LECTURE NOTE

CIRCUIT THEORY (TH2)

3RD SEM

ELECTRONICS & TELECOMMUNICATION ENGG.

PREPARED BY

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BOSE, CUTTACK

Th.2 CIRCUIT THEORY

(Common to ETC, AE&I)

Theory	: 4 Periods per week	I.A.	:20 Marks
Total Periods	: 60 Periods	Term End Exam	: 80 Marks
Examination	: 3 Hours	TOTAL MARKS	: 100 Marks

Chapter wise Distribution of periods with Total periods

Sl. No.	Topics	Periods
1.	CIRCUIT ELEMENTS & ENERGY SOURCES	06
2.	NETWORK THEOREMS (Applications in dc circuits)	12
3.	Power Relation in AC circuits & Transient Response of passive circuits(DC)	12
4.	RESONANCE AND COUPLED CIRCUITS	10
5.	LAPLACE TRANSFORM AND ITS APPLICATIONS	08
6.	Two Port Network Analysis	05
7.	FILTERS & ATTENUATORS	07
	TOTAL	60

Rationale:

A circuit may be called a DC circuit or AC circuit based on type of component and sources used. It play an important role in many diverse field of science and Engineering. Circuit laws provide a basis for analysing the circuit. The circuit analysis is the process of determining the values of the unknown quantities in a Dc or Ac circuit. The Circuit Theory will cover some the basics of electric circuit theory, circuit analysis, and will touch on circuit design. Topics covered include AC and DC circuits, passive circuit components, phasors, and RLC circuits. The focus is on students of an electrical/Electronic engineering under diploma program. Hobbyists would benefit more from reading Electronics instead.

Objective:

After completion of this course the student will be able to:

1. Classification of different Network elements.

2. Understanding Star & Delta connections and their interconnection.
3. Know different Network theorems.
4. Know the AC fundamentals.
5. Know Different Network Functions & Laplace Transform.
6. Know different filters and attenuations.
7. Know the Resonance & coupled circuits.

Detailed Contents:

Unit-1: CIRCUIT ELEMENTS& ENERGY SOURCES

1.1 Circuit elements (Resistance, Inductance, Capacitance), Scope of network analysis & synthesize
1.2 Voltage Division & Current Division, Energy Sources
1.3 Electric charge, electric current, Electrical energy, Electrical potential, R-L-C parameters, Active& Passive Elements.
1.4 Energy Sources, Current and voltage sources and their transformation & mutual inductance
1.5 Star – Delta transformation

Unit-2: NETWORK THEOREMS (Applications in dc circuits)

2.1 Nodal & Mesh Analysis of Electrical Circuits with simple problem.

2.2 Thevenin's Theorem, Norton's Theorem, Maximum Power transfer Theorem, Superposition Theorem, Millman Theorem, Reciprocity Theorem-Statement, Explanation & applications

2.3 Solve numerical problems of above.

Unit-3: Power Relation in AC circuits & Transient Response of passive circuits

3.1 Definition of frequency, Cycle, Time period, Amplitude, Average value, RMS value, Instantaneous power & Form factor, Apparent power, Reactive power, power Triangle of AC Wave.
3.2 Phasor representation of alternating quantities
3.3 Single phase Ac circuits-Behaviors of A.C. through pure Resistor, Inductor & Capacitor.
3.4 DC Transients-Behaviors of R-L, R-C, R-L-C series circuit & draw the phasor diagram and voltage triangle
3.5 Define Time Constant of the above Circuit
3.6 Solve numerical simple problems of above Circuit.

Unit-4: RESONANCE AND COUPLED CIRCUITS

4.1 Introduction to resonance circuits & Resonance tuned circuit,
4.2 Series& Parallel resonance
4.3 Expression for series resonance, Condition for Resonance, Frequency of Resonance, Impedance, Current, Voltage, power, Q Factor and Power Factor of Resonance, Bandwidth in term of Q.
4.4 Parallel Resonance (RL, RC& RLC)& derive the expression

4.5 Comparisons of Series & Parallel resonance& applications
4.6 simple problems of above Circuit

Unit-5: LAPLACE TRANSFORM AND ITS APPLICATIONS

5.1 Laplace Transformation, Analysis and derive the equations for circuit parameters of Step response of R-L, R-C &R-L-C
5.2 Analysis and derive the equations for circuit parameters of Impulse response of R-L, R-C, R-L-C

Unit-6: Two Port Network Analysis

6.1 Network elements, ports in Network (One port, two port),
6.2 Network Configurations (T & pie).
6.3 Open circuit (Z-Parameter)& Short Circuit(Y-Parameter) Parameters- Calculate open & short Circuit Parameters for Simple Circuits & its conversion
6.4 h- parameter (hybrid parameter) Representation
6.5 Define T-Network & pie – Network

Unit-7: FILTERS& ATTENUATORS

7.1 Ideal &Practical filters and its applications, cut off frequency, passband and stop band.
7.2 Classify filters- low pass, high pass, band pass, band stop filters & study their Characteristics.
7.3 Butterworth Filter Design
7.4 Attenuation and Gain, Bel , Decibel & neper and their relations.
7.5 Attenuators& its applications. Classification-T- Type & PI – Type attenuators

Coverage of Syllabus upto Internal Exams (I.A.)

Chapter 1, 2,3, 4

Books Recommended

1. Circuit Theory by A.Chakbarti, Dhanpat Rai & Co Publication
2. Network Theory bySmarajitGhosh, PHI Learning Private Limited
3. Circuit Theory by Ravish S Salivahanan& S Pravin Kumar, Vikas Publication
4. Circuit and Networks by Nagsarkar, Oxford Publication

UNIT 1 – CIRCUIT ELEMENTS & ENERGY SOURCES

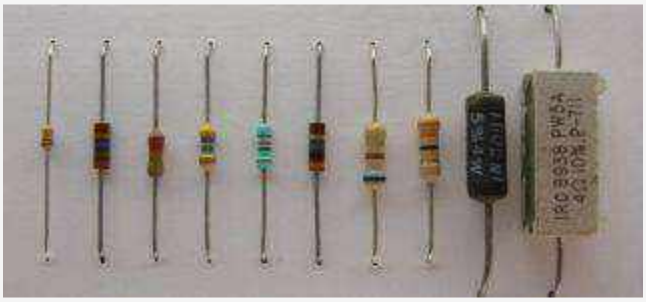
1.1 Circuit Elements-

An electronic circuit is composed of individual components which are electronic such as resistors, transistors, capacitors , inductors and diodes connected by wires which are conductive or traces through which the electric current can flow. A circuit which is electronic can be usually categorized as analogue. The analogue electronic circuit is the one in which voltage or current may vary continuously with time which corresponds to the information that is being represented. The circuit is constructed from two building fundamental blocks: parallel or the series circuits.

In a series circuit, the same current passes through a series of components. In a circuit which is parallel, all the components are connected to the same voltage and the current divides is between the various components that are according to their resistance.

Basic Electrical Components are: Resistors, Capacitors, Light Emitting Diode (LED), Transistors, Inductors, Integrated Circuit (IC), Circuit Breaker, Fuse, Switch, Transformer, Electrical Wires & Power Cables, Battery, Relay, Motor etc.

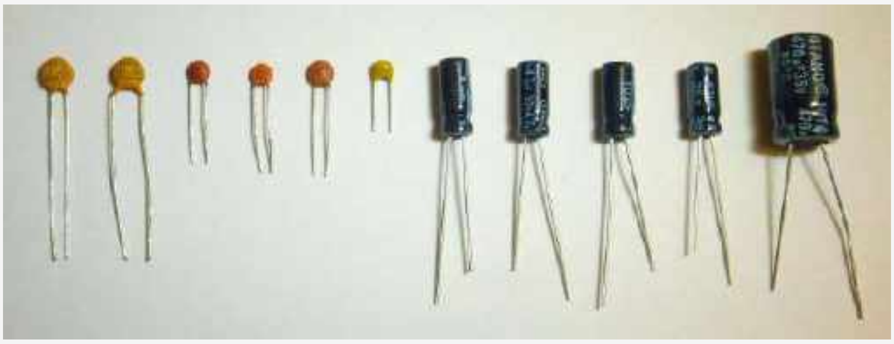
1. Resistors: Resistors control the electric currents that pass through them, as well as the voltage in each component connected to them. Without resistors, other components may not be able to handle the voltage and this may result in overloading.



The standard unit for resistance is the ohm, which is named after German physicist Georg Simon Ohm. It is defined as the resistance in a circuit with a current of 1 ampere at 1 volt. Resistance can be calculated using Ohm's law, which states that resistance equals voltage divided by current, or R = V/I (more commonly written as V = IR), where R is resistance, V is voltage and I is current.

2. Capacitors: Capacitors are passive two-terminal electronic components. They act like rechargeable batteries – they can store electrical energy, and then transmit that energy again when needed. Capacitance is the ability of a device to store electric charge, and as such, the electronic component that stores electric charge is called a capacitor. The simplest capacitor consists of two flat conducting plates separated by a small gap. The potential difference, or voltage, between the plates is proportional to the difference in the amount of the charge on the plates. This is expressed as Q = CV, where Q is charge, V is voltage and C is capacitance.

The capacitance of a capacitor is the amount of charge it can store per unit of voltage. The unit for measuring capacitance is the farad (F), named for Faraday, and is defined as the capacity to store 1 coulomb of charge with an applied potential of 1 volt. One coulomb (C) is the amount of charge transferred by a current of 1 ampere in 1 second.



Capacitive Reactance Formula

$$X_C = \frac{1}{2\pi f C}$$

- Where:
- Xc = Capacitive Reactance in Ohms, (Ω)
- π (pi) = a numeric constant of 3.142
- f = Frequency in Hertz, (Hz)
- C = Capacitance in Farads, (F)

3. Inductors: Inductors are passive two-terminal electronic components that store energy in a magnetic field when an electric current passes through them. Inductors are used to block alternating currents while allowing direct currents to pass. They can be combined with capacitors to make tuned circuits, which are used in radio and TV receivers. The unit for inductance is the henry (H), named after Joseph Henry, an American physicist. One henry is the amount of inductance that is required to induce 1 volt of electromotive force (the electrical pressure from an energy source) when the current is changing at 1 ampere per second.

Inductive Reactance Formula

$$X_L = 2\pi f L$$

- Where:
- X_L = Inductive Reactance in Ohms, (Ω)
- π (pi) = a numeric constant of 3.142
- f = Frequency in Hertz, (Hz)
- L = Inductance in Henries, (H)

4. Diodes: Diodes are semiconductor components that act as one-way switches for currents. They allow currents to pass easily in one direction but restricts currents from flowing in the opposite direction.

5. Transistors: Transistors are crucial to the printed circuit board assembly process due to their multi-functional nature. They are semiconductor devices that can both conduct and insulate and can act as switches and amplifiers. They are smaller in size, have a relatively longer life, and can operate at lower voltage supplies safely without a filament current. Transistors come in two types: bipolar junction transistor (BJT) and field effect transistors (FET).

VOLTAGE-CURRENT RELATIONSHIP OF CIRCUIT ELEMENTS-

Resistor R (Ohms Ω)	$v = Ri$	$i = \frac{v}{R}$	$P = i^2 R$
Inductor L (Henry H)	$v = L\frac{di}{dt}$	$i = \frac{1}{2}\int v dt + i_0$ where i_0 is the initial current in inductor	$P = L\frac{di}{dt}$
Capacitor C (Farad F)	$v = \frac{1}{C}\int i dt + v_0$ where v_0 is the initial voltage across capacitor	$i = c\frac{dv}{dt}$	$P = cv\frac{dv}{dt}$

Scope of network analysis & synthesize: Network analysis is a process through which we calculate various electrical parameters of a circuit element connected in an electrical network. It uses mathematical tools to analyze a circuit. Using number of methods, the networks are studied and the response is obtained. Such a response is unique for a given network and known excitation. So obtaining a response for a known network and known excitation is called network analysis. **In the network synthesis, the procedure is exactly opposite to the analysis**.

Network synthesis is a design technique for linear electrical circuits. Synthesis starts from a prescribed impedance function of frequency or frequency response and then determines the possible networks that will produce the required response. The technique is to be compared to network analysis in which the response (or other behaviour) of a given circuit is calculated.

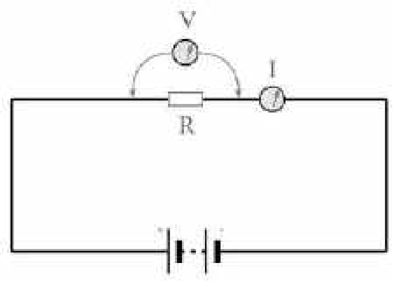
OHM'S LAW – It states that the current flowing in a circuit is directly proportional to the applied potential difference and inversely proportional to the resistance in the circuit at constant temperature. $I \alpha V$

$V=IR$

Where: V = voltage expressed in Volts

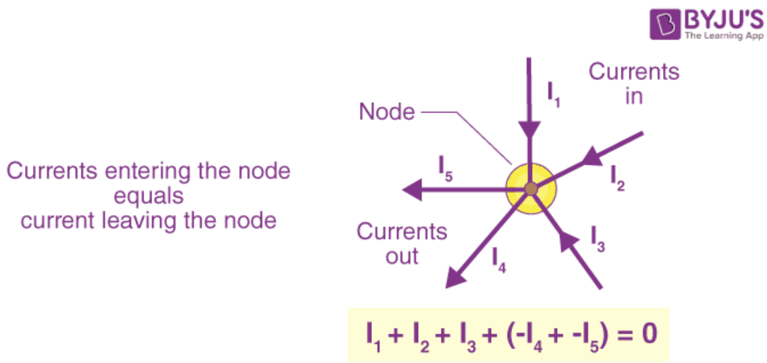
I = current expressed in Amps

R = resistance expressed in Ohms

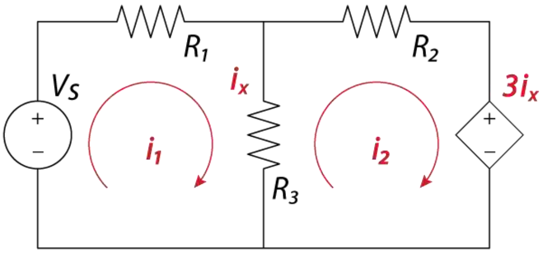


KIRCHHOFF'S LAW -

- **Kirchhoff's current law** (1st Law) states that the current flowing into a node (or a junction) must be equal to the current flowing out of it. This is a consequence of charge conservation.



- **Kirchhoff's voltage law** (2nd Law) states that in any complete loop within a circuit, the sum of all voltages across components which supply electrical energy (such as cells or generators) must equal the sum of all voltages across the other components in the same loop. *The sum of voltages around a loop is* *zero.* This law is a consequence of both charge conservation and the conservation of energy.

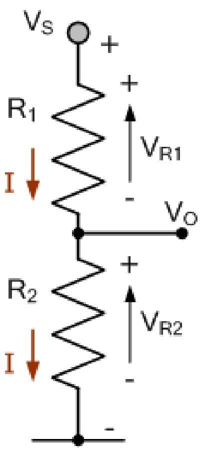


In loop 1, $Vs - I_1R_1 - (I_1 - I_2) R_3 = 0$

In loop 2, $I_2R_2 - 3I_x - (I_2 - I_1) R_3 = 0$

1.2 Voltage Division

Voltage Divider circuits are used to produce different voltage levels from a common voltage source but the current is the same for all components in a series cicruit



So solving for the current (I) flowing through the series network gives us:

$$V_S = V_{R1} + V_{R2} \quad \text{(KVL)}$$

$$V_{R1} = I \times R_1 \quad \text{and} \quad V_{R2} = I \times R_2$$

$$\text{Then: } V_S = I \times R_1 + I \times R_2$$

$$\therefore V_S = I(R_1 + R_2)$$

$$\text{So: } I = \frac{V_S}{(R_1 + R_2)}$$

The current flowing through the series network is simply I = V/R following Ohm's Law. Since the current is common to both resistors, ($I_{R1} = I_{R2}$) we can calculate the voltage dropped across resistor, R_2 in the above series circuit as being:

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{V_S}{(R_1 + R_2)}$$

$$\therefore V_{R2} = V_S\left(\frac{R_2}{R_1 + R_2}\right)$$

Likewise for resistor R_1 as being:

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{V_S}{(R_1 + R_2)}$$

$$\therefore V_{R1} = V_S\left(\frac{R_1}{R_1 + R_2}\right)$$

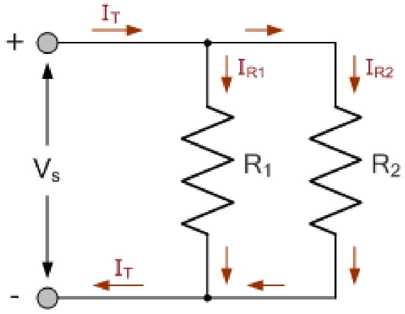
Voltage Dividers Equation

$$V_{R(x)} = V_S\left(\frac{R_X}{R_T}\right)$$

Where: $V_{R(x)}$ is the voltage drop across the resistor, R_X is the value of the resistor, and R_T is the total resistance of the series network. This voltage divider equation can be used for any number of series resistances connected together because of the proportional relationship between each resistance, R and its corresponding voltage drop, V.

Current Division

Current Divider circuits have two or more parallel branches for currents to flow through but the voltage is the same for all components in the parallel circuit.



So solving for the voltage (V) across the parallel combination gives us:

$$I_T = I_{R1} + I_{R2}$$

$$I_{R1} = \frac{V}{R_1} \quad \text{and} \quad I_{R2} = \frac{V}{R_2}$$

$$I_T = \frac{V}{R_1} + \frac{V}{R_2} = V\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\therefore V = I_T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]^{-1} = I_T\left[\frac{R_1 R_2}{R_1 + R_2}\right]$$

Solving for I_{R1} gives:

$$I_{R1} = \frac{V}{R_1} = I_T\left[\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}}\right]$$

$$\therefore I_{R1} = I_T\left(\frac{R_2}{R_1+R_2}\right)$$

Likewise, solving for I_{R2} gives:

$$I_{R2} = \frac{V}{R_2} = I_T\left[\frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}}\right]$$

$$\therefore I_{R2} = I_T\left(\frac{R_1}{R_1+R_2}\right)$$

1.3 Electrical charge, Current, electric potential, Energy and Power

Electric Charge is the property of subatomic particles that causes it to experience a force when placed in an electric and magnetic field." Electric charges are of two types: Positive and Negative, commonly carried by charge carriers protons and electrons. Coloumb is the unit of electric charge. "One coulomb is the quantity of charge transferred in one second."

Mathematically, the definition of a coloumb is represented as: Q = I.t

In the equation, Q is the electric charge, I is the electric current and t is the time.

Electric current is the flow of electric charge in a circuit. More specifically, the electric current is the rate of charge flow past a given point in an electric circuit. The charge can be negatively charged electrons or positive charge carriers including protons, positive ions or holes. The magnitude of the electric current is measured in coulombs per second, the common unit for this being the Ampere or amp.

Voltage is defined as the potential difference between two points in an electric circuit that imparts one joule (J) of energy per coulomb (C) of charge that passes through the circuit. **V = Change in Potential Energy/Charge = ΔU/Q**. It's unit is volt.

Electrical Energy supplies the power required to produce work or an action within an electrical circuit and is given in joules per second. This action can take many forms, such as thermal, electromagnetic, mechanical, electrical, etc. *Electrical energy* can be both created from batteries, generators, dynamos, and photovoltaics, etc. or stored for future use using fuel cells, batteries, capacitors or magnetic fields, etc. Thus electrical energy can be either created or stored.

Electrical Power is the product of the two quantities, *Voltage* and *Current* and so can be defined as the rate at which work is performed in expending energy. It is the

rate at which work is performed during one second. If voltage, (V) equals Joules per Coulombs (V = J/C) and Amperes (I) equals charge (*coulombs*) per second (A = Q/t), then we can define electrical power (P) as being the totality of these two quantities. This is because electrical power can also equal voltage times amperes, that is:
P = V*I.

The Watt

$$V = \frac{J}{C} \quad \text{and} \quad I = \frac{Q}{t}$$

$$\text{As:} \quad Q = 1C$$

$$\text{If:} \quad P = V \times I = \frac{J}{C} \times \frac{Q}{t} = \frac{J}{C} \times \frac{C}{t}$$

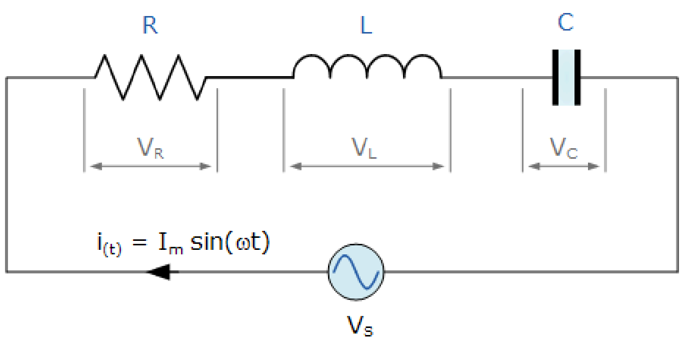
$$\text{Then:} \quad P = \frac{J}{\cancel{C}} \times \frac{\cancel{C}}{t} = \frac{J}{t}$$

1 watt (W) = 1 joule/second (J/s)

Active & Passive elements –

1. Active components are those that **deliver, produce or transfer energy** or power in the form of a voltage or current. Passive components are those that **utilise or store energy** in the form of voltage or current.
2. Examples of the active components are diodes, transistors, SCR, integrated circuits, etc. similarly, examples of the passive components are resistor, capacitor, and inductor.
3. Active components are capable of providing the **power gain**, whereas passive components are not capable of providing the power gain.
4. Active components can control the **flow of current**, but passive components cannot control the flow of the current.
5. Active components are energy donors, whereas passive components are energy acceptors.

Series RLC Circuit Analysis: Series RLC circuits consist of a resistance, a capacitance and an inductance connected in series across an alternating supply.



Element Impedance-

Circuit element	Resistance(R)	Reactance(X)	Impedance (Z)
Resistor	R	0	$Z_R = R$ $= R\angle 0^0$
Inductor	0	ωL	$Z_L = j\omega L$ $= \omega L\angle +90^0$
Capacitor	0	$\frac{1}{\omega C}$	$Z_C = \frac{1}{j\omega C}$

Instantaneous Voltages for a Series RLC Circuit

$$\text{KVL:}\quad V_S - V_R - V_L - V_C = 0$$

$$V_S - IR - L\frac{di}{dt} - \frac{Q}{C} = 0$$

$$\therefore V_S = IR + L\frac{di}{dt} + \frac{Q}{C}$$

$$V_S^2 = V_R^2 + (V_L - V_C)^2$$

$$V_S = \sqrt{V_R^2 + (V_L - V_C)^2}$$

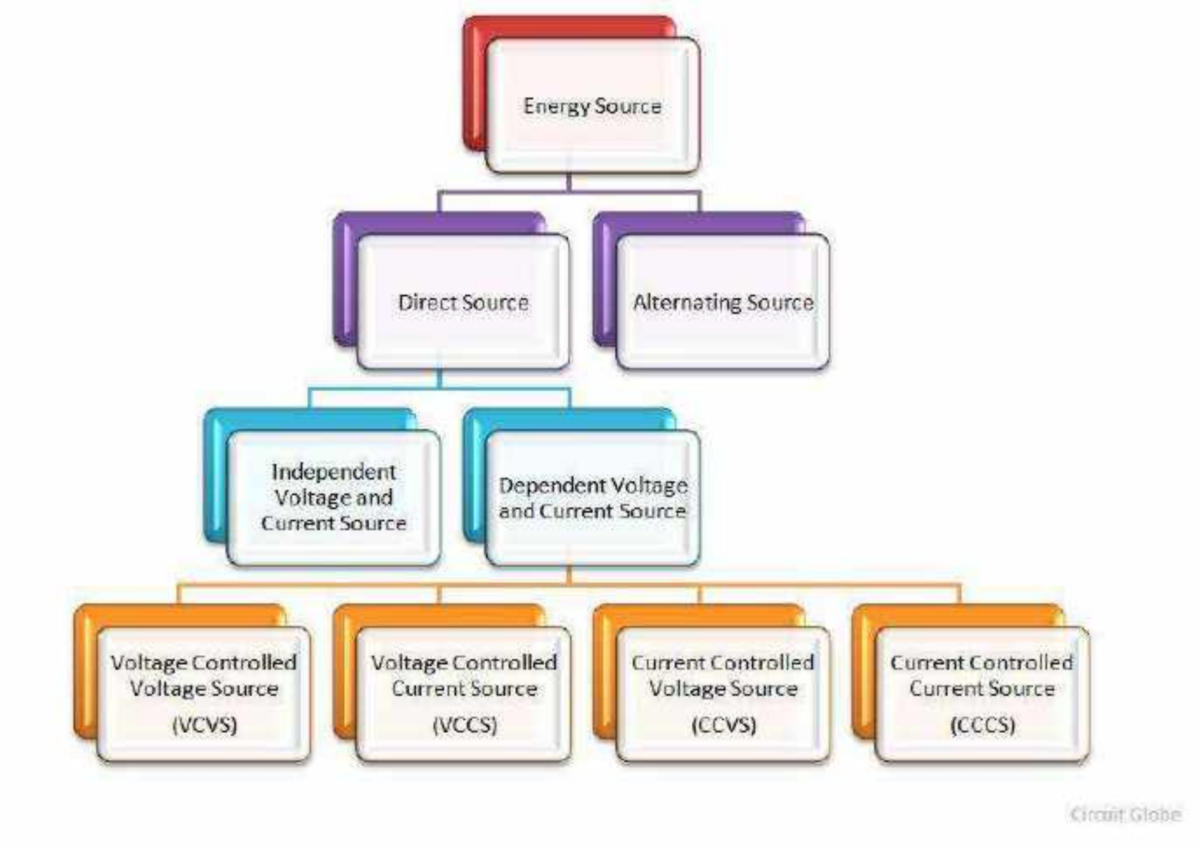
$$V_R = I.R \qquad V_L = I.X_L \qquad V_C = I.X_C$$

$$V_S = \sqrt{(I.R)^2 + (I.X_L - I.X_C)^2}$$

$$V_S = I.\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore V_S = I \times Z \qquad \text{where: } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

1.4 Energy Sources



<u>Independent Voltage and Current Source</u>

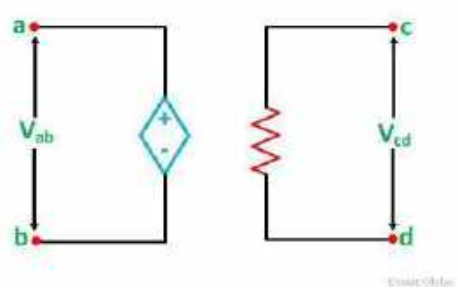
Independent sources are that which does not depend on any other quantity in the circuit. They are two-terminal devices and has a constant value, i.e. the voltage across the two terminals remains constant irrespective of all circuit conditions.

The strength of voltage or current is not changed by any variation in the connected network the source is said to be either independent voltage or independent current source. In this, the value of voltage or current is fixed and is not adjustable.

<u>Dependent Voltage and Current Source:</u> The sources whose output voltage or current is not fixed but depends on the voltage or current in another part of the circuit is called Dependent or Controlled source. They are four-terminal devices.

When the strength of voltage or current changes in the source for any change in the connected network, they are called dependent sources.

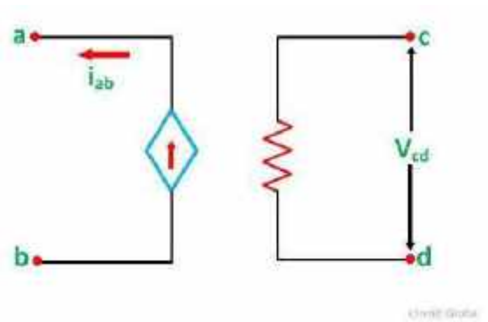
The dependent sources are further categorised as:
Voltage Controlled Voltage Source (VCVS) - In **voltage-controlled voltage source,** the voltage source is dependent on any element of the circuit.



In the above figure, the voltage across the source terminal V_{ab} is dependent on the voltage across the terminal V_{cd},

$$V_{ab} \propto V_{cd} \quad \text{or}$$
$$V_{ab} = kV_{cd}$$

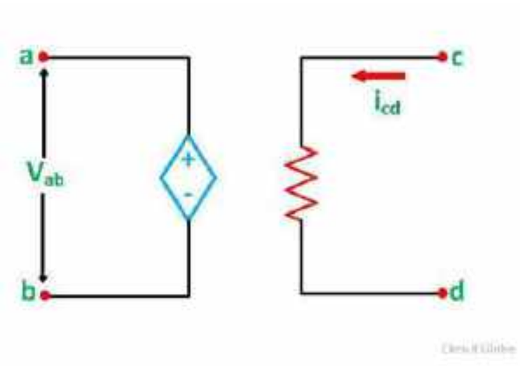
Voltage Controlled Current Source (VCCS) - In the **voltage controlled current source**, the current of the source i_{ab} depends on the voltage across the terminal cd (V_{cd}) as shown in the figure below:



$$i_{ab} \propto V_{cd}$$
$$i_{ab} = \eta V_{cd}$$

Where η is a constant known as **transconductance** and its unit is mho.

Current Controlled Voltage Source (CCVS) - In the **current controlled voltage source** voltage source of the network depends upon the current of the network as shown in the figure below

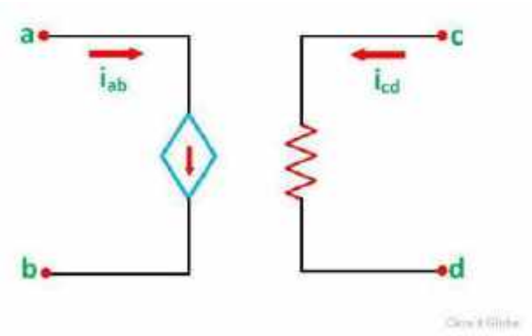


Here the voltage of source V_{ab} depends on the current of the branch cd

$$V_{ab} \propto i_{cd}$$
$$V_{ab} = r\, i_{cd}$$

Where r is a constant.

