

Flexural capacity, $M_u = 0.87\,f_y\,A_{st}\left(d - 0.42\,x_u\right)$

$$= 0.87 \times 250 \times 603\left(400 - 0.42 \times 79.2\right) \times 10^{-6}$$

$$= 48.1\,kN.m$$

Strain in concrete = 0.0035

Strain in steel, $\varepsilon_s = \varepsilon_{cu} \times \dfrac{\left(d - x_u\right)}{x_u} = 0.0035 \times \dfrac{\left(400 - 79.2\right)}{79.2}$

$$= 0.01418$$

Case II:

Area of steel, $A_{st} = 3 \times 201 = 603\,mm^2$

Depth of neutral axis $0.36\,f_{ck}\,x_u\,b = 0.87\,f_y\,A_{st}$

$$0.36 \times 25 \times 230\,x_u = 0.87 \times 250 \times 603$$

$$2070x_u = 131152.5$$

$$x_u = 63.4\,mm$$

Limiting depth of neutral axis, $x_{u,\max} = 0.53 \times d = 0.53 \times 400 = 212\,mm$

Since, $x_u < x_{u,\max}$, the section is under-reinforced.

Flexural capacity, $M_u = 0.87 f_y A_{st} \left(d - 0.42 x_u\right)$

$$= 0.87 \times 250 \times 603\left(400 - 0.42 \times 63.4\right) \times 10^{-6}$$

$$= 48.97\,kN.m$$

Strain in concrete = 0.0035

Strain in steel, $\varepsilon_s = \varepsilon_{cu} \times \dfrac{\left(d - x_u\right)}{x_u} = 0.0035 \times \dfrac{\left(400 - 63.4\right)}{63.4}$

$$= 0.01858$$

Comment:

Increase in the moment of resistance due to increase in the grade of concrete,

$$= \frac{48.97 - 48.1}{48.1} \times 100 = 1.808\%$$

Increase in moment of resistance due to increase in the grade of concrete is insignificant. However, the strain in steel has increased considerably. This results in more ductile section, which results more deflection before failure.

Problem 7.

A singly reinforced beam of 5 m span is simply supported and carries a characteristic dead load of 18 kN/m and live load of 12 kN/m in addition to its self weight. Design the beam for flexure at mid span. Also, find out the section where half the reinforcement can be curtailed theoretically. The materials are M20 grade concrete and CTD bars of grade Fe 415.

Solution:

$Span, l = 5\,m$	$f_y = 415\,N/mm^2$
$f_{ck} = 20\,N/mm^2$	$Q_{\lim} = 2.76$

Assume the width of the beam, b = 230 mm.

Depth may be assumed to be $\frac{1}{8}$ to $\frac{1}{10}$ of span, say D = 600 mm.

Load calculation:

Dead load = 18 kN/m

Live load = 12 kN/m

Self weight = $0.23 \times 0.6 \times 25 = 4\,kN/m$

Total load = 34 kN/m

Maximum bending moment, $M_{\max} = \dfrac{34 \times 5^2}{8} = 106.25\,kN.m$

Factored moment, $M_u = 1.5 \times 106.25 = 159.375\,kN.m$

Assuming one layer of 20 mm # bars

$$d = D - 30\,(\text{cover}) - \frac{\phi}{2}$$

$$= 600 - 30 - \frac{20}{2} = 560\,mm$$

$$\frac{M_u}{bd^2} = \frac{159.375 \times 10^6}{230 \times 560^2} = 2.21 < 2.76$$

The section is under-reinforced.

Area of steel, $A_{st} = 0.5\left[\dfrac{1 - \sqrt{1 - \dfrac{4.6}{f_{ck}} \times \dfrac{M_u}{bd^2}}}{f_y / f_{ck}}\right] bd$

$$= 0.5\left[\frac{1 - \sqrt{1 - \dfrac{4.6}{20} \times 2.21}}{415/20}\right] 230 \times 560$$

$$= 927.32\,mm^2$$

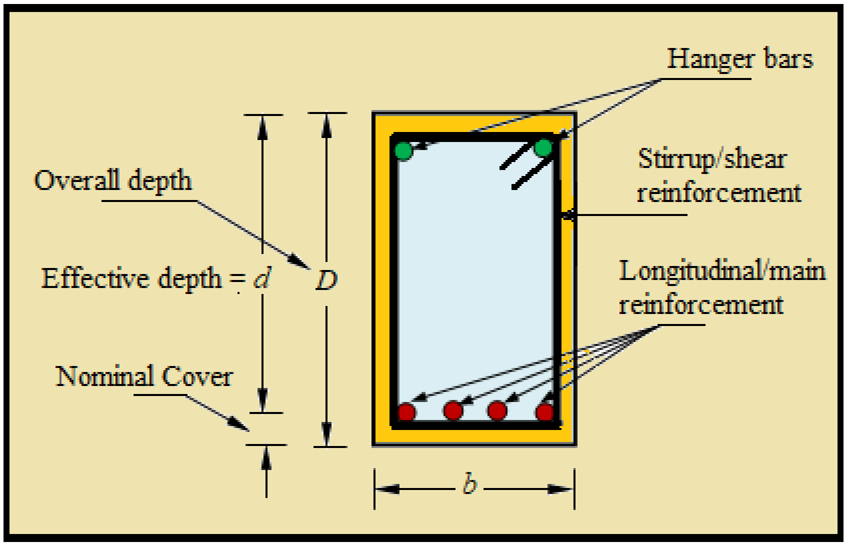
Limiting percentage of steel, $p_{t,\lim} = 50\left[\dfrac{1 - \sqrt{1 - \dfrac{4.6}{f_{ck}} \times Q_{\lim}}}{f_y / f_{ck}}\right]$

$$= 50\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 2.76}}{415/20}\right] = 0.96$$

$$A_{st,\lim} = \left(\frac{p_{t,\lim}}{100}\right) b\,d = \frac{0.96}{100}\times 230\times 560 = 1236.48\,mm^2$$

Provide 2-20 mm # + 2-16 mm # = 2(314+201) = 1030 mm^2

Thus, $A_{st,reqd} < A_{st,provided} < A_{st,\lim}$ (Ok)



Half the bars are required to be curtailed.

$$A_{st}\ (remaining) = \frac{1030}{2} = 515\,mm^2$$

$$p_t = \frac{100\times A_{st,remaining}}{b\,d} = \frac{100\times 515}{230\times 560} = 0.4$$

So, $$0.4 = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}\times\frac{M_u}{b\,d^2}}}{415/20}\right]$$

