

BHUBANANANDA ODISHA SCHOOL OF ENGINEERING, CUTTACK

DEPARTMENT OF CIVIL ENGINEERING



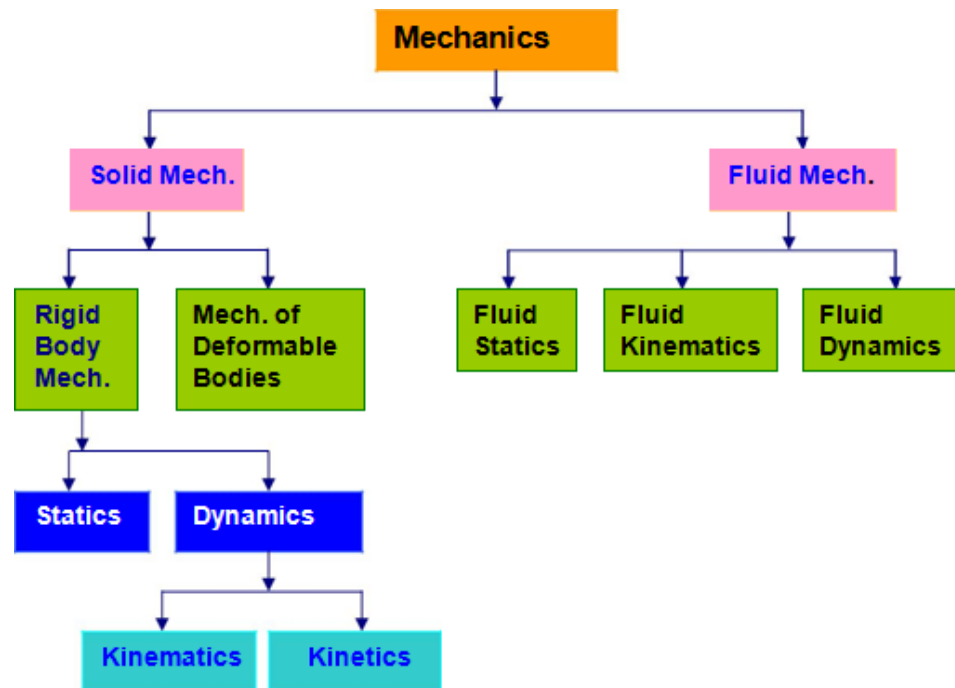
LECTURE NOTE ON: HYDRAULICS AND IRRIGATION

(TH-1) 4TH SEMESTER

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CIVIL ENGINEERING DEPT BOSE, CUTTACK

Introduction to Fluids Mechanics

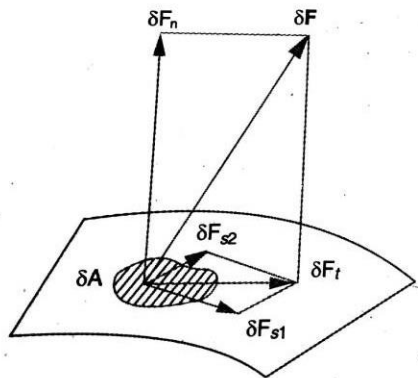


Fluid mechanics is the study of fluids either in motion (fluid *dynamics*) or at rest (fluid *statics*) and the interaction of fluids with solids or other fluids at the boundaries. Both gases and liquids are classified as fluids, and the number of fluid engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few. The study of fluid mechanics is categorized as:

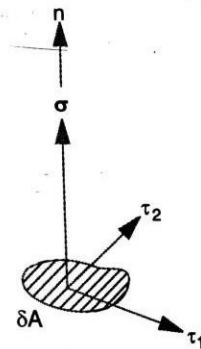
- *Hydrodynamics*: It deals with the study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds).
- *Hydraulics*: It deals with the liquid flows in pipes and open channels.
- *Gasdynamics*: It deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds.
- *Aerodynamics*: It deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds.

Definition of stress Let us consider a small element δA on the surface of a body upon which a force is acting as δF

The force can be resolved into two components, δF_n and δF_t .



(a) NORMAL AND SHEAR FORCES



(b) NORMAL AND SHEAR STRESSES

δF_n = Normal force acting on the area

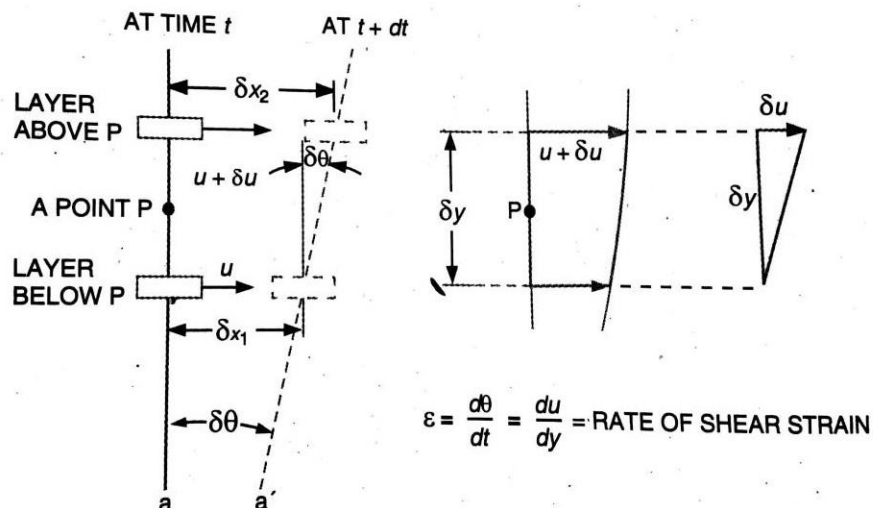
δF_t = horizontal force to the area, i.e., (tangential force or shear force)

Normal force $\sigma = \lim_{\delta A \rightarrow 0} \left(\frac{\delta F_n}{\delta A} \right)$

Tangential or shear stress $\tau = \lim_{\delta A \rightarrow 0} \left(\frac{\delta F_t}{\delta A} \right)$

Rate of strain

The rate of strain at a point in a fluid refers to the rate of shearing of the adjacent layer at that point. This may be interpreted in terms of the rate of change of angle θ of a line $a-a'$ joining the identified fluid elements in layer just above and just below the point P. Consider the changes that take place over an interval of time δt .



This element in the layer below P, moving with velocity u , translates a distance δx_1 to the dotted position such that $\delta x_1 = u \cdot \delta t$

The element above P moving with a velocity $(u + \delta u)$ translates a distance δx_2 to the dotted positions such that $\delta x_2 = (u + \delta u) \cdot \delta t$

The line element a-a therefore, shifts to a new position a'-a' such that the inclination $\delta\theta$

$$\delta\theta \equiv \tan\delta\theta = \frac{\delta x_2 - \delta x_1}{\delta y} = \frac{(u + \delta u) \cdot \delta t - u \cdot \delta t}{\delta y}$$

The rate of shear strain is given by $\epsilon = \frac{d\theta}{dt} = \lim_{\delta t \rightarrow 0} \frac{\delta\theta}{\delta t} = \frac{du}{dt}$

Solids and Fluids:

- **Solid:** A solid can resist a shear stress by a static deflection; The amount of deflection is proportional to the magnitude of applied stress up to some limiting condition. When the applied shear force is constant, a solid stops deforming.

The molecules of a solid are more closely packed. Hence the force of attraction between the molecules of solid is more larger than that of a fluid.

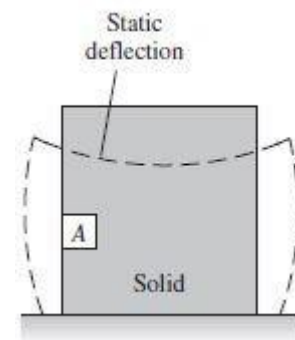


Fig.1 Deformation in a solid

- **Fluid:** Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied. When constant shear force is applied, the fluid continues to deform and stops at certain strain rate. In fluids the applied stress is proportional to rate of deformation. A fluid may be either liquid or gas.

- ✓ **Liquid:** A liquid is composed of relatively close-packed molecules with strong cohesive forces which tends to retain its volume and will form a free surface in a gravitational field if unconfined from above.

Examples: water, oil, mercury, gasoline, alcohol etc.

- ✓ **Gas:** A gas, expands until it encounters the walls of the container and fills the entire available space. This is because the gas molecules are widely spaced, and the cohesive forces between them are very small. A gas has no definite volume, and when left to itself without confinement, it forms an atmosphere that is essentially hydrostatic.

Examples: air,
hydrogen,

helium,
steam etc.,

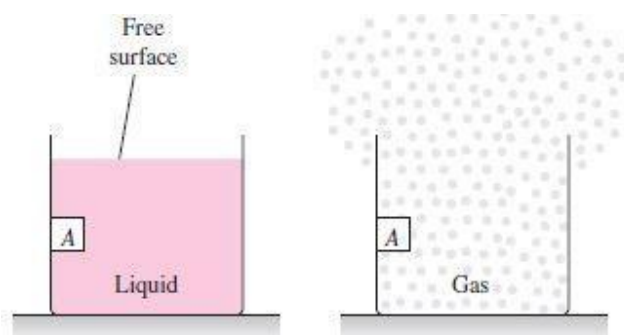


Fig.2 Liquid and Gas

Solids can resist tangential stress under static conditions whereas fluids can do it only under dynamic conditions.

Basic Fluid Properties:

Characteristics of a fluid which are independent of motion are called basic properties.

1. **Density (ρ):** Density of a fluid is its mass per unit volume.

Units: kg/m^3 in SI system

The density of most gases is proportional to pressure and inversely proportional to temperature. Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible. Thus liquid flows are analytically treated as incompressible.

The heaviest common liquid is Mercury with density, $\rho = 13,580 \frac{\text{kg}}{\text{m}^3}$ and the lightest gas is Hydrogen with density, $\rho = 0.0838 \frac{\text{kg}}{\text{m}^3}$.

2. **Specific weight (ω):** It is the weight of the fluid to the volume occupied with it.

$$\text{Specific weight, } \omega = \rho \times g = \frac{m \times g}{V} \frac{\text{N}}{\text{m}^3}$$

3. **Specific volume (V):** It is the volume occupied by the unit mass of fluid and is the reciprocal of density.