Current Controlled Current Source (CCCS) - In the **Current Controlled Current Source**, the current source is dependent on the current of the branch another branch as shown in the figure below



So,

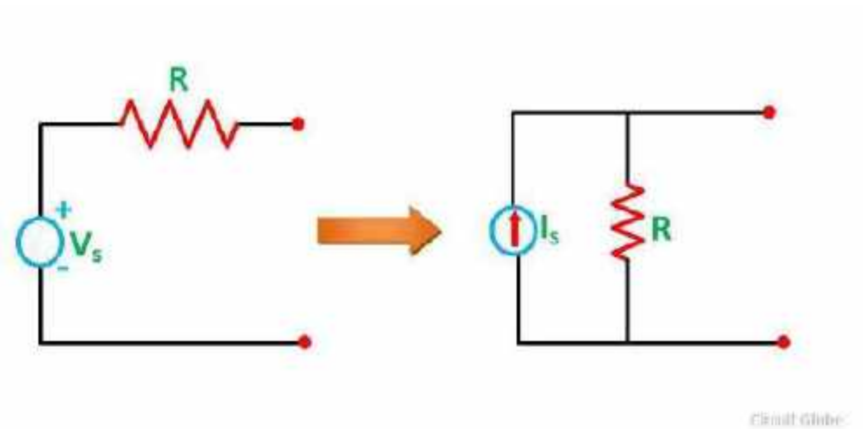
$$i_{ab} \propto i_{cd}$$
$$i_{ab} = \beta i_{cd}$$

Where β is a constant

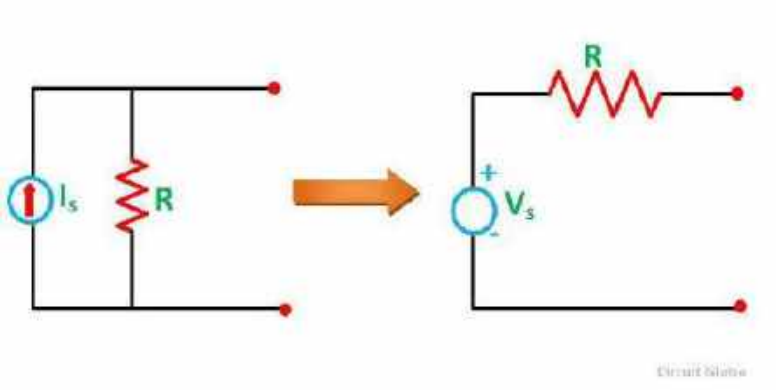
Alternating Sources - In the network applications, there are other types of sources also where voltage or current vary with time sinusoidally or exponentially etc. are termed as alternating sources.

Source Transformation

Source Transformation simply means replacing one source by an equivalent source. A practical voltage source can be transformed into an equivalent practical current source and similarly a practical current source into voltage source. The voltage and current source are mutually transferable.



Where $I_S = V_S / R$



Where, $V_S = I_S / R$

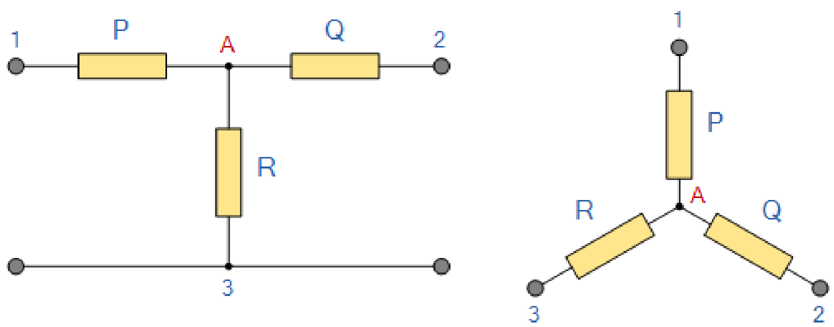
Mutual Inductance is defined as the measure of the relation between the change of current flow in one circuit to the electric potential generated in another by mutual induction. Like **inductance**, mutual inductance is measured in henry.

1.5 <u>Star-Delta Transformation:</u>

Star-Delta Transformations and Delta-Star Transformations allow us to convert impedances connected together in a 3-phase configuration from one type of connection to another. If a 3-phase, 3-wire supply or even a 3-phase load is connected in one type of configuration, it can be easily transformed or changed it into an equivalent configuration of the other type by using either the **Star Delta Transformation** or **Delta Star Transformation** process.

A resistive network consisting of three impedances can be connected together to form a T or "Tee" configuration but the network can also be redrawn to form a **Star** or Y type network as shown below.

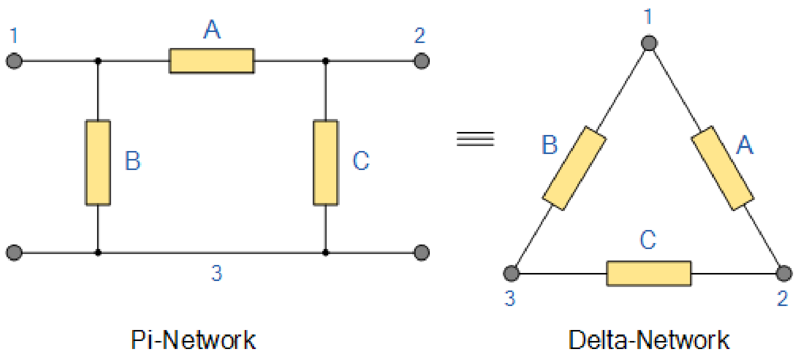
T-connected and Equivalent Star Network



T-Network Star-Network

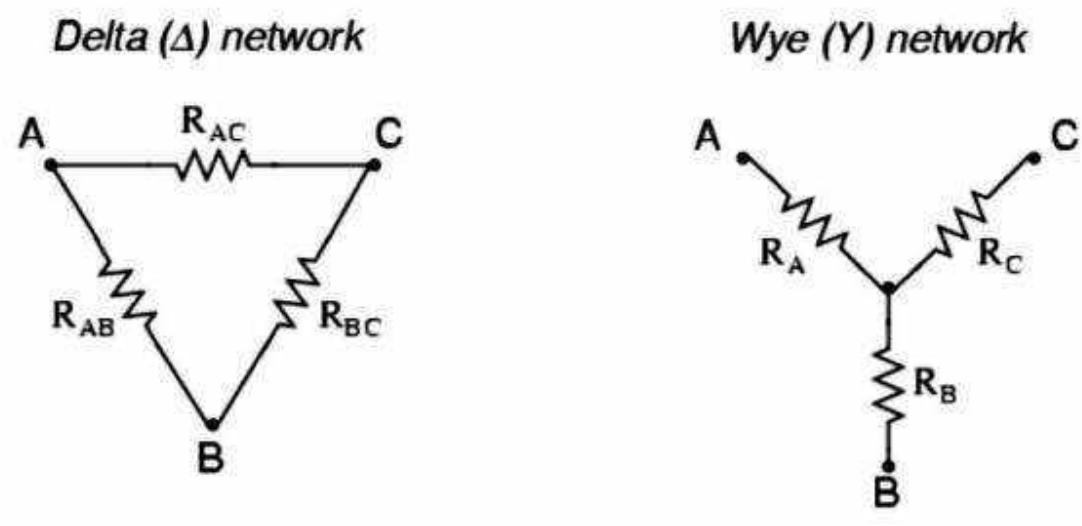
Similarly, a Pi or π type resistor network can be converted into an electrically equivalent **Delta** or Δ type network as shown below.

Pi-connected and Equivalent Delta Network



For a delta connected resistive network converted into an equivalent star connected network - If the three resistors in the delta network are all equal in value then the resultant resistors in the equivalent star network will be equal to one third the value of the delta resistors. This gives each resistive branch in the star network a value of: $R_{STAR} = (R_{DELTA})/3$.

For a star connected resistive network converted into an equivalent delta connected network - If all the resistors in the star network are all equal in value then the resultant resistors in the equivalent delta network will be three times the value of the star resistors and equal, giving: $R_{DELTA} = 3*R_{STAR}$



To convert a Delta (Δ) to a Wye (Y)

$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

To convert a Wye (Y) to a Delta (Δ)

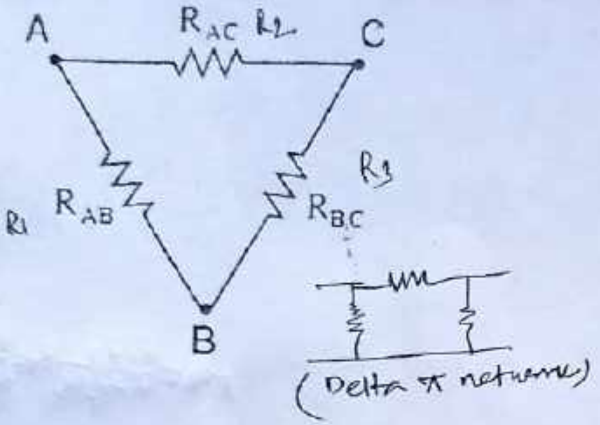
$$R_{AB} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_C}$$

$$R_{BC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_A}$$

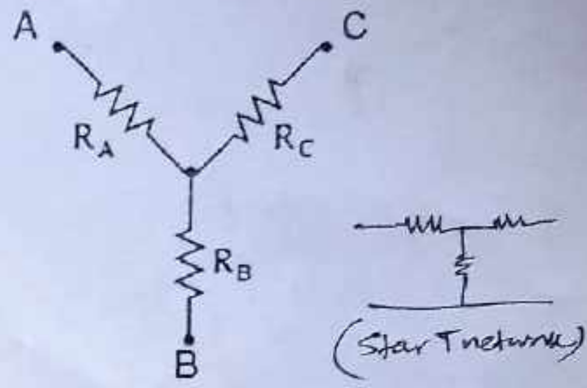
$$R_{AC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_B}$$

For a star connected resistive network converted into an equivalent delta connected network - If all the resistors in the star network are all equal in value then the resultant resistors in the equivalent delta network will be three times the value of the star resistors and equal, giving: $R_{DELTA} = 3 * R_{STAR}$

Delta (Δ) network



Wye (Y) network



To convert a Delta (Δ) to a Wye (Y)

$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

To convert a Wye (Y) to a Delta (Δ)

$$R_{AB} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_C}$$

$$R_{BC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_A}$$

$$R_{AC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_B}$$

* Line voltage is the voltage across line conductors while phase voltage is the voltage between line conductor and neutral. phase voltage is the voltage across particular phase winding of the transformer.

Star

$V_L = \sqrt{3} V_{ph}$

$I_L = I_{ph}$.

$V_{RY} = V_R - V_Y$

$V_{BR} = V_B - V_R$

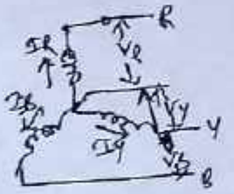
$V_{YB} = V_Y - V_B$

$V_R = V_Y = V_B = V_{ph}$

$V_{RY} = V_{BY} = V_{YB} = V_L$

$I_R = I_Y = I_B = I_{ph}$.

I_L



$V_L = V_{ph}$

$V_{RY}, V_{RB}, V_{YB} = V_L$

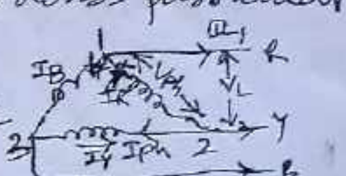
$I_{L1} = I_R - I_B$

$I_{L2} = I_Y - I_R$

$I_{L3} = I_B - I_Y$

$I_p = I_R = I_Y = I_B$

$I_L = \sqrt{3} I_{ph}$.



Delta to star network

$R_{AB} = R_{AB} \| (R_{AC} + R_{BC})$

$= R_1 \| (R_2 + R_3)$

$= \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3}$

$R_{BC} = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3}$

$R_{AC} = \frac{R_2 (R_1 + R_3)}{R_1 + R_2 + R_3}$

According to star,

$R_{AB} = R_A + R_B$

$R_{BC} = R_B + R_C$

$R_{AC} = R_A + R_C$

$\Rightarrow R_{AB} + R_{BC} + R_{AC} = 2(R_A + R_B + R_C) = \frac{2(R_1R_2 + R_1R_3 + R_2R_3)}{R_1 + R_2 + R_3}$

$\Rightarrow R_A + R_B + R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$

$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_2 + R_3} - \frac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$

$= \frac{R_1 R_2}{R_1 + R_2 + R_3}$

$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}$

$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$

$Star = \frac{R_{delta}}{3}$ if $R_1 = R_2 = R_3$

Star to Delta network

$R_A R_B = \frac{R_1 R_2}{R_1 + R_2 + R_3} \times \frac{R_1 R_3}{R_1 + R_2 + R_3}$

$R_B R_C = \frac{R_1 R_3}{R_1 + R_2 + R_3} \times \frac{R_2 R_3}{R_1 + R_2 + R_3}$

$R_C R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3} \times \frac{R_1 R_2}{R_1 + R_2 + R_3}$

$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2 R_3 (R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2}$

$= \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3}$ --- ①

$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}, R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}, R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$ ②

Dividing eqn ① by ②

$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$

$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$

$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$

UNIT 2 – NETWORK THEOREMS

2.1 Nodal & Mesh Analysis of Electrical circuits with simple problems:

There are two basic methods that are used for solving any electrical network: Nodal analysis and Mesh analysis. In Nodal analysis, we will consider the node voltages with respect to Ground. Hence, Nodal analysis is also called as Node-voltage method.

Procedure of Nodal Analysis

Follow these steps while solving any electrical network or circuit using Nodal analysis.

- **Step 1 −** Identify the **principal nodes** and choose one of them as **reference node**. We will treat that reference node as the Ground.
- **Step 2 −** Label the **node voltages** with respect to Ground from all the principal nodes except the reference node.
- **Step 3 −** Write **nodal equations** at all the principal nodes except the reference node. Nodal equation is obtained by applying KCL first and then Ohm's law.
- **Step 4 −** Solve the nodal equations obtained in Step 3 in order to get the node voltages.

Example 1: In the circuit of figure 7, find the current in 1Ω resistor.

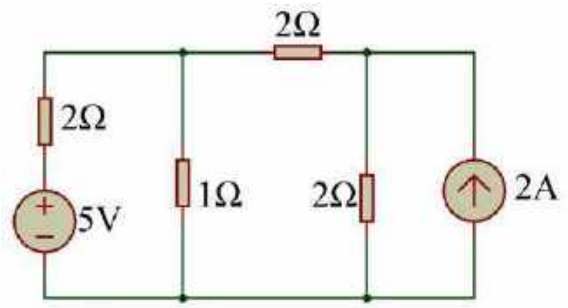


Figure: 7

Solution:

Let us first convert the current source of figure 7 to voltage source and draw the equivalent network (figure 8). Let the +ve voltage at node (1) be v_1 V.

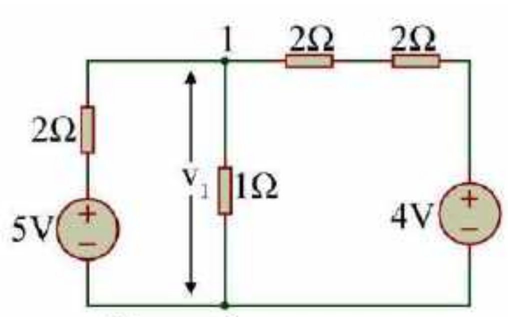


Figure: 8

∴Using nodal analysis,

$$\frac{v_1}{1} + \frac{v_1 - 5}{2} + \frac{v_1 - 4}{4} = 0$$

or, $v_1 = 2V$

Hence, the current through 1Ω resistor is

$$\frac{v_1}{1} = 2A$$

Example 2: Find V_1 and V_2 in figure 9.

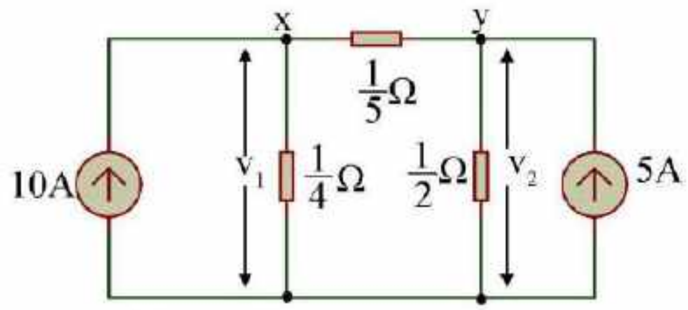


Figure: 9

Solution:

At node "x",

$$10 = \frac{V_1}{1/4} + \frac{V_1 - V_2}{1/5} = 4V_1 + 5V_1 - 5V_2$$

or, $9V_1 - 5V_2 = 10$ …..(1)

At node "y",

$$5 = \frac{V_2}{1/2} + \frac{V_2 - V_1}{1/5} = 2V_2 + 5V_2 - 5V_1$$

or, $-5V_1 + 7V_2 = 5$ …..(2)

Solving (1) and (2),

$$V_2 = 2.5V; V_1 = 2.5V$$

Example 3: Using Nodal method find the current through the resistors in the circuit configuration of figure 3.

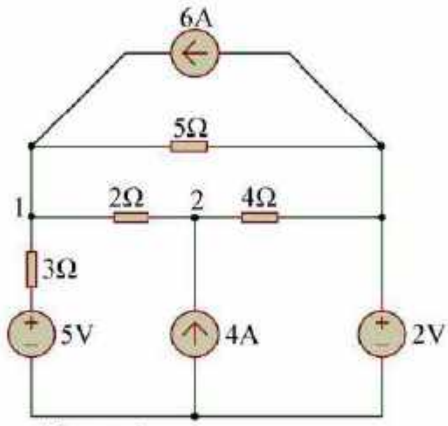


Figure: 3

Solution:

Naming the respective nodes of the circuit as (1) and (2) and assuming the voltages to be v_1 (+ve) and v_2 (+ve) respectively at these nodes, nodal equation at nodes (1) and (2) are as follows:

For node (1),

$$\frac{v_1-5}{3}+\frac{v_1-v_2}{2}+\frac{v_1-2}{5}=6$$

or, $$v_1(\frac{1}{3}+\frac{1}{2}+\frac{1}{5})-\frac{v_2}{2}-\frac{2}{5}-\frac{5}{3}-6=0$$

or, $$\frac{31}{30}v_1-\frac{v_2}{2}-\frac{121}{15}=0$$ .....(a)

For node (2),

$$\frac{v_2-v_1}{2}+\frac{v_2-2}{4}=4$$

or, $$v_2(\frac{1}{2}+\frac{1}{4})-\frac{v_1}{2}-\frac{1}{2}=4$$

or, $$\frac{3}{4}v_2-\frac{v_1}{2}-\frac{9}{2}=0$$ .....(b)

Solving (a) and (b),

v_1=15.76V while v_2=16.51V

∴ Current through 3Ω resistor

$$=\frac{v_1-5}{3}=\frac{15.76-5}{3}\approx 3.6A$$

Current through 2Ω resistor

$$=\frac{v_1-v_2}{2}=\frac{15.76-16.51}{2}=-0.375A$$

[i.e., following from node (2) to node (1)].

Current through 5Ω resistor

$$=\frac{v_1-2}{5}=\frac{15.76-2}{5}=2.76A$$

Current through 4Ω resistor

$$=\frac{v_2-2}{4}=\frac{16.51-2}{4}=3.63A$$

Figure 4 confirms the KCL both at nodes (1) and nodes (2)

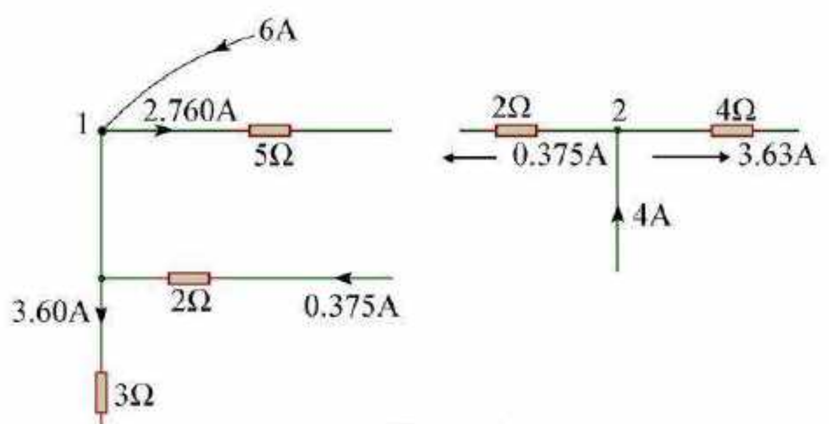


Figure: 4

<u>MESH ANALYSIS</u> is defined as the method in which the current flowing through a planar circuit is calculated. A planar circuit is defined as the circuits that are drawn on the plane surface in which there are no wires crossing each other. Therefore, a mesh analysis can also be known as loop analysis or mesh-current method.

What is Branch?

A branch is defined as the path that connects two nodes such that it contains a circuit element. If the branch belongs to only one mesh, then the branch current and the mesh current will be equal to each other.

<u>Procedure of Mesh Analysis:</u> The following steps are to be followed while solving the given electrical network using mesh analysis:

Step 1: To identify the meshes and label these mesh currents in either clockwise or counterclockwise direction.

Step 2: To observe the amount of current that flows through each element in terms of mesh current.

Step 3: Writing the mesh equations to all meshes using Kirchhoff's voltage law and then Ohm's law.

Step 4: The mesh currents are obtained by following Step 3 in which the mesh equations are solved.

Hence, for a given electrical circuit the current flowing through any element and the voltage across any element can be determined using the node voltages.

What is Super Mesh Analysis?

Super mesh analysis is used for solving huge and complex circuits in which two meshes share a common component as a source of current.

What is the difference between loop and mesh?

The difference between loop and mesh is that a loop is a closed path in a circuit in which none of the nodes repeat more than once. While a mesh is a closed path in a circuit in which no other paths are present.

What are the limitations of mesh analysis?

The following are the disadvantages of mesh analysis:

- Mesh analysis is useful only when the circuit is planar.
- As the number of meshes increases, the number of equations increases, which makes it inconvenient for solving.

Which Kirchhoff's law is used in mesh analysis?
Kirchhoff's voltage law is used in mesh analysis.

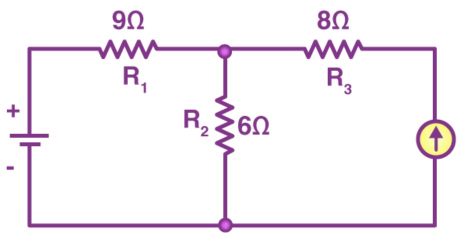
What is the difference between mesh and nodal analysis?

The difference between mesh and nodal analysis is that nodal analysis is an application of Kirchhoff's current law, which is used for calculating the voltages at each node in an equation. While mesh analysis is an application of Kirchhoff's voltage law which is used for calculating the current.

Example of Mesh Analysis

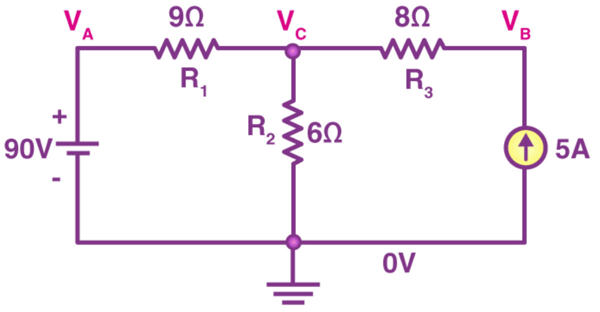
Example 1:

In the given circuit 90v is the battery value, 5A is the current source and the three resistors are 9 ohms, 6 ohms, and 8 ohms. Using mesh analysis, determine the current across each resistor and potential difference.

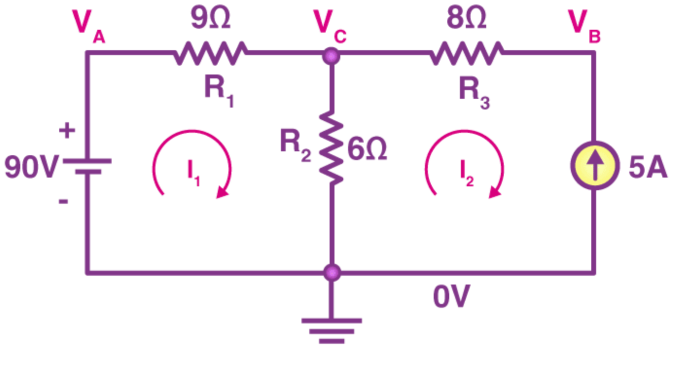


Solutions:-

Let's first determine the ground and then the potential be V_A, V_B, and V_C as shown in the figure.



Let I_1 and I_2 be the currents flowing through the two loops in the clockwise direction as shown in the figure.



Therefore,

$+V_B - V_1 - V_2 = 0$

$90 - I_1R_1 - R_2(I_1 - I_2) = 0$

$90 - 9I_1 - 6(I_1 - I_2) = 0$

$-15I_1 + 6I_2 = -90$

$5I_1 - 2I_2 = 30$ (this is obtained by dividing the equation with -3)

Substituting I_2 as -5 since the direction of I_2 is opposite to the actual direction of current

Therefore,

$I_1 = 4A$

So, through R_1, 4A current is flowing and through R_3, 5A current is flowing.

Now the potential difference at $V_A = 90V$

At V_B, the potential difference is $V_2 = I_2 - R_2$

Therefore, $V_B = 54V$

At V_C, the potential difference is $V_3 = I_3 - R_3$

$V_C - 54 = 40$

$V_C = 94V$

Example: 2 Determine the node voltages and the current through the resistors using mesh method for the network shown in figure 3.

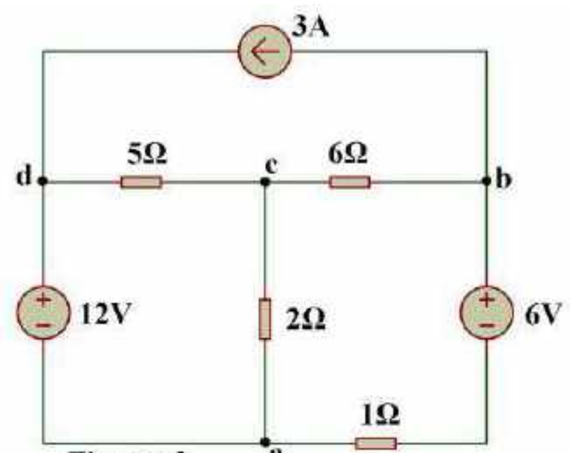


Figure: 3

Solution: The circuit of figure 3 is redrawn with the loop currents in the three loops (figure 4).

For loop-1,

$$(i_1 + i_3)5 + (i_1 - i_2)2 = 12$$

or, $$(i_1 + 3)5 + (i_1 - i_2)2 = 12$$

or, $$7i_1 - 2i_2 = -3 \quad \text{.....(i)}$$

For loop-2,

$$(i_2 - i_1)2 + (i_2 + i_3)6 + i_2.1 = -6$$

or, $(i_2 - i_1)2 + (i_2 + 3)6 + i_2 = -6$

or, $-2i_1 + 9i_2 = -24$ …..(ii)

Solving equation (i) and (ii),

$i_1 = -1.27A;$ $i_2 = -2.95A$

Current through 5Ω resistor

$$= (i_1 + i_3) = -1.27 + 3 = 1.73A$$

Current through 2Ω resistor

$$= (i_1 - i_2) = -1.27 - (-2.95) = 1.68A$$

Current through 6Ω resistor

$$= (i_2 + i_3) = -2.95 + 3 = 0.05A$$

Current through 1Ω resistor

$$= i_2 = -2.95A$$

Also,

voltage at node "a" = 0V

voltage at node "b" $= i_2 \times 1 + 6 = -2.95 \times 1 + 6 = 3.05V$

voltage at node "c" $= (i_1 - i_2)2 = 1.68 \times 2 = 3.36V$

voltage at node "d" = 12V.

Example: 3 Find v by mesh method such that the current through the 5V source is zero (figure 7).

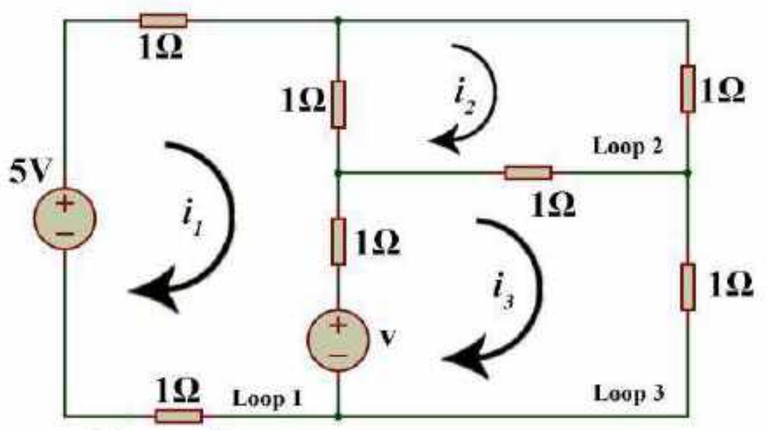


Figure: 7

Solution:

In loop-1,

$$-5 + i_1 \times 1 + (i_1 - i_2)1 + (i_1 - i_3)1 + v = 0$$

or, $3i_1 - i_2 - i_3 = -v + 5$ …..(1)

In loop-2,

$$i_2 \times 1 + (i_2 - i_3)1 + (i_2 - i_1)1 = 0$$

or, $3i_2 - i_1 - i_3 = 0$

or, $-i_1 + 3i_2 - i_3 = 0$ …..(2)

In loop-3,

$$(i_3 - i_2)1 + i_3 \times 1 - v + (i_3 - i_1)1 = 0$$

or, $-i_1 - i_2 + 3i_3 = v$ …..(3)

However, as per question,

$i_1 = 0$ [i_1 being the current through the 5v source]

∴ The three equations (1), (2) and (3) become

$-i_2 - i_3 + v = 5$ …..(4)

$3i_2 - i_3 = 0$ …..(5)

$-i_2 + 3i_3 = v$ …..(6)

From equations (4) and (6), we get

$-4i_3 = 5 - 2v$ …..(7)

However, from the equations (5) and (6),

$i_3 = \frac{3}{8}v$ …..(8)

Using (8) and (7),

$$-4 \times (\frac{3}{8}v) = 5 - 2v$$

$$v = 10V$$

The voltage v should be 10V.

2.2 & 2.3 SUPERPOSITION THEOREM

Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance. The superposition theorem is used to solve the network where two or more sources are present and connected. It is only applicable to the circuit which is valid for the ohm's law (i.e., for the linear circuit).

Example 1: Find I in the circuit shown in figure 1.

