DEPARTMENT OF MATHEMATICS & SC

ENGINEERING MATHEMATICS

DIPL OMA

LEARNING MATERIAL

ON

VECTOR

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Unit-1 – Vector

Note:-

Q.1. **What is vector?**

> A physical quanity with specified magnitude as well as definite direction.

2. **How vector Represented ?**

> A vector generally represented by a directed line segment, say $\vec{AB}$.

> 'A' is called initial point and B is called the terminal point.

3. **Zero vector**

> A vector of zero magnitude is a zero vector.
i.e. which has the same initial & terminal point,

> It is denoted as "$\vec{0}$".

> The direction of zero vector is indeterminate.
i.e. infinitely many direction

> zero vector is also called null vector.

4. **Unit Vector**

> vector of unit magnitude

> $\hat{i}, \hat{j}, \hat{k}$ are unit vector along x, y, & Z - axis.

> unit vector along ang vector $\vec{a}$ is denoted by $\hat{a}$. and $\boxed{\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}}$

5. **Equal Vector**

Two vector are said to be equal if they have the same magnitude, direction & represent the same physical quanity.

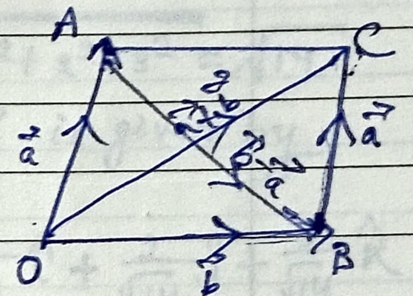
6. **Position vector of a point**

> Let 'O' be a fixed origin, then the position vector of a point P is the vector $\overrightarrow{OP}$.

> If $\vec{a}$ & $\vec{b}$ are position vectors of two point A and B, then $\overrightarrow{AB} = \vec{b} - \vec{a}$.

7. **Addition of vectors**

If two vector $\vec{a}$ and $\vec{b}$ are represented by $\overrightarrow{OA}$ & $\overrightarrow{OB}$, then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB



properties

> $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ (commutative)

> $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$ (associative)

> $\vec{a}+\vec{0} = \vec{a} = \vec{0}+\vec{a}$

> $\vec{a}+(-\vec{a}) = \vec{0} = (-\vec{a})+\vec{a}$

8. **multiplication of a vector by a scalar**

If $\vec{a}$ is a vector & m is a scalar; then $m\vec{a}$ is a vector parallel to $\vec{a}$ whose modulus is $|m|$ times than $\vec{a}$, this multiplication is called scalar multiplication. If $\vec{a}$ & $\vec{b}$ are vector & m, n are scalar, then,

$m(\vec{a}) = (\vec{a})m = m\vec{a}$

$m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$

$(m+n)\vec{a} = m\vec{a} + n\vec{a}$

$m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$

9. **Representation of vector in component form**

Let 'O' be the origin and let $P(x, y, z)$ be any point in space. Let $\hat{i}, \hat{j}, \hat{k}$ be unit vectors along x, y, & z-axis respectively. Let the position vector of P be $\vec{r}$, then $\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$

x, y, z, are called scalar components of $\vec{r}$

$x\hat{i}, y\hat{j}, z\hat{k}$ are called its vector components.

10. **Magnitude of vector**

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Then $|\vec{r}| = \sqrt{x^2+y^2+z^2}$

Solved Example

example-1

Find a unit vector in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$.

solution :- $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \Rightarrow |\vec{a}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

Unit vector in the direction of $\vec{a}$ is given by

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

Example-2

Find a unit vector in the direction of $\vec{AB}$, where A (1, 2, 3) and B (4, 5, 6) are given points.

solution :-

we have position vector of $A = \hat{i} + 2\hat{j} + 3\hat{k}$

position vector of $B = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$$\therefore \vec{AB} = (\text{P.V. of } B) - (\text{P.V. of } A)$$
$$= (4\hat{i} + 5\hat{j} + 6\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$
$$= 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$|\vec{AB}| = \sqrt{3^2+3^2+3^2} = \sqrt{27}$$

$\therefore$ Unit vector in the direction of $\vec{AB}$ = $\boxed{\frac{\vec{AB}}{|\vec{AB}|} = \hat{AB}}$

$$= \frac{3\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{27}} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{3\sqrt{3}} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$$

Example - 3

If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$, find a unit vector in the direction of $\vec{a} + \vec{b}$.