Units: m^3/kg in SI system

4. **Specific gravity (G):** It is the ratio of density of fluid to the density of standard fluid. It is also referred as relative density.

In general, water is the standard fluid for liquids while air is for gases.

$$\text{For liquids, } G = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} \quad \text{For gases, } G = \frac{\rho_{\text{gas}}}{\rho_{\text{air}}}$$

5. **State relations for liquids:**

Liquids are nearly incompressible and have a single, reasonably constant specific heat. Thus an idealized state relation for a liquid is

$$\rho \approx \text{constant} \quad c_p \approx c_v \approx \text{constant} \quad dh \approx c_p dT$$

The density of a liquid usually decreases slightly with temperature and increases moderately with pressure. If we neglect the temperature effect, an empirical pressure– density relation for a liquid is given as

$$\frac{p}{p_a} \approx (B+1) \left| \frac{(\rho/\rho_a)^n}{1 - B} \right|$$

where B and n are dimensionless parameters that vary slightly with temperature and p_a and ρ_a are standard atmospheric values. Water can be fitted approximately to the values B=3000 and n=7.

Viscosity: It is a quantitative measure of a fluid's resistance to flow. It determines the strain rate in a fluid that is generated by a given applied shear stress. This property is due to the cohesive forces of attraction between the fluid molecules.

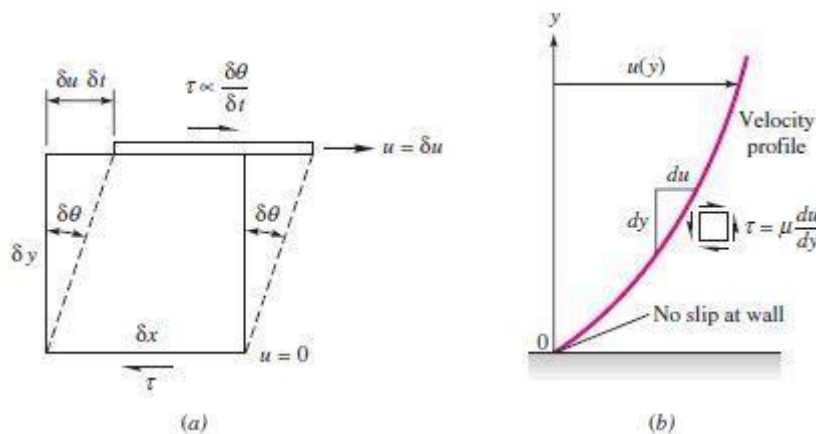


Fig.3(a) Fluid element straining at rate $\frac{d\theta}{dt}$

Fig.3(b) Newtonian shear distribution in a shear layer near a wall.

Consider a fluid element sheared in one plane by a single shear stress τ , as in fig. 3(a). The shear strain angle $\delta\theta$ will continuously grow with time as long as the stress τ is maintained, the upper surface moving at speed δu larger than the lower. Such common fluids as water, oil, and air show a linear relation between applied shear and resulting strain rate:

$$\tau \propto \frac{\delta\theta}{\delta t}$$

From fig. 3(a), we can write, $\tan \delta\theta = \frac{\delta u \delta t}{\delta y}$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient:

$$\frac{\delta \theta}{\delta t} = \frac{\delta u}{\delta y}$$

Then, the applied shear is also proportional to the velocity gradient for the common linear fluids. The constant of proportionality is the viscosity coefficient μ :

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$

μ -absolute viscosity or dynamic viscosity.

Unit for Absolute viscosity: N-sec/m² or Pa-sec

Common unit is poise. 1 poise = 0.1 pa-sec.

From the fig. 3(b), it has to be observed that at the solid boundary, i.e. $y=0$, the fluid velocity is zero indicating the no-slip condition where the velocity gradient is zero. With increase in the distance from the solid boundary, the velocity increases gradually i.e. velocity gradient increases.

Kinematic viscosity: It is the ratio of dynamic viscosity to the density of the fluid.

$$\gamma = \frac{\mu}{\rho}$$

Units: It is expressed in m²/sec or cm²/sec. 1

cm²/sec = 1 stoke

Parameter	Liquids	Gases
Pressure	For liquids, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures.	For gases, dynamic viscosity is independent of pressure (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure.
Temperature	The viscosity of liquids decreases with temperature. This is because in a liquid the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly.	the viscosity of gases increases with temperature. This is because, in a gas the intermolecular forces are negligible, making gas molecules moving randomly at high velocities at high temperatures. This results in

	As a result, the energized liquid molecules can move more freely.	more molecular collisions per unit volume per unit time and therefore in greater resistance to flow.
Governing equations	Viscosity is given as: $\ln \left(\frac{\mu}{\mu_0} \right) \approx a + b \left(\frac{T}{T_0} \right)^c + c \left(\frac{T}{T_0} \right)^d$	Viscosity is given as: $\mu \approx \mu_0 \left(\frac{T}{T_0} \right)^n \dots \dots \text{Powerlaw}$ $\mu \approx \mu_0 \frac{(T_0 + s)^3}{(T + s)} \dots \text{Sutherlandlaw}$

- **Newtonian fluids**: These are the fluids for which the rate of deformation is proportional to the shear stress.

Examples: Air, water, kerosene

- **Non-Newtonian fluids**: These are the fluids for which the rate of deformation is not

directly proportional to the shear stress i.e. $\tau = k \left(\frac{du}{dy} \right)^n$, where k - consistency index,

n - flow behavior index. This equation can be rewritten as

$$\tau = k \left(\frac{du}{dy} \right)^n = \eta \left(\frac{du}{dy} \right) \quad \text{where apparent viscosity, } \eta = k \left(\frac{du}{dy} \right)^{n-1}$$

Non-Newtonian fluids are classified as time independent and time dependent fluids.

Time independent fluids

1. ***Pseudoplastic fluids***: Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$).

Examples: Polymer solutions, colloidal suspensions, paper pulp in water, gelatine, blood, milk

2. ***Dilatant fluids***: Fluids in which the apparent viscosity increases with increasing deformation rate ($n > 1$).

Examples: Suspension of starch, sugar in starch

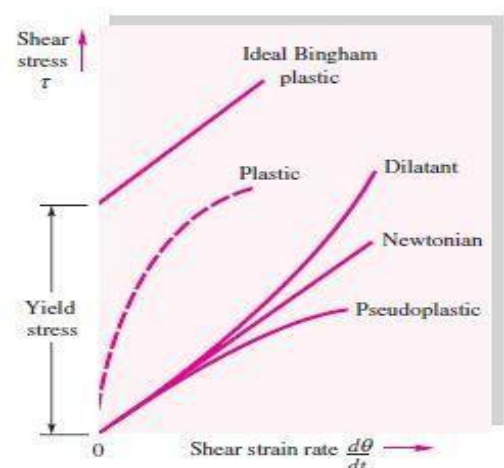


Fig.4 Stress vs Strain rate

3. ***Bingham plastic***: It is a fluid which behaves as a solid until minimum yield stress

(τ_y) is exceeded and subsequently exhibits a linear relationship between stress

and rate of deformation. The relationship is given as $\tau = \tau_y + \mu_p \left(\frac{du}{dy} \right)$

Examples: Clay suspensions, drilling muds

Time dependent fluids

1. *Thixotropic fluids*: Fluids whose apparent viscosity decreases with time under a constant applied stress.

Examples: Most of the paints

2. *Rheopectic fluids*: Fluids whose apparent viscosity increases with time under a constant applied stress.

3. *Viscoelastic fluids*: These fluids partially return to their original shape after deformation, when the applied stress is released.

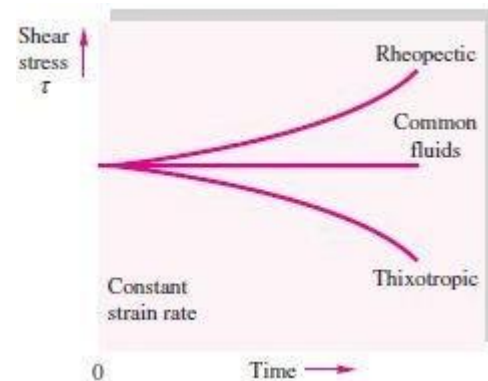


Fig.5 Shear stress vs time

Surface Tension

Molecules deep within the liquid repel each other because of their close packing. Molecules at the surface are less dense and attract each other. Since half of the neighboring molecules are missing, the mechanical effect is that the surface is in tension. This tensile force acts parallel to the surface and towards the interior of the liquid. The magnitude of this force per

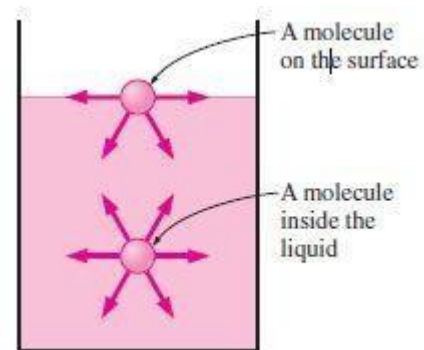


Fig.6 Attractive forces in liquid

unit length is called surface tension, σ_s .

Units: In SI system unit is N/m.

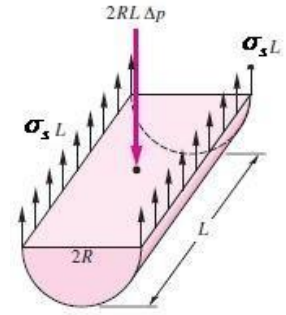
This force (surface tension) is balanced by the repulsive forces from the molecules below the surface that are being compressed. The resulting compression effect causes the liquid to minimize its surface area. This is the reason for the tendency of the liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.

This effect is also called **surface energy** expressed in J/m^2 , which represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

Applications: Surface tension determines the size of the liquid droplets that form.

1. *Pressure inside a cylindrical jet:*

Consider a jet of water exiting as a cylinder with radius 'R' and length 'L'. When one half of the cylinder is considered, as shown in the fig. 7, two forces can be observed. One force is surface tension acting along the circumference and



another due to the internal pressure (Δp in Fig. 7 Section of liquid cylinder excess of atmosphere pressure).