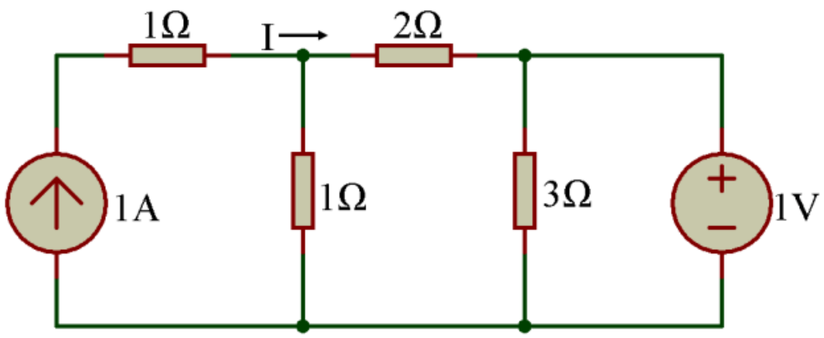


Figure: 1

Solution: Principle of Superposition is applied by taking 1V source only at first (figure2)

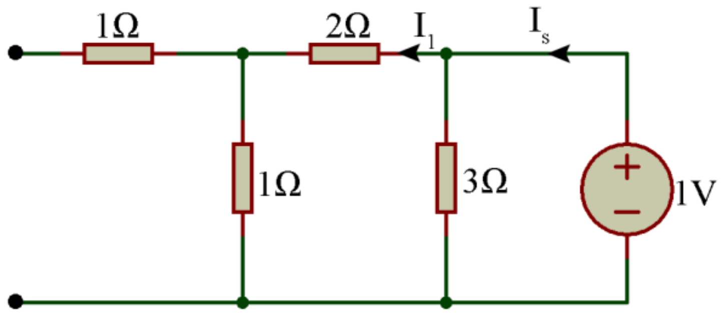


Figure: 2

$$I_s = \frac{1V}{[(1+2)||3]\Omega} = \frac{1}{1.5}A$$

$$I_1 = I_s\frac{3}{3+2+1} = \frac{1}{1.2}\times\frac{3}{6} = \frac{1}{3}A$$ [by current division formula]

Next, let us assume the current source only (figure 3)

$$I_2 = 1\times\frac{1}{1+2} = \frac{1}{3}A$$ [by current division formula]

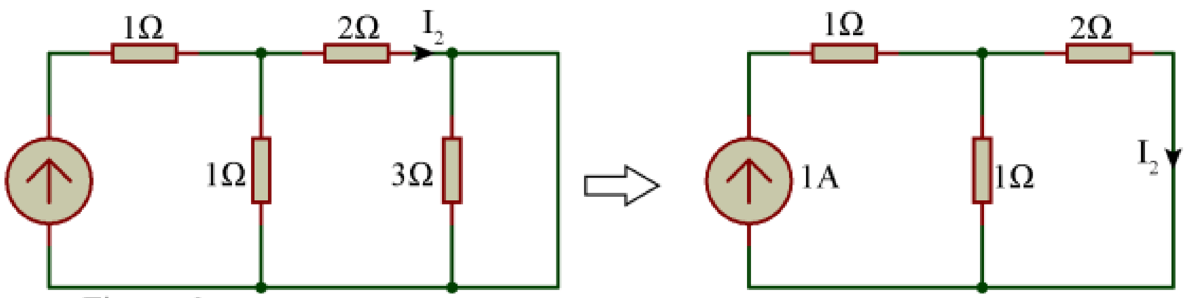


Figure: 3

It may be observed that utilising the principle of Superposition, the net response can be obtained when both the sources (1A and 1V) are present.
The current through 2Ω resistor is obtained as

$$I = (I_1 - I_2) = \frac{1}{3} - \frac{1}{3} = 0$$ [I_1 and I_2 being directed reverse].

Example 2: Find v_L in the circuit of figure 7 using Superposition theorem.

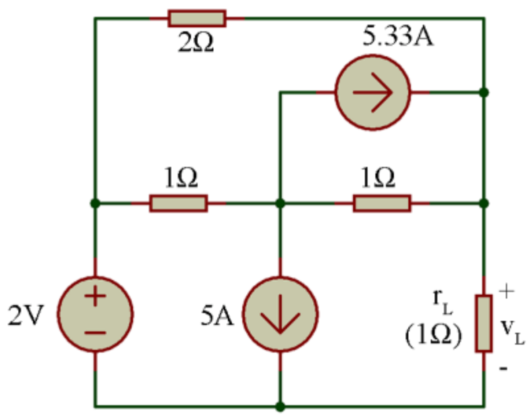


Figure: 7

Solution:
Let us first take the 2V source deactivating the current sources (figure 8).

$$i_1 = \frac{2}{\frac{2\times 2}{2+2}+1} = 1A$$

∴ v_1 (drop across r_L due to 2V source) = 1 × 1 = 1V

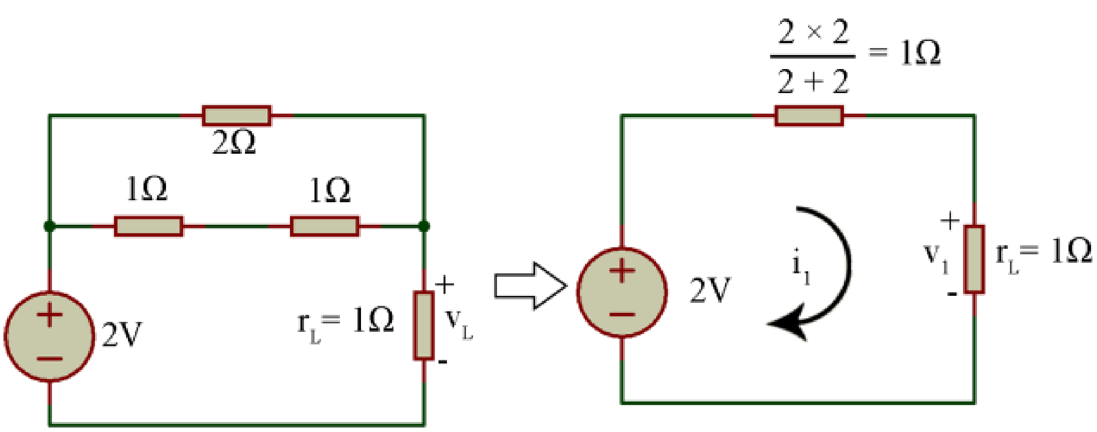


Figure: 8

Next, taking the lower current source only (figure 9).

$$i_2 = (-5)\frac{1}{1+1+\frac{2}{3}} = (-5)\frac{3}{8} = -\frac{15}{8}A.$$

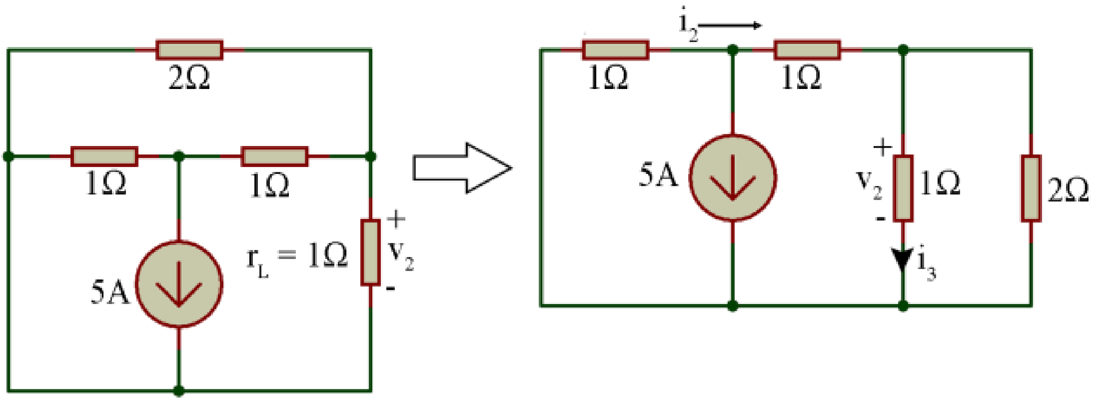


Figure: 9

$$\therefore i_3 = -(\frac{15}{8})\frac{2}{1+2} = -(\frac{15}{8})\frac{2}{3} = -\frac{5}{4}A$$

This gives

$$v_2 = -\frac{5}{4} \times 1 = -\frac{5}{4}V.$$

In figure 10,

$$i_4 = 5.33\frac{1}{\frac{2}{3}+2} = 3A$$

[with 5.33A source]

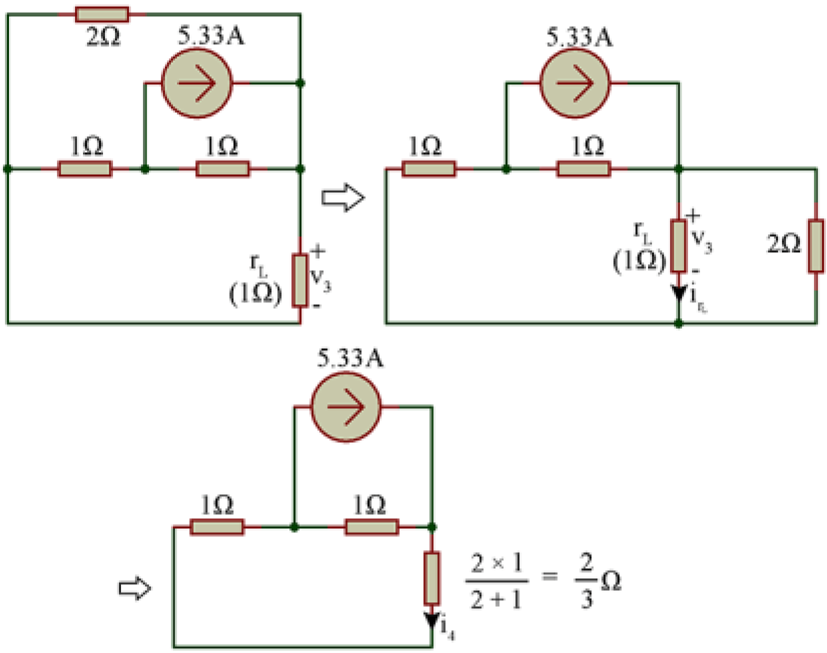


Figure: 10

This gives

$$i_{r_L} = 3\frac{2}{2+1} = 2A.$$

$$\therefore v_3 = 2 \times 1 = 2V$$

By superposition,

$$v_L = v_1 + v_2 + v_3 = 1 + (-\frac{5}{4}) + 2 = \frac{7}{4}V$$

$$= 1.75V.$$

Example 3: Find i_o and i from the circuit of figure 11 using Superposition Theorem.

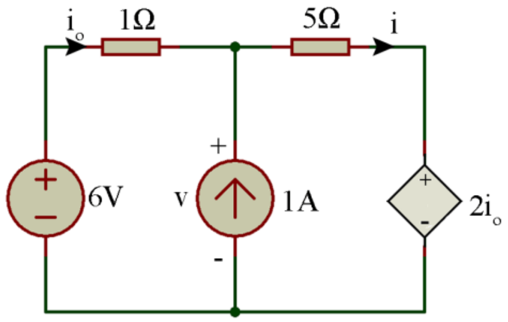


Figure: 11

Solution:

Assuming only 6V source to be active, with reference to figure 12(a).

$$-6 + (1+5)i_o{}' + 2i_o{}'' = 0$$

$$8i_o{}' = 6 \;\; \therefore i_o{}' = \frac{3}{4}A$$

$$\therefore i_o{}' = i' = \frac{3}{4}A$$

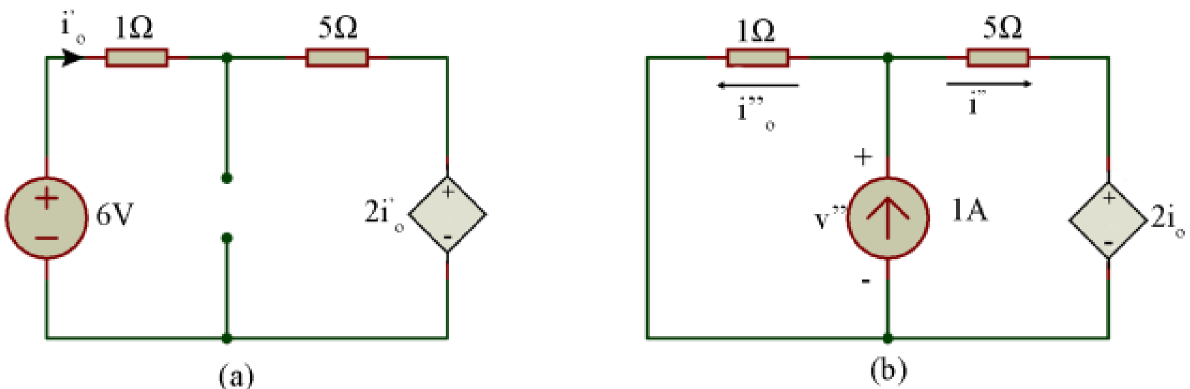


Figure: 12

Next, assuming 1A source active source only, with reference to figure 12(b).

$$1 = i_o'' + i''$$

$$= \frac{v''}{1} + \frac{v'' - 2i_o''}{5} = 1.2v'' - 0.4i_o''$$

But $i_o'' = \frac{v''}{1}$

∴ We finally get,

$$1 = 1.2i_o'' - 0.4i_o'' = 0.8i_o''$$

i.e., $i_o'' = 1.25A$ and $i'' = \frac{v'' - 2i_o''}{5}$

$$= -\frac{i_o''}{5} = -0.25A$$

Using the principle of Superposition,

$$i_o = i_o' - i_o'' = \frac{3}{4} - 1.25 = -0.5A$$

$$i = i_o' + i'' = \frac{3}{4} - 0.25 = 0.5A$$

Example 4: Find the power loss in 5Ω resistor by Superposition Theorem in figure 19.

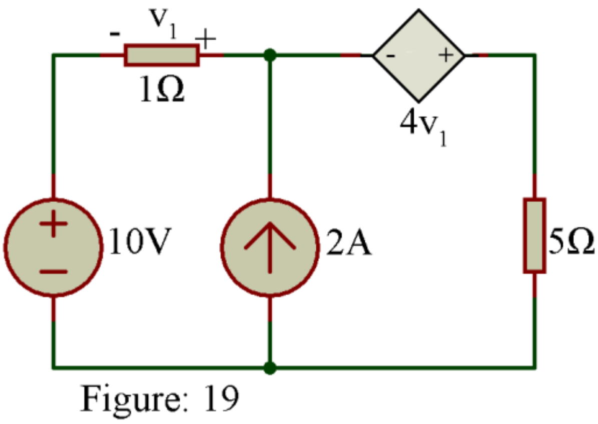


Figure: 19

Solution:

Assuming the 10V source first (figure 20), KVL yields

$$-10 - v_1 - 4v_1 + 5I_1 = 0$$

or, $5I_1 = 5v_1 + 10$ …(1)

But $v_1 = -1 \times I_1$ …(2)

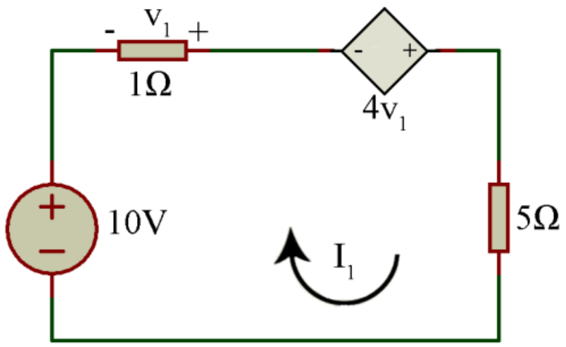


Figure: 20

Using (2) in (1),

$$10I_1 = 10$$

$$I_1 = \frac{10}{10} = 1$$

Next, taking the current source only, referring figure 21, at node (1),

$$2 = \frac{v_1}{1} + \frac{v_1 + 4v_1}{5} = v_1 + 0.2v_1 + 0.8v_1$$

$$v_1 = 1V.$$

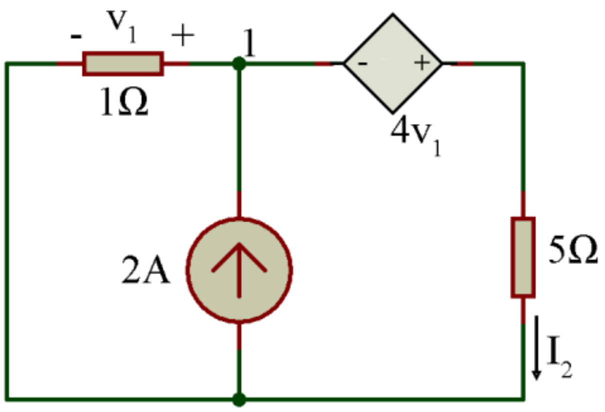


Figure: 21

$$\therefore I_2 = \frac{v_1 + 4v_1}{5} = 1A$$

Hence, the current through 5Ω resistor is

$$I = I_1 + I_2 = 2A$$

∴ Power loss in 5Ω resistor is $(2)^2 \times 5 = 20$W.

THEVENIN'S THEOREM

Thevenin's Theorem states that any complicated network across its load terminals can be substituted by a voltage source with one resistance in series. This theorem helps in the study of the variation of current in a particular branch when the resistance of the branch is varied while the remaining network remains the same.

A more general statement of Thevenin's Theorem is that any linear active network consisting of independent or dependent voltage and current source and the network elements can be replaced by an equivalent circuit having a voltage source in series with a resistance where the voltage source being the open-circuited voltage across the open-circuited load terminals and the resistance being the internal resistance of the source. In other words, the current flowing through a resistor connected across any two terminals of a network by an equivalent circuit having a voltage source V_{th} in series with a resistor R_{th}. Where V_{th} is the open-circuit voltage between the required two terminals called the Thevenin voltage and the R_{th} is the equivalent resistance of the network as seen from the two-terminal with all other sources replaced by their internal resistances called Thevenin resistance.

Example: 1 In the network of figure 1, find the current through the 10Ω resistor utilizing Thevenin's Theorem.

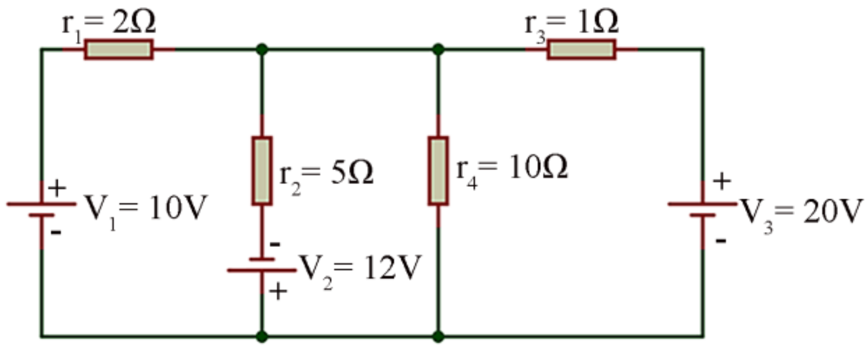


Figure: 1

Solution:

Let the resistance r_4 (10Ω) be removed and the circuit is exhibited in figure 2.

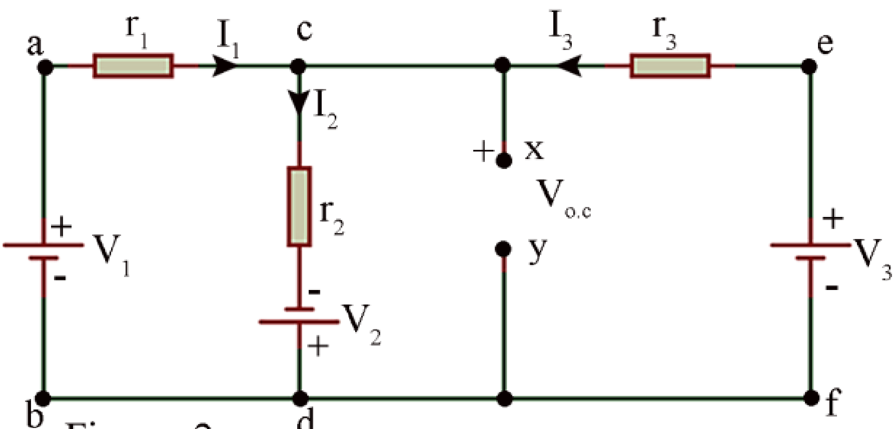


Figure: 2

At node C, application of KCL yields

$$I_1 + I_3 - I_2 = 0$$

or, $\frac{V_1 - V_{o.c}}{r_1} + \frac{V_3 - V_{o.c}}{r_3} - \frac{V_{o.c} + V_2}{r_2} = 0$

[assuming the open circuit voltage across the terminal x-y in figure 2 to be $V_{o.c}$; obviously, the potential at C node is $V_{o.c}$]

i.e. $\frac{10 - V_{o.c}}{2} + \frac{20 - V_{o.c}}{1} - \frac{V_{o.c} + 12}{5} = 0$

or, $-0.5V_{o.c} - V_{o.c} - 0.2V_{o.c} = 2.4 - 20 - 5$

or, $1.7V_{o.c} = +22.6V$

$\therefore$ $V_{o.c} = 13.29V$

Next, the independent voltage sources are removed by short circuits (figure 3)

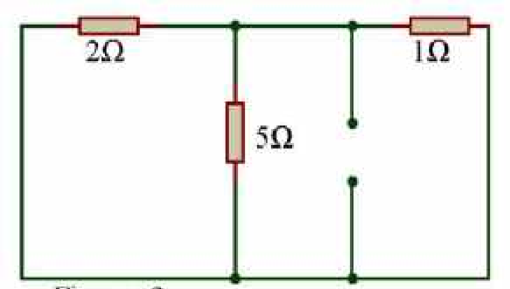


Figure: 3

Here,

$$\frac{1}{R_{th}} = \frac{1}{2} + \frac{1}{5} + \frac{1}{1}$$

or, $\frac{1}{R_{th}} = \frac{10}{17}\Omega$

Thevenin's equivalent circuit being shown in the figure 4,

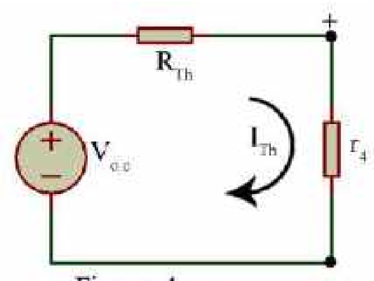


Figure: 4

I_4 current through $r_4 = I_{th} = \frac{V_{o.c}}{R_{th} + r_4}$

i.e., $I_4 = \frac{13.29}{\frac{10}{17} + 10} = 1.26A$

Thus current through r_4 is 1.26A**.**

Example: 2

In the circuit of figure 5, find the power loss in the 1Ω resistor by Thevenin's Theorem.

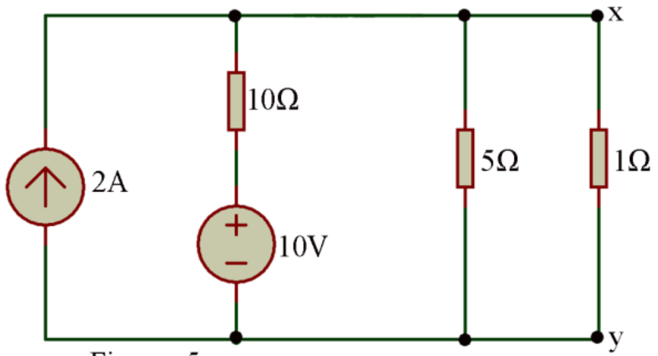


Figure: 5

Solution:

Let us first remove the 1Ω resistor from x-y terminal as shown in figure 6.

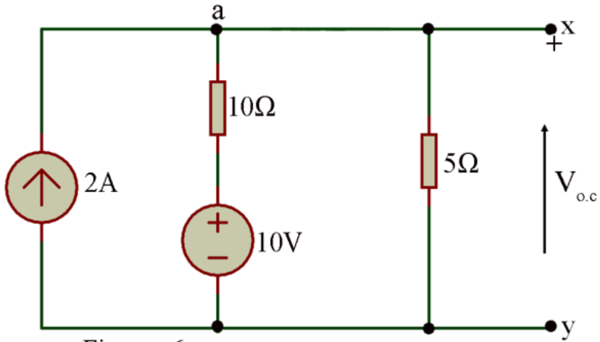


Figure: 6

Application of KCL at node "a" results

$$\frac{V_{o.c}}{5} + \frac{V_{o.c} - 10}{10} = 2$$

or, $0.2V_{o.c} + 0.1V_{o.c} - 1 = 2$

or, $V_{o.c} = 10V$

Figure 7 represents the circuit with independent sources deactivated.

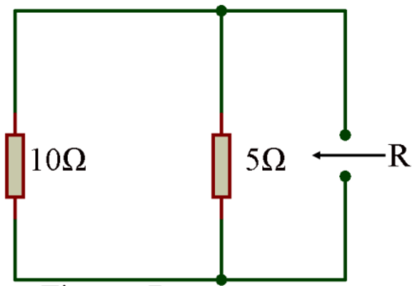


Figure: 7

$$R_{th} = \frac{5 \times 10}{5 + 10} = 3.33\Omega$$

Thevenin's equivalent circuit being shown in figure 8.

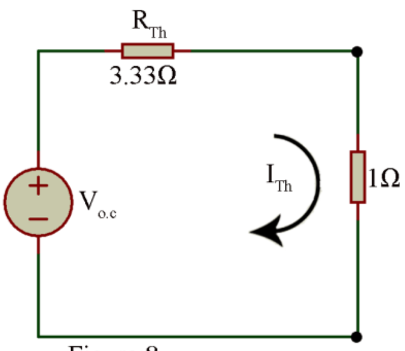


Figure:8

Here, current through 1Ω resistor is then given by

$$I_{th} = \frac{v_{o.c}}{R_{th} + 1} = \frac{10}{3.33 + 1} = 2.31A.$$

∴Power loss in 1Ω resistor

$$= 2.31^2 \times 1 = 5.33W$$

Example: 4

In the network of figure 11, find $V_{x\text{-}y}$ and R_{int} (across x-y) using Thevenin's theorem.

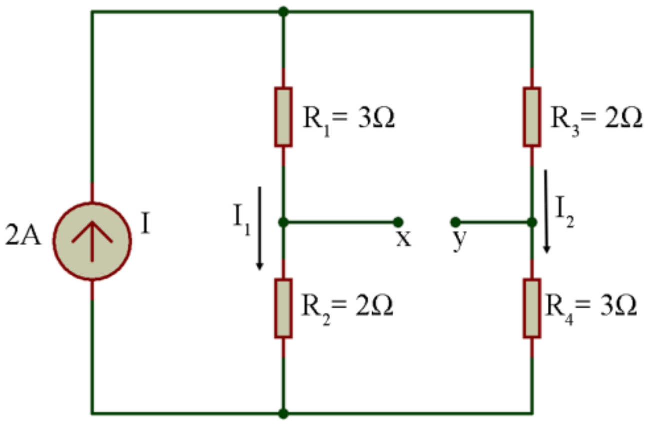


Figure: 11

Solution:

In fig. 11,

$$I_1 = I\frac{R_3 + R_4}{(R_1 + R_2) + (R_3 + R_4)}$$ [by current division formula]

$$= 2 \times \frac{2 + 3}{(2 + 3) + (3 + 2)} = 1A$$

and $I_2 = I \times \frac{R_1 + R_2}{(R_1 + R_2) + (R_3 + R_4)}$

$$= 2 \times \frac{3+2}{(3+2)+(2+3)} = 1A$$

Thus, the voltage drop across $R_2 = 1 \times 2 = 2V$

and voltage drop across $R_4 = 1 \times 3 = 3V$

$$\therefore V_{x-y} = V_x - V_y = 2V - 3V = -1V$$

i.e., y is at higher potential.

To find R_{int}, across x-y, current source is removed and $(R_1 + R_2)$ is in parallel to $(R_3 + R_4)$

$\therefore R_{int} = (R_1 + R_2)||(R_3 + R_4)$ (figure 12)

$$= 5||5 = 2.5\Omega.$$

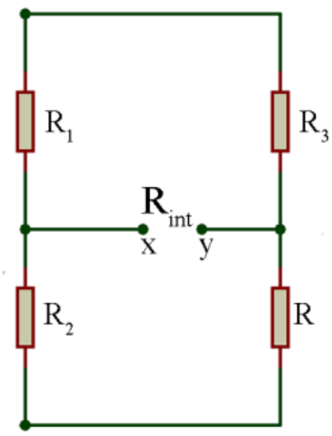


Figure: 12

NORTON'S THEOREM

Norton's Theorem states that – A linear active network consisting of the independent or dependent voltage source and current sources and the various circuit elements can be substituted by an equivalent circuit consisting of a current source in parallel with a resistance. The current source being the short-circuited current across the load terminal and the resistance being the internal resistance of the source network.

The Norton's theorems reduce the networks equivalent to the circuit having one current source, parallel resistance and load**. Norton's theorem** is the converse of Thevenin's Theorem. It consists of the equivalent current source in parallel with a resistance instead of an equivalent voltage source in series with a resistance as in Thevenin's theorem.

Example: 1

Find Norton's equivalent circuit to the left of terminal x-y in the network of figure 1.

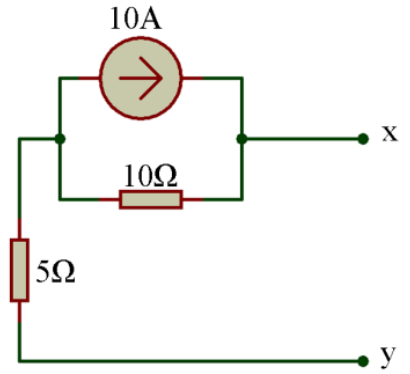


Figure: 1

Solution:

Let us first short the terminals x-y (figure 2).

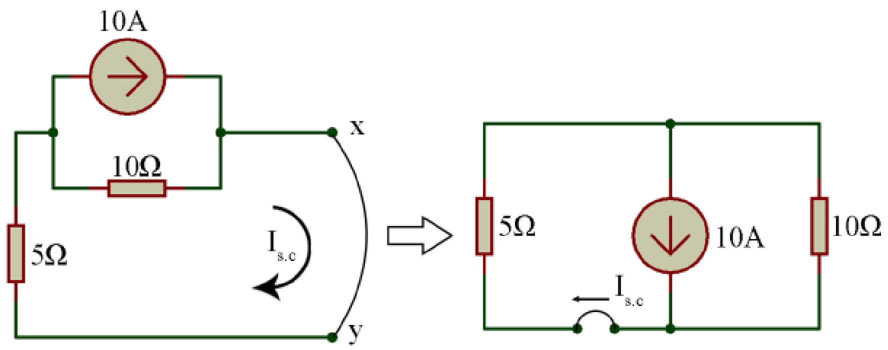


Figure: 2

Here, $I_{s.c}$ is the current through 5Ω resistor.

$$\therefore I_{s.c} = 10 \times \frac{10}{10+5} = 6.67A$$

[by current divider rule]

To determine the equivalent resistance of the circuit of figure 1, looking through x-y, the constant source is deactivated as shown in figure 3 (a).

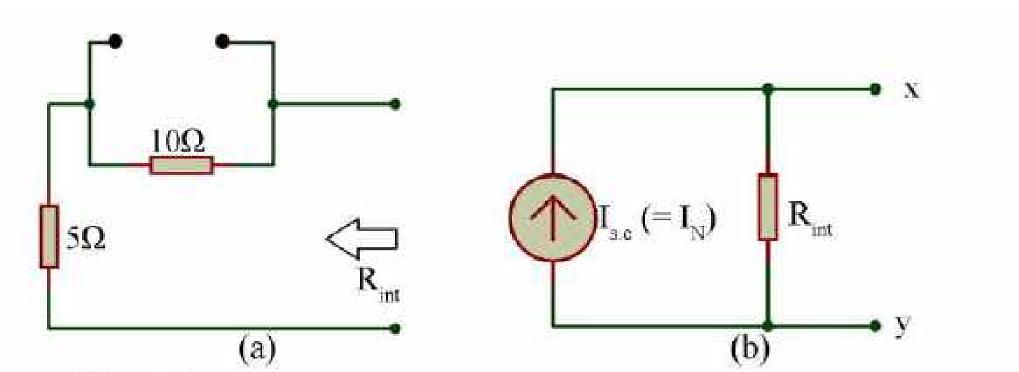


Figure: 3

Here,

$$R_{int} = 10 + 5 = 15\Omega$$

Norton's equivalent circuit has been shown in figure 3(b).