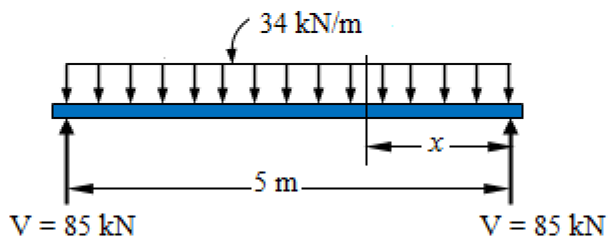
$$\frac{0.4\times 415}{50\times 20} = 1-\sqrt{1-0.23\frac{M_u}{b\,d^2}}$$

$$\sqrt{1-0.23\frac{M_u}{bd^2}}=1-0.166=0.834$$

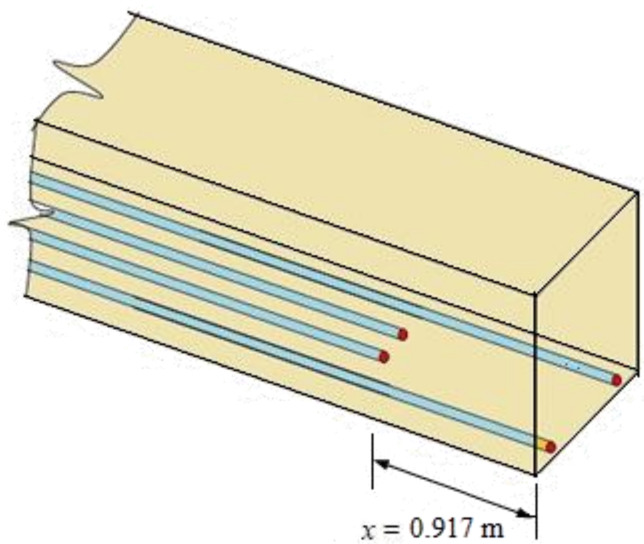
$$\frac{M_u}{bd^2}=\frac{1-0.834^2}{0.23}=1.3237$$

$$M_u=1.3237\times230\times560^2\times10^{-6}=95.475\,kN.m$$

$$M=\frac{M_u}{1.5}=\frac{95.475}{1.5}=63.65\,kN.m$$



Reaction at support $=\frac{34\times5}{2}=85\,kN$



Bending Moment at section $XX,\ M_x=85x-\frac{34x^2}{2}=63.65$

$$x^2-5x+3.744=0$$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4\times1\times3.744}}{2\times1}$$

$$x = 0.917\,m \text{ or } 4.083\,m$$

Thus, half the bars can be curtailed theoretically at 0.917 m from either end.

Note: This distance is theoretical one. The actual curtailment in practice is done by observing a few rules which will be discussed in detail later.

Problem. 8

A simply supported rectangular beam of 8 m span carries a uniformly distributed load of 23 kN/m inclusive of its self weight. Determine the reinforcement for flexure. The materials are M30 grade concrete and TMT bars of grade Fe 415.

Solution:

$Span, l = 8\,m$	$f_y = 415\,N/mm^2$
$f_{ck} = 30\,N/mm^2$	$Q_{\lim} = 2.76 kN.m$

$$Q_{u,\lim} = 0.36\ f_{ck}\frac{x_{u,\max}}{d}\left(1 - 0.42\frac{x_{u,\max}}{d}\right)$$

$$= 0.36 \times 30 \times 0.48(1 - 0.42 \times 0.48) = 4.14$$

$$p_{t,\lim} = 50\left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_{u,\lim}}{bd^2}}}{f_y/f_{ck}}\right]$$

$$= 50\left[\frac{1 - \sqrt{1 - \frac{4.6}{30} \times 4.14}}{415/30}\right] = 1.430$$

Maximum bending moment, $M_{\max} = \dfrac{23 \times 8^2}{8} = 184\,kN.m$

Factored moment, $M_u = 1.5 \times 184 = 276\,kN.m$

Assume, b = 250 mm

$$d_{reqd} = \sqrt{\frac{M_{u,\lim}}{Q_{\lim} b}} = \sqrt{\frac{276 \times 10^6}{4.14 \times 250}} = 516.4\,mm$$

Increase the depth by 10% and add 60 mm effective cover.

$$D = 1.1 \times 516.4 + 60 = 628\,mm$$

Taking D as nearest multiple of 25 mm, D = 650 mm

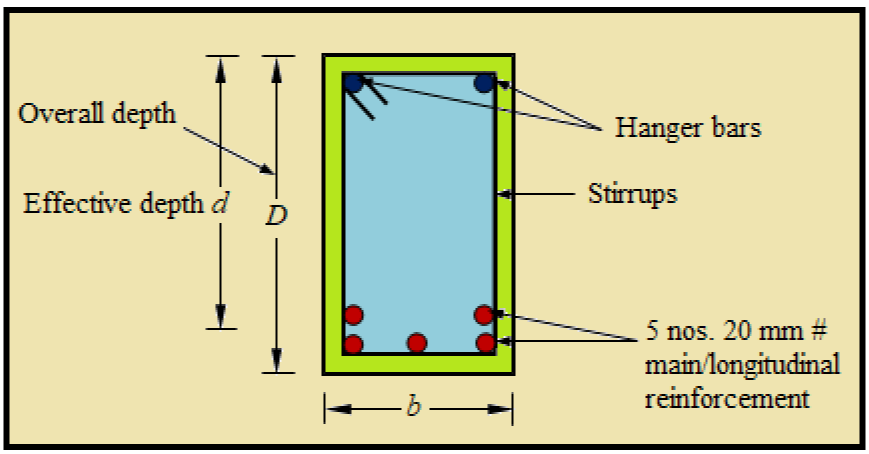
$$d = 650 - 30\,\text{(clear cover) - 20 (dia of bar) - 10 (upto centroid of two layers)}$$

$$= 590\,mm$$

$$\frac{M_u}{bd^2} = \frac{276 \times 10^6}{250 \times 590^2} = 3.17 < 4.14$$

$$p_t = 50\left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{bd^2}}}{f_y / f_{ck}}\right]$$

$$= 50\left[\frac{1 - \sqrt{1 - \frac{4.6}{30} \times 3.17}}{415/30}\right] = 1.023$$



$$A_{st} = \left(\frac{p_t}{100}\right) b\,d = \frac{1.023}{100} \times 250 \times 590 = 1508.925\,mm^2$$

$$A_{st,\lim} = \left(\frac{p_{t,\lim}}{100}\right) b\,d = \frac{1.43}{100} \times 250 \times 590 = 2109.25\,mm^2$$

Provide 5-20 mm # = 5 x 314 = 1570 mm^2

Thus, $A_{st,reqd} < A_{st,provided} < A_{st,\lim}$ (Ok)

Problem. 9

Determine the main reinforcement for a simply supported one-way slab of span 3 m carrying a uniformly distributed load of 5.5 kN/m^2 inclusive of self-weight. Thickness of slab is 100 mm. T materials are M20 grade and HYSD reinforcement of grade Fe 415. Clear cover to main bars may be taken as 15 mm.

Solution:

Slab is designed as a beam of 1 m width. Consider 1 m width of slab.

Load on the slab = 5.5 x 1 = 5.5 kN/m.