Surface tension force along the length of the cylinder, $F_t = \sigma_s(L+L) = 2L\sigma_s$

Force due to internal pressure, $F_p = \Delta p(L \times 2R) = 2LR(\Delta p)$

For equilibrium, both forces should be same i.e. $F_p = F_t \Rightarrow 2LR(\Delta p) = 2L\sigma_s$

Excess pressure inside a cylindrical jet, $\Delta p = \frac{\sigma_s}{R}$

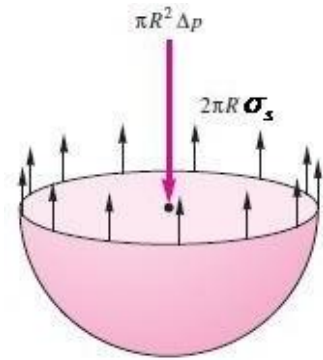
2. *Pressure inside a spherical droplet:*

Consider a spherical droplet of diameter 'R' subject to internal pressure (Δp in excess of atmosphere pressure) balanced by surface tension acting along the circumference.

Surface tension force along the circumference,

$$F_t = \sigma_s(2\pi R)$$

Force due to internal pressure, $F_p = \Delta p(\pi R^2)$ Fig. 8 Section of spherical droplet



For equilibrium, both forces should be same i.e.

$$F_p = F_t \Rightarrow \Delta p(\pi R^2) = (2\pi R)\sigma_s$$

Excess pressure inside a spherical droplet, $\Delta p = \frac{2\sigma_s}{R}$

3. *Pressure inside a soap bubble:*

In the case of a soap bubble, it has two interfaces with air; one at inner surface and another at outer surface. Hence the excess pressure is given as

$$(\Delta p)_{\text{soap bubble}} \approx 2(\Delta p)_{\text{spherical bubble droplet}} = \frac{4\sigma_s}{R}$$

Bulk Modulus and Compressibility

Bulk modulus, K is defined as the ratio of the change in pressure to the rate of change of volume due to the change in pressure or the ratio of change in pressure to the volumetric strain.

$$K = \frac{-dp}{\left(\frac{dv}{v}\right)}$$

where dp is the change in pressure causing a change in volume dv when the original volume was v . The unit is the same as that of pressure.

Also, we can express volumetric strain in terms of relative change in density, $dv/v = -dp/\rho$.

Bulk modulus can also be expressed in terms of change of density as $K = \frac{-dp}{\left(\frac{\Delta v}{v}\right)} = \frac{dp}{\left(\frac{dp}{\rho}\right)}$.

The negative

sign indicates that if the pressure increases, then the volume of the substance decreases.

This definition can be applied to liquids as such, without any modifications. In the case of gases, the value of compressibility will depend on the process law for the change of volume and will be different for different processes.

The bulk modulus for liquids depends on both pressure and temperature. The value increases with pressure as dv will be lower at higher pressures for the same value of dp . With temperature the bulk modulus of liquids generally increases, reaches a maximum and then decreases. For water the maximum is at about 50°C . The value is in the range of 2000 MN/m^2 .

Pressure

Pressure is a measure of force distribution over any surface associated with the force. Pressure is a surface phenomenon and it can be physically visualized or calculated only if the surface over which it acts is specified. **Pressure may be defined as the force acting along the normal direction on unit area of the surface.**

$$P = \frac{dF}{dA}$$

The force dF in the normal direction on the area dA due to the pressure P is $dF = P dA$.

The unit of pressure in the SI system is N/m^2 , called as Pascal (Pa). As the magnitude is small kN/m^2 (kPa) and MN/m^2 (Mpa) are more popularly used. The atmospheric pressure is

approximately 10^5 N/m^2 and is designated as “bar”. This is also a popular unit of pressure. In the metric system the popular unit of pressure is kg-f/cm^2 . This is approximately equal to the atmospheric pressure or 1 bar.

Pascal's Law

A fluid element when isolated from the surroundings experiences two types of external forces: (a) Body force and (b) Surface force

Body force: These forces act throughout the body of the fluid element and are distributed over the entire mass or the volume of the element.

Ex: Gravitational force, Magnetic force etc.,

Surface force: It includes all the forces exerted on the fluid element by its surroundings through direct contact at the surface.

Ex: Normal force and shear force

For a fluid element at rest, shear force is zero (shear stress = 0) and hence only normal stresses exist.

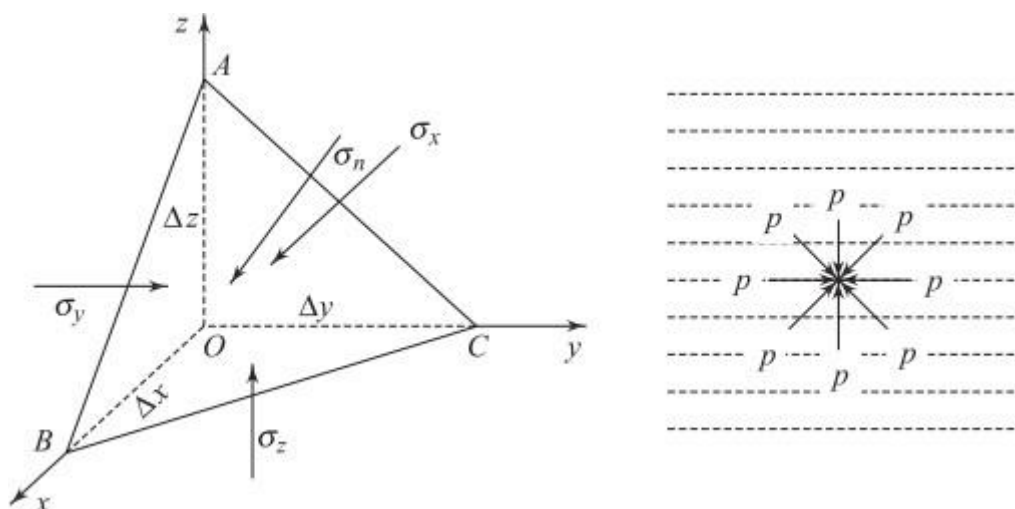


Fig.9 State of stress on a fluid element at rest, state of normal stress on a fluid element at rest

Consider a stationary fluid element of tetrahedral shape with three of its faces coinciding with the coordinate planes as shown in the figure. Hence the nature of normal stress acting on the fluid element will be of compressive in nature. Considering gravity as the only external body force, the equations of static equilibrium for the tetrahedral fluid element are written as:

$$\Sigma F_x = \sigma_x \left(\frac{\Delta y \Delta z}{2} \right) - \sigma_n \Delta A \cos \alpha = 0$$

$$\Sigma F_y = \sigma_y \left(\frac{\Delta x \Delta z}{2} \right) - \sigma_n \Delta A \cos \beta = 0$$

$$\Sigma F_z = \sigma_z \left(\frac{\Delta x \Delta y}{2} \right) - \sigma_n \Delta A \cos \gamma - \frac{\rho g}{6} (\Delta x \Delta y \Delta z) = 0$$

Where ΣF_x , ΣF_y and ΣF_z are the net forces acting on the fluid element in positive X, Y

and Z directions respectively, $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are the direction cosines of the normal to the plane of area ΔA .

Therefore,

$$\Delta A \cos \alpha = (\Delta y \cdot \Delta z)/2$$

$$\Delta A \cos \beta = (\Delta x \cdot \Delta z)/2$$

$$\Delta A \cos \gamma = (\Delta x \cdot \Delta y)/2.$$

Substituting these equations in force equilibrium equations gives,

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

This equation concludes that the normal stress at any point in a fluid at rest are directed towards the point from all directions and are of equal magnitude. These stresses are defined as hydrostatic pressure or thermodynamics pressure. This is known as Pascal's law of hydrostatics. With convention of the positive sign for the tensile stress, the above statement can be analytically expressed as

$$\sigma_x = \sigma_y = \sigma_z = -P$$

Pressure Measurement

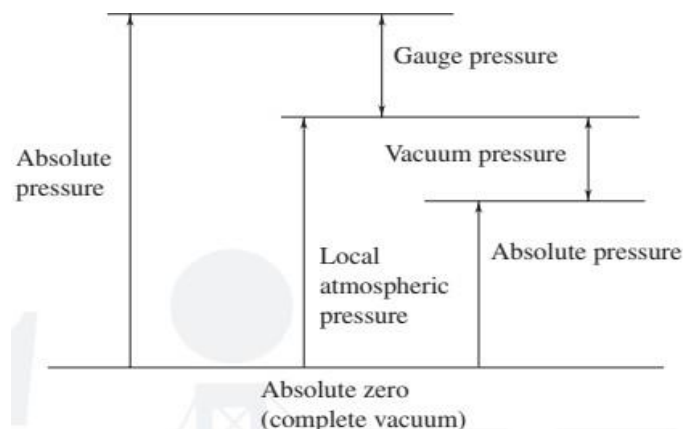


Fig. Scale of Pressure

- Pressure is usually expressed with reference to either absolute zero pressure (complete vacuum) or local atmospheric pressure.

- The absolute pressure is the pressure expressed as a difference between its value and the absolute zero pressure.
- When the pressure is expressed as a difference between its value and the local atmospheric pressure, it is known as gauge pressure.

$$P_{\text{gauge}} = P - P_{\text{atm}}$$

- If the pressure P is less than the local atmospheric pressure, the gauge pressure becomes negative and is called vacuum pressure.
- At sea level, the international standard atmospheric pressure is given as 101.325 kN/m^2 .

Pressure is generally measured using a sensing element which is exposed on one side to the pressure to be measured and on the other side to the surrounding atmospheric pressure or other reference pressure.

Piezometer

It is used to measure the gauge pressure i.e. system pressure greater than the local atmospheric pressure. It contains a vertical tube with an opening at the top to atmosphere. Thus when connected to a pipe in which some fluid is flowing, there occurs a rise in the liquid level in the vertical tube called as pressure head.

From the Hydrostatic law, the Pressure at a point 'A' is

$P_A = \text{External pressure} + \text{Pressure due to rise of liquid level in the tube}$

$$P_A = P_{\text{atm}} + \rho gh$$

$$\text{Gauge pressure} = P_A - P_{\text{atm}} = \rho gh$$

Barometer

It is used to measure the atmospheric pressure. Mercury is used as the working fluid because of its very small vapour pressure at normal temperature and its high density for a relatively short column to be obtained.