Solution :- Let $\vec{c} = \vec{a} + \vec{b}$, then,

$$\vec{c} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (2\hat{i} + 3\hat{j} + \hat{k})$$
$$= (3\hat{i} + 5\hat{j} - 2\hat{k})$$

$$|\vec{c}| = \sqrt{3^2 + 5^2 + (-2)^2} = \sqrt{38}$$

Hence, the required vector is given by

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{i} + 5\hat{j} - 2\hat{k}}{\sqrt{38}} = \frac{3}{\sqrt{38}}\hat{i} + \frac{5}{\sqrt{38}}\hat{j} - \frac{2}{\sqrt{38}}\hat{k}$$

Assignment - 1

Q1. Write down the magnitude and a unit vector in the direction of the following vectors.

(i) $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$ (ii) $\vec{b} = 5\hat{i} - 4\hat{j} - 3\hat{k}$

Q2. Show that the point A, B, C with position vector $\vec{a} = (3\hat{i} - 4\hat{j} - 4\hat{k})$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$ respectively form the vertices of a right-angled triangle.

Q3. Find the unit vector in the direction of the vector $\vec{a} + 2b - c$ if $\vec{a} = \hat{i} - 3\hat{j} - 5\hat{k}$, $\vec{b} = -3\hat{i} + 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + \hat{k}$.

11. **Scalar Product of vectors** (Dot product)

> Let $\vec{a}$ and $\vec{b}$ be two vectors and let θ be the angle between them, then the scalar product, or dot product, of $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}\cdot\vec{b}$, is defined as $\boxed{\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta}$.

N.B.. the scalar product of two vectors is a scalar.

12. > **Angle between two vectors in terms of scalar product**

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}\right)}$$

13. N.B.- (i) If $\vec{a}$ and $\vec{b}$ are like vectors $\theta = 0$

$$\vec{a}\cdot\vec{b} = ab\cos 0 = ab \quad (\cos 0^{\circ} = 1)$$

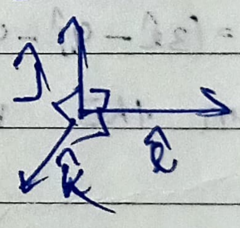
$$\boxed{\vec{a}\cdot\vec{a} = a^2}$$

(ii) If $\boxed{\vec{a} \perp \vec{b}}$, $\theta = 90$ $\Rightarrow \boxed{\vec{a}\cdot\vec{b} = 0}$

$$\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{b}\cos 90 = 0$$

(iii) $\boxed{\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0}$

$$\boxed{\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1}$$



14. > **Working Rule**

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{a}\cdot\vec{b} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\cdot(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$

$$= a_1\hat{i}\cdot(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) + a_2\hat{j}\cdot(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) + a_3\hat{k}\cdot(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$

$$= a_1 b_1\,\hat{i}\cdot\hat{i} + a_2 b_2\,\hat{j}\cdot\hat{j} + a_3 b_3\,\hat{k}\cdot\hat{k}$$

$$\boxed{\vec{a}\cdot\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3}$$

Examples

Q1. Find the angle between the vectors $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$?