Maximum bending moment, $M_{\max} = \dfrac{5.5\times 3^2}{8} = 6.19\,kN.m$

Factored moment, $M_u = 1.5\times 6.19 = 9.28\,kN.m$

Using 10 mm bars, $d = 100-15-5 = 80\,mm$

Width, $b = 1000\,mm$

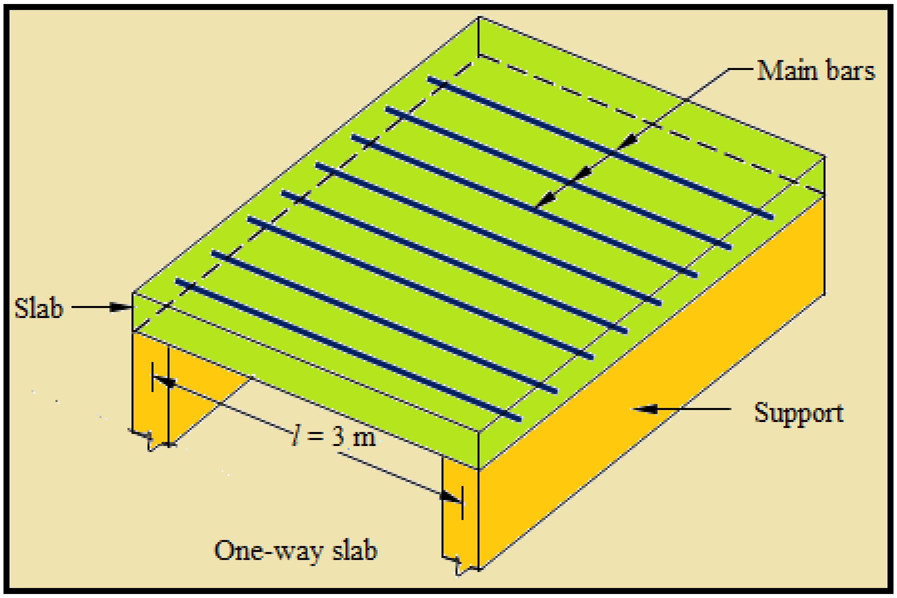
$$\frac{M_u}{bd^2} = \frac{9.28\times 10^6}{1000\times 80^2} = 1.45$$

$$p_t = 50\left[\frac{1-\sqrt{1-\dfrac{4.6}{f_{ck}}\times\dfrac{M_u}{bd^2}}}{f_y/f_{ck}}\right]$$

$$= 50\left[\frac{1-\sqrt{1-\dfrac{4.6}{20}\times 1.45}}{415/20}\right] = 0.442$$

$$A_{st} = \left(\frac{p_t}{100}\right)b\,d = \frac{0.442}{100}\times 1000\times 80 = 353.6\,mm^2$$

Area of 8 mm diameter bar, $A_{\phi} = 50\,mm^2$



No. of 8 mm diameter bars per meter $= \dfrac{353.6}{50} = 7.072$ say 8

Spacing of bars, $s = \dfrac{1000 \times A_\phi}{A_{st}} = \dfrac{1000 \times 50}{353.6} = 141.40\, say\, 140\, mm\, c/c$

Problem. 10

A singly reinforced concrete beam of 200 mm width and 400 mm effective depth is reinforced with 4 bars of 16 mm diameter of Fe 415 steel. Take M20 grade of concrete. Use IS code method to

a) Find the moment of resistance and redesign the beam if necessary.

b) Determine the actual stresses when the section is subjected to the limiting moment of resistance.

Solution:

a) ***Moment of resistance***

$4 - 16\, mm\, \varphi$	$f_y = 415\, N/mm^2$
$b = 200\, mm$	$f_{ck} = 20\, N/mm^2$
$d = 400\, mm$	$p_{t,\lim} = 0.96$

Area of steel, $A_{st} = 4 \times 201 = 804 mm^2$

Depth of neutral axis $\quad 0.36 f_{ck}\, x_u\, b = 0.87 f_y\, A_{st}$

$$0.36 \times 20 \times 200\, x_u = 0.87 \times 415 \times 804$$

$$1400 x_u = 290284.2$$

$$x_u = 207.34\, mm$$

Limiting depth of neutral axis, $\quad x_{u,\max} = 0.48 \times d = 0.48 \times 400 = 192\, mm$

Since, $x_u > x_{u,\max}$, the section is over-reinforced.

The section is over-reinforced and the code recommends that such a beam should be redesigned keeping $x_u = x_{u,\max} = 192\, mm$

Moment of resistance, $M_{u,\lim} = 0.36 f_{ck}\, b\, x_{u,\max} \left(d - 0.42\, x_{u,\max}\right)$

$$= 0.36 \times 20 \times 200 \times 192 (400 - 0.42 \times 192) \times 10^{-6}$$

$$= 88.30\ kN.m$$

Area of steel, $\quad A_{st,\lim} = \dfrac{p_{t,\lim}}{100} \times b\, d = \dfrac{0.96}{100} \times 200 \times 100 = 768\, mm^2$

Alternatively

Area of steel, $\quad A_{st,\lim} = 0.5 \left[\dfrac{1 - \sqrt{1 - \dfrac{4.6}{f_{ck}} \times Q_{\lim}}}{f_y / f_{ck}} \right] b\, d$

$$= 0.5 \left[\frac{1 - \sqrt{1 - \dfrac{4.6}{20} \times 2.76}}{415/20} \right] 200 \times 400 = 762.76\, mm^2$$

b) *Actual stresses*

$$A_{st} = 4 \times 201 = 804\, mm^2, \quad M_{u,\lim} = 88.30\, kN.m$$

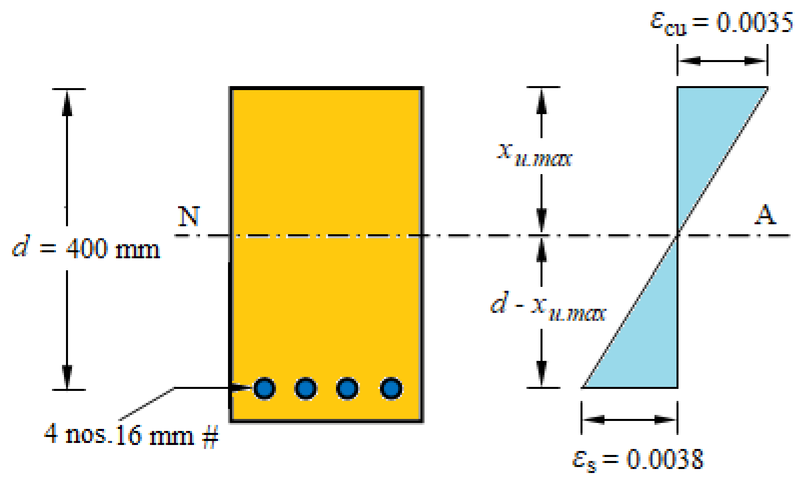
$$x_u = 207.34\, mm, \qquad x_{u,\max} = 192\, mm$$

The actual stress in the steel, f_{st}, when the section is subjected to $M_{u,\lim}$ is determined by equating the total tension and total compression.

$$A_{st}\, f_{st} = 0.36\, f_{ck}\, x_u\, b$$

$$804\, f_{st} = 0.36 \times 20 \times 192 \times 200$$

$$f_{st} = \frac{276480}{804} = 343.88\, N/mm^2$$



(a) Section (b) Strain diagram

The corresponding strain in steel,

Strain in concrete = 0.0035

Strain in steel, $\varepsilon_s = \varepsilon_{cu} \times \frac{(d - x_u)}{x_u} = 0.0035 \times \frac{(400 - 192)}{192}$

$$= 0.00379$$

Check:

Lever arm, $Z = d - 0.42\, x_{u,\max} = 400 - 0.42 \times 192 = 319.36\, mm$

Moment of resistance, $M_u = Compressive\ force \times Z = 0.36\, f_{ck} x_{u,\max} b \times z$

$$= 0.36 \times 20 \times 192 \times 200 \times 319.36 \times 10^{-6}$$

$$= 88.3\, kN.m$$

Moment of resistance, $M_u = Tensile\ force \times Z = f_{st}\, A_{st} \times z$

$$= 804 \times 343.88 \times 316.36 \times 10^{-6}$$

$$= 88.3\, kN.m$$

Problem. 11

A beam simply supported over a span of 7 m carries a live load of 20 kN/m. Design the beam for flexure, using M20 concrete and HYSD reinforcement of grade Fe 415. Take width equal to half the effective depth. Assume unit weight of concrete as 25 kN/mm^3.