Here,

$$I_N = 6.67A; R_{int} = 15\Omega.$$

Example: 2

Find the current through R_L in the circuit of figure 7 using Norton's Theorem.

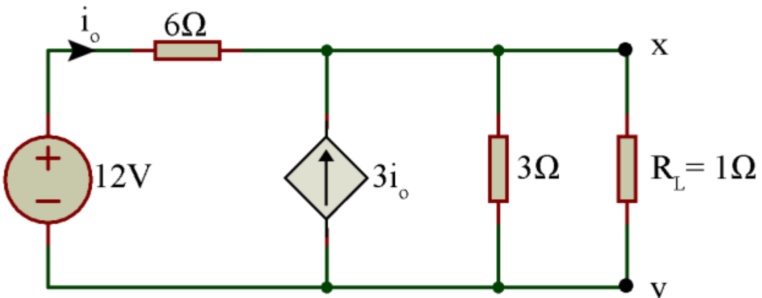


Figure: 7

Solution:

Let us first remove R_L from x-y terminals and short x-y(figure 8).

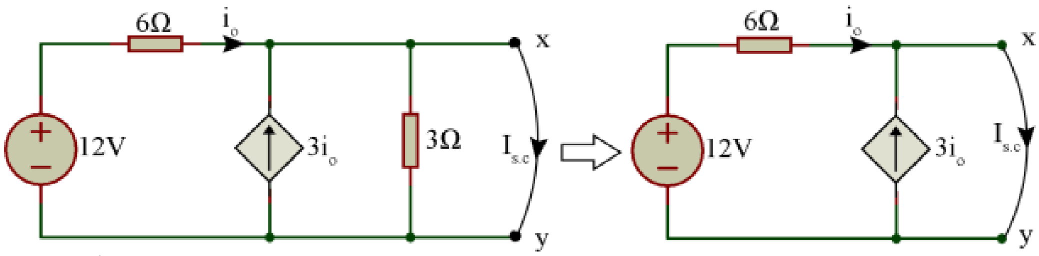


Figure: 8

It is evident from the figure that

$$I_{s.c} = 3i_o + i_o = 4i_o$$

But $$i_o = \frac{12}{6} = 2A$$

$$\therefore I_{s.c} = 4 \times 2 = 8A (= I_N)$$

Let us now remove the short circuit and the circuit is open circuited at x-y (figure 9).

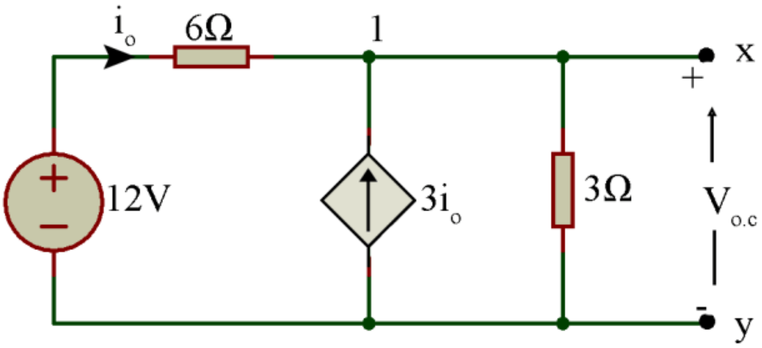


Figure: 9

Nodal analysis at node 1 gives

$$i_o + 3i_o - \frac{v_{o.c}}{3} = 0$$

or, $$4i_o - \frac{v_{o.c}}{3} = 0$$

or, $$4(\frac{12 - v_{o.c}}{6}) - \frac{v_{o.c}}{3} = 0$$

or, $$8 - \frac{2v_{o.c}}{3} - \frac{v_{o.c}}{3} = 0$$

or, $$v_{o.c} = 8V$$

This gives $$R_{int} = \frac{v_{o.c}}{I_{s.c}} = \frac{8}{8} = 1\Omega$$

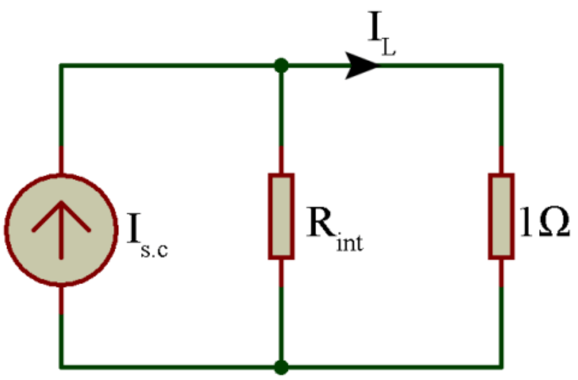


Figure: 10

Norton's equivalent circuit being shown in figure 10.

I_L (current through 1Ω resistor)

$$= I_{s.c}\frac{R_{int}}{R_{int} + R_L} = 8\frac{1}{1+1} = 4A.$$

Example: 3

Find the current through 3Ω resistor in the circuit of figure 25 using Norton's theorem. Verify the result using Thevenin's theorem.

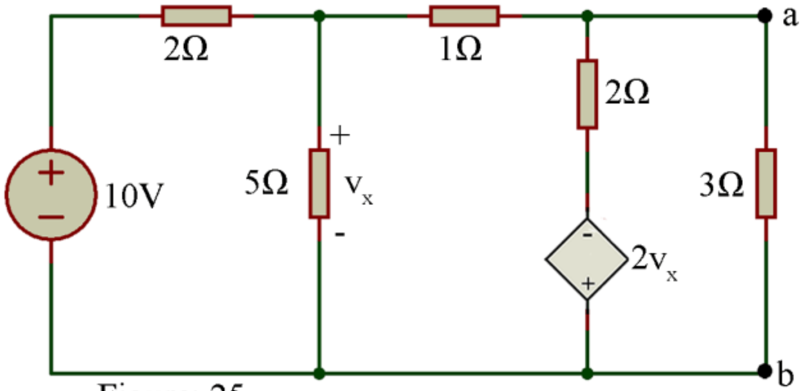


Figure: 25

Solution:

Let us remove the 3Ω resistor first and short terminals a-b (figure 26)

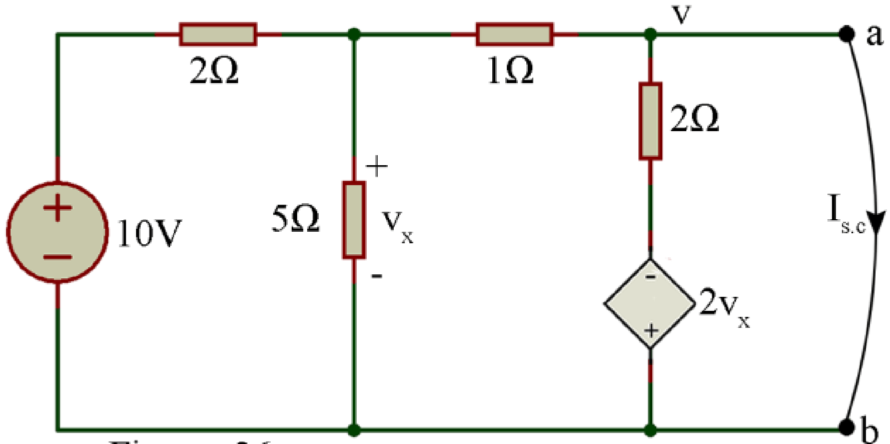


Figure: 26

At node a, nodal analysis gives

$$\frac{v - v_x}{1} + \frac{v + 2v_x}{2} + I_{s.c} = 0$$

[Assuming node a voltage to be v]

But v = 0, since a-b terminals are shorted. Thus, the nodal equation finally becomes

$$-v_x + v_x + I_{s.c} = 0 \text{ i.e. } I_{s.c} = 0$$

Next, terminal a-b is open circuited (figure 27)

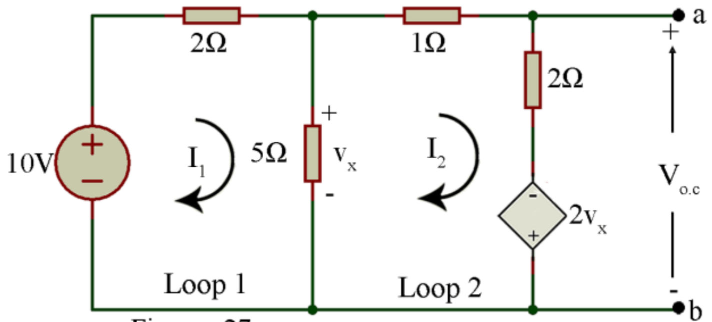


Figure: 27

In loop-1, KVL yields

$$-10 + 2I_1 + 5(I_1 - I_2) = 0$$

or, $7I_1 - 5I_2 = 10$ ...(1)

In loop-2, KVL yields

$$-v_x + I_2 \times 1 + 2I_2 - 2v_x = 0$$

or, $-3v_x + 3I_2 = 0 \text{ or } v_x = I_2$ ...(2)

But $v_x = (I_1 - I_2)5$ ...(3)

Substituting (2) in (3), we get

$$I_2 = 5I_1 - 5I_2 \text{ or } 5I_1 - 6I_2 = 0$$

or, $I_1 = \frac{6}{5}I_2$

Then from (1).

$$7(\frac{6}{5}I_2) - 5I_2 = 10$$

or, $3.4I_2 = 10 \text{ or } I_2 = 2.94A$

and $I_1 = 3.528A$

This gives v_x = 2.94V and $2v_x$ (the dependent source) as 5.88V.

$\therefore$ $V_{o.c}$ = drop across 2Ω in loop 2 $= I_2 \times 2 - 5.88 = 0$

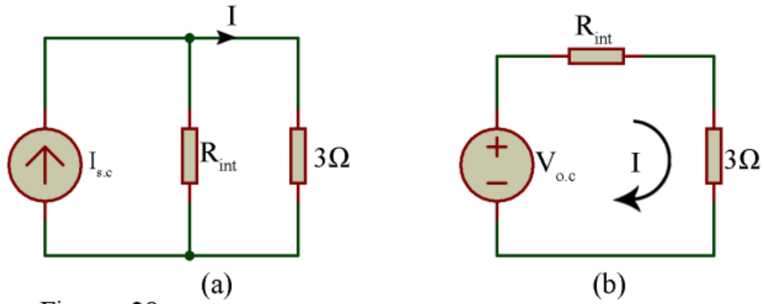


Figure: 28

Thus from Norton's equivalent circuit (figure 28(a))

$$I = I_{s.c}\frac{R_{int}}{R_{int}+3} = 0 \quad [\because I_{s.c} = 0]$$

While from Thevenin's equivalent circuit (figure 28(b)),

$$I = \frac{V_{o.c}}{R_{int}+3} = 0 \quad [V_{o.c} = 0]$$

MAXIMUM POWER TRANSFER THEOREM

Maximum Power Transfer Theorem states that – A resistive load, being connected to a DC network, receives maximum power when the load resistance is equal to the internal resistance known as (Thevenin's equivalent resistance) of the source network as seen from the load terminals. The Maximum Power Transfer theorem is used to find the load resistance for which there would be the maximum amount of power transfer from the source to the load.

The maximum power transfer theorem is applied to both the DC and AC circuit. The only difference is that in the AC circuit the resistance is substituted by the impedance. The maximum power transfer theorem finds their applications in communication systems which receive low strength signal. It is also used in speaker for transferring the maximum power from an amplifier to the speaker.

Example: 1
Find the value of R in the circuit of figure 1 such that maximum power transfer takes place. What is the amount of this power?

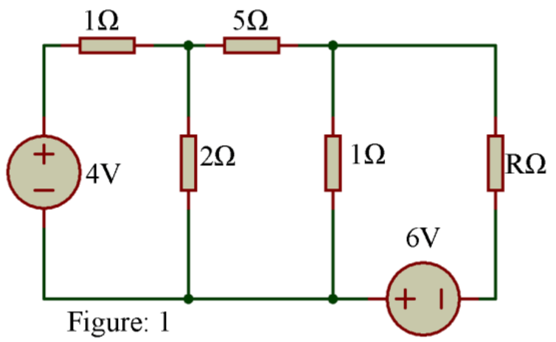


Figure: 1

Solution:
Let R be replaced first and the open circuit voltage be $V_{o.c}$ (figure 2).

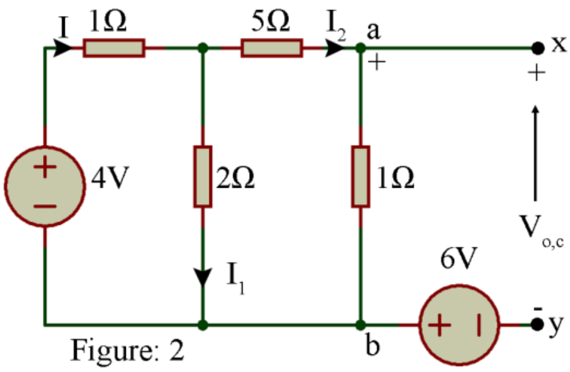


Figure: 2

Here

$$I = \frac{4V}{[(5+1)||2+1]\Omega} = \frac{4}{\frac{5}{2}} = \frac{8}{5}A$$

$$\therefore I_2 = I\frac{2}{2+5+1} = \frac{8}{5} \times \frac{1}{4} = \frac{2}{5}A$$

The drop across a-b branch is then

$$V_{a-b} = \frac{2}{5} \times 1 = \frac{2}{5}V$$

Obviously,

$$V_{o.c} = V_{a-b} + 6V = \frac{2}{5} + 6 = \frac{32}{5}V$$

or, $V_{o.c} = 6.4V$

To find internal resistance of the circuit across x-y, with reference to figure 3,

$$R_{Th} = 1||2 + 5||1 = \frac{17}{3}||1 = \frac{17}{20}\Omega$$

$$= 0.85\Omega$$

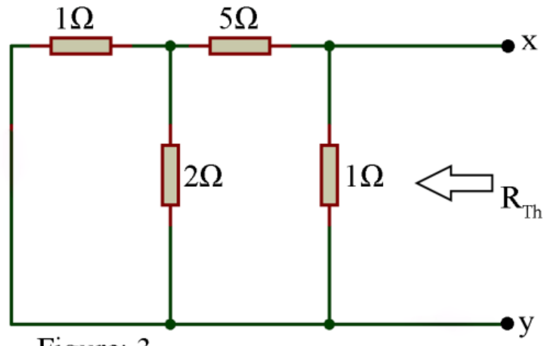


Figure: 3

As per maximum power transfer theorem,

$$R = R_{Th} = 0.85\Omega$$

and P_{max} (max.power) $= \frac{V_{o.c}^2}{4R} = \frac{6.4^2}{4 \times 0.85} \approx 12W$

Example: 2

What should be the value of R such that maximum power transfer can take place from the rest of the network to R in figure 4 ? Obtain the amount of this power.

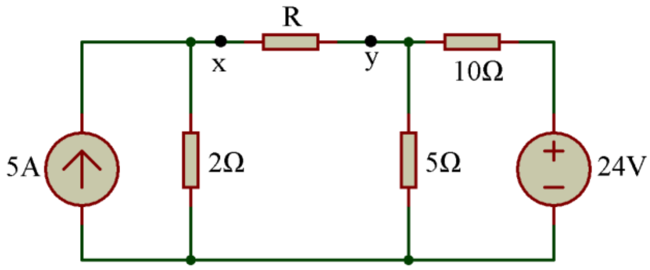


Figure: 4

Solution:

Let us first convert the "I" source to "V" source and remove R from x-y terminal, the voltage at these terminals being $V_{o.c}$.

With reference to figure 5,

$$i = \frac{24}{15} = 1.6A$$

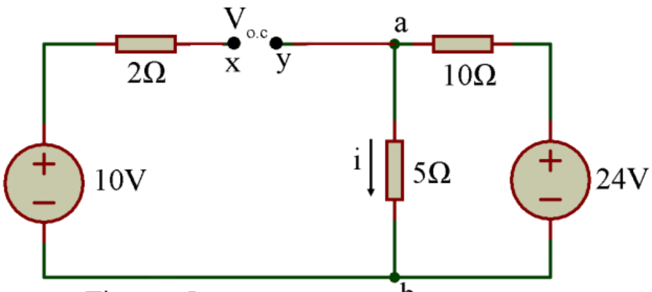


Figure: 5

$$\therefore V_{a-b} = \text{ drop across } 5\Omega = 1.6 \times 5 = 8V$$

Thus in the left loop,

$$-10 + v_{o.c} + 8 = 0$$

or, $v_{o.c} = 2V$

Again, with reference to figure 6, R_{Th} (internal resistance of the circuit looking through x-y) is obtained as

$$R_{Th} = \frac{10 \times 5}{10 + 5} + 2 = 5.33\Omega$$

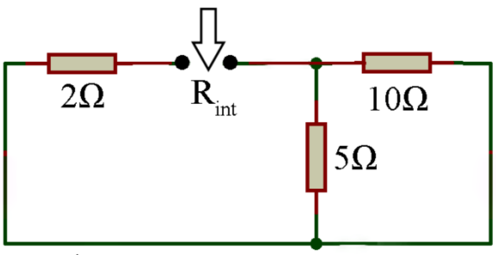


Figure: 6

As per maximum power transfer theorem,

$$R = R_{Th} = 5.33\Omega$$

and $P_{max} = \frac{V_{o.c}{}^2}{4R} = \frac{2^2}{4 \times 5.33} = 188mW.$

Example: 3

What is the value of R such that maximum power transfer takes place from the sources to R in the circuit of figure 19? Determine the amount of maximum power.

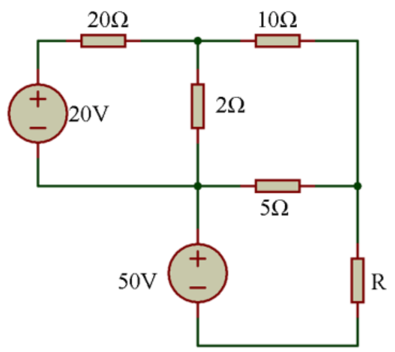


Figure: 19

Solution:

Replacing R and naming the loop currents, with reference to figure 20, at loop-1,

$$-20 + I_1(20 + 2) - 2I_2 = 0$$

or, $22I_1 - 2I_2 = 20$

or, $11I_1 - I_2 = 10$ …(1)

At loop-2

$$(10 + 5 + 2)I_2 - 2I_1 = 0$$

or, $17I_2 - 2I_1 = 0$

or, $I_1 = \frac{17}{2} I_2 = 8.5 I_2$ …(2)

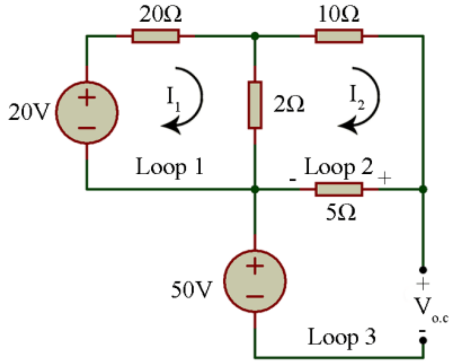


Figure: 20

Using (2) in (1),

$$11(8.5I_2) - I_2 = 10 \text{ or } 92.5I_2 = 10$$

ie., $I_2 = \dfrac{10}{92.5} = 0.108A$

Thus, drop across 5Ω,

i.e., $V_{5\Omega} = 0.108 \times 5 = 0.54V$

∴ In loop-3, we find that

$$-50 - 0.54 + V_{o.c} = 0$$

or, $V_{o.c} = 50.54V$

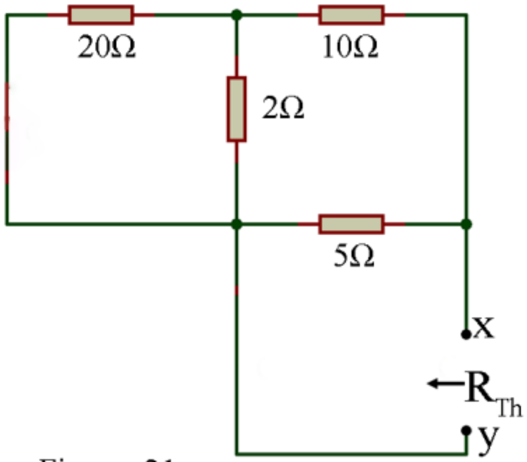


Figure: 21

To find internal resistance across x-y we find

$$R_{Th} = [(20||2) + 10||5]\Omega$$

$$= \frac{(\frac{20 \times 2}{20 + 2} + 10)5}{\frac{20 \times 2}{20 + 2} + 10 + 5}$$

$$= \frac{11.82 \times 5}{16.82} \approx 3.5\Omega$$

As per maximum power theorem,

$$R = R_{Th} = 3.5\Omega$$

$$P_{max} = \frac{V_{oc}{}^2}{4R} = \frac{(50.54)^2}{4 \times 3.5} = 182.44W$$

Example: 4

What is amount of maximum power transfer to R in the circuit of figure 22?

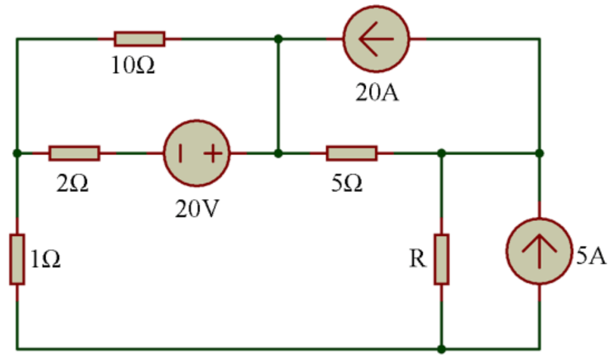


Figure: 22

Solution:
R is replaced by open circuit in figure 23.

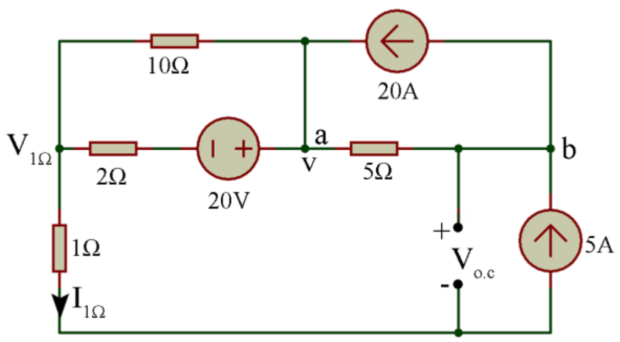


Figure: 23

Obviously, the current $I_{1\Omega}$ through the 1Ω resistor is 5A (downwards).

$$\therefore \text{Drop across it}(V_{1\Omega}) = 5 \times 1 = 5V$$

Also, at node a, assuming the node voltage to be v volts, nodal analysis at a gives

$$\frac{v - V_{1\Omega}}{10} + \frac{v - 20 - V_{1\Omega}}{2} + \frac{v - V_{o.c}}{5} = 2$$

or, $0.1v - 0.1V_{1\Omega} + 0.5v - 10 - 0.5V_{1\Omega} + 0.2v - 0.2V_{o.c} = 2$

or, $0.8v - 0.6V_{1\Omega} = 12 + 0.2V_{o.c}$

or, $0.8v - 0.6 \times 5 = 12 + 0.2V_{o.c}$

i.e., $0.8v - 0.2V_{o.c} = 12 + 3 = 15$ …(1)

Again, at node b, nodal analysis gives

$$\frac{V_{oc} - v}{5} + 2 = 5$$

or, $0.2V_{o.c} - 0.2v = 3$ …(2)

Solving for (1) and (2),

$$0.2V_{oc} - 0.2v = 3$$

$$\therefore V_{oc} = 45V \quad \text{[from(2)]}$$

Again, with reference to figure 24,

$$R_{Th} = [(10||2) + 1] + 5 = (\frac{20}{12} + 1) + 5 = 7.67\Omega$$

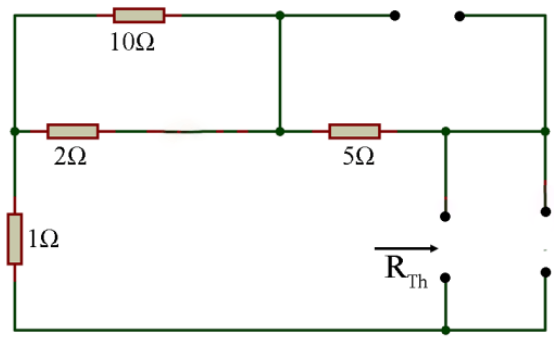


Figure: 24

Following the theorem of maximum power transfer

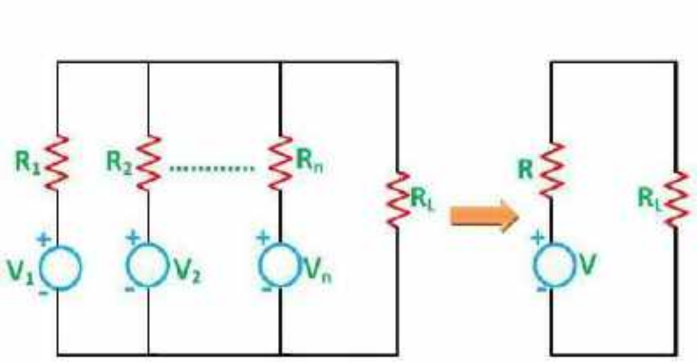
$$R = R_{Th} = 7.67\Omega$$

and $P_{max} = \frac{{V_{o.c}}^2}{4R} = \frac{45^2}{4 \times 7.67} \approx 66W$

<u>Millman's Theorem</u>

The **Millman's Theorem** states that when a number of voltage sources (V_1, V_2, V_3……V_n) are in parallel having internal resistance (R_1, R_2, R_3……….R_n) respectively, the arrangement can be replaced by a single equivalent voltage source V in series with an equivalent series resistance R.

The utility of **Millman's Theorem** is that the number of parallel voltage sources can be reduced to one equivalent source. It is applicable only to solve the parallel branch with one resistance connected to one voltage source or current source. It is also used in solving network having an unbalanced bridge circuit.



As per Millman's Theorem

$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \dots \dots \dots \pm V_n G_n}{G_1 + G_2 + \dots \dots . + G_n} \quad \text{and}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + ,,,,,,, + G_n}$$

Steps for Solving Millman's Theorem

Step 1 – Obtain the conductance (G_1, G_2,….) of each voltage source (V_1, V_2,….).
Step 2 – Find the value of equivalent conductance G by removing the load from the network.
Step 3 – Now, apply Millman's Theorem to find the equivalent voltage source V by the equation shown below

$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \dots \dots \dots \pm V_n G_n}{G_1 + G_2 + \dots \dots . + G_n}$$

Step 4 – Determine the equivalent series resistance (R) with the equivalent voltage sources (V) by the equation

$$R = \frac{1}{G}$$

Step 5 – Find the current I_L flowing in the circuit across the load resistance R_L by the equation

$$I_L = \frac{V}{R + R_L}$$

Example – 1
A circuit is given as shown in fig-c. Find out the voltage across 2 Ohm resistance and current through the 2 ohm resistance.

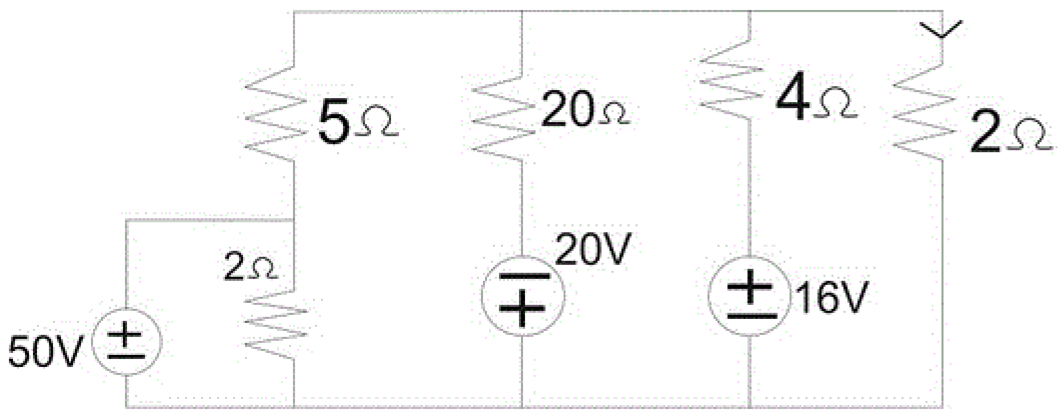


Fig-c

Answer : We can go through any solving method to solve this problem but the most effecting and time saving method will be none another than **Millman's theorem**. Given circuit can be reduced to a circuit shown in fig-d where equivalent voltage V_E can be obtained by millman's theorem and that is

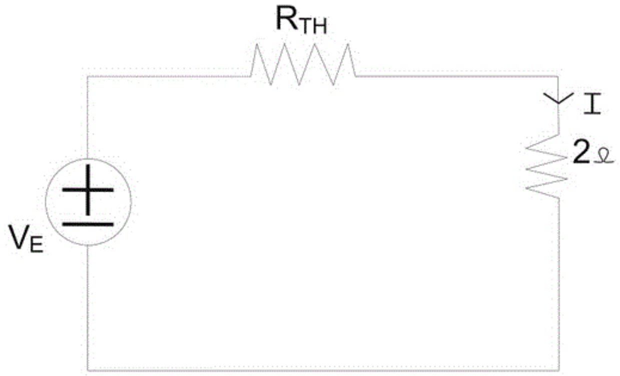


fig-d

$$V_E = \frac{\frac{50}{5} - \frac{20}{20} + \frac{16}{4}}{\frac{1}{5} + \frac{1}{20} + \frac{1}{4}} = 26\ V$$

Equivalent resistance or Thevenin resistance can be found by shorting the voltage sources as shown in fig – e.