Solution :-

$$|\vec{a}| = \sqrt{3^2 + (-2)^2 + (1)^2} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{(1)^2 + (-2)^2 + (3)^2} = \sqrt{14}$$

Now $\vec{a}\cdot\vec{b} = 3\cdot 1 + (-2)(-2) + (1)(3)$

$= 3 + 4 + 3 = 10$

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right) = \cos^{-1}\left(\frac{10}{\sqrt{14}\sqrt{14}}\right) = \cos^{-1}\left(\frac{10}{14}\right)$$

$$\Rightarrow \boxed{\theta = \cos^{-1}\frac{5}{7}}$$

Q2. Find the value of λ for which the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} + \lambda\hat{j} - 3\hat{k}$ are $\perp$ to each other.

Solution :- $\vec{a} \perp \vec{b} \Rightarrow \vec{a}\cdot\vec{b} = 0$

$\Rightarrow (3\hat{i} + \hat{j} - 2\hat{k})\cdot(\hat{i} + \lambda\hat{j} - 3\hat{k}) = 0$

$\Rightarrow 3\times 1 + 1\times\lambda + (-2)(-3) = 0$

$\Rightarrow 3 + \lambda + 6 = 0$

$\Rightarrow \lambda + 9 = 0$

$\Rightarrow \boxed{\lambda = -9}$

15. Scalar project and vector projection

(i) Scalar projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ (scalar quanity)

(ii) vector projection of $\vec{a}$ on $\vec{b} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

N.B: $[(S.P) \text{ of } \vec{a} \text{ on } \vec{b}] \times \hat{b} = V.P \text{ of } \vec{a} \text{ on } \vec{b}$

into

Example 1·1

Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

Solution :- scalar projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$\vec{a}\cdot\vec{b} = (2\hat{i}+3\hat{j}+2\hat{k})(\hat{i}+2\hat{j}+\hat{k})$$
$$= 2\times1 + 3\times2 + 2\times1$$
$$= 2+6+2$$
$$= 10$$

$$|\vec{b}| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$$

$$S.P = \frac{10}{\sqrt{6}}$$

vector projection of $\vec{a}$ on $\vec{b} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\hat{b}$

$$= \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$$
$$= \frac{10}{(\sqrt{6})^2}(\hat{i}+2\hat{j}+\hat{k})$$
$$= \frac{\cancel{10}}{\cancel{6}}(\hat{i}+2\hat{j}+\hat{k})$$
$$= \frac{5}{3}(\hat{i}+2\hat{j}+\hat{k})$$

vector projection of $\vec{a}$ on $\vec{b} = \dfrac{5\hat{i}}{3} + \dfrac{10}{3}\hat{j} + \dfrac{5\hat{k}}{3}$.

Assignment - 2

Q1. Find the value of λ for which $\vec{a}$ & $\vec{b}$ are perpendicular, where $\vec{a} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = -\lambda\hat{i} + 3\hat{j} + 3\hat{k}$.

Q2. Find the angle between the vector $\vec{a}$ and $\vec{b}$ when, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.

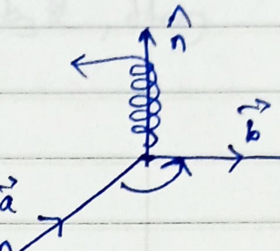
Q3- Let $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$, find the projection of (i) $\vec{a}$ on $\vec{b}$ and (ii) $\vec{b}$ on $\vec{a}$,

16. **Vector (cross) product:**

> Let $\vec{a}$ and $\vec{b}$ be two non-zero, non-parallel vectors, and let θ be the angle between them such that $0 < \theta < \pi$.

Then the vector product is defined as

- $\boxed{\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}}$.



>> $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, such that $\vec{a}, \vec{b}, \hat{n}$ form a right-handed system.