When an inverted tube is dipped into a vessel containing fluid like mercury, there occurs a rise in level in the tube above the surface level in the vessel, as shown in the figure.

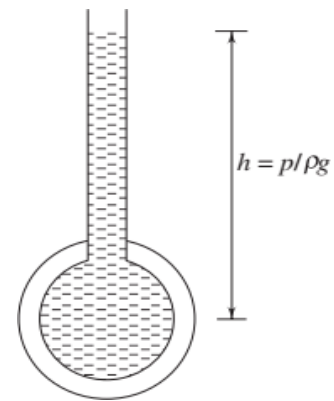


Fig. Piezometer

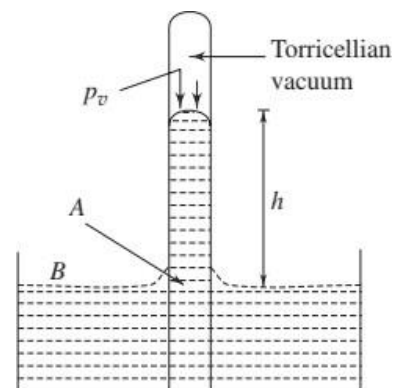


Fig. Barometer

At the level above the mercury in the tube, vacuum is created thus forming mercury vapour having pressure equal to the vapour pressure of mercury at its existing temperature.

From the figure, we can observe that points A and B lie on the same horizontal plane. Therefore we can write, $P_B = P_{atm} = P_v + \rho gh$.

The vapour pressure will be small in comparison to atmospheric pressure and hence it is neglected. Hence $P_B = P_{atm} = \rho gh$.

Bourdon Tube Pressure Gauge:

In the Bourdon gauge a tube of elliptical section bent into circular shape is exposed on the inside to the pressure to be measured and on the outside to atmospheric pressure. The tube will tend to straighten under pressure. The end of the tube will move due to this action and will actuate

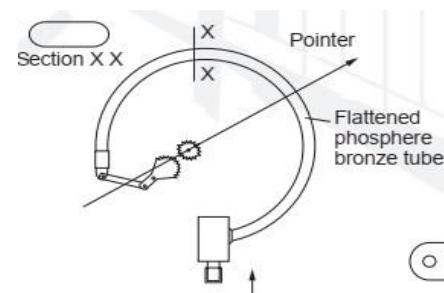


Fig. 9 Bourdon Tube

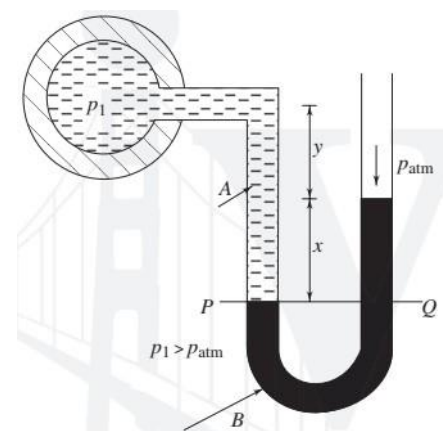
Pressure Gauge pressure. Vacuum also can be measured by such a gauge. Under vacuum the tube will tend to bend further inwards and as in the case of pressure, will actuate the pointer to indicate the vacuum pressure. The scale is obtained by calibration with known pressure source.

Manometers: Manometer is a device to measure pressure or mostly difference in pressure using a column of liquid to balance the pressure. It is a basic instrument and is used extensively in flow measurement. It needs no calibration. Very low pressures can be measured using micromanometers.

The basic principle of operation of manometers is that at the same level in continuous fluid at rest, the pressure is the same. The pressure due to a constant density liquid (ρ) column of height h is equal to ρgh .

Simple U-tube Manometer

It is a transparent 'U' tube as shown in figure. One of its ends is connected to a pipe or a container having fluid (A) whose pressure is to be measured while the other end is open to atmosphere. The lower part of the tube contains a liquid immiscible with the fluid A and is of greater density than that of A. This fluid is called manometric fluid. The pressure at two points P and Q in a horizontal plane within



the continuous expanse of the same fluid must be equal. Fig. 10 Manometer to measure gauge

pressure

Pressure at the point P = Pressure at the point Q

$$P_1 + \rho_A g(y+x) = P_{atm} + \rho_B g x$$

$$P_1 - P_{atm} = g x (\rho_B - \rho_A) - \rho_A g y$$

When the pressure of the fluid in the container is lower than the atmospheric pressure, the liquid levels in the manometer would be adjusted as shown in the figure.

Hence it becomes

Pressure at the point P = Pressure at the point Q

$$\begin{aligned} P_1 + \rho_A g y + \rho_B g x \\ = P_{atm} - P_1 - \rho_A g y - \rho_B g x \end{aligned}$$

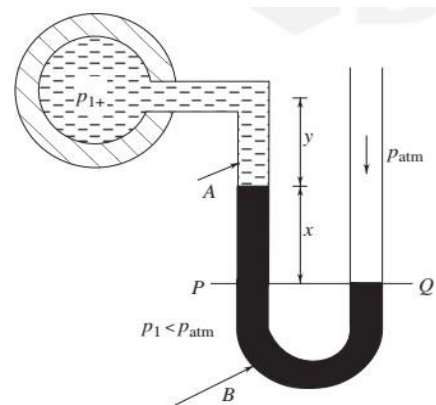


Fig.11 Manometer measuring vacuum

Differential U-tube manometer

A manometer is frequently used to measure the pressure difference, in course of flow, across a restriction in a horizontal pipe as shown in figure.

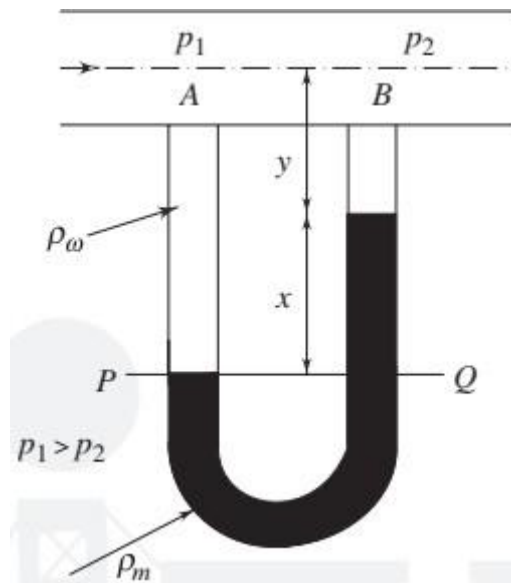


Fig.12 Manometer measuring pressure differential

It is important that the axis of each connecting tube at A and B to be perpendicular to the direction of the flow and also for the edges of the connection to be smooth. From the principle of hydrostatics, Pressure at the point P = Pressure at the point Q

$$P_1 + \rho_w g(y+x) = P_2 + \rho_w g y + \rho_m g x$$

$$P_1 - P_2 = g x (\rho_m - \rho_w)$$

Where ρ_m is the density of manometric fluid and ρ_w is the density of working fluid. Other configurations of manometers are:

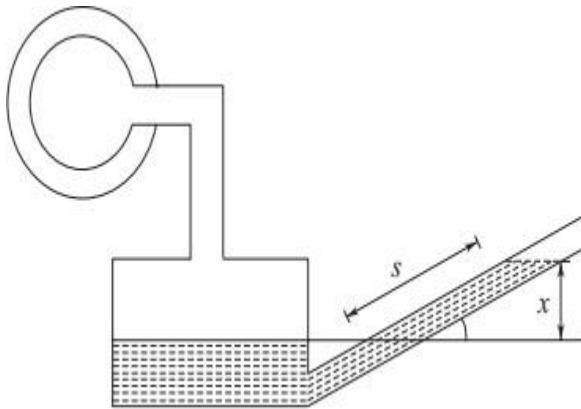


Fig.13 Inclined tube manometer

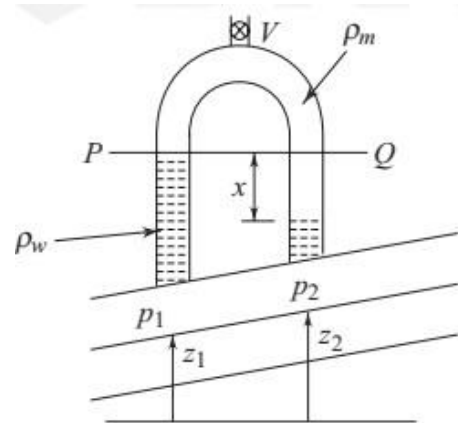


Fig.14 Inverted tube manometer

FLUIDMECHANICS

UNIT-II

FLUIDKINEMATICS

Introduction

Fluid Kinematics gives the geometry of fluid motion. It is a branch of fluid mechanics, which describes the fluid motion, and its consequences without consideration of the nature of forces causing the motion. Fluid kinematics is the study of velocity as a function of space and time in the flow field. From velocity, pressure variations and hence, forces acting on the fluid can be determined.

Velocityfield

Velocity at a given point is defined as the instantaneous velocity of the fluid particle, which at a given instant is passing through the point. It is represented by $V=V(x,y,z,t)$. Vectorially, $V=ui+vj+wk$ where u,v,w are three scalar components of velocity in x,y and z directions and (t) is the time. Velocity is a vector quantity and velocity field is a vector field.