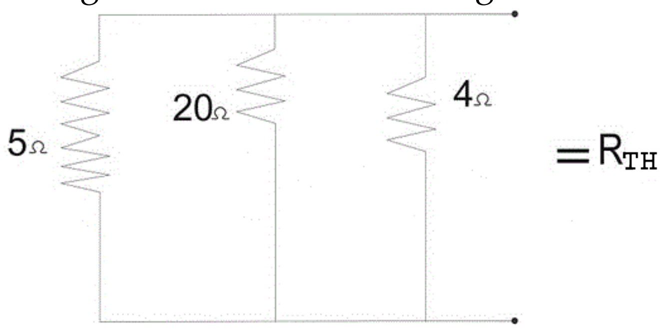


fig-e

$$R_{TH} = \frac{1}{\frac{1}{5} + \frac{1}{20} + \frac{1}{4}} = 2\ \Omega$$

Now we can easily found the required current through 2 Ohm load resistance by Ohm's law.

$$I_{2\Omega} = \frac{26}{2+2} = 6.5\ A$$

Voltage across load is,

$$V_L = I_{2\Omega} \times 2 = 6.5 \times 2 = 13\ V$$

Example 2 :

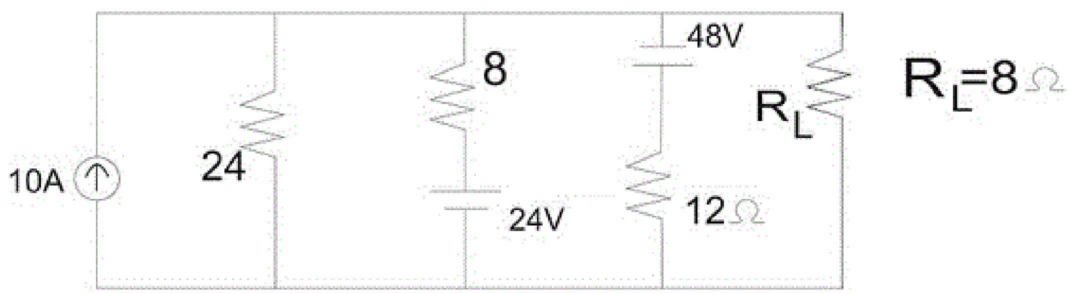


fig- h

A circuit is given as shown in fig-h. Find out the current through load resistance where $R_L = 8\ \Omega$.

Answer : This problem may seem to be difficult to solve and time consuming but it can easily be solved in a very less time with the help of **Millman's Theorem**. The given circuit can be reduced in a circuit as shown in fig – i.

Where, V_E can be obtained with the help of Millman's theorem,

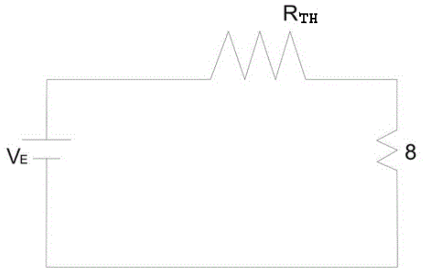


fig-I

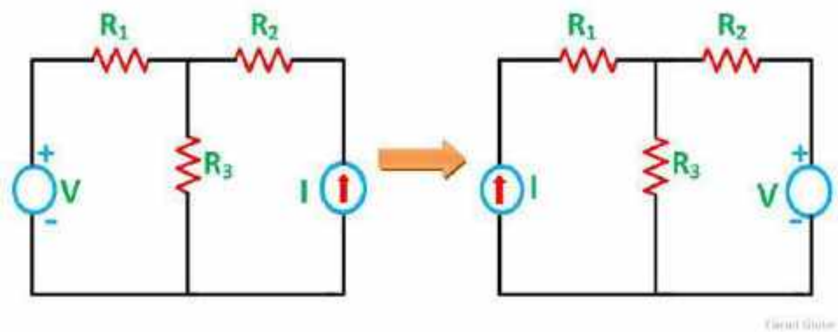
$$V_E = \frac{10 + \frac{0}{24} + \frac{24}{8} - \frac{48}{12}}{\frac{1}{24} + \frac{1}{8} + \frac{1}{12}} = 36\ V \quad \& \quad R_{TH} = 6\ \Omega$$

Therefore, current through load resistance 8 Ω is,

$$I_{8\Omega} = \frac{36}{6+8} = 2.57\ A$$

Reciprocity Theorem

Reciprocity Theorem states that in any branch of a bilateral linear network or circuit, the current at one point in a circuit due to a voltage at a second point is the same as the current at the second point due to the same voltage at the first. The reciprocity theorem is valid for almost all passive networks.



The limitation of this theorem is that it is applicable only to single-source networks and not in the multi-source network.

Example 1: Show the application of reciprocity theorem in the network of fig 1.

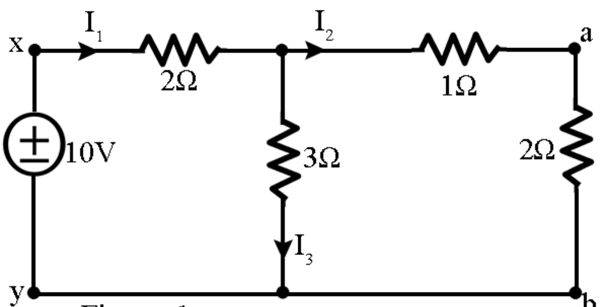


Figure: 1

Solution:

With the reference to figure 1, the equivalent resistance across x-y is given by

$$R_{eq} = [(2+1)||3] + 2 = 3.5\Omega$$

$$\therefore I_1 = \frac{10}{3.5} = 2.86A$$

$$I_2 = 2.86 \times \frac{3}{3+3} = 1.43A$$

$$I_3 = 2.86 - 1.43 = 1.43A$$

with reference to figure 2,

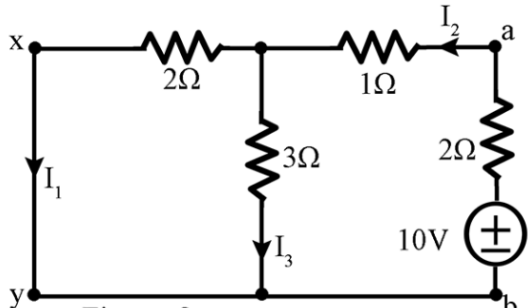


Figure: 2

$$R_{eq} = (2||3) + 1 + 2 = \frac{6}{5} + 3 = \frac{21}{5} = 4.2\Omega$$

$$\therefore I_2 = \frac{10V}{4.2\Omega} = 2.381A$$

This gives

$$I_1 = I_2 \frac{3}{3+2} = 2.381 \times \frac{3}{5} = 1.43A$$

Hence we observe that when the sources was in branch x-y as in figure 1, the a-b branch current is 1.43A; again when the source is in branch a-b (figure 2), the x-y branch current becomes 1.43A. This proves the reciprocity theorem.

Example 2: Verify the reciprocity theorem in the network given below (figure 7).

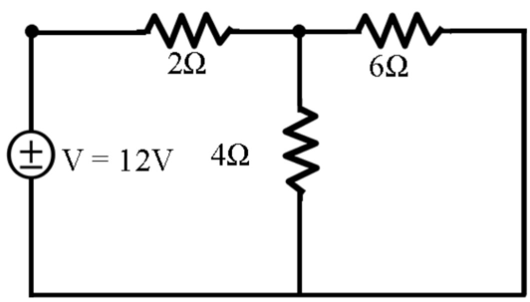


Figure: 7

Solution:
In order to verify reciprocity theorem, the network is drawn as;

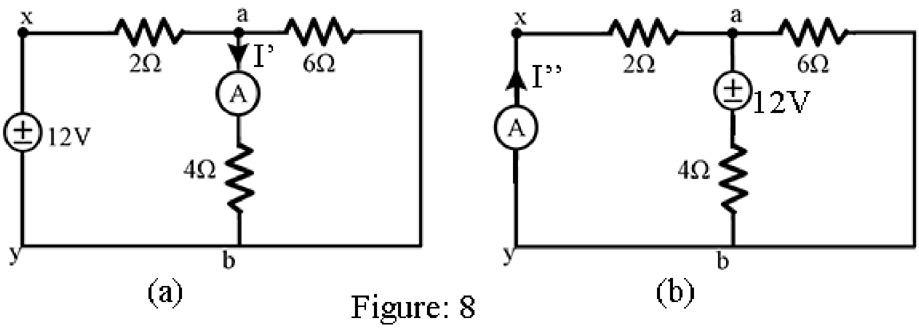


Figure: 8

Considering figure 8(a),
Equivalent resistance (R_{eq}) is given by,

$$R_{eq} = (4||6) + 2$$

$$= \frac{4 \times 6}{4 + 6} + 2 = 4.4\Omega$$

Current supplied by battery is given by

$$= \frac{12}{4.4} = \frac{30}{11}A = 2.727A$$

Ammeter current, I' is given by

$$I' = \frac{\frac{30}{11}}{4 + 6} \times 6 = \frac{18}{11}A = 1.636A$$

Now, considering figure 8(b),
Equivalent resistance (R_{eq}) is given by,

$$R_{eq} = (2||6) + 4$$

$$= \frac{2 \times 6}{2 + 6} + 4 = 5.5\Omega$$

Current supplied by battery is given by

$$= \frac{12}{5.5} = \frac{24}{11}A = 2.182A$$

Ammeter current, I" is given by

$$I'' = \frac{\frac{24}{11}}{2 + 6} \times 6 = \frac{18}{11}A = 1.636A$$

Hence, we observed that when the source was in branch xy as in figure 8(a), the ab branch current was 1.636 A and when the source was in branch ab as in figure 8(b), the xy branch current was 1.636 A. This proves the reciprocity theorem.

CHAPTER 3 – POWER RELATION IN AC CIRCUITS & TRANSIENT RESPONSE OF PASSIVE CIRCUITS

3.1 AC Waveform and AC Circuit Theory - AC Sinusoidal Waveforms are created by rotating a coil within a magnetic field. Alternating voltages and currents form the basis of AC Theory. An alternating function or AC Waveform is defined as one that varies in both magnitude and direction in more or less an even manner with respect to time. It is a "Bi-directional" waveform. An AC function can represent either a power source or a signal source with the shape of an AC waveform generally following that of a mathematical sinusoid being defined as: $A(t) = A_{max} \sin(2\pi f t)$.

AC Waveform Characteristics

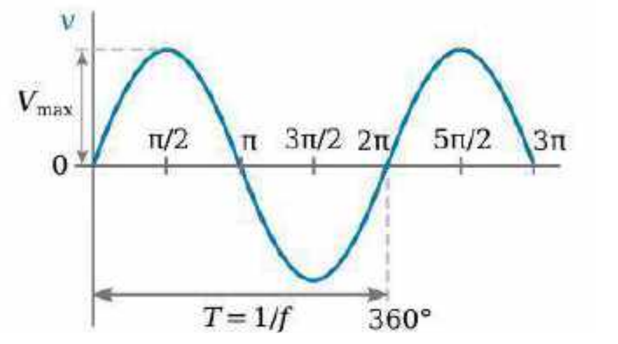
• **Cycle** of a wave is one complete evolution of its shape until the point that it is ready to repeat itself. It comprises of positive and negative half cycles.

• **Time Period** (T) is defined as the time taken by an alternating quantity to complete one cycle. Its SI unit is second (s). This can also be called the Periodic Time of the waveform for sine waves, or the Pulse Width for square waves.

• **Frequency** (f) is defined as the number of cycles completed within one second. Frequency is the reciprocal of the time period, (f = 1/T). The unit of frequency is Hertz, (Hz).

Definition of Frequency Prefixes

Prefix	Definition	Written as	Periodic time
Kilo	Thousand	kHz	1ms
Mega	Million	MHz	1us
Giga	Billion	GHz	1ns
Terra	Trillion	THz	1ps



• **Amplitude** (A) is the magnitude or intensity of the signal waveform measured in volts or amps.

Peak Amplitude - The peak value is the greatest value of either voltage or current that the waveform reaches during each half cycle measured from the zero baseline. For pure sinusoidal waveforms, this peak value will always be the same for both half cycles (i.e. +Vm or -Vm).

Instantaneous Amplitude - The value of alternating quantity (emf, voltage or current) at any particular instant is called the instantaneous value and is designated by *e* for emf, *v* for voltage and *i* for current. Expression for instantaneous value of ac voltage (assumed sinusoidal), is given as

$$v = V_{max}\ sin\ \omega t = V_{max}\ sin\ 2\pi\ ft$$

The Average Value of an AC Waveform

The average or mean value of a continuous DC voltage will always be equal to its maximum peak value as a DC voltage is constant. This average value will only change if the duty cycle of the DC voltage changes. In a pure sine wave if the average value is calculated over the full cycle, the average value would be equal to zero as the positive and negative halves will cancel each other out. So the average or mean value of an AC waveform is calculated or measured over a half cycle only. The average value of alternating voltage or current is defined as the average of all values of voltage or current over a positive half-cycle or negative half-cycle.

DERIVATON -

The instantaneous value of sinusoidal alternating current is given by the equation i = I_m sinωt or i = I_m sinθ (where θ = ωt). The sum of all currents over a half-cycle is given by area of positive half-cycle (or negative half-cycle). Therefore,

$$I_{av} = \frac{\text{Area of positive half-cycle (or negative half-cycle)}}{\text{Base length of half-cycle}} \qquad (4.37)$$

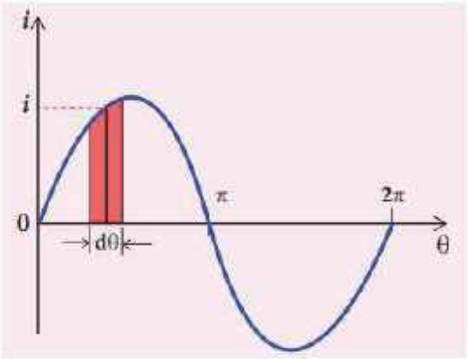


Figure 4.41 Sine wave of an alternating current

Consider an elementary strip of thickness dθ in the positive half-cycle of the current wave. Let i be the mid-ordinate of that strip.

Area of the elementary strip = i dθ

Area of positive half-cycle

$$= \int_0^{\pi} i\,d\theta = \int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= I_m\left[-\cos\theta\right]_0^{\pi} = -I_m\left[\cos\pi - \cos 0\right] = 2I_m$$

The base length of half-cycle = π

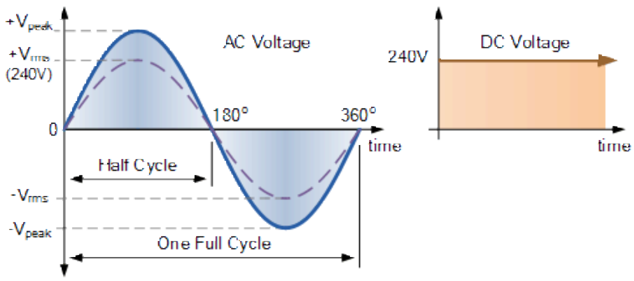
Average value of AC, $I_{av} = 2I_m/\pi$

For a pure sinusoidal waveform this average or mean value will always be equal to 0.637*I_{max} or 0.637*V_{max}.

The RMS Value of an AC Waveform

Root mean square or R.M.S. value of Alternating current is defined as that value of steady current, which would generate the same amount of heat in a given resistance in given time, as it is done by A.C. current, when maintained across the same resistance for the same time. The effective value or RMS value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle. It is denoted by I_{RMS}. For alternating voltages, the RMS value is given by V_{RMS}.

RMS Voltage Equivalent



DERIVATON - The alternating current i = I_m sinωt or i = I_m sinθ. The sum of the squares of all currents over one cycle is given by the area of one cycle of squared wave. Therefore,

$$I_{RMS} = \sqrt{\frac{\text{Area of one cycle of squared wave}}{\text{Base length of one cycle}}} \quad (4.39)$$

An elementary area of thickness dθ is considered in the first half-cycle of the squared current wave. Let i^2 be the mid-ordinate of the element.

Area of the element = $i^2 d\theta$

$$\text{Area of one cycle of squared wave} = \int_0^{2\pi} i^2 d\theta$$

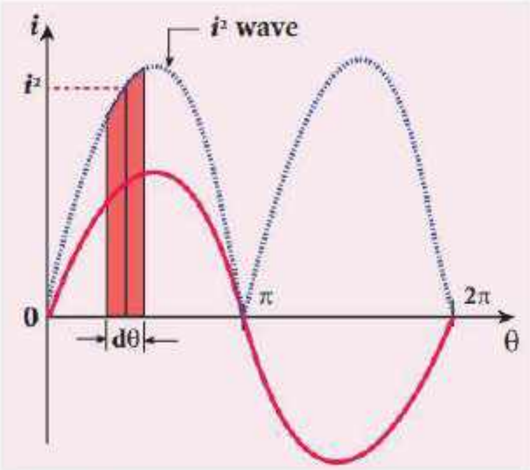


Figure 4.42 Squared wave of AC

$$= \int_0^{2\pi} I_m^2 \sin^2\theta d\theta = I_m^2 \int_0^{2\pi} \sin^2\theta d\theta \quad (4.40)$$

$$= I_m^2 \int_0^{2\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$\text{since } \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{I_m^2}{2}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta\right]$$

$$= \frac{I_m^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2}\left[\left(2\pi - \frac{\sin 2\times 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{I_m^2}{2} \times 2\pi = I_m^2 \pi \quad [\because \sin 0 = \sin 4\pi = 0]$$

Substituting this in equation (4.39), we get

$$I_{RMS} = \sqrt{\frac{I_m^2 \pi}{2\pi}} = \frac{I_m}{\sqrt{2}} \quad [\text{Base length of one cycle is } 2\pi]$$

$$I_{rms} = 0.707 I_m \quad (4.41)$$

For a pure sinusoidal waveform this effective or R.M.S. value will always be equal to $0.707 * I_{max}$ or $0.707 * V_{max}$.

Form Factor and Crest Factor

Form Factor is the ratio between the average value and the RMS value and is given as.

$$\text{Form Factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{0.707 \times V_{max}}{0.637 \times V_{max}}$$

For a pure sinusoidal waveform, the Form Factor will always be equal to 1.11.

Crest Factor is the ratio between the R.M.S. value and the Peak value of the waveform and is given as.

$$\text{Crest Factor} = \frac{\text{Peak value}}{\text{R.M.S. value}} = \frac{V_{max}}{0.707 \times V_{max}}$$

For a pure sinusoidal waveform, the Crest Factor will always be equal to 1.414.

Sinusoidal Waveform Conversion Table

Convert from	Multiply by	Or Multiply By	To Get Value
Peak	2	$(\sqrt{2})^2$	Peak-to-Peak
Peak-to-Peak	0.5	½	Peak
Peak	0.707	$1/(\sqrt{2})$	RMS
Peak	0.637	$2/\pi$	Average
Average	1.570	$\pi/2$	Peak
Average	1.111	$\pi/(2\sqrt{2})$	RMS
RMS	1.414	$\sqrt{2}$	Peak
RMS	0.901	$(2\sqrt{2})/\pi$	Average

Power relations in AC circuit

The rate of doing work or the amount of energy transferred by a circuit per unit time is known as power in AC circuits. It is used to calculate the total power required to supply a load.

Active Power (P) – It is the real or true power which is actually utilised or consumed in the circuit. The product of voltage (RMS) and current (RMS) with the cosine of the angle between them, in an AC circuit is termed as active power. It is measured in watts (W). For a pure inductive and pure capacitive circuit, the active power being zero.

Active Power, $P = VI \cos \Phi$

Reactive Power (Q) – It is the power which continuously bounces back and forth between source and load. This power is not useful in the circuit. The product of voltage (RMS) and current (RMS) with sine of the angle between them, in an AC circuit is known as reactive power. It is measured in VAR (Volt-Ampere Reactive).

Reactive Power, $Q = VI \sin \Phi$

Apparent Power (S) – It is the total power of the circuit. The product of voltage (RMS) and current (RMS) is called as the apparent power. It is measured in VA (Volt-Ampere).

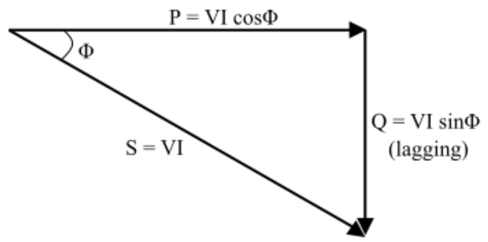
Apparent Power, S = VI

<u>Power Triangle</u>

The power triangle is the geometrical representation of the apparent power (S), active power (P) and reactive power (Q) in an AC circuit.

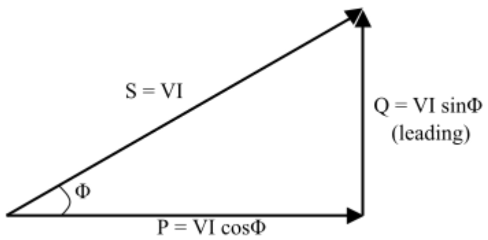
Case 1 – Inductive Circuit

In an inductive circuit, the current lags behind the voltage. Therefore, the reactive power lags behind the active power i.e. an inductor absorbs lagging reactive power and delivers leading reactive power. Hence, the power triangle for a Inductive circuit,



Case 2 – Capacitive Circuit

In a capacitive circuit, the circuit current leads the voltage across the capacitor. Therefore, the reactive power leads ahead of active power i.e. a capacitor absorbs leading reactive power whereas delivers lagging reactive power. Hence, the power triangle for a capacitive circuit,



<u>Power Factor in AC circuit</u>

The power factor of an AC circuit can be defined as the ratio of the active power to the apparent power (total power).

Power Factor, PF = Active Power/Apparent Power = VI cos Φ / VI = cos Φ

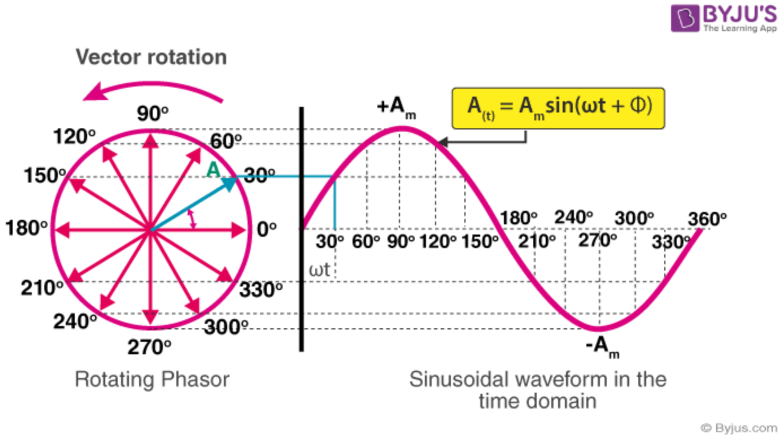
Hence, the power factor of an AC circuit is the cosine of phase angle between the voltage and current. It should be noted that, the phase angle (Φ) is the angle between the phase voltage and phase current, not between the line voltage and line current. It is dimensionless and its value is between 0 to 1.

Also, cosΦ = R/Z

Where, R= resistance in the circuit and Z= impedance of the circuit.

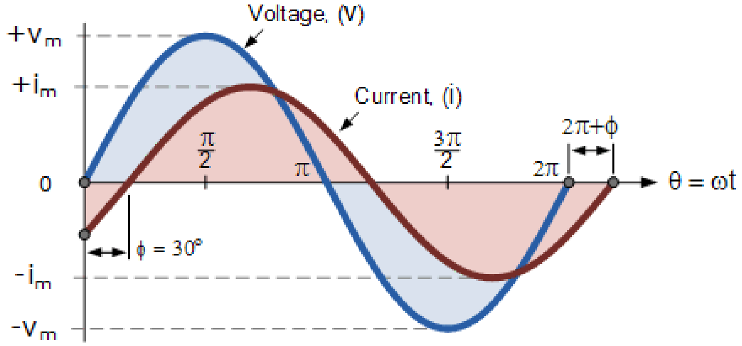
3.2 <u>Phasor representation of alternating quantities</u>

A phasor is a vector that is used to represent a sinusoidal function. It rotates about the origin with an angular speed ω. The vertical component of phasors represents the quantities that are sinusoidally varying for a given equation, such as v and i. Here, the magnitude of the phasors represents the peak value of the voltage and the current.



(Relation between phasor and sinusoidal representation of the function w.r.t time)

Phasor Diagrams are a graphical way of representing the magnitude and directional relationship between two or more alternating quantities.

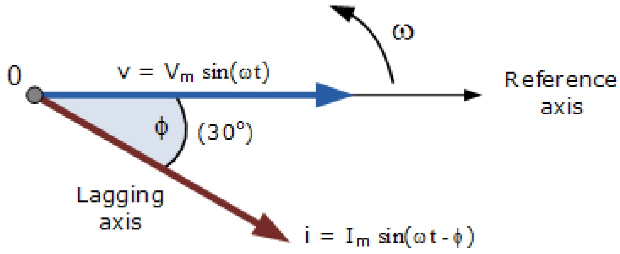


The generalised mathematical expression to define these two sinusoidal quantities will be written as:

$$v_{(t)} = V_m \sin(\omega t)$$

$$i_{(t)} = I_m \sin(\omega t - \phi)$$

The current, i is lagging the voltage, v by angle Φ i.e. 30°. So the difference between the two phasors representing the two sinusoidal quantities is angle Φ and the resulting phasor diagram will be.



3.3 Single phase AC circuits-

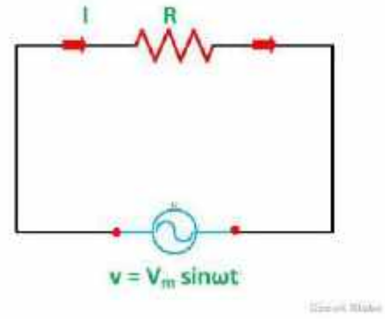
AC through pure resistive circuit

The circuit containing only a pure resistance of R ohms in the AC circuit is known as **Pure Resistive AC Circuit**. The presence of inductance and capacitance does not exist in a purely resistive circuit. In the purely resistive circuit, the power is dissipated by the resistors and the phase of the voltage and current remains same i.e., both the voltage and current reach their maximum value at the same time.

Let the alternating voltage applied across the circuit be given by the equation,

$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots(1)$$

Then the instantaneous value of current flowing through the resistor shown in the figure below will be:



$$i = \frac{v}{R} = \frac{V_m}{R}\sin\omega t \ldots\ldots\ldots(2)$$

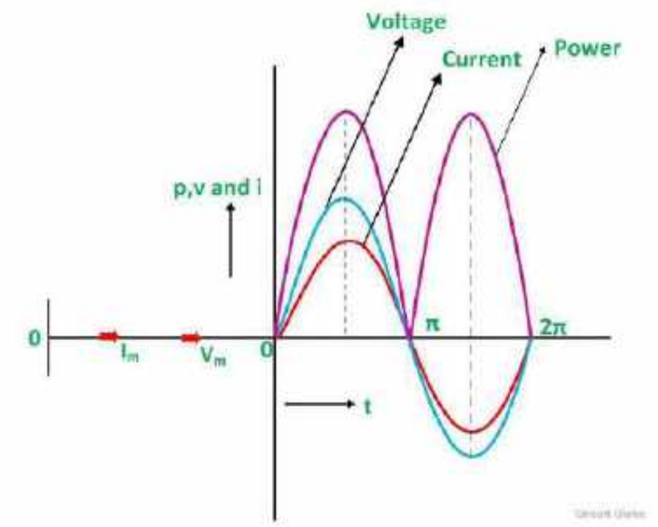
The value of current will be maximum when ωt= 90° or sinωt = 1

Putting the value of sinωt in equation (2) we will get

$$i = I_m \sin\omega t \ldots\ldots\ldots(3)$$

Phase Angle and Waveform of Resistive Circuit

From equation (1) and (3), it is clear that there is no phase difference between the applied voltage and the current flowing through a purely resistive circuit, i.e. phase angle between voltage and current is **zero**. Hence, in an AC circuit containing pure resistance, the current is in phase with the voltage as shown in the waveform figure below.



(Waveform and Phasor Diagram of Pure Resistive Circuit)

Power in Pure Resistive Circuit -
The instantaneous power in a purely resistive circuit is given by the equation shown below:
Instantaneous power, p= vi

$$p = (V_m \sin\omega t)(I_m \sin\omega t)$$

$$p = \frac{V_m I_m}{2} 2\sin^2\omega t = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}(1 - \cos 2\omega t)$$

$$p = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos 2\omega t$$

The average power consumed in the circuit over a complete cycle is given by

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} - \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\omega t \ldots \ldots (4)$$

As the valve of cosωt is zero.
So, putting the value of cosωt in equation (4) the value of power will be given by

$$P = V_{r.m.s} I_{r.m.s} - 0$$

Where,

- P – average power
- $V_{r.m.s}$ – root mean square value of supply voltage
- $I_{r.m.s}$ – root mean square value of the current

Hence, the power in a purely resistive circuit is given by:

$$P = VI$$

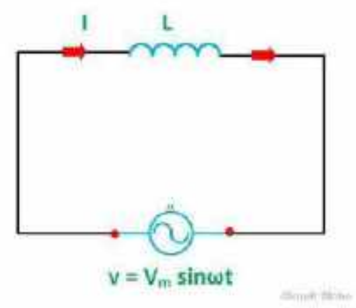
The voltage and the current in the purely resistive circuit are in phase with each other having **no phase difference** with phase angle zero. The alternating quantity reaches their peak value at the interval of the same time period that is the rise and fall of the voltage and current occurs at the same time.

AC through Pure inductive Circuit

The circuit which contains only inductance (L) and not any other quantities like resistance and capacitance in the circuit is called a **Pure inductive circuit.** In this type of circuit, the current lags behind the voltage by an angle of 90 degrees.

The inductor is a type of coil which reserves electrical energy in the magnetic field when the current flow through it. When the current flowing through inductor changes then time-varying magnetic field causes emf which obstruct the flow of current. The inductance is measured in **Henry**. The opposition of flow of current is known as the **inductive reactance**.