Solution:

$Span, l = 7\,m$	$f_y = 415\,N/mm^2$
$f_{ck} = 20\,N/mm^2$	$Q_{\lim} = 2.76$

Depth may be assumed to be $\frac{1}{8}$ to $\frac{1}{10}$ of span, say D = 600 mm for calculation of self weight of the beam.

Width of the beam, $b \approx 0.5D = 0.5 \times 600 = 300\,mm$

Load calculation:

Live load = 20 kN/m

Self weight = $0.3 \times 0.6 \times 25 = 4.5$ kN/m

Total load = 24.5 kN/m

Maximum bending moment, $M_{max} = \frac{24.5 \times 7^2}{8} = 150.0625\,kN.m$

Factored moment, $M_u = 1.5 \times 150.0625 = 225.09\,kN.m$

After the above step, there can be two approaches:

Approach I:

Assuming one layer of 20 mm # bars

$$d = D - 30\,(\text{cover}) - \frac{\phi}{2}$$

$$= 600 - 30 - \frac{20}{2} = 560\,mm$$

Width, $b = 0.5d = 0.5 \times 560 = 280\,mm$

$$\frac{M_u}{bd^2} = \frac{225.09 \times 10^6}{280 \times 560^2} = 2.563 < 2.76$$

The section is under-reinforced.

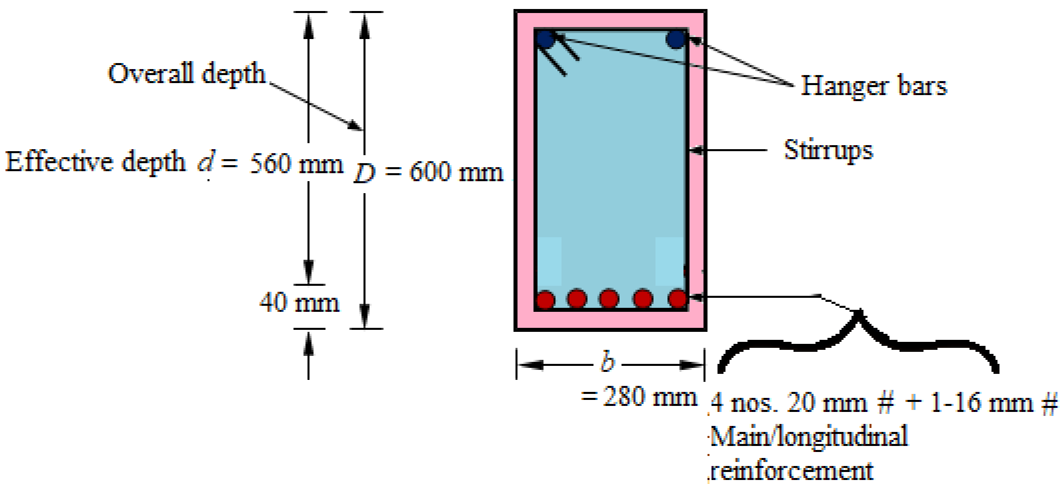
Area of steel, $A_{st} = 0.5\left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{bd^2}}}{f_y / f_{ck}}\right] bd$

$$= 0.5\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 2.563}}{415/20}\right]280\times 560$$

$$= 1357.508\,mm^2$$

Limiting percentage of steel, $p_{t,\lim} = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}\times Q_{\lim}}}{f_y/f_{ck}}\right]$

$$= 50\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 2.76}}{415/20}\right] = 0.96$$



$$A_{st,\lim} = \left(\frac{p_{t,\lim}}{100}\right)b\,d = \frac{0.96}{100}\times 280\times 560 = 1505.28\,mm^2$$

Provide 4-20 mm # + 1-16 mm # = 4 x 314 + 201 = 1457 mm^2

Thus, $A_{st,reqd} < A_{st,provided} < A_{st,\lim}$ (Ok)

Approach II:

Let d = effective depth required

Width, $b = 0.5\,d$

$$M_{u,\lim} = Q_{\lim} bd^2 = 2.76 \times 0.5d \times d^2$$

$$1.38d^3 = 225.09 \times 10^6$$

$$d = 546.38\,mm$$

Increase the depth by 5% and add 40 mm effective cover.

$$D = 1.05 \times 546.38 + 40 = 613.70\,mm$$

Taking D = 620 mm

$$d = 620 - 30\,(\text{clear cover}) - \frac{20}{2}\,(\text{dia of bar})$$

$$= 580\,mm$$

Width, $b = 0.5d = 0.5 \times 580 = 290\,mm$

Load calculation:

Live load = 20 kN/m

Self weight = $0.29 \times 0.62 \times 25 = 4.495 \cong 4.5\,\text{kN/m}$

Total load = 24.5 kN/m

Maximum bending moment, $M_{\max} = \dfrac{24.5 \times 7^2}{8} = 150.0625\,kN.m$

Factored moment, $M_u = 1.5 \times 150.0625 = 225.09\,kN.m$

$$\frac{M_u}{bd^2} = \frac{225.09 \times 10^6}{290 \times 580^2} = 2.307 < 2.76$$

The section is under-reinforced.

Area of steel, $A_{st} = 0.5 \left[\dfrac{1 - \sqrt{1 - \dfrac{4.6}{f_{ck}} \times \dfrac{M_u}{bd^2}}}{f_y / f_{ck}} \right] bd$

$$= 0.5 \left[\frac{1 - \sqrt{1 - \dfrac{4.6}{20} \times 2.307}}{415/20} \right] 290 \times 580$$