Velocityandacceleration

Let V = Resultant velocity at any point in a fluid flow. (u,v,w) are the velocity components in x,y and z directions which are functions of space coordinates and time.

$$u=u(x,y,z,t); v=v(x,y,z,t); w=w(x,y,z,t).$$

$$\text{Resultant velocity} = V = ui + vj + wk$$

$$|V| = (u^2 + v^2 + w^2)^{1/2}$$

$$\text{Let } a_x, a_y, a_z \text{ are the total accelerations in the } x, y, z \text{ directions respectively}$$

$$a_x = \left[\frac{du}{dt} \right] = \left[\frac{\partial u}{\partial x} \right] \left[\frac{\partial x}{\partial t} \right] + \left[\frac{\partial u}{\partial y} \right] \left[\frac{\partial y}{\partial t} \right] + \left[\frac{\partial u}{\partial z} \right] \left[\frac{\partial z}{\partial t} \right] + \left[\frac{\partial u}{\partial t} \right]$$

$$a_x = \left[\frac{du}{dt} \right] = u \left[\frac{\partial u}{\partial x} \right] + v \left[\frac{\partial u}{\partial y} \right] + w \left[\frac{\partial u}{\partial z} \right] + \left[\frac{\partial u}{\partial t} \right]$$

Similarly,

$$a_y = \left[\frac{dv}{dt} \right] = u \left[\frac{\partial v}{\partial x} \right] + v \left[\frac{\partial v}{\partial y} \right] + w \left[\frac{\partial v}{\partial z} \right] + \left[\frac{\partial v}{\partial t} \right]$$

$$a_z = \left[\frac{dw}{dt} \right] = u \left[\frac{\partial w}{\partial x} \right] + v \left[\frac{\partial w}{\partial y} \right] + w \left[\frac{\partial w}{\partial z} \right] + \left[\frac{\partial w}{\partial t} \right]$$

1. Convective Acceleration Terms – The first three terms in the expressions for a_x, a_y, a_z .

Convective acceleration is defined as the rate of change of velocity due to change of position of the fluid particles in a flow field

2. Local Acceleration Terms- The 4th term, $[\partial () / \partial t]$ in the expressions for a_x , a_y , a_z . Local or temporal acceleration is the rate of change of velocity with respect to time at a given point in a flow field.

Material or Substantial Acceleration = Convective Acceleration + Local or Temporal Acceleration

In a steady flow, temporal or local acceleration is zero. In uniform flow, convective acceleration is zero.

For steady flow, $[\partial u / \partial t] = [\partial v / \partial t] = [\partial w / \partial t] = 0$ $a_x =$

$$[du/dt] = u[\partial u / \partial x] + v[\partial u / \partial y] + w[\partial u / \partial z]$$

$$a_y = [dv/dt] = u[\partial v / \partial x] + v[\partial v / \partial y] + w[\partial v / \partial z]$$

$$a_z = [dw/dt] = u[\partial w / \partial x] + v[\partial w / \partial y] + w[\partial w / \partial z]$$

$$\text{Acceleration Vector} = a_x i + a_y j + a_z k; |A| = [a_x^2 + a_y^2 + a_z^2]^{1/2}$$

Method of describing fluid motion

Two methods of describing the fluid motion are: (a) Lagrangian method and (b) Eulerian method.

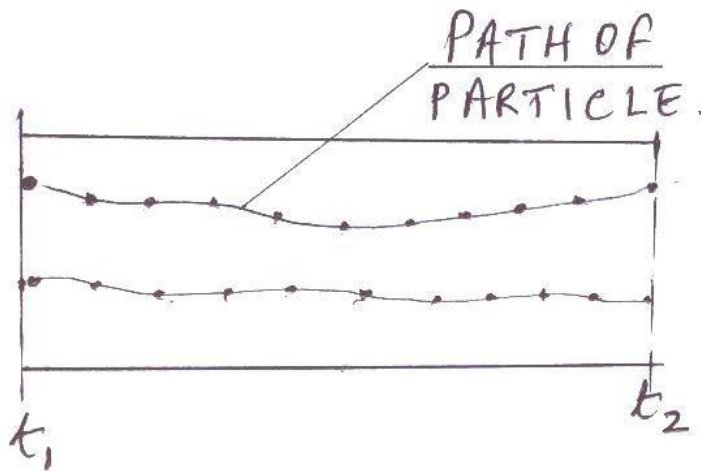


Fig. Lagrangian method

A single fluid particle is followed during its motion and its velocity, acceleration etc. are described with respect to time. Fluid motion is described by tracing the kinematics behavior of each and every individual particle constituting the flow. We follow individual fluid particle as it moves through the flow. The particle is identified by its position at some instant and the time elapsed since that instant. We identify and

follow small, fixed masses of fluid. To describe the fluid flow where there is a relative motion, we need to follow many particles and to resolve details of the flow; we need a large number of particles. Therefore, Lagrangian method is very difficult and not widely used in Fluid Mechanics.

Eulerian method

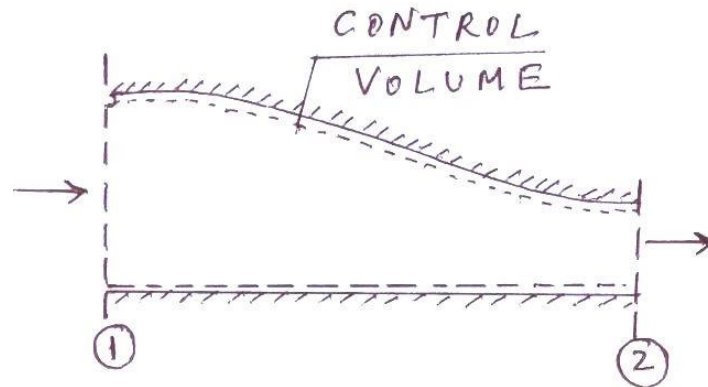


Fig. Eulerian Method

The velocity, acceleration, pressure etc. are described at a point or at a section as a function of time. This method is commonly used in Fluid Mechanics. We look for field description, for Ex.; seek the velocity and its variation with time at each and every location in a flow field. Ex., $V = V(x, y, z, t)$. This is also called control volume approach. We draw an imaginary box around a fluid system. The box can be large or small, and it can be stationary or in motion.

Types of fluid flow

1. Steady and Un-steady flows
2. Uniform and Non-uniform flows
3. Laminar and Turbulent flows
4. Compressible and Incompressible flows
5. Rotational and Irrotational flows
6. One, Two and Three dimensional flows

1. Steady and unsteady flow

Steady flow is the type of flow in which the various flow parameters and fluid properties at any point do not change with time. In a steady flow, any property may vary from point to point in the field, but all properties remain constant with time at every point. $[\partial V / \partial t]_{x,y,z} = 0$; $[\partial p / \partial t]_{x,y,z} = 0$. Ex.: $V = V(x,y,z)$; $p = p(x,y,z)$. Time is a criterion.

Unsteady flow is the type of flow in which the various flow parameters and fluid properties at any point change with time. $[\partial V / \partial t]_{x,y,z} \neq 0$; $[\partial p / \partial t]_{x,y,z} \neq 0$. Eg.: $V = V(x,y,z,t)$, $p = p(x,y,z,t)$ or $V = V(t)$, $p = p(t)$. Time is a criterion.

2. Uniform and non-uniform flows

Uniform Flow is the type of flow in which velocity and other flow parameters at any instant of time do not change with respect to space. Eg., $V = V(x)$ indicates that the flow is uniform in 'y' and 'z' axis. $V = V(t)$ indicates that the flow is uniform in 'x', 'y' and 'z' directions. Space is a criterion.

Uniform flow field is used to describe a flow in which the magnitude and direction of the velocity vector are constant, i.e., independent of all space coordinates throughout the entire flow field (as opposed to uniform flow at a cross section). That is, $[\partial V / \partial s]_{t=\text{constant}} = 0$, that is 'V' has unique value in entire flow field.

Non-uniform flow is the type of flow in which velocity and other flow parameters at any instant change with respect to space.

$[\partial V / \partial s]_{t=\text{constant}}$ is not equal to zero. Distance or space is a criterion.

3. Laminar and turbulent flows

Laminar Flow is a type of flow in which the fluid particles move along well-defined paths or stream-lines. The fluid particles move in laminas or layers gliding smoothly over one another. The behavior of fluid particles in motion is a criterion.

Turbulent Flow is a type of flow in which the fluid particles move in zigzag way in the flow field. Fluid particles move randomly from one layer to another. Reynolds number is a criterion. We can assume that for a flow in pipe, for Reynolds No. less than 2000, the flow is laminar; between 2000-4000, the flow is transitional; and greater than 4000, the flow is turbulent.

4. Compressible and incompressible flows

Incompressible Flow is a type of flow in which the density (ρ) is constant in the flow field. This assumption is valid for flow Mach numbers within 0.25. Mach number

is used as a criterion. Mach Number is the ratio of flow velocity to velocity of sound waves in the fluid medium

Compressible Flow is the type of flow in which the density of the fluid changes in the flow field. Density is not constant in the flow field. Classification of flow based on Mach number is given below:

$M < 0.25$ – Low speed M

$< \text{unity}$ – Subsonic

$M \text{ around unity}$ – Transonic M

$> \text{unity}$ – Supersonic

$M \gg \text{unity}$, (say 7) – Hypersonic

5. Rotational and irrotational flows

Rotational flow is the type of flow in which the fluid particles while flowing along stream-lines also rotate about their own axis.

Ir-rotational flow is the type of flow in which the fluid particles while flowing along stream-lines do not rotate about their own axis.

6. One, two and three dimensional flows

The number of space dimensions needed to define the flow field completely governs dimensionality of flow field. Flow is classified as one, two and three-dimensional depending upon the number of space co-ordinates required to specify the velocity fields.

One-dimensional flow is the type of flow in which flow parameters such as velocity is a function of time and one space coordinate only.

For Ex., $V = V(x, t)$ – 1-D, unsteady; $V = V(x)$ – 1-D, steady

Two-dimensional flow is the type of flow in which flow parameters describing the flow vary in two space coordinates and time.

For Ex., $V = V(x, y, t)$ – 2-D, unsteady; $V = V(x, y)$ – 2-D, steady

Three-dimensional flow is the type of flow in which the flow parameters describing the flow vary in three space coordinates and time.