The circuit containing pure inductance is shown below:



Circuit Diagram of pure Inductive Circuit
Let the alternating voltage applied to the circuit is given by the equation:

$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots (1)$$

As a result, an alternating current i flows through the inductance which induces an emf in it. The equation is shown below:

$$e = -L \frac{di}{dt}$$

The emf which is induced in the circuit is equal and opposite to the applied voltage. Hence, the equation becomes,

$$v = -e \ldots \ldots \ldots (2)$$

Putting the value of e in equation (2) we will get the equation as

$$v = -\left(-L\frac{di}{dt}\right) \quad \text{or}$$

$$V_m \mathrm{Sin}\omega t = L\frac{di}{dt} \quad \text{or}$$

$$di = \frac{V_m}{L}\sin\omega t\, dt \quad \ldots\ldots\ldots\ldots(3)$$

Integrating both sides of the equation (3), we will get

$$\int di = \int \frac{V_m}{L}\sin\omega t\, dt \quad \text{or}$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t) \quad \text{or}$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - {}^{\pi}/_{2}\right) = \frac{V_m}{X_L}\sin\left(\omega t - {}^{\pi}/_{2}\right) \quad \ldots\ldots\ldots..(4)$$

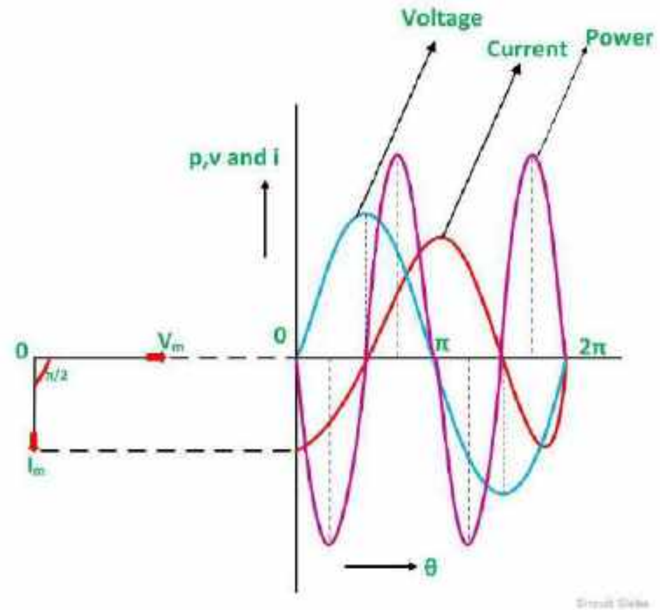
where, $X_L = \omega L$ is the opposition offered to the flow of alternating current by a pure inductance and is called inductive reactance.
The value of current will be maximum when sin (ωt – π/2) = 1. Therefore,

$$I_m = \frac{V_m}{X_L} \ldots\ldots\ldots\ldots(5)$$

Substituting this value in I_m from the equation (5) and putting it in equation (4) we will get

$$i = I_m\sin\left(\omega t - {}^{\pi}/_{2}\right)$$

Phasor Diagram and Power Curve of Inductive Circuit
The current in the pure inductive AC circuit lags the voltage by 90 degrees. The waveform, power curve and phasor diagram of a purely inductive circuit is shown below



(Phasor Diagram and Waveform of Pure Inductive Circuit)

When the values of voltage and current are at its peak as a positive value, the power is also positive and similarly, when the voltage and current give negative waveform the power will also become negative. This is because of the phase difference between voltage and current.

When the voltage drops, the value of the current changes. When the value of current is at its maximum or peak value of the voltage at that instance of time will be zero, and therefore, the voltage and current are out of phase with each other by an angle of 90 degrees.

The phasor diagram is also shown on the left-hand side of the waveform where current (I_m) lag voltage (V_m) by an angle of π/2.

Power in Pure Inductive Circuit

Instantaneous power in the inductive circuit is given by

$$p = vi$$

$$P = (V_m \sin\omega t)(I_m \sin(\omega t + \pi/2)$$

$$P = V_m I_m \sin\omega t \cos\omega t$$

$$P = \frac{V_m I_m}{2} 2 \sin\omega t \cos\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\omega t \text{ or}$$

$$P = 0$$

Hence, the average power consumed in a purely inductive circuit is zero.

The average power in one alteration, i.e., in a half cycle is zero, as the negative and positive loop is under power curve is the same.

In the purely inductive circuit, during the first quarter cycle, the power supplied by the source, is stored in the magnetic field set up around the coil. In the next quarter cycle, the magnetic field diminishes and the power that was stored in the first quarter cycle is returned to the source. This process continues in every cycle, and thus, no power is consumed in the circuit.

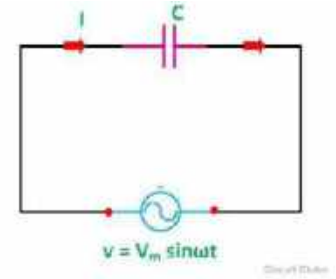
AC through Pure Capacitor Circuit

The circuit containing only a pure capacitor of capacitance C farads is known as a **Pure Capacitor Circuit**. The capacitor stores electrical power in the electric field, their effect is known as the capacitance. It is also called the **condenser**. The capacitor consists of two conductive plates which are separated by the dielectric medium. The dielectric material is made up of glass, paper, mica, oxide layers, etc. In pure AC capacitor circuit, the current leads the voltage by an angle of 90 degrees.

When the voltage is applied across the capacitor, then the electric field is developed across the plates of the capacitor and no current flow between them. If the variable voltage source is applied across the capacitor plates then the ongoing current flows through the source due to the charging and discharging of the capacitor.

Explanation and derivation of Capacitor Circuit

A capacitor consists of two insulating plates which are separated by a dielectric medium. It stores energy in electrical form. The capacitor works as a storage device, and it gets charged when the supply in **ON** and gets discharged when the supply is **OFF**. If it is connected to the direct supply, it gets charged equal to the value of the applied voltage.



(Circuit Diagram of pure Capacitor Circuit)

Let the alternating voltage applied to the circuit is given by the equation:

$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots(1)$$

Charge of the capacitor at any instant of time is given as:

$$q = Cv \ldots\ldots\ldots\ldots(2)$$

Current flowing through the circuit is given by the equation:

$$i = \frac{d}{dt} q$$

Putting the value of q from the equation (2) in equation (3) we will get

$$i = \frac{d}{dt}(Cv) \ldots\ldots\ldots\ldots(3)$$

Now, putting the value of v from the equation (1) in the equation (3) we will get

$$i = \frac{d}{dt} C V_m \text{Sin}\omega t = C V_m \frac{d}{dt} \sin\omega t \quad \text{or}$$

$$i = \omega C V_m \cos\omega t = \frac{V_m}{1/\omega C} \sin(\omega t + \pi/2) \quad \text{or}$$

$$i = \frac{V_m}{X_C} \sin(\omega t + \pi/2) \ldots\ldots\ldots\ldots(4)$$

Where Xc = 1/ωC is the opposition offered to the flow of alternating current by a pure capacitor and is called **Capacitive Reactance.**
The value of current will be maximum when sin(ωt + π/2) = 1. Therefore, the value of maximum current I_m will be given as:

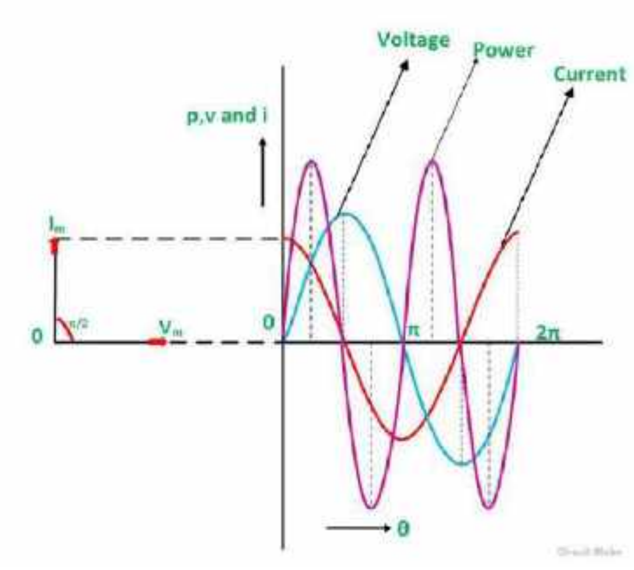
$$I_m = \frac{V_m}{X_C}$$

Substituting the value of I_m in the equation (4) we will get:

$$i = I_m \sin(\omega t + \pi/2)$$

Phasor Diagram and Power Curve

In the pure capacitor circuit, the current flowing through the capacitor leads the voltage by an angle of 90 degrees. The phasor diagram and the waveform of voltage, current and power are shown below:



(Phasor Diagram and Waveform of Pure Capacitor Circuit)

When the voltage is increased, the capacitor gets charged and reaches or attains its maximum value and, therefore, a positive half cycle is obtained. Further when the voltage level decreases the capacitor gets discharged, and the negative half cycle is formed.

If you examine the curve carefully, you will notice that when the voltage attains its maximum value, the value of the current is zero that means there is no flow of current at that time.

When the value of voltage is decreased and reaches a value π, the value of voltage starts getting negative, and the current attains its peak value. As a result, the capacitor starts discharging. This cycle of charging and discharging of the capacitor continues.

The values of voltage and current are not maximised at the same time because of the phase difference as they are out of phase with each other by an angle of 90 degrees.

The phasor diagram is also shown in the waveform indicating that the current (I_m) leads the voltage (V_m) by an angle of π/2.

Power in Pure Capacitor Circuit

Instantaneous power is given by p = vi

$$P = (V_m \sin\omega t)(I_m \sin(\omega t + \pi/2)$$

$$P = V_m I_m \sin\omega t \cos\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\,\omega t \quad \text{or}$$

$$P = 0$$

Hence, from the above equation, it is clear that the average power in the capacitive circuit is zero.

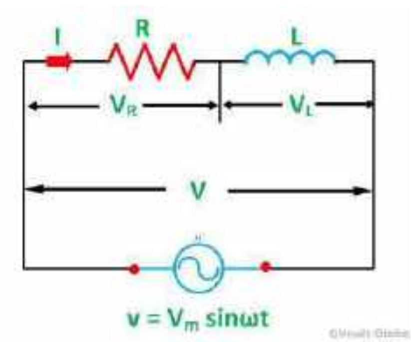
The average power in a half cycle is zero as the positive and negative loop area in the waveform shown are same.

In the first quarter cycle, the power which is supplied by the source is stored in the electric field set up between the capacitor plates. In the another or next quarter cycle, the electric field diminishes, and thus the power stored in the field is returned to the source. This process is repeated continuously and, therefore, no power is consumed by the capacitor circuit.

3.4 AC through RL Series Circuit

A circuit that contains a pure resistance R ohms connected in series with a coil having a pure inductance of L (Henry) is known as **RL Series Circuit**. When an AC supply voltage V is applied, the current, I flows in the circuit.

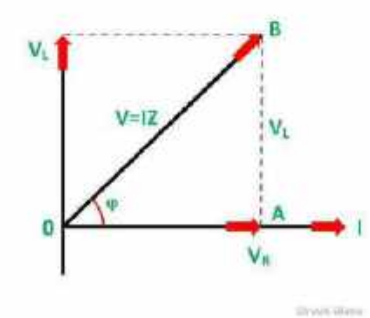
So, I_R and I_L will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other. The circuit diagram of RL Series Circuit is shown below:



Where,

- V_R – voltage across the resistor R
- V_L – voltage across the inductor L
- V – Total voltage of the circuit

Phasor Diagram of the RL Series Circuit



Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below which are followed to draw the phasor diagram step by step:

- Current I is taken as a reference.
- The Voltage drop across the resistance $V_R = I_R$ is drawn in phase with the current I.
- The voltage drop across the inductive reactance $V_L = IX_L$ is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.

Now, In right-angle triangle OAB

$V_R = I_R$ and $V_L = IX_L$ where $X_L = 2\pi fL$

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$

$$I = L = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$

Where, Z is the total opposition offered to the flow of alternating current by an RL Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

Phase Angle

In RL Series circuit the current lags the voltage by 90 degrees angle known as phase angle. It is given by the equation:

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

Power in R L Series Circuit

If the alternating voltage applied across the circuit is given by the equation:

$$v = V_m \operatorname{Sin}\omega t \ldots\ldots\ldots\ldots\ldots (1)$$

The equation of current I is given as:

$$i = I_m \sin(\omega t - \varphi) \ldots\ldots\ldots\ldots (2)$$

Then the instantaneous power is given by the equation:

$$p = v\,i \ldots\ldots\ldots (3)$$

Putting the value of v and i from the equation (1) and (2) in the equation (3) we will get

$$P = (V_m \operatorname{Sin}\omega t) x\, I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t - \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}[\cos\varphi - \cos(2\omega t - \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below:

$$P = \text{average of } \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s}\cos\varphi = V\,I\cos\varphi$$

Where cosϕ is called the power factor of the circuit.

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \ldots\ldots\ldots\ldots (4)$$

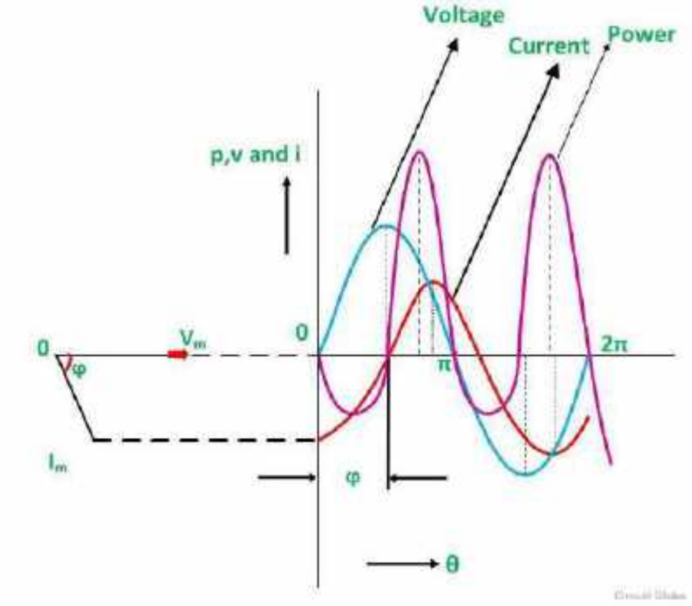
The power factor is defined as the ratio of resistance to the impedance of an AC Circuit. Putting the value of V and cosϕ from the equation (4) the value of power will be:

$$P = (IZ)(I)(R/Z) = I^2 R \ldots\ldots\ldots (5)$$

From equation (5) it can be concluded that the inductor does not consume any power in the circuit.

Waveform and Power Curve of the RL Series Circuit

The **waveform** and **power curve** of the RL series circuit is shown below:



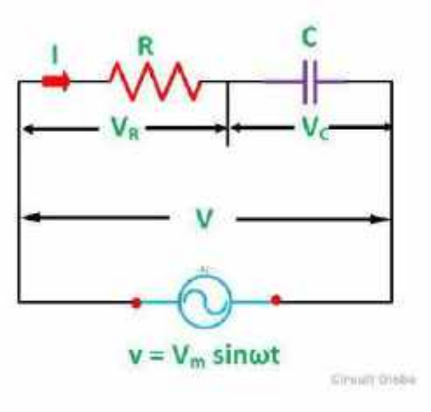
The various points on the power curve are obtained by the product of voltage and current.

If you analyze the curve carefully, it is seen that the power is negative between angle 0 and ϕ and between 180 degrees and (180 + ϕ) and during the rest of the cycle the power is positive. The current lags the voltage and thus they are not in phase with each other.

AC through RC Series Circuit

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit.** A sinusoidal voltage is applied and current I flows through the resistance (R) and the capacitance (C) of the circuit.

The RC Series circuit is shown in the figure below:

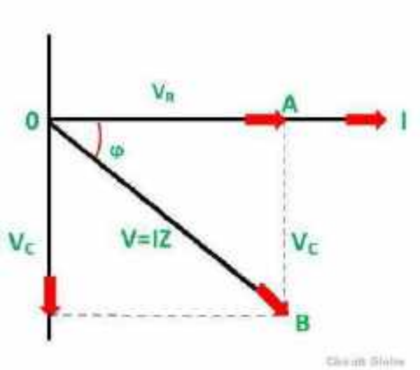


Where,

- V_R – voltage across the resistance R
- V_C – voltage across capacitor C
- V – total voltage across the RC Series circuit

Phasor Diagram of RC Series Circuit

The phasor diagram of the RC series circuit is shown below:



Steps to draw a Phasor Diagram

The following steps are used to draw the phasor diagram of RC Series circuit

- Take the current I (r.m.s value) as a reference vector
- Voltage drop in resistance VR = IR is taken in phase with the current vector
- Voltage drop in capacitive reactance VC = IXC is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees (in the pure capacitive circuit)
- The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).

Now, $\mathbf{V_R = I_R}$ and $\mathbf{V_C = IX_C}$

Where X_C = I/2πfC

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + X_C^2}$$

Where, Z is the total opposition offered to the flow of alternating current by an RC series circuit and is called **impedance** of the circuit. It is measured in ohms (Ω).

Phase angle

From the phasor diagram shown above, it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the **phase angle**.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Power in RC Series Circuit

If the alternating voltage applied across the circuit is given by the equation

$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

Then,

$$i = I_m \sin(\omega t + \varphi) \ldots\ldots\ldots\ldots(2)$$

Therefore, the instantaneous power is given by p = vi

Putting the value of v and i from the equation (1) and (2) in p = vi

$$P = (V_m \text{Sin}\omega t) x\, I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t + \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}[\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t + \varphi)$$

The average power consumed in the circuit over a complete cycle is given by:

$$P = \text{average of } \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t + \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s}\cos\varphi = V\,I\cos\varphi$$

Where cosϕ is called the **power factor** of the circuit.

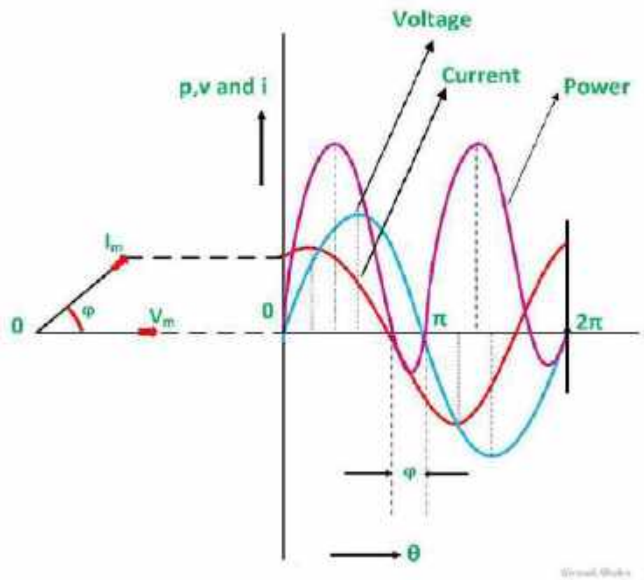
$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \ldots\ldots\ldots\ldots(3)$$

Putting the value of V and cosϕ from the equation (3) the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R \ldots\ldots\ldots(4)$$

From the equation (4) it is clear that the power is actually consumed by the resistance only and the capacitor does not consume any power in the circuit.

Waveform and Power Curve of the RC Series Circuit
The waveform and power curve of the RC circuit is shown below:

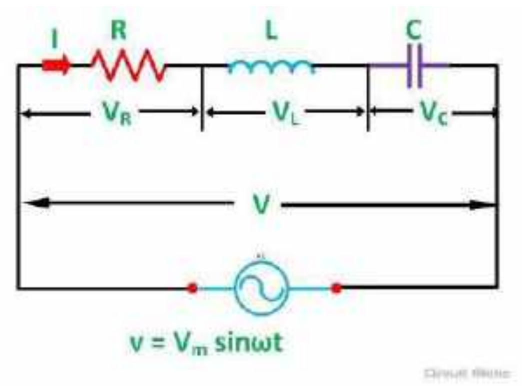


The various points on the power curve are obtained from the product of the instantaneous value of voltage and current. The power is negative between the angle (180° – ϕ) and 180° and between (360° -ϕ) and 360° and in the rest of the cycle, the power is positive. Since the area under the positive loops is greater than that under the negative loops, therefore the net power over a complete cycle is **positive**.

AC through RLC Series Circuit

When a pure resistance of R ohms, a pure inductance of L Henry and a pure capacitance of C farads are connected together in series combination with each other then **RLC Series Circuit** is formed. As all the three elements are connected in series so, the current flowing through each element of the circuit will be the same as the total current I flowing in the circuit.

The **RLC Circuit** is shown below :



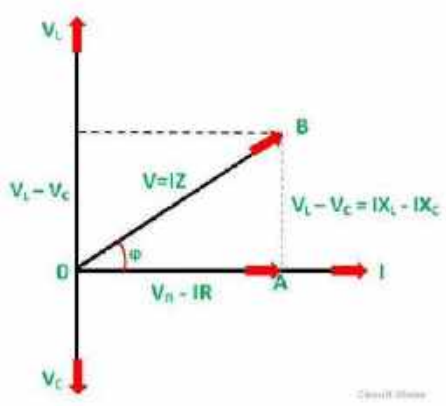
When the AC voltage is applied through the RLC Series circuit the resulting current I flows through the circuit, and thus the voltage across each element will be:

- V_R = IR that is the voltage across the resistance R and is in phase with the current I.
- $V_L = IX_L$ that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.

- $V_C = IX_C$ that is the voltage across capacitor C and it lags the current I by an angle of 90 degrees.

Phasor Diagram of RLC Series Circuit

The phasor diagram of the RLC series circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as a capacitive circuit.



Steps to draw the Phasor Diagram of the RLC Series Circuit

- Take current I as the reference as shown in the figure above
- The voltage across the inductor L that is V_L is drawn leads the current I by a 90-degree angle.
- The voltage across the capacitor c that is V_c is drawn lagging the current I by a 90-degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees.
- The two vector V_L and V_C are opposite to each other.

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

It is the total opposition offered to the flow of current by an RLC Circuit and is known as **Impedance** of the circuit.

Phase Angle

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

Power in RLC Series Circuit

The product of voltage and current is defined as power.

$$P = VI\cos\varphi = I^2R$$

Where cosϕ is the power factor of the circuit and is expressed as:

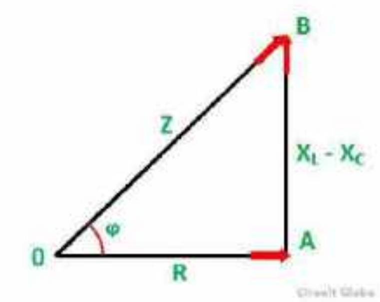
$$\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$$

The three cases of RLC Series Circuit

- When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of the power factor is **unity**.

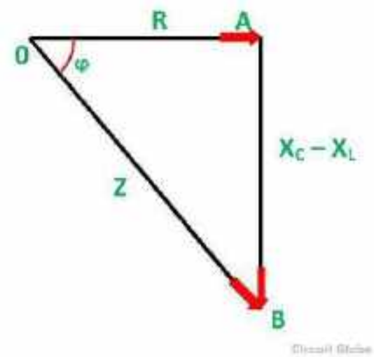
Impedance Triangle of RLC Series Circuit

When the quantities of the phasor diagram are divided by the common factor I then the right angle triangle is obtained known as impedance triangle. The impedance triangle of the RL series circuit, when ($X_L > X_C$) is shown below:



If the inductive reactance is greater than the capacitive reactance than the circuit reactance is inductive giving a **lagging phase angle**.

Impedance triangle is shown below when the circuit acts as an RC series circuit ($X_L < X_C$)



When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as capacitive and the phase angle will be leading.

Applications of RLC Series Circuit

The following are the application of the RLC circuit:

- It acts as a variable tuned circuit
- It acts as a low pass, high pass, bandpass, bandstop filters depending upon the type of frequency.
- The circuit also works as an oscillator
- Voltage multiplier and pulse discharge circuit

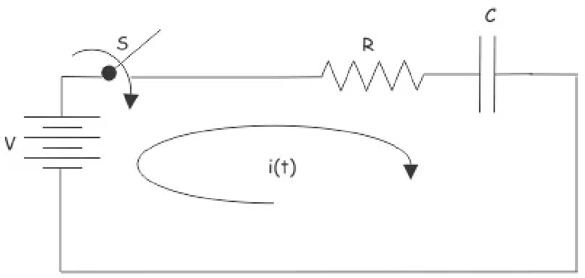
3.5 <u>Time Constant</u>

The time constant is commonly used to characterize the response of an RL, RC or RLC circuit.

Time Constant of an RC Circuit

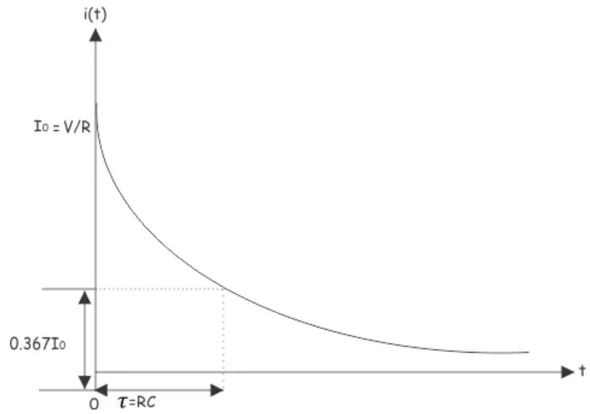
Time constant is the duration in seconds during which the current through a capacitor circuit becomes 36.7 percent of its initial value. This is numerically equal to the product of resistance and capacitance value of the circuit. The time **constant** is normally denoted by τ (tau). So,

$$\tau = RC$$



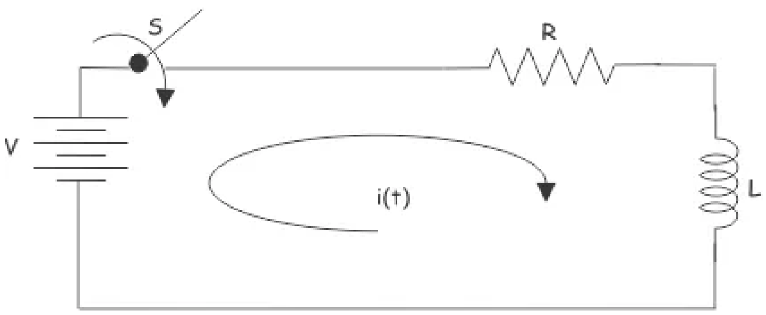
Applying Kirchhoff Voltage Law in that single mesh circuit, we get,

$$V = Ri(t) + \frac{1}{C}\int i(t)dt$$



$$I_{t=RC} = \frac{V}{R}e^{-1}$$
$$= 0.367\frac{V}{R} = 0.367I_0 \ or\ 36.7\% \ of\ I_0$$

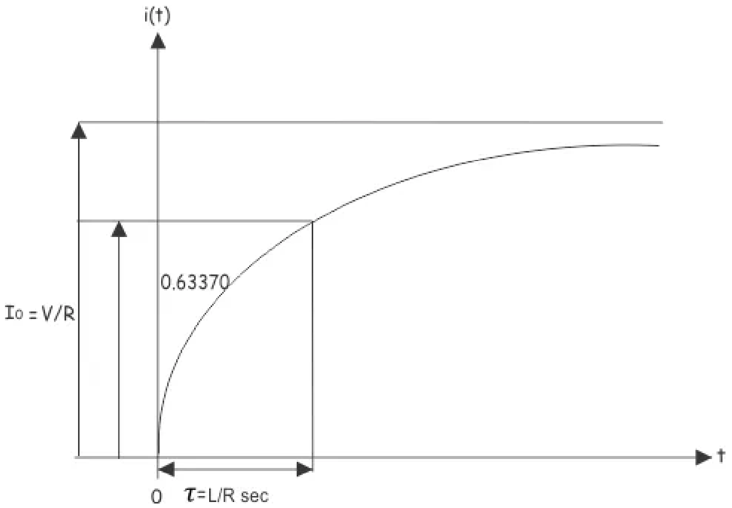
Time Constant of an RL Circuit



Applying Kirchhoff Voltage Law in the above circuit. We get,

$$V = Ri(t) + L\frac{di(t)}{dt}$$

The time constant of an LR circuit is the ratio of inductance to the resistance of the circuit.



At the RL circuit, at time = L/R sec, the current becomes 63.3% of its final steady-state value. The L/R is known as the time constant of an LR circuit.

Time constant for RLC Circuit:

In RLC circuit, we have both RL and RC time constant combined, which makes a problem calculating the time constant. So we calculate what we call the Q-Factor (quality factor).

τ for Series RLC Circuit:

$$Q\ factor = \frac{1}{R}\sqrt{\frac{L}{C}}$$

τ for Parallel RLC Circuit:

$$Q\ factor = R\sqrt{\frac{C}{L}}$$

3.6 <u>Numerical Example</u>

Q1. A 240 V, 50 Hz AC supply is applied a coil of 0.08 H inductance and 4 Ω resistance connected in series with a capacitor of 8 μF. Calculate (i) Impedance, (ii) Circuit current, (iii) Phase angle, (iv) power factor, (v) Power consumed, (vi) Q factor of the circuit at resonant frequency.

Q2. When a series *RLC* circuit is subject to 48 V, V_R is 15 V, and V_L is 22 V. What is the voltage across the capacitor?

Q3. In an ac circuit, V=100 sin(wt+30^0) and I=5 sin (wt-30^0). Find apparent power, reactive power and power factor of the circuit?

Q4. A voltage v(t)=150 sin 10^3 t is applied a series circuit where R=40 ohm, L=0.13 H, C=10 microfarad. Find (i) current in the circuit, (ii) power supplied by the source, (iii) Reactive power supplied by the source, (iv) power factor of the circuit (v) reactive power of the capacitor, (vi) reactive power of the inductor.

DC Transient Behaviors -

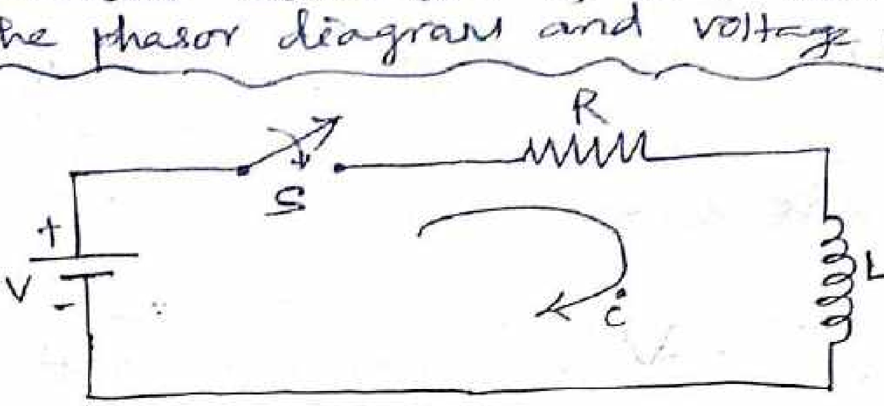
* The purely resistive device will allow instantaneous change in current and voltage. An inductive device will not allow sudden change in current whereas a capacitive device will not allow sudden change in voltage.
* When switching operation is performed in inductive or capacitive devices, the current and voltage in the device will take a certain time to change from pre-switching value to steady value after switching.
* The state of the circuit from the instant of switching to attainment of steady state is called transient and this time duration is called transient period. The current and voltage of circuit elements during transient period is called transient response.
* The switching instant is taken as time origin i.e. $t=0$. The time $t=0^-$ is used to denote the time instant just prior to switching and the time $t=0^+$ is used to denote the time instant immediately after switching.
*
$$i_R(0^+) \neq i_R(0^-) \text{ and } V_R(0^+) \neq V_R(0^-)$$
$$i_L(0^+) = i_L(0^-) \text{ and } V_L(0^+) \neq V_L(0^-)$$
$$i_C(0^+) \neq i_C(0^-) \text{ and } V_C(0^+) \neq V_C(0^-)$$

Behavior of passive elements in transient state in comparison to steady state :-

Element	DC steady state $s=0$	AC steady state $s=j\omega$	Transient state $t \to 0^+$
R	R	i in phase with V	R
L	s.c.	i lags V by 90°	o.c.
C	o.c.	i leads V by 90°	s.c.

$t \to 0^+$ is considered as transient solution.
$t \to \infty$ is steady state solution.