$$= 1298.214\,mm^2$$

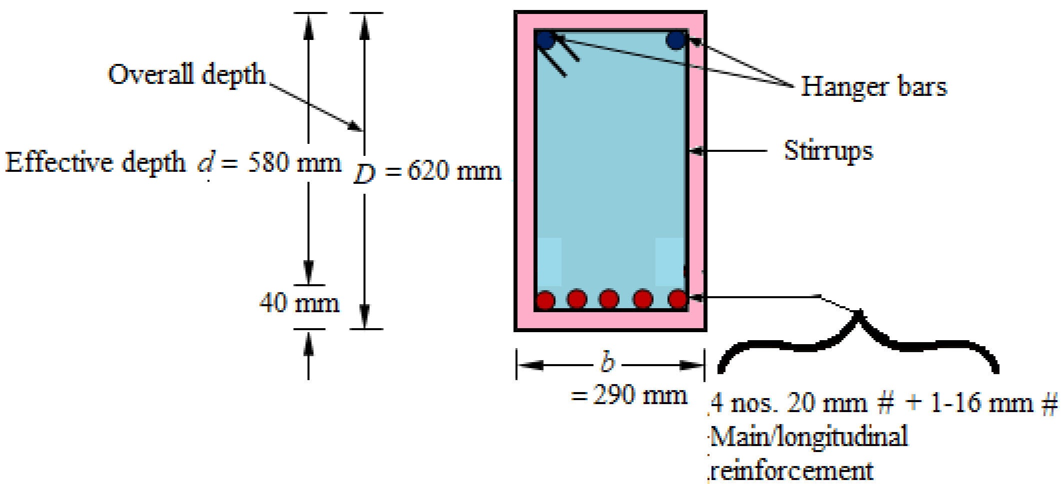
Limiting percentage of steel, $p_{t,\lim} = 50\left[\dfrac{1-\sqrt{1-\dfrac{4.6}{f_{ck}}\times Q_{\lim}}}{f_y/f_{ck}}\right]$

$$= 50\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 2.76}}{415/20}\right] = 0.96$$

$$A_{st,\lim} = \left(\frac{p_{t,\lim}}{100}\right)bd = \frac{0.96}{100}\times 290\times 580 = 1614.72\,mm^2$$

Provide 4-20 mm # + 1-16 mm # = 4 x 314 + 201 = 1457 mm^2

Thus, $A_{st,reqd} < A_{st,provided} < A_{st,\lim}$ (Ok)



Problem. 12

A singly reinforced beam has width equal to 300 mm and total depth is equal to 650 mm with a cover of 40 mm to the centre of reinforcement. Design the beam for flexure, if it is subjected to a working moment of 160 kN.m. Use M20 concrete and HYSD reinforcement of grade Fe 415. (Use SP 16).

Solution:

$b = 300\,mm$ | $f_y = 415\,N/mm^2$

$D = 650\,mm$ | $f_{ck} = 20\,N/mm^2$

Working moment, $M_w = 160\,kN.m$

Design moment, $M_u = 1.5 \times 160 = 240\,kN.m$

$$M_{u,\lim} = 0.36\, f_{ck} \frac{x_{u,\max}}{d} b\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) b\, d^2$$

$$= 0.36 \times 20 \times 0.48\,(1 - 0.42 \times 0.48) \times 300 \times 610^2 \times 10^{-6}$$

$$= 308.02\,kN.m > M_u$$

The beam is under-reinforced.

$$\frac{M_u}{bd^2} = \frac{225.09 \times 10^6}{290 \times 580^2} = 2.307$$

From Table 2 of SP 16

$\frac{M_u}{bd^2}$	p_t
2.30	0.757
2.32	0.765

For $\frac{M_u}{bd^2} = 2.307$, $\quad p_t = 0.757 + \frac{0.765 - 0.757}{2.32 - 2.3} \times (2.307 - 2.3) = 0.7598$

Area of steel, $A_{st} = \frac{p_t \times b\,d}{100} = \frac{0.7598 \times 300 \times 610}{100} = 1319.434\,mm^2$

From Table 2 of SP 16, $p_{t,\lim} = 0.955$

$$A_{st} = \frac{p_{t,\lim} \times b\,d}{100} = \frac{0.955 \times 300 \times 610}{100} = 1747.65\,mm^2$$

Provide 5 nos. 20 mm # bars = 5 x 314 = 1570 mm^2.

Thus, $\quad A_{st,reqd} < A_{st,provided} < A_{st,\lim}$ (Ok)

FLEXURE - Reinforcement Percentage, p_t for Singly Reinforced Sections as in Table 1, 2, 3, 4 & E of SP 16 are reproduced as follows.

TABLE 1. FLEXURE-REINFORCEMENT PERCENTAGE, p_t FOR SINGLY REINFORCED SECTIONS

$f_{ck} = 15$ N/mm^2

M_u/bd^2	f_y, N/mm^2					M_u/bd^2	f_y, N/mm^2				
N/mm^2	240	250	415	480	500	N/mm^2	240	250	415	480	500
0·30	0·147	0·141	0·085	0·074	0·071	1·50	0·829	0·796	0·480	0·415	0·398
0·35	0·172	0·166	0·100	0·086	0·083	1·52	0·842	0·809	0·487	0·421	0·404
0·40	0·198	0·190	0·114	0·099	0·095	1·54	0·856	0·821	0·495	0·428	0·411
0·45	0·224	0·215	0·129	0·112	0·107	1·56	0·869	0·834	0·503	0·434	0·417
0·50	0·250	0·240	0·144	0·125	0·120	1·58	0·882	0·847	0·510	0·441	0·423
0·55	0·276	0·265	0.159	0·138	0·132	1·60	0·896	0·860	0·518	0·448	0·430
0·60	0·302	0·290	0.175	0·151	0·145	1·62	0·909	0·873	0·526	0·455	0·436
0·65	0·329	0·316	0.190	0·164	0·158	1·64	0·923	0·886	0·534	0·461	0·443
0·70	0·356	0·342	0.206	0·178	0·171	1·66	0·936	0·899	0·542	0·468	0·449
0·75	0·383	0·368	0.221	0·191	0·184	1·68	0·950	0·912	0·550	0·475	0·456
0·80	0·410	0·394	0·237	0·205	0·197	1·70	0·964	0·925	0·558	0·482	0·463
0·82	0·421	0·405	0·244	0·211	0·202	1·72	0·978	0·939	0·566	0·489	0·469
0·84	0·433	0·415	0·250	0·216	0·208	1·74	0·992	0·952	0·574	0·496	0·476
0·86	0·444	0·426	0·257	0·222	0·213	1·76	1·006	0·966	0·582	0·503	0·483
0·88	0·455	0·437	0·263	0·227	0·218	1·78	1·020	0·980	0·590	0·510	0·490
0·90	0·466	0·448	0·270	0·233	0·224	1·80	1·035	0·993	0·598	0·517	0·497
0·92	0·477	0·458	0·276	0·239	0·229	1·82	1·049	1·007	0·607	0·525	0·504
0·94	0·489	0·469	0·283	0·244	0·235	1·84	1·064	1·021	0·615	0·532	0·511
0·96	0·500	0·480	0·289	0·250	0·240	1·86	1·078	1·035	0·624	0·539	0·518
0·98	0·512	0·491	0·296	0·256	0·246	1·88	1·093	1·049	0·632	0·546	0·525
1·00	0·523	0·502	0·303	0·262	0·251	1·90	1·108	1·063	0·641	0·554	0·532
1·02	0·535	0·513	0·309	0·267	0·257	1·92	1·123	1·078	0·649	0·561	0·539
1·04	0·546	0·524	0·316	0·273	0·262	1·94	1·138	1·092	0·658	0·569	0·546
1·06	0·558	0·536	0·323	0·279	0·268	1·96	1·153	1·107	0·667	0·576	0·553
1·08	0·570	0·547	0·329	0·285	0·273	1·98	1·168	1·121	0·676	0·584	0·561
1·10	0·581	0·558	0·336	0·291	0·279	2·00	1·184	1·136	0·685	0·592	
1·12	0·593	0·570	0·343	0·297	0·285	2·02	1·199	1·151	0·693		
1·14	0·605	0·581	0·350	0·303	0·290	2·04	1·215	1·166	0·703		
1·16	0·617	0·592	0·357	0·309	0·296	2·06	1·231	1·181	0·712		
1·18	0·629	0·604	0·364	0·315	0·302	2·08	1·247	1·197			
1·20	0·641	0·615	0·371	0·321	0·308	2·10	1·263	1·212			
1·22	0·653	0·627	0·378	0·327	0·314	2·12	1·279	1·228			
1·24	0·665	0·639	0·385	0·333	0·319	2·14	1·295	1·243			
1·26	0·678	0·650	0·392	0·339	0·325	2·16	1·312	1·259			
1·28	0·690	0·662	0·399	0·345	0·331	2·18	1·328	1·275			
1·30	0·702	0·674	0·406	0·351	0·337	2·20	1·345	1·291			
1·32	0·715	0·686	0·413	0·357	0·343	2·22	1·362	1·308			
1·34	0·727	0·698	0·420	0·364	0·349	2·24	1·379				
1·36	0·740	0·710	0·428	0·370	0·355						
1·38	0·752	0·722	0·435	0·376	0·361						
1·40	0·765	0·734	0·442	0·382	0·367						
1·42	0·778	0·747	0·450	0·389	0·373						
1·44	0·790	0·759	0·457	0·395	0·379						
1·46	0·803	0·771	0·465	0·402	0·386						
1·48	0·816	0·784	0·472	0·408	0·392						