17. >> **Sine Angle between two vectors:**

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

$$\Rightarrow |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta \quad [\because |\hat{n}| = 1 \text{ as a unit vector}]$$

$$\Rightarrow \boxed{\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}}$$

18. **Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$**

$$\boxed{\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}}$$

19. **Properties**

① $\vec{b}\times\vec{a} = -(\vec{a}\times\vec{b})$ (not commutative)

② $m(\vec{a}\times\vec{b}) = (m\vec{a})\times\vec{b} = \vec{a}\times(m\vec{b})$

③ $\vec{a}\parallel\vec{b} \Rightarrow \boxed{\vec{a}\times\vec{b} = 0}$

- N.B. $\boxed{\begin{array}{l}\hat{i}\times\hat{j} = \hat{j}\times\hat{k} = \hat{k}\times\hat{i} = 1 \\ \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0\end{array}}$

20. **Application of cross product:**

(i) Area of parallelogram with two adjacent sides $\vec{a}$ & $\vec{b}$ is given by $|\vec{a}\times\vec{b}|$ sq units

(ii) Area of a parallelogram with diagonal $\vec{u}$ & $\vec{v}$ = $\frac{1}{2}|\vec{u}\times\vec{v}|$ sq units

(iii) Area of $\triangle ABC$; where $\vec{AB} = \vec{c}$, $\vec{BC} = \vec{a}$, $\vec{CA} = \vec{b}$ is $\frac{1}{2}|\vec{a}\times\vec{b}|$ or $\frac{1}{2}|\vec{b}\times\vec{c}|$ or $\frac{1}{2}|\vec{c}\times\vec{a}|$ sq units.

21. $\boxed{\text{working Rule of vector product}}$ (in terms of components)

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$\boxed{\text{solved Examples}}$

Example-1 If $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$. find $\vec{a}\times\vec{b}$ and $|\vec{a}\times\vec{b}|$.

solution :-

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix}$$

$$= (-2+20)\hat{i} - (-6+24)\hat{j} + (15-6)\hat{k}$$

$$= 18\hat{i} - 18\hat{j} + 9\hat{k}.$$

$$|\vec{a}\times\vec{b}| = \sqrt{18^2 + (-18)^2 + 9^2} = \sqrt{729} = 27$$

Example-2 :- find a Unit vector $\perp^{r}$ to each one of the vectors $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$.

Solution :-

We know that $\vec{a}\times\vec{b}$ is a vector $\perp^{r}$ to each one of $\vec{a}$ and $\vec{b}$.

so Unit vector $\perp^{r}$ to both $\vec{a}$ & $\vec{b}$ is $\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$$\text{Now } \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ 2 & 2 & -1 \end{vmatrix} = (1-6)\hat{i} - (-4-6)\hat{j} + (8+2)\hat{k}$$

$$= -5\hat{i} + 10\hat{j} + 10\hat{k}.$$

$$|\vec{a}\times\vec{b}| = \sqrt{(-5)^2 + (10)^2 + (10)^2} = \sqrt{225} = 15$$

Hence the required unit vector $= \dfrac{-5\hat{i}+10\hat{j}+10\hat{k}}{15}$

$$= \frac{5}{15}(-\hat{i}+2\hat{j}+2\hat{k})$$
$$= \frac{1}{3}(-\hat{i}+2\hat{j}+2\hat{k})$$

Example-3

Find the sine angle between the vectors $\vec{a} = 2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b} = \hat{i}+3\hat{j}+2\hat{k}$.

Solution :- $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 3 & 2 \end{vmatrix} = (-2-9)\hat{i}-(4-3)\hat{j}+(6+1)\hat{k}$$
$$= -11\hat{i}-\hat{j}+7\hat{k}.$$

$$|\vec{a}\times\vec{b}| = \sqrt{(-11)^2+(-1)^2+7^2} = \sqrt{171} = 3\sqrt{19}$$

$$|\vec{a}| = \sqrt{2^2+(-1)^2+(3)^2} = \sqrt{14}$$
$$|\vec{b}| = \sqrt{1^2+3^2+2^2} = \sqrt{14}$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\because \sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{3\sqrt{19}}{\sqrt{14}\sqrt{14}} = \frac{3}{14}\sqrt{19} \text{ (Ans).}$$

Example-4 :-

Find the area of the parallelogram whose adjacent sides are represented by the vector $(3\hat{i}+\hat{j}-2\hat{k})$ and $\hat{i}-3\hat{j}+4\hat{k}$.