DC Transient Behaviors of R-L series ckt & draw the phasor diagram and voltage triangle



Applying KVL when switch is closed at t=0,

$$iR + L\frac{di}{dt} = V$$

$$\Rightarrow \frac{di}{dt} = \frac{V-Ri}{L}$$

$$\Rightarrow \frac{di}{V-Ri} = \frac{dt}{L}$$

Integrating both sides gives

$$\int \frac{1}{L}dt = \int \frac{di}{V-Ri}$$

$$\Rightarrow \frac{t}{L} = \int \frac{di}{-Ri+V} = -\frac{1}{R}\ln(V-Ri) + A \quad \text{--- (1)}$$

[Let, $x = -Ri + V$

$$\frac{dx}{di} = -R$$

$$di = \frac{dx}{-R}$$

$$\int \frac{di}{-Ri+V} = \int \frac{dx}{-Rx} = -\frac{1}{R}\int\frac{dx}{x} = -\frac{1}{R}\ln x + A$$

$$= -\frac{1}{R}\ln(V-Ri) + A \;]$$

Where, A = constant.

At $t = 0^-$ when switch is opened

current $i = 0$

Putting the value of i in eq^n (1)

$$\frac{t}{L} = -\frac{1}{R}\ln(V-Ri) + A$$

$$\Rightarrow \frac{0}{L} = -\frac{1}{R}\ln V + A$$

$$\Rightarrow \boxed{A = \frac{1}{R}\ln V}$$

Putting the value of A in eq^n (1)

$$\frac{t}{L} = -\frac{1}{R}\ln(V - Ri) + A$$

$$= -\frac{1}{R}\ln(V-Ri) + \frac{1}{R}\ln V$$

[redacted] $$\frac{t}{L} = -\frac{1}{R}\left[\ln(V-Ri) - \ln V\right]$$

$$\Rightarrow -\left(\frac{R}{L}\right)t = \ln\left(\frac{V-Ri}{V}\right) \quad \left[\because \ln A - \ln B = \ln\frac{A}{B}\right]$$

$$\Rightarrow e^{-\frac{R}{L}t} = \frac{V-Ri}{V}$$

$$\Rightarrow V \cdot e^{-\frac{R}{L}t} = V - Ri$$

$$\Rightarrow iR = V - V e^{-\left(\frac{R}{L}\right)t}$$

$$= V\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$$

$$\Rightarrow \boxed{i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

This is the current in the circuit at any [redacted] instant.

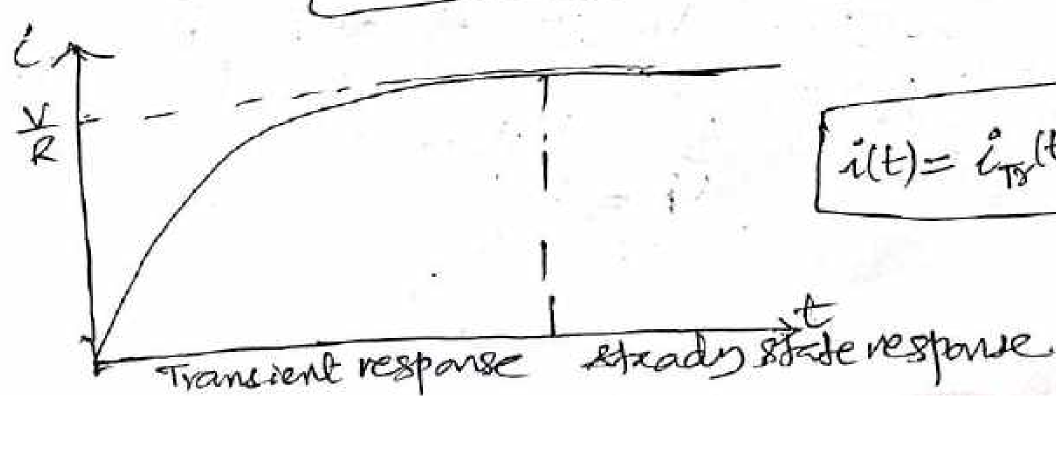
$$V_R = iR = \frac{V}{R}\left(1 - e^{-R/L\, t}\right) \times R$$

$$\boxed{V_R = V\left(1 - e^{-\frac{R}{L}t}\right)}$$

$$V_L = V - V_R = V - V\left(1 - e^{-\frac{R}{L}t}\right)$$

$$= V - V + V e^{-\frac{R}{L}t}$$

$$\Rightarrow \boxed{V_L = V e^{-\frac{R}{L}t}}$$

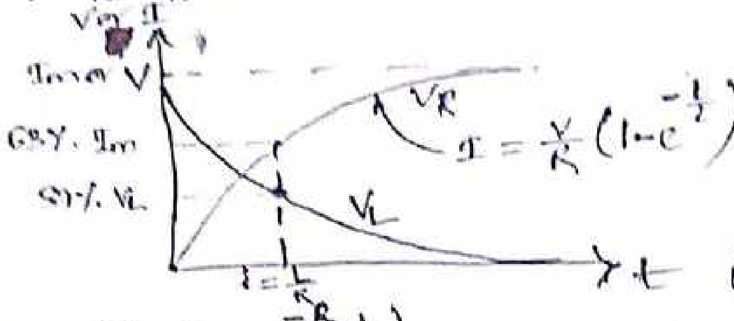


$$\boxed{i(t) = i_{Tr}(t) + i_{ss}(t)}$$

$$i = \frac{V}{R}\left[1 - e^{(-R/L)t}\right]$$

$$= \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Where, τ is the time constant and $\boxed{\tau = \frac{L}{R}}$



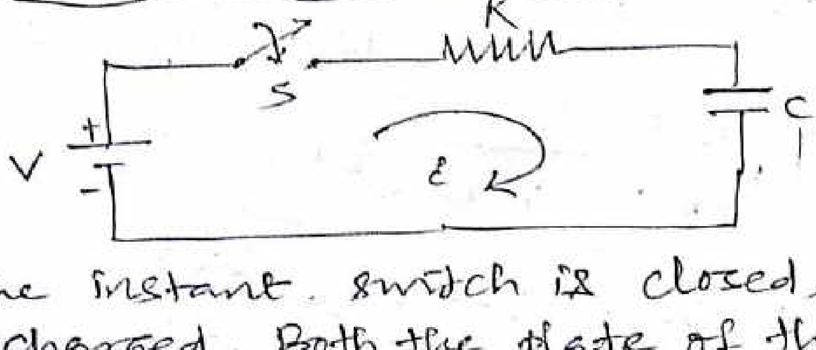
(Characteristics curve)

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

The term $\left(\frac{L}{R}\right)$ is called time constant and it is defined as the time taken by the current to reach 63.2% of its maximum steady state value (i.e. 63% of I_M).

The term $\left(\frac{V}{R}\right)$ represents the final steady state value of the current in the circuit.

- DC transient behaviors of R-C series ckt & draw the phasor diagram and voltage triangle -



At the instant switch is closed, the capacitor is uncharged. Both the plate of the capacitor are at same potential, hence Vc across capacitor is zero. The capacitor behaves as short ckt.

$$V = V_R \quad \text{and} \quad I_0 = \frac{V}{R}$$

The capacitor starts charging and Vc increases.

Then, $V = V_R + V_C$

$$\Rightarrow V_R = V - V_C$$

$$\Rightarrow iR = V - V_C$$

$$\Rightarrow i = \frac{V - V_C}{R}$$

When capacitor is fully charged, then

$$V = V_c \text{ and } V_R = 0$$

and charging current becomes zero. Now the capacitor becomes open ckt.

By KVL, $V = V_R + V_c$

$$= iR + V_c$$

$$= \frac{dq}{dt} R + V_c \quad \left(\because i = \frac{dq}{dt}\right)$$

$$= R \frac{d}{dt} C V_c + V_c \quad (\because q = CV)$$

$$= RC \frac{dV_c}{dt} + V_c$$

$$\Rightarrow V - V_c = RC \frac{dV_c}{dt}$$

$$\Rightarrow \frac{1}{RC} dt = \frac{dV_c}{V - V_c}$$

Integrating both sides,

$$\int \frac{1}{RC} dt = \int \frac{dV_c}{V - V_c}$$

$$\Rightarrow \frac{t}{RC} = -\ln(V - V_c) + B \text{ --- (1)}$$

where, B = Integration constant

[Let $x = V - V_c$

$\frac{dx}{dV_c} = -1$

$dV_c = -dx$

$\int \frac{dV_c}{V - V_c} = \int \frac{-dx}{x}$

$= -\ln x + B$

$= -\ln(V - V_c) + B$]

When $t = 0$, $q = 0$

$$V_c = \frac{q}{c} = \frac{0}{c} = 0$$

Applying this to eq^n (1)

$$\frac{t}{RC} = -\ln(V - V_c) + B$$

$$\Rightarrow \frac{0}{RC} = -\ln V + B$$

$$\Rightarrow 0 = -\ln V + B$$

$$\Rightarrow B = \ln V$$

Putting the value of B in eq^n - (1)

$$\Rightarrow \frac{t}{RC} = -\ln(V - V_c) + \ln V$$

$$\Rightarrow -\frac{t}{RC} = \ln(V - V_c) - \ln V$$

$$= \ln\left(\frac{V - V_c}{V}\right)$$

$\left[\because \ln A - \ln B = \ln \frac{A}{B}\right]$

$$e^{-\frac{t}{RC}} = \frac{V - V_c}{V}$$

$$\Rightarrow V - V_c = V e^{-\frac{t}{RC}}$$

$$\Rightarrow \boxed{V_c = V - V e^{-\frac{t}{RC}} = V\left(1 - e^{-\frac{t}{RC}}\right)}$$

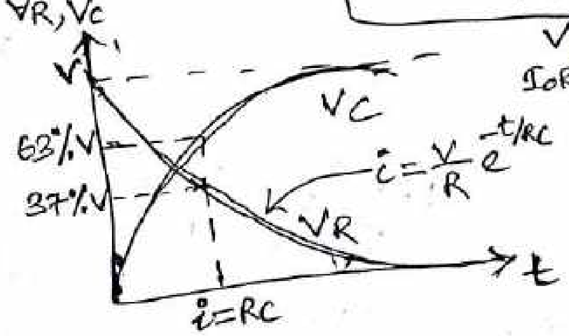
$$V_R = V - V_c$$

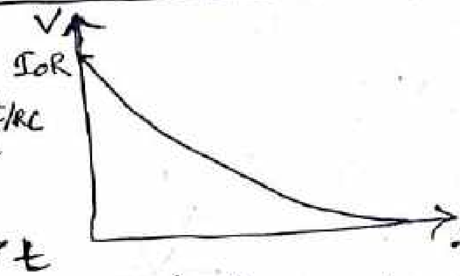
$$= V - V\left(1 - e^{-\frac{t}{RC}}\right)$$

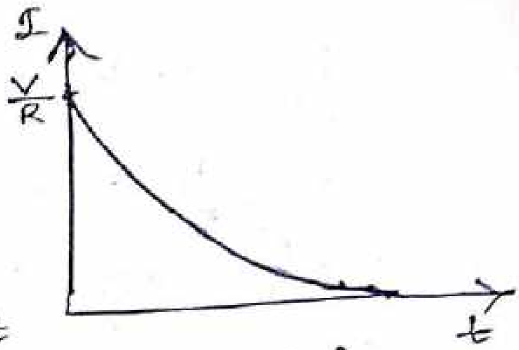
$$\Rightarrow \boxed{V_R = V e^{-\frac{t}{RC}}}$$

current in the circuit at any instant

$$\boxed{i = \frac{V_R}{R} = \frac{V}{R} e^{-t/RC}}$$





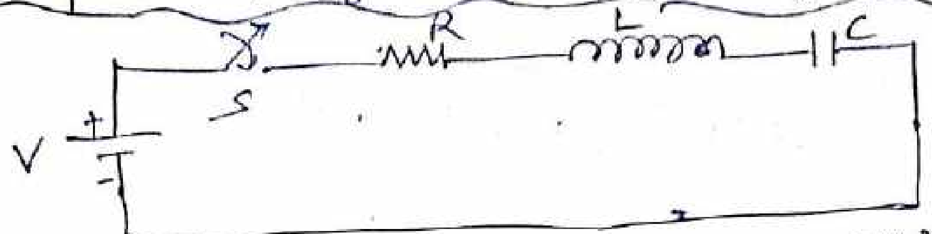


(Characteristics curve)

$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

The term RC is called time constant and it is defined as the time taken by the capacitor to charge through the resistor from an initial charge voltage of zero to 63.2% of the value of an applied DC voltage.

Dc Transient Behaviour of R-L-C series ckt & draw the phasor diagram and voltage triangle



At $t=0$, the transient current $i(t)$ in the circuit can be found by applying KVL,

$$R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt = V$$

Differentiating both side,

$$R\frac{di}{dt} + L\frac{d^2 i(t)}{dt^2} + \frac{i(t)}{C} = 0 \quad \text{---} (1)$$

Let $m = \frac{d}{dt}$

$$R\,m\,i(t) + Lm^2 i(t) + \frac{i(t)}{C} = 0$$

$$i(t)\left[Lm^2 + Rm + \frac{1}{C}\right] = 0$$

$$\Rightarrow Lm^2 + Rm + \frac{1}{C} = 0$$

$$\Rightarrow m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$$

The roots of this eqn

$$m_1, m_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4.1\frac{1}{LC}}}{2\times 1}$$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

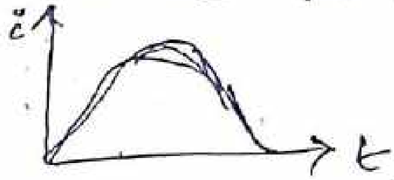
Let $\alpha = -\frac{R}{2L}$ and $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Then, $m_1 = \alpha + \beta$ and $m_2 = \alpha - \beta$

Here, β may be positive, negative or zero.

<u>Case-I</u> - If β is positive, when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

The roots are real and unequal, and give over damped response as shown in fig below.



Eq^n ① becomes,

$$[m-(\alpha+\beta)]\,[m-(\alpha-\beta)]\,i = 0$$

The solution for the above equation is

$$i = c_1 e^{(\alpha+\beta)t} + c_2 e^{(\alpha-\beta)t}$$

Where c_1 & c_2 are constants.

<u>Case II</u> If β is negative, when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

The roots are complex conjugate, and gives the underdamped response as shown in fig below.

Eq^n ① becomes

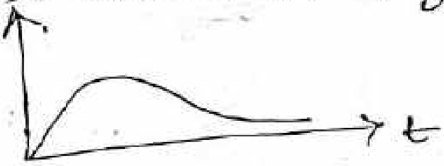
$$[m-(\alpha+j\beta)]\,[m-(\alpha-j\beta)]\,i = 0$$

The solution for the above equation is

$$i = e^{\alpha t}\left[c_1 \cos\beta t + c_2 \sin\beta t\right]$$

<u>Case III</u> If β is zero, when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

The roots are equal and gives the critically damped response as shown in fig below.



Eq^n ① becomes

$$(m-\alpha)(m-\alpha)\,i = 0$$

The solution for the above equation is

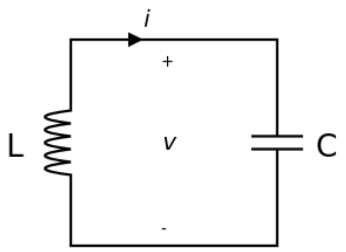
$$i = e^{\alpha t}(c_1 + c_2 t)$$

CHAPTER 4 – RESONANCE AND COUPLED CIRCUITS

4.1 Introduction to resonance circuits and Resonance tuned circuit –

Electrical resonance occurs in an electric circuit at a particular frequency called resonant frequency when the impedances or admittances of circuit elements cancel each other. Resonance in electrical circuits consisting of passive and active components represents a particular state of the circuit when the current or voltage in the circuit is maximum or minimum with respect to the magnitude of excitation at particular frequency, the circuit impedance being either minimum or maximum at the power factor unity. The phenomenon of resonance is observed in both series or parallel ac circuits comprising of R, L and C and excited by an ac source.

An LC circuit, called a resonant circuit, tank circuit, or tuned circuit, is an electric circuit consisting of an inductor and a capacitor connected together. The circuit can act as an electrical resonator, oscillators, filters, tuners and frequency mixers.



Resonance effect

Resonance occurs when an LC circuit is driven from an external source at an angular frequency ω_0 at which the inductive and capacitive reactances are equal in magnitude. The frequency at which this equality holds for the particular circuit is called the resonant frequency. The resonant frequency of the LC circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

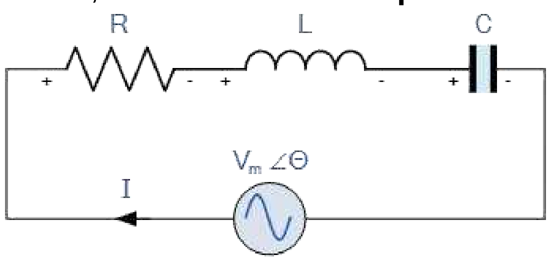
where L is the inductance in henry, C is the capacitance in farad and ω_0 is the angular frequency in radians per second.

The equivalent frequency in units of hertz is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.$$

4.2 Series resonance –

In the RLC series circuit, when the circuit current is in phase with the applied voltage, the circuit is said to be in **Series Resonance.** The resonance condition arises in the series RLC circuit when the inductive reactance is equal to the capacitive reactance. In other words, $X_L = X_C$. The frequency at which this occurs is called the **Resonant Frequency** (f_r) of the circuit. A series resonant circuit has the capability to draw heavy current and power from the mains; it is also called **acceptor circuit**.



4.3 <u>Expression for series resonance</u> -

At the resonance, X = 0 or $X_L - X_C = 0$ or $X_L = X_C$
The Impedance will be:

$$Z_r = \sqrt{R^2 + (X_L - X_C)^2} \dots\dots\dots\dots (1)$$

Where Zr is the resonance impedance of the circuit.
Putting the value of $X_L - X_C = 0$ in equation (1) we will get:
Zr = R
Current I = V/ Zr = V/R
Since at resonance the opposition to the flow of current is only resistance (R) of the circuit. At this condition, the circuit draws the maximum current.

$$X_L = X_C \Rightarrow 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

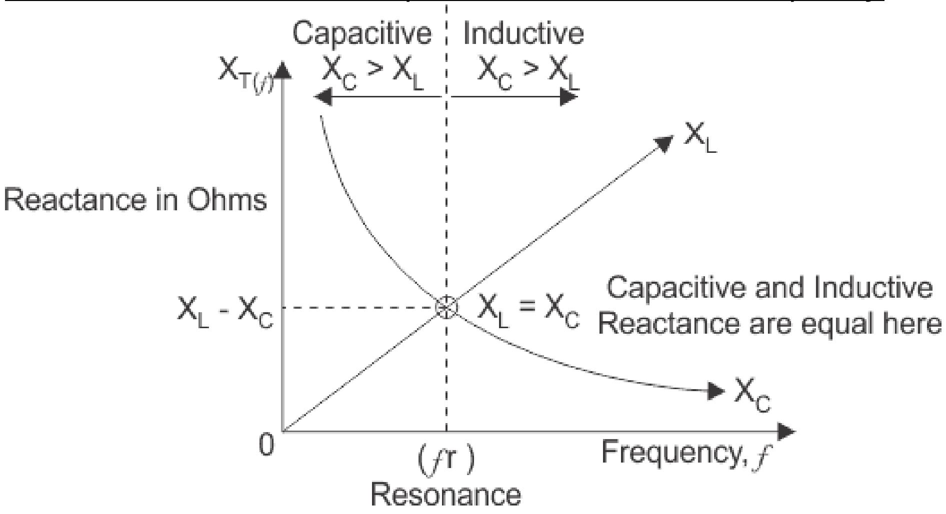
$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}} \text{ (Hz)} \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \text{ (rads)}$$

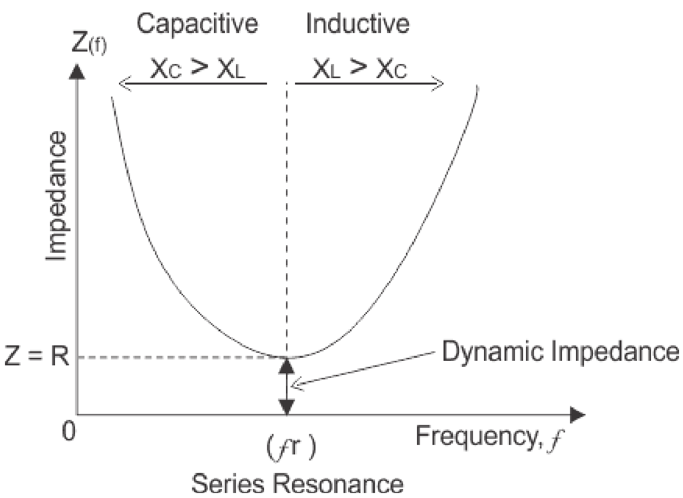
Where fr is the resonant frequency in hertz when inductance L is measured in Henry and capacitance C in Farads.

The **resonant frequency** condition arises in the series circuit when the inductive reactance is equal to the capacitive reactance. If the supply frequency is changed the value of $X_L = 2\pi fL$ and $X_C = 1/2\pi fC$ is also changed. When the frequency increases, the value of X_L increases, whereas the value of X_C decreases. Similarly, when the frequency decreases, the value of X_L decreases and the value of X_C increases. At point P when (XL = XC), the resonant frequency condition is obtained.

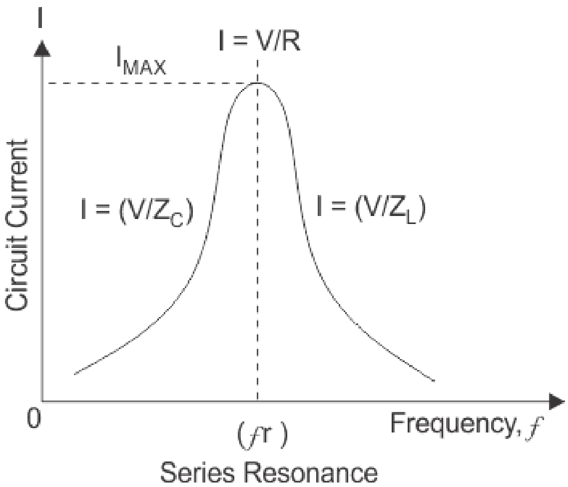
<u>Inductive Reactance and Capacitive Reactance Vs Frequency</u>



Variation of Impedance Vs Frequency



Resonant Current Vs Frequency



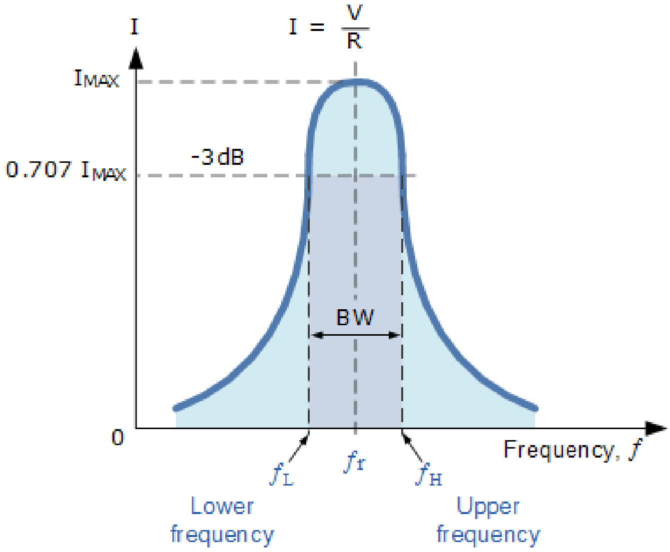
In series RLC circuit, the total voltage is the phasor sum of voltage across resistor, inductor and capacitor. In series RLC circuit current, I = V / Z. At **resonance in series RLC circuit**, both inductive and capacitive reactance cancel each other and as the current flowing through all the elements is same, the voltage across inductor and capacitor is equal in magnitude and opposite in direction and thereby they cancel each other. So, in a series resonant circuit, voltage across resistor is equal to supply voltage i.e $V = V_r$. But at resonance current I = V / R, therefore the current at resonant frequency is maximum and the impedance of circuit is resistance only and is minimum.

At resonance the power absorbed by the circuit must be at its maximum value as $\mathbf{P = I^2Z}$ = $\mathbf{I^2R}$.

Bandwidth & Quality factor of a Series Resonance Circuit

The half power points called -3dB points give us a current value that is 70.7% of its maximum resonant value which is defined as: $0.5(I^2 R) = (0.707 \times I)^2 R$. Then the point corresponding to the lower frequency at half the power is called the “lower cut-off frequency”, labelled f_L with the point corresponding to the upper frequency at half power being called the “upper cut-off frequency”, labelled f_H.

The distance between these two points, i.e. ($f_H - f_L$) is called the **Bandwidth**, (BW) and is the range of frequencies over which at least half of the maximum power.



The frequency response of the circuits current magnitude above, relates to the "sharpness" of the resonance in a series resonance circuit. The sharpness of the peak is measured quantitatively and is called the **Quality factor, Q** of the circuit. Quality factor is defined as the ratio of the voltage across the inductor or capacitor to the applied voltage.

$Q = V_L / V = V_C / V$

$Q = V_L / V = I_O X_L / I_O R = X_L / R$ (for the coil)

$Q = V_C / V = I_O X_C / I_O R = X_C / R$ (for the capacitor)

The quality factor is **also the ratio of resonant frequency to bandwidth**. It relates the maximum or peak energy stored in the circuit (the reactance) to the energy dissipated (the resistance) during each cycle of oscillation. The higher the circuit Q, the smaller the bandwidth.

$Q = f_r / BW$.

As the bandwidth is taken between the two -3dB points, the **selectivity** of the circuit is a measure of its ability to reject any frequencies either side of these points. A more selective circuit will have a narrower bandwidth whereas a less selective circuit will have a wider bandwidth.

Then the relationship between resonance, bandwidth, selectivity and quality factor for a series resonance circuit being defined as:

1). Resonant Frequency, (f_r)

$$X_L = X_C \quad \Rightarrow \quad \omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r^2 = \frac{1}{LC} \quad \therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

2). Current, (I)

$$\text{at } \omega_r \quad Z_T = \text{min}, \quad I_S = \text{max}$$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{max}}{\sqrt{R^2 + \left(\omega_r L - \frac{1}{\omega_r C}\right)^2}}$$

3). Lower cut-off frequency, (f_L)

$$\text{At half power, } \frac{P_m}{2},\ I = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

$$Z = \sqrt{2}R,\ X = -R\ (\text{capacitive})$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

4). Upper cut-off frequency, (f_H)

$$\text{At half power, } \frac{P_m}{2},\ I = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

$$Z = \sqrt{2}R,\ X = +R\ (\text{inductive})$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

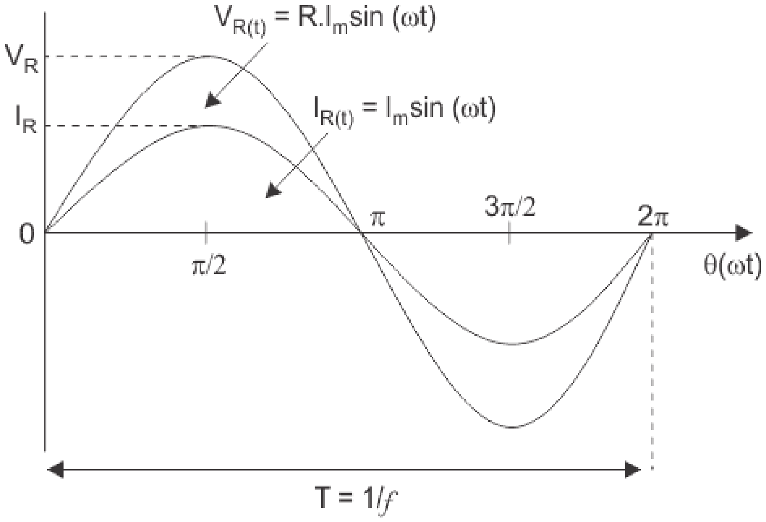
5). Bandwidth, (BW)

$$BW = \frac{f_r}{Q},\quad f_H - f_L,\quad \frac{R}{L}\ (\text{rads})\quad \text{or}\quad \frac{R}{2\pi L}\ (\text{Hz})$$

6). Quality Factor, (Q)

$$Q = \frac{\omega_r L}{R} = \frac{X_L}{R} = \frac{1}{\omega_r CR} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Power Factor at Resonance



At resonance, the circuit behaves like a pure resistive circuit. As in pure resistive circuit, voltage and the circuit current are in same phase i.e V_r, V and I are in same phase direction.

Therefore, the phase angle between voltage and current is zero and the power factor is unity.

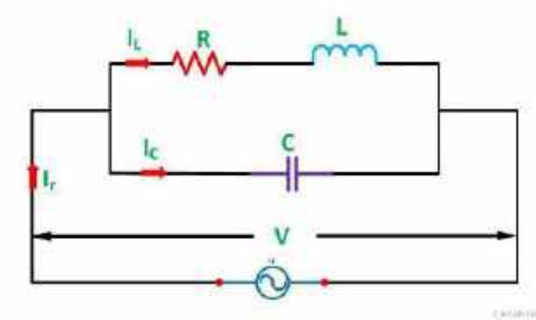
Effects of Series Resonance

The following effects of the series resonance condition are given below:

- At resonance condition, XL = XC the impedance of the circuit is minimum and is reduced to the resistance of the circuit. i.e Zr = R
- At the resonance condition, as the impedance of the circuit is minimum, the current in the circuit is maximum. i.e Ir = V/Zr = V/R
- As the value of resonant current Ir is maximum hence, the power drawn by the circuit is also maximized. i.e $Pr = I^2R$
- At the resonant condition, the current drawn by the circuit is very large or we can say that the maximum current is drawn. Therefore, the voltage drop across the inductance L i.e ($V_L = IX_L = I \times 2\pi frL$) and the capacitance C i.e ($V_C = IX_C = I \times I/2\pi frC$) will also be very large.

Parallel Resonance

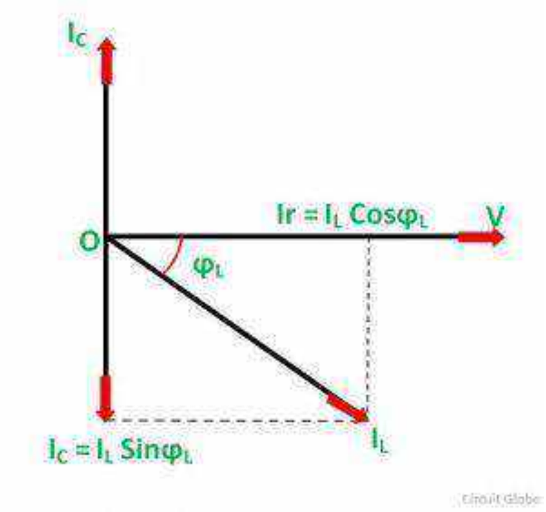
Parallel Resonance means when the circuit current is in phase with the applied voltage of an AC circuit containing an inductor and a capacitor connected together in parallel. Since the parallel resonant circuit can draw a very small current and power from the mains, therefore, it is also called as **Rejector Circuit.**



At resonance, the capacitive current must be equal to the inductive part of the coil current i.e. the imaginary components of I_L & I_C must cancel each other. The circuit current Ir will only be in phase with the supply voltage when the following condition given below in the equation is satisfied.

$$I_C = I_L \, \text{Sin}\varphi_L$$

Phasor Diagram



At the Resonance condition, the circuit draws the minimum current as under this (resonance) condition the reactive component of current is suppressed.