* Figures in the red boxes indicate limiting percentage of reinforcement for different grades of reinforcement.

NOTE - Blanks indicate inadmissible reinforcement percentage (see Table E)

TABLE 2. FLEXURE-REINFORCEMENT PERCENTAGE, p_t FOR SINGLY REINFORCED SECTIONS

$f_{ck} = 20\ \mathrm{N/mm^2}$

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
0·30	0·146	0·140	0·085	0·073	0·070
0·35	0·171	0·164	0·099	0·086	0·082
0·40	0·196	0·188	0·114	0·098	0·094
0·45	0·222	0·213	0·128	0·111	0·106
0·50	0·247	0·237	0·143	0·123	0·119
0·55	0·272	0·262	0·158	0·136	0·131
0·60	0·298	0·286	0·172	0·149	0·143
0·65	0·324	0·311	0·187	0·162	0·156
0·70	0·350	0·336	0·203	0·175	0·168
0·75	0·376	0·361	0·218	0·188	0·181
0·80	0·403	0·387	0·233	0·201	0·193
0·85	0·430	0·412	0·248	0·215	0·206
0·90	0·456	0·438	0·264	0·228	0·219
0·95	0·483	0·464	0·280	0·242	0·232
1·00	0·511	0·490	0·295	0·255	0·245
1·05	0·538	0·517	0·311	0·269	0·258
1·10	0·566	0·543	0·327	0·283	0·272
1·15	0·594	0·570	0·343	0·297	0·285
1·20	0·622	0·597	0·359	0·311	0·298
1·25	0·650	0·624	0·376	0·325	0·312
1·30	0·678	0·651	0·392	0·339	0·326
1·35	0·707	0·679	0·409	0·354	0·339
1·40	0·736	0·707	0·426	0·368	0·353
1·45	0·765	0·735	0·443	0·383	0·367
1·50	0·795	0·763	0·460	0·397	0·382
1·55	0·825	0·792	0·477	0·412	0·396
1·60	0·855	0·821	0·494	0·427	0·410
1·65	0·885	0·850	0·512	0·443	0·425
1·70	0·916	0·879	0·530	0·458	0·440
1·75	0·947	0·909	0·547	0·473	0·454
1·80	0·978	0·939	0·565	0·489	0·469
1·85	1·009	0·969	0·584	0·505	0·484
1·90	1·041	1·000	0·602	0·521	0·500
1·95	1·073	1·030	0·621	0·537	0·515
2·00	1·106	1·062	0·640	0·553	0·531
2·02	1·119	1·074	0·647	0·559	0·537
2·04	1·132	1·087	0·655	0·566	0·543
2·06	1·145	1·099	0·662	0·573	0·550
2·08	1·159	1·112	0·670	0·579	0·556
2·10	1·172	1·125	0·678	0·586	0·562
2·12	1·185	1·138	0·685	0·593	0·569
2·14	1·199	1·151	0·693	0·599	0·575
2·16	1·212	1·164	0·701	0·606	0·582
2·18	1·226	1·177	0·709	0·613	0·588
2·20	1·239	1·190	0·717	0·620	0·595

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
2·22	1·253	1·203	0·725	0·627	0·602
2·24	1·267	1·216	0·733	0·633	0·608
2·26	1·281	1·230	0·741	0·640	0·615
2·28	1·295	1·243	0·749	0·647	0·621
2·30	1·309	1·256	0·757	0·654	0·628
2·32	1·323	1·270	0·765	0·661	0·635
2·34	1·337	1·283	0·773	0·668	0·642
2·36	1·351	1·297	0·781	0·675	0·648
2·38	1·365	1·311	0·790	0·683	0·655
2·40	1·380	1·324	0·798	0·690	0·662
2·42	1·394	1·338	0·806	0·697	0·669
2·44	1·408	1·352	0·814	0·704	0·676
2·46	1·423	1·366	0·823	0·711	0·683
2·48	1·438	1·380	0·831	0·719	0·690
2·50	1·452	1·394	0·840	0·726	0·697
2·52	1·467	1·408	0·848	0·734	0·704
2·54	1·482	1·423	0·857	0·741	0·711
2·56	1·497	1·437	0·866	0·748	0·719
2·58	1·512	1·451	0·874	0·756	0·726
2·60	1·527	1·466	0·883	0·764	0·733
2·62	1·542	1·481	0·892	0·771	0·740
2·64	1·558	1·495	0·901	0·779	0·748
$\boxed{2·66}$	1·573	1·510	0·910	0·786	$\boxed{0·755}$
2·68	1·588	1·525	0·919	0·794	
2·70	1·604	1·540	0·928		
2·72	1·620	1·555	0·937		
2·74	1·636	1·570	0·946		
$\boxed{2·76}$	1·651	1·585	$\boxed{0·955}$		
2·78	1·667	1·601			
2·80	1·683	1·616			
2·82	1·700	1·632			
2·84	1·716	1·647			
2·86	1·732	1·663			
2·88	1·749	1·679			
2·90	1·766	1·695			
2·92	1·782	1·711			
2·94	1·799	1·727			
2·96	1·816	1·743			
$\boxed{2·98}$	1·833	$\boxed{1·760}$			

* Figures in the red boxes indicate limiting percentage of reinforcement for different grades of reinforcement.