Solution :-

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = (4-6)\hat{i}-(12+2)\hat{j}+(-9-1)\hat{k}$$
$$= -2\hat{i}-14\hat{j}-10\hat{k}$$

Required area $= |\vec{a} \times \vec{b}|$

$= \sqrt{(-2)^2 + (-14)^2 + (-10)^2}$ sq units

$= \sqrt{300}$ sq units $= 10\sqrt{3}$ sq units.

Example-5 :-

Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d_1} = (2\hat{i} - \hat{j} + \hat{k})$ and $d_2 = (3\hat{i} + 4\hat{j} - \hat{k})$

solution:- Given that $\vec{d_1} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{d_2} = 3\hat{i} + 4\hat{j} - \hat{k}$.

vector area of parallelogram is $\frac{1}{2}(d_1 \times d_2)$

$$\text{Now} = (d_1 \times d_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$$

$= (1-4)\hat{i} - (-2-3)\hat{j} + (8+3)\hat{k}$.

$= -3\hat{i} + 5\hat{j} + 11\hat{k}$.

Required area $= \frac{1}{2}|d_1 \times d_2|$

$= \frac{1}{2}\sqrt{(-3)^2 + (5)^2 + (11)^2}$ sq units

$= \frac{1}{2}\sqrt{155}$ sq units.

Example-6 :-

Using vector method, find the area of the triangle whose vertices are $A(1,1,1)$, $B(1,2,3)$ and $C(2,3,1)$.

Solution :-

position vector of $A = \hat{i} + \hat{j} + \hat{k}$

position vector of $B = \hat{i} + 2\hat{j} + 3\hat{k}$.

position vector of $C = 2\hat{i} + 3\hat{j} + \hat{k}$.

$\therefore \vec{AB} = (\text{p.v of } B) - (\text{p.v of } A)$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (\hat{i} + \hat{j} + \hat{k}) = \hat{j} + 2\hat{k}$

And $\vec{AC}$ = (p. v of C) − (p. v of A)

$= (2\hat{i} + 3\hat{j} + \hat{k}) - (\hat{i} + \hat{j} + \hat{k})$

$= \hat{i} + 2\hat{j}$.

Area of $\Delta ABC = \frac{1}{2}|\vec{AB} \times \vec{AC}|$

Now $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix}$

$= (0-4)\hat{i} + (2-0)\hat{j} + (0-1)\hat{k} = -4\hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow \frac{1}{2}|(\vec{AB} \times \vec{AC})| = \frac{1}{2}|-4\hat{i} + 2\hat{j} - \hat{k}|$

$= \frac{1}{2}\sqrt{(-4)^2 + (2)^2 + (-1)^2}$

$= \frac{1}{2}\sqrt{16 + 4 + 1}$

$= \frac{1}{2}\sqrt{21} = \frac{\sqrt{21}}{2}$ sq units (Ans)

Assignment − 3

Q1. Find unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$, when.

(i) $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

(ii) $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$.

Q2. Find the area of the parallelogram whose adjacent sides are represented by the vectors.

(i) $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$.

(ii) $\vec{a} = 2\hat{i} + \hat{j}$ and $\vec{b} = \hat{j} - 3\hat{k}$.

Q3. Find the area of the parallelogram whose diagonals are represented by the vectors.

(i) $\vec{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$.

(ii) $\vec{d_1} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{d_2} = 3\hat{i} + 4\hat{j} - \hat{k}$.

Q4. Using vector method find the area of ΔABC, whose vertices are $A(3,-1,2)$, $B(1,-1,-3)$ and $C(4,-3,1)$.