For Ex., $V = V(x, y, z, t)$ – 3-D, unsteady; $V = V(x, y, z)$ – 3-D, steady

Flow patterns

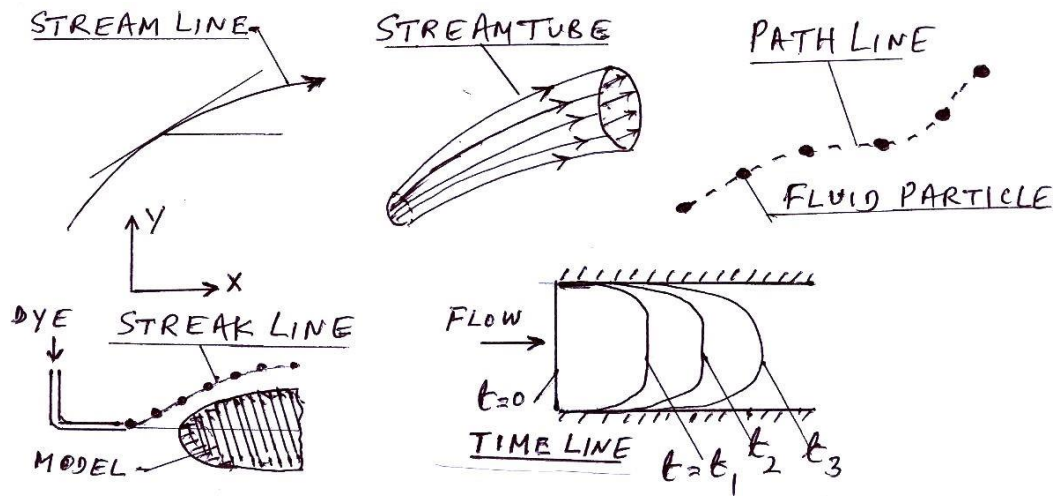


Fig.FlowPatterns

Fluid Mechanics is a visual subject. Patterns of flow can be visualized in several ways. Basic types of line patterns used to visualize flow are streamline, path line, streak line and time line.

- (a) **Stream line** is a line, which is everywhere tangent to the velocity vector at a given instant.
- (b) **Pathline** is the actual path traversed by a given particle.
- (c) **Streakline** is the locus of particles that have earlier passed through a prescribed point.
- (d) **Timeline** is a set of fluid particles that form a line at a given instant.

Streamline is convenient to calculate mathematically. Other three lines are easier to obtain experimentally. Streamlines are difficult to generate experimentally. Streamlines and Time lines are instantaneous lines. Path lines and streak lines are generated by passage of time. In a steady flow situation, streamlines, path lines and streak lines are identical. In Fluid Mechanics, the most common mathematical result for flow visualization is the streamline pattern – It is a common method of flow pattern presentation.

Streamlines are everywhere tangent to the local velocity vector. For a streamline, $(dx/u) = (dy/v) = (dz/w)$. Stream tube is formed by a closed collection of streamlines. Fluid within the stream tube is confined there because flow cannot cross streamlines. Stream tube walls need not be solid, but may be fluid surfaces

Continuity Equation

Rate of flow or discharge (Q) is the volume of fluid flowing per second. For incompressible fluids flowing across a section,

Volume flow rate, $Q = A \times V \text{ m}^3/\text{s}$ where A = cross sectional area and V = average velocity.

For compressible fluids, rate of flow is expressed as mass of fluid flowing across a section per second.

Mass flow rate (m) = $(\rho AV) \text{ kg/s}$ where ρ = density.

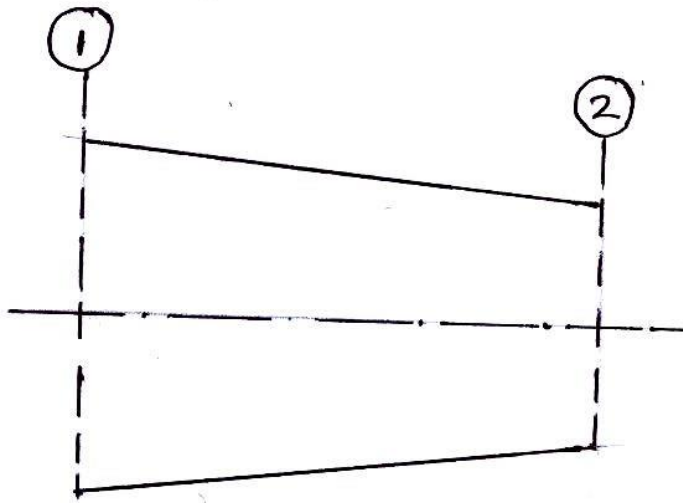


Fig. Continuity Equation

Continuity equation is based on Law of Conservation of Mass. For a fluid flowing through a pipe, in a steady flow, the quantity of fluid flowing per second at all cross-sections is a constant.

Let v_1 = average velocity at section [1],

ρ_1 = density of fluid at [1],

A_1 = area of flow at [1];

Let v_2, ρ_2, A_2 be corresponding values at section [2].

Rate of flow at section [1] = $\rho_1 A_1 v_1$

Rate of flow at section [2] = $\rho_2 A_2 v_2$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This equation is applicable to steady compressible or incompressible fluid flows and is called Continuity Equation. If the fluid is incompressible, $\rho_1 = \rho_2$ and the continuity equation reduces to $A_1 v_1 = A_2 v_2$

For steady, one-dimensional flow with one inlet and one outlet,

$$\rho_1 A_1 v_1 - \rho_2 A_2 v_2 = 0$$

For control volume with N inlets and outlets

$$\sum_{i=1}^N (\rho_i A_i v_i) = 0 \text{ where inflows are positive and outflows are negative.}$$

Velocities are normal to the areas. This is the continuity equation for steady one-dimensional flow through a fixed control volume

When density is constant, $\sum_{i=1}^N (A_i v_i) = 0$

Continuity equation in 3-dimensions

(Differential form, Cartesian co-ordinates)

Consider infinitesimal control volume as shown of dimensions dx , dy and dz in x , y , and z directions

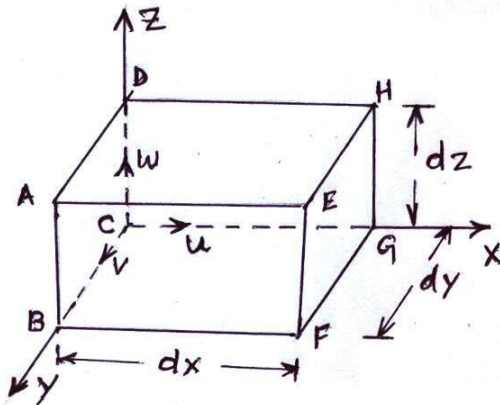


Fig. Continuity Equation in Three Dimensions

u, v, w are the velocities in x, y, z directions.

Mass of fluid entering the face ABCD =

Density $\times$ velocity in x -direction $\times$ Area ABCD = $\rho u dy dz$

Mass of fluid leaving the face EFGH = $\rho u dy dz + \left[\frac{\partial(\rho u dy dz)}{\partial x} \right] (dx)$

Therefore, net rate of mass efflux in x -direction = $-\left[\frac{\partial(\rho u dy dz)}{\partial x} \right] (dx) =$

$-\left[\frac{\partial(\rho u)}{\partial x} \right] (dx dy dz)$

Similarly, the net rate of mass efflux in y -

direction = $-\left[\frac{\partial(\rho v)}{\partial y} \right] (dx dy dz)$

direction = $-\left[\frac{\partial(\rho w)}{\partial z} \right] (dx dy dz)$

The rate of accumulation of mass within the control volume = $\frac{\partial(\rho dV)}{\partial t} = \rho \frac{\partial}{\partial t} (dV)$

where dV = Volume of the element = $dx dy dz$ and dV is invariant with time.

From conservation of mass, the net rate of efflux = Rate of accumulation of mass within the control volume.

$$-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right](dx dy dz) = \rho \frac{\partial}{\partial t}(dx dy dz) \text{ OR } \rho \frac{\partial}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

This is the continuity equation applicable for

- (a) Steady and unsteady flows
- (b) Uniform and non-uniform flows
- (c) Compressible and incompressible flows.

For steady flows, $(\partial/\partial t) = 0$ and $[\partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho w)/\partial z] = 0$. If the fluid is incompressible, $\rho = \text{constant}$ $[\partial u/\partial x + \partial v/\partial y + \partial w/\partial z] = 0$. This is the continuity equation for 3-D flows.

For 2-D flows, $w = 0$ and $[\partial u/\partial x + \partial v/\partial y] = 0$

Velocity potential and stream function

Velocity Potential Function is a Scalar Function of space and time co-ordinates such that its negative derivatives with respect to any direction give the fluid velocity in that direction.

$\Phi = \Phi(x, y, z)$ for steady flow.

$u = -(\partial\Phi/\partial x)$; $v = -(\partial\Phi/\partial y)$; $w = -(\partial\Phi/\partial z)$ where u, v, w are the components of velocity in x, y and z directions.

In cylindrical co-ordinates, the velocity potential function is given by $u_r = (\partial\Phi/\partial r)$, $u_\theta = (1/r)(\partial\Phi/\partial\theta)$

The continuity equation for an incompressible flow in steady state is $(\partial u/\partial x + \partial v/\partial y + \partial w/\partial z) = 0$

Substituting for u, v and w and simplifying, $(\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2) = 0$

Which is a Laplace Equation. For 2-D Flow, $(\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2) = 0$

If any function satisfies Laplace equation, it corresponds to some case of steady incompressible fluid flow.

Irrotational flow and velocity potential

Assumption of Ir-rotational flow leads to the existence of velocity potential. Consider the rotation of the fluid particle about an axis parallel to z-axis. The rotation component is defined as the average angular velocity of two infinitesimal linear segments that are mutually perpendicular to each other and to the axis of rotation.

Consider two line segments $\delta x, \delta y$. The particle at $P(x, y)$ has velocity components u, v in the x-y plane.

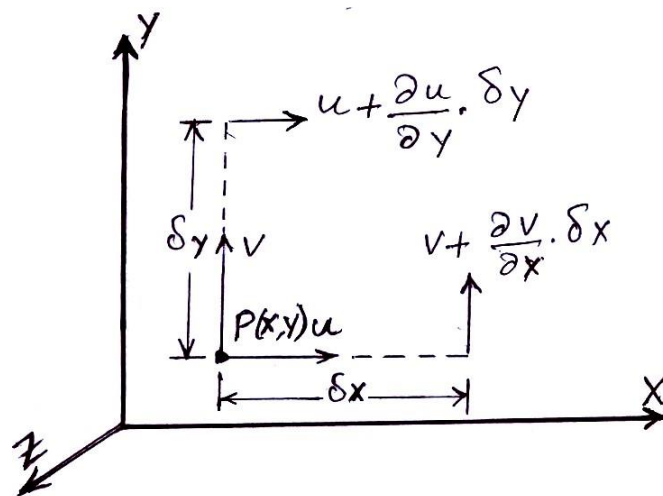


Fig. Rotation of a fluid particle.