NOTE - Blanks indicate inadmissible reinforcement percentage (see Table E)

TABLE 3. FLEXURE-REINFORCEMENT PERCENTAGE, p_t FOR SINGLY REINFORCED SECTIONS

f_{ck} = 25 N/mm^2

M_u/bd^2 N/mm^2	f_y, N/mm^2 240	250	415	480	500
0·30	0·146	0·140	0·084	0·073	0·070
0·35	0·171	0·164	0·099	0·085	0·082
0·40	0·195	0·188	0·113	0·098	0·094
0·45	0·220	0·211	0·127	0·110	0·106
0·50	0·245	0·236	0·142	0·123	0·118
0·55	0·271	0·260	0·156	0·135	0·130
0·60	0·296	0·284	0·171	0·148	0·142
0·65	0·321	0·309	0·186	0·161	0·154
0·70	0·347	0·333	0·201	0·174	0·167
0·75	0·373	0·358	0·216	0·186	0·179
0·80	0·399	0·383	0·231	0·199	0·191
0·85	0·425	0·408	0·246	0·212	0·204
0·90	0·451	0·433	0·261	0·225	0·216
0·95	0·477	0·458	0·276	0·239	0·229
1·00	0·504	0·483	0·291	0·252	0·242
1·05	0·530	0·509	0·307	0·265	0·255
1·10	0·557	0·535	0·322	0·279	0·267
1·15	0·584	0·561	0·338	0·292	0·280
1·20	0·611	0·587	0·353	0·306	0·293
1·25	0·638	0·613	0·369	0·319	0·306
1·30	0·666	0·639	0·385	0·333	0·320
1·35	0·693	0·666	0·401	0·347	0·333
1·40	0·721	0·692	0·417	0·360	0·346
1·45	0·749	0·719	0·433	0·374	0·359
1·50	0·777	0·746	0·449	0·388	0·373
1·55	0·805	0·773	0·466	0·403	0·387
1·60	0·834	0·800	0·482	0·417	0·400
1·65	0·862	0·828	0·499	0·431	0·414
1·70	0·891	0·856	0·515	0·446	0·428
1·75	0·920	0·883	0·532	0·460	0·442
1·80	0·949	0·911	0·549	0·475	0·456
1·85	0·979	0·940	0·566	0·489	0·470
1·90	1·009	0·968	0·583	0·504	0·484
1·95	1·038	0·997	0·601	0·519	0·498
2·00	1·068	1·026	0·618	0·534	0·513
2·05	1·099	1·055	0·635	0·549	0·527
2·10	1·129	1·084	0·653	0·565	0·542
2·15	1·160	1·114	0·671	0·580	0·557
2·20	1·191	1·143	0·689	0·596	0·572
2·25	1·222	1·173	0·707	0·611	0·587
2·30	1·254	1·204	0·725	0·627	0·602
2·35	1·285	1·234	0·743	0·643	0.617
2·40	1·317	1·265	0·762	0·659	0·632
2·45	1·350	1·296	0·781	0·675	0·648
2·50	1·382	1·327	0·799	0·691	0·663

M_u/bd^2 N/mm^2	f_y, N/mm^2 240	250	415	480	500
2·55	1·415	1·358	0·818	0·708	0·679
2·60	1·448	1·390	0·837	0·724	0·695
2·65	1·482	1·422	0·857	0·741	0·711
2·70	1·515	1·455	0·876	0·758	0·727
2·75	1·549	1·487	0·896	0·775	0·744
2·80	1·584	1·520	0·916	0·792	0·760
2·85	1·618	1·554	0·936	0·809	0·777
2·90	1·653	1·587	0·956	0·827	0·794
2·95	1·689	1·621	0·977	0·844	0·811
3·00	1·724	1·655	0·997	0·862	0·828
3·05	1·760	1·690	1·018	0·880	0·845
3·10	1·797	1·725	1·039	0·898	0·863
3·15	1·834	1·760	1·061	0·917	0·880
3·20	1·871	1·796	1·082	0·936	0·898
3·25	1·909	1·832	1·104	0·954	0·916
3·30	1·947	1·869	1·126	0·973	0·935
$\boxed{3·32}$	1·962	1·884	1·135	0·981	$\boxed{0·942}$
3·34	1·978	1·899	1·144	0·989	
3·36	1·993	1·914	1·153		
3·38	2·009	1·929	1·162		
3·40	2·025	1·944	1·171		
3·42	2·040	1·959	1·180		
$\boxed{3·44}$	2·056	1·974	$\boxed{1·189}$		
3·46	2·072	1·989			
3·48	2·088	2·005			
3·50	2·104	2·020			
3·52	2·120	2·036			
3·54	2·137	2·051			
3·56	2·153	2·067			
3·58	2·170	2·083			
3·60	2·186	2·099			
3·62	2·203	2·115			
3·64	2·219	2·131			
3·66	2·236	2·147			
3·68	2·253	2·163			
3·70	2·270	2·179			
$\boxed{3·72}$	2·287	$\boxed{2·196}$			
3·74	2·304				

* Figures in the red boxes indicate limiting percentage of reinforcement for different grades of reinforcement.

NOTE - Blanks indicate inadmissible reinforcement percentage (see Table E)

TABLE 4. FLEXURE-REINFORCEMENT PERCENTAGE, p_t FOR SINGLY REINFORCED SECTIONS

f_{ck} = 30 N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
0·30	0·145	0·140	0·084	0·073	0·070
0·35	0·170	0·163	0·098	0·085	0·082
0·40	0·195	0·187	0·113	0·097	0·093
0·45	0·219	0·211	0·127	0·110	0·105
0·50	0·244	0·235	0·141	0·122	0·117
0·55	0·269	0·259	0·156	0·135	0·129
0·60	0·294	0·283	0·170	0·147	0·141
0·65	0·320	0·307	0·185	0·160	0·153
0·70	0·345	0·331	0·200	0·172	0·166
0·75	0·370	0·356	0·214	0·185	0·178
0·80	0·396	0·380	0·229	0·198	0·190
0·85	0·422	0·405	0·244	0·211	0·202
0·90	0·447	0·429	0·259	0·224	0·215
0·95	0·473	0·454	0·274	0·237	0·227
1·00	0·499	0·479	0·289	0·250	0·240
1·05	0·525	0·504	0·304	0·263	0·252
1·10	0·552	0·529	0·319	0·276	0·265
1·15	0·578	0·555	0·334	0·289	0·277
1·20	0·604	0·580	0·350	0·302	0·290
1·25	0·631	0·606	0·365	0·315	0·303
1·30	0·658	0·631	0·380	0·329	0·316
1·35	0·685	0·657	0·396	0·342	0·329
1·40	0·712	0·683	0·411	0·356	0·342
1·45	0·739	0·709	0·427	0·369	0·355
1·50	0·766	0·735	0·443	0·383	0·368
1·55	0·793	0·762	0·459	0·397	0·381
1·60	0·821	0·788	0·475	0·410	0·394
1·65	0·849	0·815	0·491	0·424	0·407
1·70	0·876	0·841	0·507	0·438	0·421
1·75	0·904	0·868	0·523	0·452	0·434
1·80	0·932	0·895	0·539	0·466	0·448
1·85	0·961	0·922	0·556	0·480	0·461
1·90	0·989	0·950	0·572	0·495	0·475
1·95	1·018	0·977	0·589	0·509	0·488
2·00	1·046	1·005	0·605	0·523	0·502
2·05	1·075	1·032	0·622	0·538	0·516
2·10	1·104	1·060	0·639	0·552	0·530
2·15	1·134	1·088	0·656	0·567	0·544
2·20	1·163	1·116	0·673	0·581	0·558
2·25	1·192	1·145	0·690	0·596	0·572
2·30	1·222	1·173	0·707	0·611	0·587
2·35	1·252	1·202	0·724	0·626	0·601
2·40	1·282	1·231	0·742	0·641	0·615
2·45	1·312	1·260	0·759	0·656	0·630
2·50	1·343	1·289	0·777	0·671	0·645

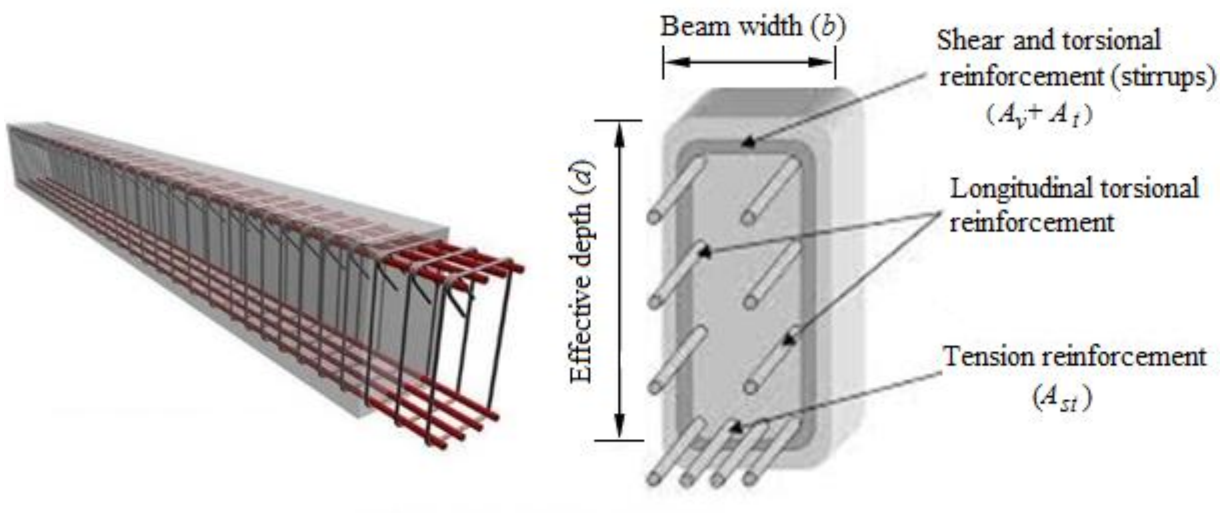
M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
2·55	1·374	1·319	0·794	0·687	0·659
2·60	1·404	1·348	0·812	0·702	0·674
2·65	1·435	1·378	0·830	0·718	0·689
2·70	1·467	1·408	0·848	0·733	0·704
2·75	1·498	1·438	0·866	0·749	0·719
2·80	1·530	1·469	0·885	0·765	0·734
2·85	1·562	1·499	0·903	0·781	0·750
2·90	1·594	1·530	0·922	0·797	0·765
2·95	1·626	1·561	0·940	0·813	0·781
3·00	1·659	1·592	0·959	0·829	0·796
3·05	1·691	1·624	0·978	0·846	0·812
3·10	1·725	1·656	0·997	0·862	0·828
3·15	1·758	1·687	1·017	0·879	0·844
3·20	1·791	1·720	1·036	0·896	0·860
3·25	1·825	1·752	1·055	0·913	0·876
3·30	1·859	1·785	1·075	0·930	0·892
3·35	1·893	1·818	1·095	0·947	0·909
3·40	1·928	1·851	1·115	0·964	0·925
3·45	1·963	1·884	1·135	0·981	0·942
3·50	1·998	1·918	1·156	0·999	0·959
3·55	2·034	1·952	1·176	1·017	0·976
3·60	2·069	1·986	1·197	1·035	0·993
3·65	2·105	2·021	1·218	1·053	1·011
3·70	2·142	2·056	1·239	1·071	1·028
3·75	2·178	2·091	1·260	1·089	1·046
3·80	2·215	2·127	1·281	1·108	1·063
3·85	2·253	2·163	1·303	1·126	1·081
3·90	2·291	2·199	1·325	1·145	1·099
$\boxed{3\cdot95}$	2·329	2·236	1·347	1·164	$\boxed{1\cdot118}$
4·00	2·367	2·273	1·369	1·184	
4·05	2·406	2·310	1·391		
$\boxed{4\cdot10}$	2·445	2·348	$\boxed{1\cdot414}$		
4·15	2·485	2·386			
4·20	2·525	2·424			
4·25	2·566	2·463			
4·30	2·607	2·502			
4·35	2·648	2·542			
4·40	2·690	2·583			
$\boxed{4\cdot45}$	2·733	$\boxed{2\cdot623}$			

* Figures in the red boxes indicate limiting percentage of reinforcement for different grades of reinforcement.

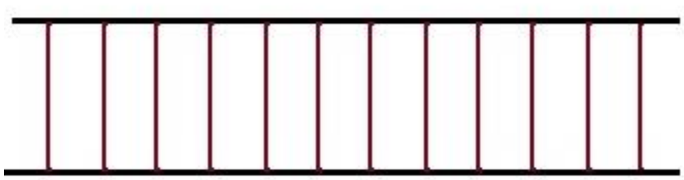
NOTE - Blanks indicate inadmissible reinforcement percentage (see Table E)

TABLE E. MAXIMUM PERCENTAGE OF TENSILE REINFORCEMENT, FOR SINGLY REINFORCEMENT RECTANGULAR SECTIONS

f_{ck}, N/mm^2	f_y, N/mm^2		
	250	415	500
15	1·32	0·72	0·57
20	1·76	0·96	0·76
25	2·20	1·19	0·94
30	2·64	1·43	1·13



(a) 3 - Dimensional view of beam

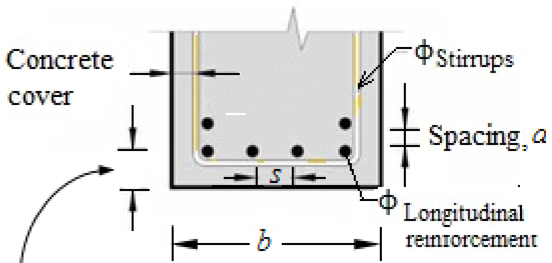


(b) Longitudinal section

(c) Cross-section

s = Minimum horizontal distance between two parallel reiforcing bars shall usually not be less than the greatsest of

- i) the diameter of the bar if diameters are equal
- ii) the diameter of the largest bar if the diameters are unequal
- iii) 5 mm more than the nominal size of coarse aggregate



a = Minimum vertical distance between the bars shall be greatest of

- i) 15 mm
- ii) two-third nominal maximum size of aggregate or
- iii) maximum size of bar

Effective cover to main reinforcement

$$= \text{Clear cover to stirrup} + \text{stirrup diameter} + \frac{\text{Dia. of main bar}}{2}$$