The angular velocities of δx and δy are sought.

The angular velocity of (δx) is $\{[v + (\partial v / \partial x) \delta x - v] / \delta x\} = (\partial v / \partial x)$ rad/sec

The angular velocity of (δy) is $\{-[u + (\partial u / \partial y) \delta y - u] / \delta y\} = -(\partial u / \partial y)$ rad/sec

Counter clockwise direction is taken positive. Hence, by definition, rotation component (ω_z) is $\omega_z = 1/2 \{(\partial v / \partial x) - (\partial u / \partial y)\}$. The other two components are $\omega_x =$

$1/2 \{(\partial w / \partial y) -$

$(\partial v / \partial z)\}$ $\omega_y = 1/2$

$\{(\partial u / \partial z) - (\partial w / \partial x)\}$

The rotation vector $\omega = i\omega_x + j\omega_y + k\omega_z$.

The vorticity vector (Ω) is defined as twice the rotation vector $= 2\omega$

Properties of potential function

$$\omega_z = 1/2 \{ (\partial v / \partial x) - (\partial u / \partial y) \}$$

$$\omega_x = 1/2 \{ (\partial w / \partial y) - (\partial v / \partial z) \}$$

$$\omega_y = 1/2 \{ (\partial u / \partial z) - (\partial w / \partial x) \};$$

Substituting $u = -(\partial \Phi / \partial x)$;

$$v = -(\partial \Phi / \partial y);$$

$$w = -(\partial \Phi / \partial z);$$

$$\text{we get } \omega_z = 1/2 \{ (\partial / \partial x)(-\partial \Phi / \partial y) - (\partial / \partial y)(-\partial \Phi / \partial x) \}$$

$$= 1/2 \{ -(\partial^2 \Phi / \partial x \partial y) + (\partial^2 \Phi / \partial y \partial x) \} = 0 \text{ since } \Phi \text{ is a continuous function.}$$

Similarly, $\omega_x = 0$ and $\omega_y = 0$

All rotational components are zero and the flow is irrotational. – Therefore, irrotational flow is also called as Potential Flow.

If the velocity potential (Φ) exists, the flow should be irrotational. If velocity potential function satisfies Laplace Equation, It represents the possible case of steady, incompressible, irrotational flow. Assumption of a velocity potential is equivalent to the assumption of irrotational flow.

Laplace equation has several solutions depending upon boundary conditions. If

Φ_1 and Φ_2 are both solutions, $\Phi_1 + \Phi_2$ is also a solution

$$\nabla^2(\Phi_1) = 0, \nabla^2(\Phi_2) = 0, \nabla^2(\Phi_1 + \Phi_2) = 0$$

Also if Φ_1 is a solution, $C\Phi_1$ is also a solution (where $C = \text{Constant}$)

Stream function (ψ)

Stream Function is defined as the scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. Stream function is defined only for two dimensional flows and 3-D flows with axial symmetry.

$$(\partial \psi / \partial x) = v;$$

$$(\partial \psi / \partial y) = -u$$

In Cylindrical coordinates

$$u_r = (1/r) (\partial \psi / \partial \theta) \text{ and}$$

$$u_\theta = -(\partial \psi / \partial r)$$

Continuity equation for 2-D flow is $(\partial u / \partial x) + (\partial v / \partial y)$

$$= 0 \Rightarrow (\partial / \partial x)(-\partial \psi / \partial y) + (\partial / \partial y)(\partial \psi / \partial x) = 0$$

$$-(\partial^2 \psi / \partial x \partial y) + (\partial^2 \psi / \partial y \partial x) = 0;$$

Therefore, continuity equation is satisfied. Hence, the existence of (ψ) means a possible case of fluid flow. The flow may be rotational or irrotational. The rotational component are:

$$\omega_z = 1/2 \{(\partial v / \partial x) - (\partial u / \partial y)\}$$

$$\omega_z = 1/2 \{(\partial / \partial x)(\partial \psi / \partial x) - (\partial / \partial y)(-\partial \psi / \partial y)\} \quad \omega_z = 1/2 \{(\partial^2 \psi / \partial x^2) + (\partial^2 \psi / \partial y^2)\}$$

For irrotational flow, $\omega_z = 0$. Hence for 2-D flow, $(\partial^2 \psi / \partial x^2) + (\partial^2 \psi / \partial y^2) = 0$ which is a Laplace equation.

Properties of stream function

1. If the Stream Function (ψ) exists, it is a possible case of fluid flow, which may be rotational or irrotational.
2. If Stream Function satisfies Laplace Equation, it is a possible case of an irrotational flow.

Equi-potential & constant stream function lines

On an equi-potential line, the velocity potential is constant, $\Phi = \text{constant}$ or $d(\Phi) = 0$. $\Phi = \Phi(x, y)$ for steady flow.

$$d(\Phi) = (\partial \Phi / \partial x) dx + (\partial \Phi / \partial y) dy.$$

$$d(\Phi) = -u dx - v dy = -(u dx + v dy) = 0. \text{ For equi-potential line, } u dx + v dy = 0$$

Therefore, $(dy/dx) = -(u/v)$ which is a slope of equi-potential lines. For lines of constant stream Function,

$$\psi = \text{Constant or } d(\psi) = 0, \psi = \psi(x, y)$$

$$d(\psi) = (\partial \psi / \partial x) dx + (\partial \psi / \partial y) dy = v dx - u dy$$

$$\text{Since } (\partial \psi / \partial x) = v; (\partial \psi / \partial y) = -u$$

Therefore, $(dy/dx) = (v/u) = \text{slope of the constant stream function line}$. This is the slope of the stream line.

The product of the slope of the equi-potential line and the slope of the constant stream function line (or stream Line) at the point of intersection = -1.

Thus, equi-potential lines and stream lines are orthogonal at all points of intersection.

Flownet

A grid obtained by drawing a series of equi-potential lines and streamlines is called a FlowNet. A FlowNet is an important tool in analyzing two-dimensional ir-rotational flow problems.

Examples of flownets

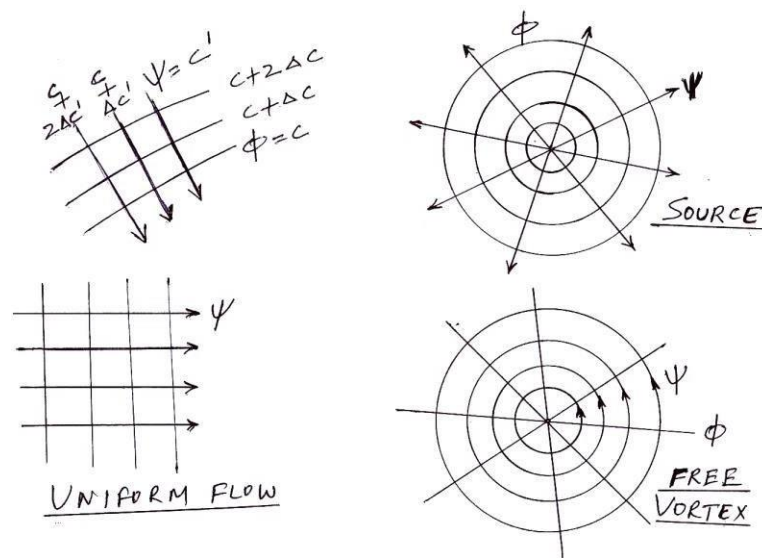


Fig. Flow Nets

Examples: Uniform flow, Line source and sink, Line vortex

Two-dimensional doublet – a limiting case of a line source approaching a line sink

Relationship between stream function and velocity Potential

$$u = -(\partial\Phi/\partial x), v = -(\partial\Phi/\partial y)$$

$$u = -(\partial\psi/\partial y), v = (\partial\psi/\partial x); \text{ Therefore, -}$$

$$(\partial\Phi/\partial x) = -(\partial\psi/\partial y) \text{ and } -(\partial\Phi/\partial y) = (\partial\psi/\partial x)$$

$$\text{Hence, } (\partial\Phi/\partial x) = (\partial\psi/\partial y) \text{ and } (\partial\Phi/\partial y) = -(\partial\psi/\partial x)$$

Types of motion

A fluid particle while moving in a fluid may undergo any one or a combination of the following four types of displacements:

1. Linear or pure translation
2. Linear deformation
3. Angular deformation
4. Rotation.

(1) Linear Translation is defined as the movement of fluid element in which fluid element moves from one position to another bodily – Two axes ab & cd and $a'b'$ & $c'd'$ are parallel (Fig. 8a)

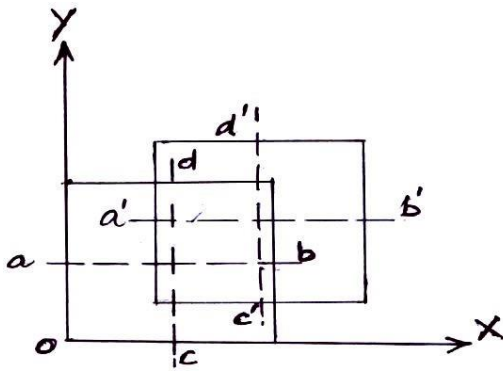


Fig. Linear translation.

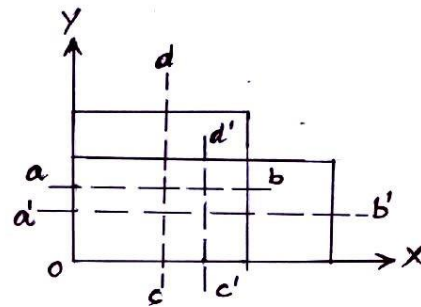


Fig. Linear deformation.

(2) Linear deformation is defined as deformation of fluid element in linear direction – axes are parallel, but length changes.

(2) Angular deformation, also called shear deformation is defined as the average change in the angle contained by two adjacent sides. The angular deformation or shear strain rate $= \frac{1}{2}(\Delta\theta_1 + \Delta\theta_2) = \frac{1}{2}(\partial v/\partial x + \partial u/\partial y)$

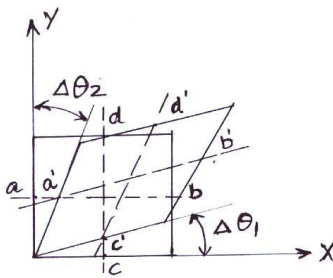


Fig Angular deformation

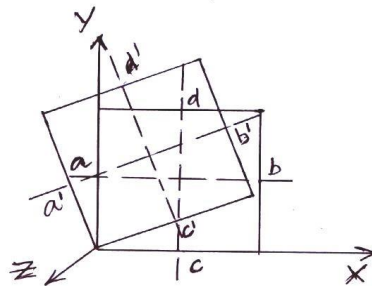


Fig. Rotation

(4) Rotation is defined as the movement of the fluid element in such a way that both its axes (horizontal as well as vertical) rotate in the same direction. Rotational components are:

FluidMechanics**Unit-III****FluidDynamics**

- Fluid Dynamics is the branch of fluid science that deals with the study of fluid motion and the associated external forces and internal resistances.
- Based on Newton's second law, it is possible to derive certain differential equations, called equations of motion or momentum conservation, which completely define the Dynamic behavior of the body.

TYPESOFFORCESINFLUID MOTION:

The different forces acting on the fluid mass in motion may be classified as follows;

- Body forces or Volume forces (Weights, Gravity Force F_g , compressibility or elastic force F_e)
- Surface forces (Pressure forces F_p , Shear Forces, viscous force F_v , Turbulent Force F_t)
- Linear Forces (Capillary or surface tension forces F_s)

If a certain mass of fluid in motion is influenced by all the above mentioned forces, then according to Newton's second law, the following equations of motion can be written.

$$M \times a = F_g + F_e + F_p + F_v + F_t + F_s \text{----- eq.(1)}$$

Where M = Mass of the fluid in flow a =

Acceleration of the flow

Further by resolving the various forces in x, y & z directions, we get $M \times a_x$

$$= F_{gx} + F_{ex} + F_{px} + F_{vx} + F_{tx} + F_{sx}$$

$$M \times a_y = F_{gy} + F_{ey} + F_{py} + F_{vy} + F_{ty} + F_{sy} \quad M \times a_z$$

$$= F_{gz} + F_{ez} + F_{pz} + F_{vz} + F_{tz} + F_{sz}$$

Reynold's equation of motion:

In most of the problems of fluids in motion, the surface tension forces and compressibility forces are not significant. Hence these forces can be neglected and eq. (1) becomes

$$M \times a = F_g + F_p + F_v + F_t \text{----- eq.(2)}$$

The above eq is known as Reynold's equation of motion.

Navier–stokes equation of motion:

Further for laminar or viscous flows, turbulent forces also become less significant. Hence eq. (2) becomes

$$M \times a = F_g + F_p + F_v \quad \text{----- eq.(3)}$$

The above eq is known as Navier – stokes equation of motion.

Euler's equation of motion:

Further if the viscous forces are also of little significance, then these forces may also be neglected. Hence eq. (3) becomes

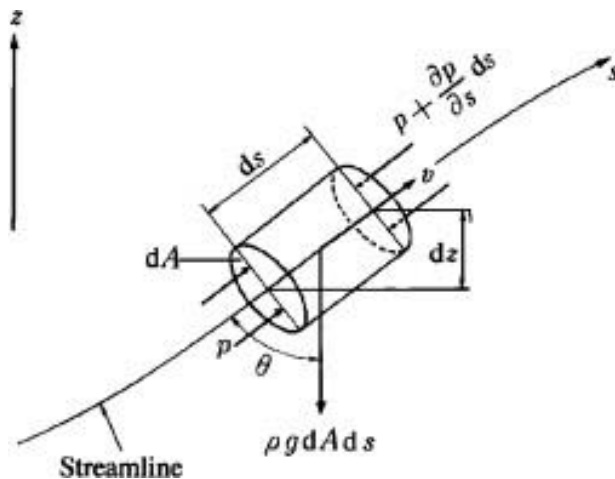
$$M \times a = F_g + F_p \quad \text{----- eq.(4)}$$

The above equation is known as Euler's equation of motion.

EULER'S EQUATION FOR 1-D FLOW:

General flows are three dimensional, but many of them may be studied as if they are one dimensional. For example, whenever a flow in a tube is considered, if it is studied in terms of mean velocity, it is a one-dimensional flow which is studied very simply. Let us apply the principle of conservation of momentum to the infinitesimal, cylindrical element of fluid having the cross-sectional area dA and length ds which lay along the streamline

- We also assume that the flow is steady, it is that all local time derivatives are equal zero, $\partial v / \partial t = 0$.
- We also assume that flow is inviscid (ideal).



Around a cylindrical element of fluid having the cross-sectional area ' dA ' and length ' ds ' is considered.

- Let p be the pressure acting on the lower face, and pressure $\{p + dp\}$ acts on the upper face a distance 'ds' away.
- The gravitational force acting on this element is its weight, $[p.g.dA.ds]$. Applying Newton's second law of motion to this element, the resultant force acting on it, and producing acceleration along the streamline is the force due to the pressure difference across the streamline and the component of any other external force (in this case only the gravitational force) along the streamline. Therefore the following equation is obtained:

- We know that the external force tending to accelerate the fluid element in the direction of the streamline

$$\begin{aligned}
 &= P.dA - (P + dP)dA \\
 &= -dP.dA
 \end{aligned}$$

• (1)

- We also know that the weight of the fluid element,

$$dW = \rho g.dA.ds$$
- From the geometry of the figure, we find that the component of the weight of the fluid element in the direction of flow,

$$\begin{aligned}
 &= \rho g.dA.ds \cos \theta \\
 &= \rho g.dA.ds \frac{dz}{ds} \\
 &= \rho g.dA.dz
 \end{aligned}$$

• (2)

- $\therefore$ Mass of the fluid element $= \rho.dA.ds$
- We see that the acceleration of the fluid element

$$\frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v \frac{dv}{ds}$$

• (3)

- Now, as per Newton's second law of motion, we know that Force = Mass * Acceleration

$$\Rightarrow (-dp.dA) - (\rho g.dA.dz) = \rho.dA.ds \times v \frac{dv}{ds}$$

- Dividing both sides by $(-\rho dA)$

$$\Rightarrow \frac{dP}{\rho} + g.dz = -v.dv$$

- or,

$$\Rightarrow \frac{dP}{\rho} + g.dz + v.dv = 0$$

• (4)

- This is the required Euler's equation for motion as in the form of a differential equation.

BERNOLLI'S EQUATION FROM EULER'S EQUATION

- Integrating the above equation,

$$\frac{1}{\rho} \int dP + \int g \cdot dz + v \cdot dv = \text{Constant}$$

$$\Rightarrow \frac{P}{\rho} + gz + \frac{v^2}{2} = \text{Constant}$$

$$\Rightarrow P + wz + \frac{wv^2}{2g} = \text{Constant}$$

$$\Rightarrow \frac{P}{w} + Z + \frac{v^2}{2g} = \text{Constant}$$

- or in other words,

$$\frac{P_1}{w_1} + Z_1 + \frac{v_1^2}{2g} = \frac{P_2}{w_2} + Z_2 + \frac{v_2^2}{2g}$$

MOMENTUM EQUATION:

It is based on law of conservation of momentum, which states that the net force acting on the fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on fluid mass 'm' is given by Newton's second law

$$F = m \times a$$

$$\text{But } a = dv/dt$$

$$\text{Therefore } F = m [dv/dt]$$

$$\Rightarrow F = [d(m \cdot v)/dt] \text{ ----- (1) momentum principle}$$

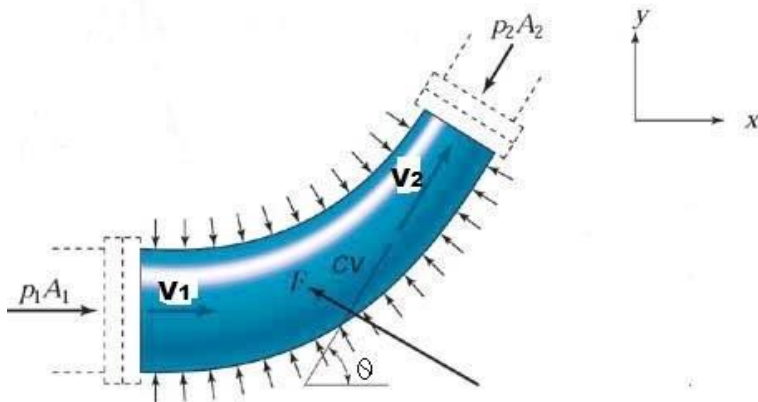
$$\Rightarrow F \times dt = d(m \cdot v) \text{ ----- (2) Impulse-momentum equation}$$

Impulse momentum equation states that the impulse of a force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum d (m.v) in the direction of force.

Application:

The Impulse momentum equation is used to determine the resultant force exerted by flowing fluid on a pipe bend.

Consider a pipe bend with two sections 1 and 2 as shown in figure.



Let V_1 and V_2 be the velocities at inlet and outlet respectively. P_1 and P_2 be the pressures at inlet and outlet respectively.

The continuity equation yields $\rho V_1 A_1 = \rho V_2 A_2 = \dot{m}$

Carrying out a force balance in x-direction, we have

$$\begin{aligned} p_1 A_1 - p_2 A_2 \cos \theta + F_x &= \rho V_2 A_2 V_2 \cos \theta - \rho V_1 A_1 V_1 \\ &= \dot{m} V_2 \cos \theta - \dot{m} V_1 = \dot{m} (V_2 \cos \theta - V_1) \end{aligned}$$

In the y-direction we have,

$$0 - p_2 A_2 \sin \theta + F_y = \rho V_2 A_2 V_2 \sin \theta - 0$$

$$\text{giving} \quad p_2 A_2 \sin \theta + F_y = \dot{m} V_2 \sin \theta$$

Thus the force components acting on the bend are

$$F_x = p_2 A_2 \cos \theta - p_1 A_1 - \dot{m} (V_2 \cos \theta - V_1)$$

$$F_y = -p_2 A_2 \sin \theta + \dot{m} V_2 \sin \theta$$