Frequency at Resonance Condition in Parallel resonance Circuit

The value of inductive reactance $\mathbf{X_L = 2\pi fL}$ and capacitive reactance $\mathbf{X_C = 1/2\pi fC}$ can be changed by changing the supply frequency. As the frequency increases, the value of X_L and consequently the value of Z_L increases. As a result, there is a decrease in the magnitude of current I_L and this I_L current lags behind the voltage V.

On the other hand, the value of capacitive reactance decreases and consequently the value of I_C increases.

At some frequency, f_r called resonance frequency.

$$I_C = I_L \operatorname{Sin}\varphi_L$$

Where,

$$I_L = \frac{V}{Z_L}$$

$$\operatorname{Sin}\varphi_L = \frac{X_L}{Z_L} \text{ and } I_C = \frac{V}{X_C}$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \text{ x } \frac{X_L}{Z_L} \text{ or } X_L X_C = Z_L^2 \text{ or}$$

$$\frac{\omega L}{\omega C} = Z_L^2 = (R^2 + X_L^2) \text{ or}$$

$$\frac{L}{C} = R^2 + (2\pi f_r L)^2 \text{ or}$$

$$2\pi f_r L = \sqrt{\frac{L}{C} - R^2} \text{ or}$$

$$f_r = \frac{1}{2\pi L} = \sqrt{\frac{L}{C} - R^2} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R is very small as compared to L, then resonant frequency will be

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

At parallel resonance line current $I_r = I_L \cos\phi$ or

$$\frac{V}{Z_r} = \frac{V}{Z_l} \times \frac{R}{Z_l} \quad \text{or} \quad \frac{1}{Z_r} = \frac{R}{Z_L^2} \quad \text{or}$$

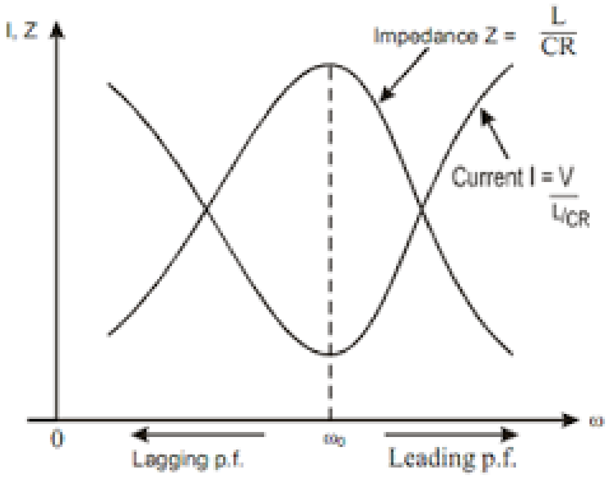
$$\frac{1}{Z_r} = \frac{R}{L/C} = \frac{CR}{L} \quad \left(\text{as } Z_L^2 = \frac{L}{C}\right)$$

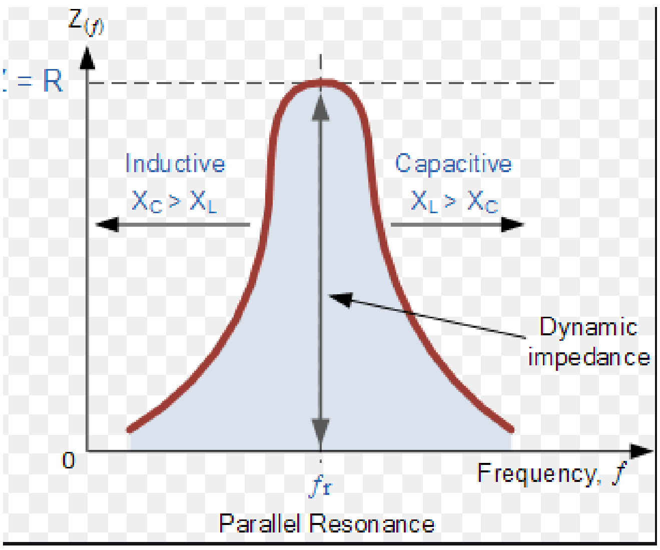
Therefore, the circuit impedance will be given as:

$$Z_r = \frac{L}{CR}$$

The current at resonance is then V / (L / CR)

Power factor of parallel resonance circuit is unity as $I_L \sin\Phi_L$ & I_C cancel each other.





Effect of Parallel Resonance:

- The circuit impedance is purely resistive because there is no frequency term present in it. If the value of inductance, capacitance and resistance is in Henry, Farads and Ohm then the value of circuit impedance Z_r will be in Ohms.
- The value of Zr will be very high because the ratio L/CR is very large at parallel resonance.

- The value of circuit current, Ir = V/Zr is very small because the value of Zr is very high.
- The current flowing through the capacitor and the coil is much greater than the line current because the impedance of each branch is quite lower than that of circuit impedance Zr.

4.4 <u>Parallel resonance (RL, RC & RLC) & derive the expression</u> -

4.5 <u>Comparision of series & parallel resonance and applications</u> –

Basis of Difference	**Series Resonance**	**Parallel Resonance**
Definition	When a resistor, inductor and capacitor are connected in series across an AC supply and the inductor and capacitor cancel the effect of each other at a particular frequency, then this condition of the series circuit is known as series resonance.	A combination of a resistor, inductor and a capacitor are connected in parallel across an AC source and the inductor and capacitor cancel the effect of each other at a specific supply frequency, then this condition of the parallel RLC circuit is known as parallel resonance.
Impedance	The impedance of a series RLC circuit becomes minimum at series resonance.	The impedance of a parallel RLC circuit becomes maximum at parallel resonance.
Admittance	The series RLC circuit offers maximum admittance at series resonance.	The admittance of the parallel RLC circuit at parallel resonance is minimum.
Current	The series resonance results in the maximum current through the circuit.	The current in circuit at parallel resonance is minimum.
Behave of the circuit	The series RLC circuit behaves as an accepter circuit at series resonance.	The parallel RLC circuit acts as a rejector circuit at parallel resonance
Magnify	The series resonance magnifies the voltage in the circuit.	The parallel resonance magnifies the current in the circuit.
Equation of effective impedance	The effective impedance of series RLC circuit at series resonance is given by, $Z=R$	The effective impedance of parallel RLC circuit at parallel resonance is given by, $Z=L/(CR)$
Quality factor (Q-factor)	For series resonance, the quality factor is given by, $Q{-}factor=\omega_0 L/R=1/\omega_0 RC$	For parallel resonance, the quality factor is given by, $Q{-}factor=R/\omega_0 L=\omega_0 RC$

Basis of Difference	Series Resonance	Parallel Resonance
Applications	The series resonance is widely used in tuning, oscillator circuits, voltage amplifiers, high frequency filters, etc.	The parallel resonance is used in current amplifiers, induction heating, filters, radio-frequency amplifiers, etc.

Applications

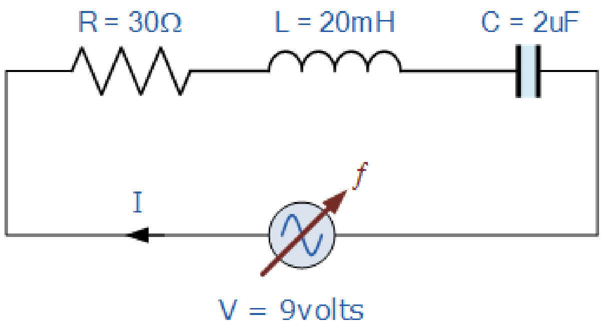
- The most common application of tank circuits is tuning radio transmitters and receivers. For example, when tuning a radio to a particular station, the LC circuits are set at resonance for that particular carrier frequency.
- A series resonant circuit provides voltage magnification. Series resonance circuits are found in various electrical and electronic circuits such as in voltage amplifiers, oscillator circuits, AC mains filters, high frequency filters, noise filters and also in radio and television tuning circuits producing a very selective tuning circuit for the receiving of the different frequency channels.
- A parallel resonant circuit provides current magnification. Parallel resonance circuits are used in current amplifiers, filter circuits, radio frequency amplifiers,
- Both parallel and series resonant circuits are used in induction heating.

4.6 Simple problems-

Series Resonance Example No1

A series resonance network consisting of a resistor of 30Ω, a capacitor of 2uF and an inductor of 20mH is connected across a sinusoidal supply voltage which has a constant output of 9 volts at all frequencies.

Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies.



1. Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times 2\times 10^{-6}}} = 796\text{Hz}$$

2. Circuit Current at Resonance, I_m

$$I = \frac{V}{R} = \frac{9}{30} = 0.3\text{A} \text{ or } 300\text{mA}$$

3. Inductive Reactance at Resonance, X_L

$$X_L = 2\pi f L = 2\pi \times 796 \times 0.02 = 100\Omega$$

4. Voltages across the inductor and the capacitor, V_L, V_C

$$V_L = V_C$$
$$V_L = I \times X_L = 300mA \times 100\Omega$$
$$V_L = 30\text{volts}$$

Note: the supply voltage may be only 9 volts, but at resonance, the reactive voltages across the capacitor, V_C and the inductor, V_L are 30 volts peak!

5. Quality factor, Q

$$Q = \frac{X_L}{R} = \frac{100}{30} = 3.33$$

6. Bandwidth, BW

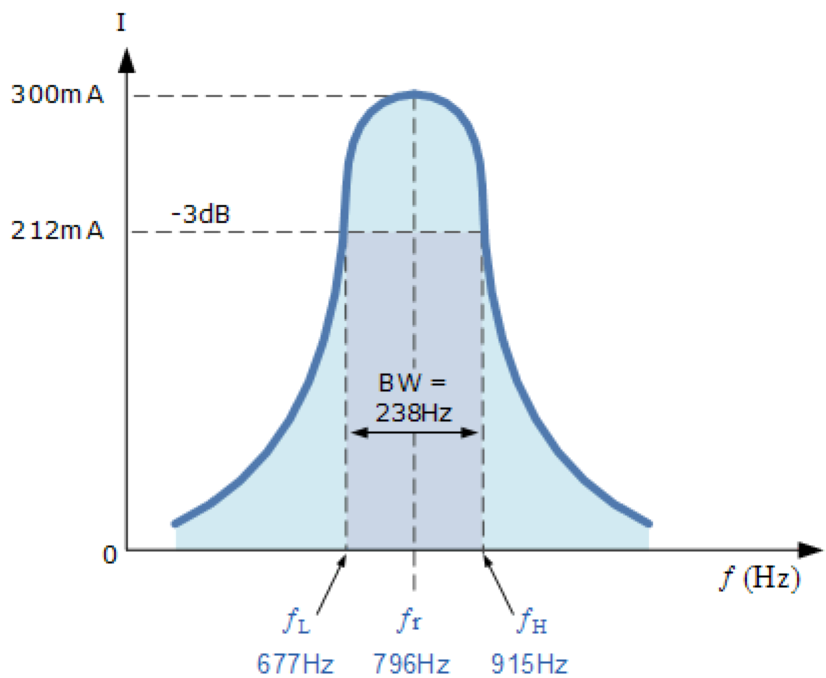
$$BW = \frac{f_r}{Q} = \frac{796}{3.33} = 238Hz$$

7. The upper and lower -3dB frequency points, f_H and f_L

$$f_L = f_r - \frac{1}{2}BW = 796 - \frac{1}{2}(238) = 677Hz$$

$$f_H = f_r + \frac{1}{2}BW = 796 + \frac{1}{2}(238) = 915Hz$$

8. Current Waveform



Series Resonance Example No2

A series circuit consists of a resistance of 4Ω, an inductance of 500mH and a variable capacitance connected across a 100V, 50Hz supply. Calculate the capacitance require to produce a series resonance condition, and the voltages generated across both the inductor and the capacitor at the point of resonance.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157.1\Omega$$

$$\text{at resonance: } X_C = X_L = 157.1\Omega$$

$$\therefore C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi.50.157.1} = 20.3\mu F$$

Voltages across the inductor and the capacitor, V_L, V_C

$$I_S = \frac{V}{R} = \frac{100}{4} = 25\text{Amps}$$

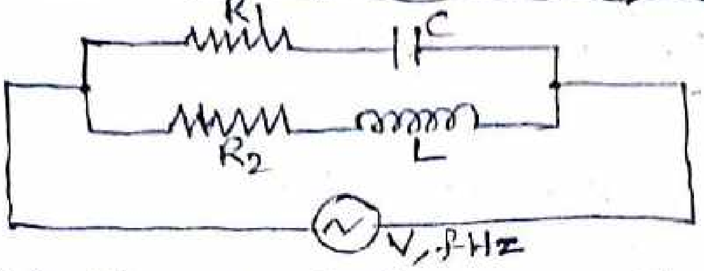
$$\text{at Resonance: } V_L = V_C$$

$$V_L = I \times X_L = 25 \times 157.1$$

$$\text{Thus } V_L = 3{,}927.5\text{volts or } 3.9\text{kV}$$

$$\text{and } V_C = 3{,}927.5\text{volts or } 3.9\text{kV}$$

Resonance between parallel R-C & R-L circuit



Let Y_1 = Admittance of $R_1 - C$ circuit $= \frac{1}{R_1 - jX_C}$

Y_2 = Admittance of $R_2 - L$ circuit $= \frac{1}{R_2 + jX_L}$

Y = Net admittance $= Y_1 + Y_2$

$$= \frac{1}{R_1 - jX_C} + \frac{1}{R_2 + jX_L} = \frac{R_1 + jX_C}{(R_1 + jX_C)(R_1 - jX_C)} + \frac{R_2 - jX_L}{(R_2 + jX_L)(R_2 - jX_L)}$$

$$= \frac{R_1 + jX_C}{R_1^2 + X_C^2} + \frac{R_2 - jX_L}{R_2^2 + X_L^2}$$

$$= \left(\frac{R_1}{R_1^2 + X_C^2} + \frac{R_2}{R_2^2 + X_L^2}\right) + j\left(\frac{X_C}{R_1^2 + X_C^2} - \frac{X_L}{R_2^2 + X_L^2}\right)$$

The circuit will be at resonance when the imaginary parts of capacitive and inductive admittances are cancelled.

i.e. $$\frac{X_C}{R_1^2 + X_C^2} = \frac{X_L}{R_2^2 + X_L^2}$$

$$\Rightarrow X_C (R_2^2 + X_L^2) = X_L (R_1^2 + X_C^2)$$

$$\Rightarrow \frac{1}{\omega_0 C}(R_2^2 + \omega_0^2 L^2) = \omega_0 L \left(R_1^2 + \frac{1}{\omega_0^2 C^2}\right)$$

If $R_1^2 = R_2^2 = \frac{L}{C}$, the above equation becomes

$$\frac{1}{\omega_0 C}\left(\frac{L}{C} + \omega_0^2 L^2\right) = \omega_0 L \left(\frac{L}{C} + \frac{1}{\omega_0^2 C^2}\right)$$

$$\Rightarrow \frac{L}{\omega_0 C^2} + \frac{\omega_0 L^2}{C} = \frac{\omega_0 L^2}{C} + \frac{L}{\omega_0 C^2}$$

$\therefore$ L.H.S = R.H.S

so, $R_1 = R_2 = \sqrt{\frac{L}{C}}$

This indicates that the circuit will be at resonance for any frequency provided

$$R_1 = R_2 = \sqrt{L/C}$$

At resonance, $\boxed{I_C \sin\phi_1 = I_L \sin\phi_2}$

where, I_C = current in R_1-C ckt

I_L = current in R_2-L ckt

ϕ_1 = power factor angle of R_1-C ckt

ϕ_2 = power factor angle of R_2-L ckt

$$I_C = \frac{V}{Z_{(R_1\text{-}C\ ckt)}} \quad \text{and} \quad I_L = \frac{V}{Z_{(R_2\text{-}L\ ckt)}}$$

$$\sin\phi_1 = \frac{X_C}{Z_{(R_1\text{-}C\ ckt)}} \quad \text{and} \quad \sin\phi_2 = \frac{X_L}{Z_{(R_2\text{-}L\ ckt)}}$$

$$\therefore \frac{V}{\sqrt{R_1^2 + \frac{1}{W_0^2 C^2}}} \times \frac{1/W_0 C}{\sqrt{R_1^2 + \frac{1}{W_0^2 C^2}}} = \frac{V}{\sqrt{R_2^2 + W_0^2 L^2}} \times \frac{W_0 L}{\sqrt{R_2^2 + W_0^2 L^2}}$$

$$\Rightarrow \frac{1/W_0 C}{R_1^2 + \frac{1}{W_0^2 C^2}} = \frac{W_0 L}{R_2^2 + W_0^2 L^2}$$

$$\Rightarrow \frac{1}{W_0 C\left(R_1^2 + \frac{1}{W_0^2 C^2}\right)} = \frac{W_0 L}{R_2^2 + W_0^2 L^2}$$

$$\Rightarrow \frac{1}{R_1^2 W_0 C + \frac{1}{W_0 C}} = \frac{W_0 L}{R_2^2 + W_0^2 L^2}$$

$$\Rightarrow R_2^2 + W_0^2 L^2 = R_1^2 W_0^2 LC + \frac{L}{C}$$

$$\Rightarrow W_0^2 (L^2 - R_1^2 LC) = \frac{L}{C} - R_2^2$$

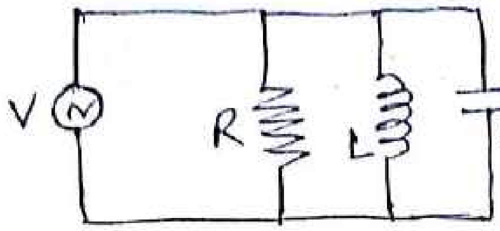
$$\Rightarrow W_0^2 LC\left(\frac{L}{C} - R_1^2\right) = \frac{L}{C} - R_2^2$$

$$\Rightarrow W_0^2 = \frac{1}{LC}\left[\frac{\frac{L}{C} - R_2^2}{\frac{L}{C} - R_1^2}\right]$$

$$\Rightarrow W_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{\frac{L}{C} - R_2^2}{\frac{L}{C} - R_1^2}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{\frac{L}{C} - R_2^2}{\frac{L}{C} - R_1^2}}$$

This is the expression of resonance of parallel R-L & R-C circuit.

Parallel resonance of R-L-C circuit



Parallel resonance occurs when the susceptance part is zero. Susceptance is the reciprocal of reactance. $B = \frac{1}{X}$

Admittance, $Y = G + jB$ $\quad [\because Z = R + jX]$

$$= \frac{1}{R} + \frac{1}{jWL} + jWC \quad \left[\because X = jWL - \frac{j}{WC}\right]$$

$$= \frac{1}{R} + j\left(WC - \frac{1}{WL}\right)$$

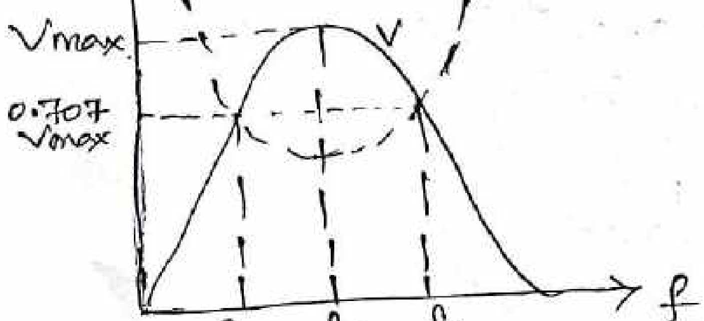
The frequency at which resonance occurs is

$$W_0 C - \frac{1}{W_0 L} = 0$$

$$\Rightarrow W_0^2 = \frac{1}{LC} \Rightarrow W_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

At resonant frequency, the current becomes minimum.

Bandwidth, $BW = f_2 - f_1$



(Voltage & Current variation with frequency)

Lower half power frequency can be obtained by

$$\omega_1 C - \frac{1}{\omega_1 L} = -\frac{1}{R}$$

$$\Rightarrow \omega_1 C - \frac{1}{\omega_1 L} + \frac{1}{R} = 0$$

$$\Rightarrow \frac{\omega_1^2 LCR - R + \omega_1 L}{\omega_1 LR} = 0$$

$$\Rightarrow \frac{\omega_1^2 LCR}{LCR} - \frac{R}{LCR} + \frac{\omega_1 L}{LCR} = 0$$

$$\Rightarrow \omega_1^2 + \frac{\omega_1}{CR} - \frac{1}{LC} = 0$$

Simplify the equation, we get

$$\omega_1 = \frac{-\frac{1}{CR} + \sqrt{\left(\frac{1}{CR}\right)^2 - 4 \times 1 \times \left(-\frac{1}{LC}\right)}}{2 \times 1}$$

$$= -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Similarly, the upper half power frequency can be obtained by

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R}$$

$$\Rightarrow \omega_2^2 - \frac{\omega_2}{CR} - \frac{1}{LC} = 0$$

Simplify the equation, we get

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth is given by

$$BW = \omega_2 - \omega_1 = \frac{1}{RC}$$

Quality factor is defined as

$$Q_0 = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0}{1/RC} = \omega_0 RC$$

CHAPTER – 5
LAPLACE TRANSFORM & ITS APPLICATIONS

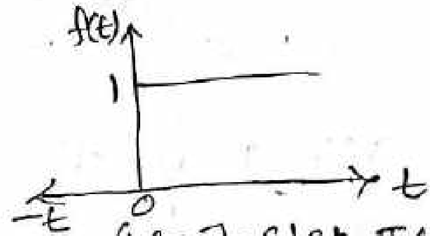
Laplace transformation is a powerful method of solving linear differential equations. As the transient response of an electric circuit can be described by a linear differential equation, laplace transform is used to solve the transient behaviour of electric circuits.

* Laplace transform of function $f(t)$ is defined as
$$F(S) = \mathcal{L}f(t) = \int_0^{\infty} f(t)\, e^{-st}\, dt$$
where, s being the transformed variable.
* S is the complex variable $(\sigma + j\omega)$.
* The operation of $\mathcal{L}f(t)$ is in the complex frequency domain or s-domain.
$$\mathcal{L}^{-1} F(S) = f(t)$$

Test signals or I/p

(i) Step function —

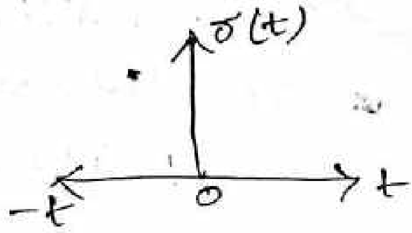
$$u(t) = 0 \text{ when } t < 0$$
$$= A \text{ when } t \geq 0$$



(Unit step I/p)

For unit step I/p,
$$u(t) = 0 \text{ when } t \leq 0$$
$$= 1 \text{ when } t > 0$$
$$\mathcal{L}u(t) = \mathcal{L}(1) = \frac{1}{s}$$

(ii) Impulse function —



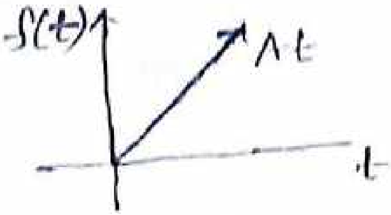
$$\delta(t) = 0 \text{ when } t \neq 0$$
$$= \infty \text{ when } t = 0$$

For unit impulse I/p,
$$\mathcal{L}\delta(t) = 1$$
$$\delta(t) = \frac{d}{dt} u(t)$$
The unit impulse function is the derivative of unit step function.

(iii) Ramp function —

$f(t) = At$ when $t \geq 0$
$= 0$ when $t < 0$

$F(s) = L f(t) = L(At)$
$= \frac{A}{s^2}$

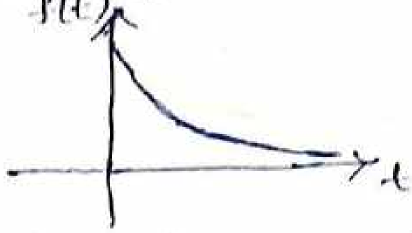


Integration of unit step function results in unit ramp function

(iv) Exponential function

$f(t) = e^{-at}$

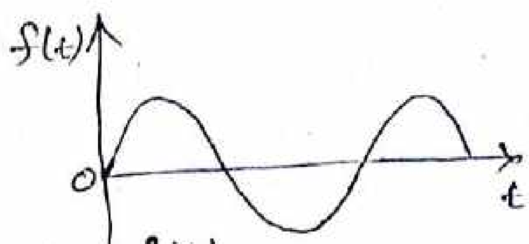
$F(s) = L f(t) = L e^{-at}$
$= \frac{1}{s+a}$



(v) Sinusoidal function

$f(t) = \sin wt$

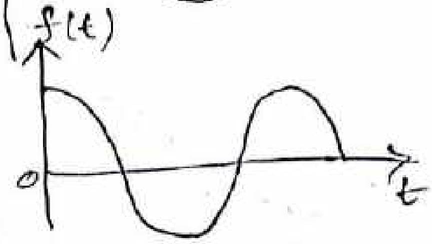
$F(s) = L \sin wt = \frac{w}{s^2+w^2}$



(vi) Cosinusoidal function

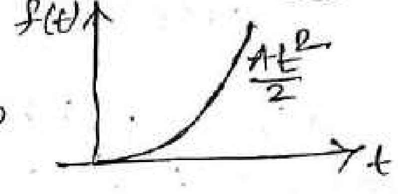
$f(t) = \cos wt$

$F(s) = L \cos wt = \frac{s}{s^2+w^2}$



(vii) Parabolic function

$f(t) = \frac{At^2}{2}$ when $t \geq 0$
$= 0$ when $t < 0$



$F(s) = L f(t) = L\left(\frac{At^2}{2}\right) = \frac{A}{s^3}$

■ Some Laplace Formula.

$L(t^n) = \frac{n!}{s^{n+1}}$

$L\, e^{-at} \sin wt = \frac{w}{(s+a)^2 + w^2}$

$L\, e^{-at} \cos wt = \frac{s+a}{(s+a)^2+w^2}$

$L\, e^{-at} t^n = \frac{n!}{(s+a)^{n+1}}$

$L\, V(t) = V(s)$

$L\, I(t) = I(s)$

$L(R) = R$

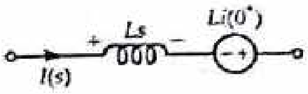
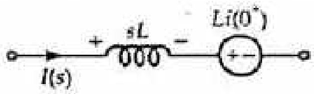
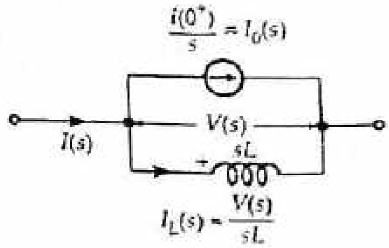
$L \frac{di(t)}{dt} = \left[s I(s) - i(0^+) \right]$

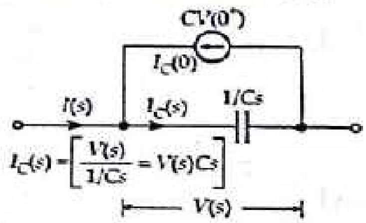
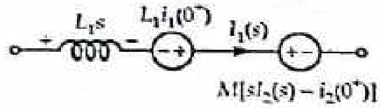
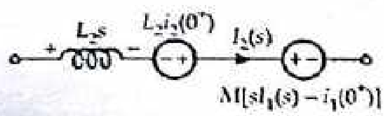
$L(L) = Ls$

$L(C) = \frac{1}{Cs}$

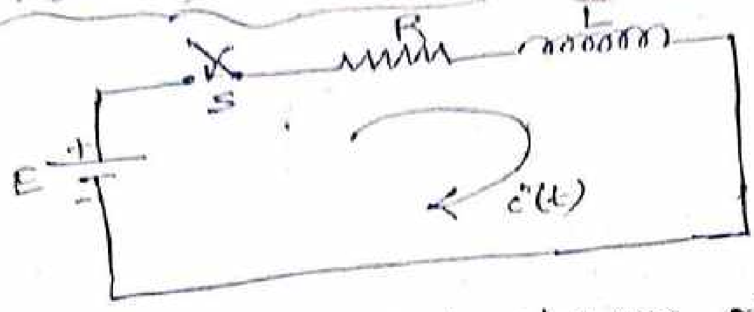
$L \int i(t)\,dt = \left[\frac{I(s)}{s} + \frac{i(0^+)}{s} \right]$

Table 9.2

	Time domain	s-domain
1.	Constant voltage source (say a battery terminal voltage) V.	$\frac{V}{s}$
2.	Time dependent voltage source, $V = f(t)$.	$V(s)$
3.	Constant current source I.	$\frac{I}{s}$
4.	Time dependent current source, $I = f(t)$.	$I(s)$
5.	(*a*) Resistance element (R)	R
	(*b*) current is $i(t)$ through R causing drop $v = Ri(t)$.	$V(s) = RI(s)$
6.	(*a*) Inductance element (L).	Ls
	(*b*) Initial current is $i(0^+)$, clockwise, circuit current being $i(t)$; $v(t) = L\frac{di(t)}{dt}$.	$V(s) = [sLI(s) - Li(0^+)]$ (Fig. 9.9)  Fig. 9.9 [If the initial current is $-i(0^+)$, the voltage equation becomes $V(s) = [sLI(s) + Li(0^+)]$ and the equivalent circuit in Laplace domain is [Fig. 9.9(*a*)].  Fig. 9.9(a)
	(*c*) Current in the inductor is given by $i(t) = \frac{1}{L}\int_0^t v(t)\,dt + i(0^+)$.	$I(s) = \frac{1}{L}\cdot\frac{V(s)}{s} + \frac{i(0^+)}{s} = I_L(s) + I_0(s)$ which gives the following circuit configuration in s-domain (Fig. 9.10) $\frac{i(0^+)}{s} = I_0(s)$.  Fig. 9.10

Time domain	s-domain
7. (*a*) Capacitance element (C) (*b*) Initial voltage is V_0, with +ve polarity at the side of the capacitor that opposes charging current $i(t)$ though it. $v(t) = V_0 + \frac{1}{C}\int_0^t i(t)\,dt$	$1/Cs$ $V(s) = \frac{V_0}{s} + \frac{1}{C}\frac{I(s)}{s} = \frac{V_0}{s} + \frac{I(s)}{Cs}$. The equivalent circuit [Fig. 9.11(*a*)] would be, in *s* domain.  Fig. 9.11(*a*)
(*c*) If the polarity of the initial voltage be $-V_0$ *i.e.*, it assists charging current $i(t)$. $v(t) = -V_0 + \frac{1}{C}\int_0^t i(t)\,dt$	$V(s) = -\frac{V_0}{s} + \frac{I(s)}{Cs}$ which gives the equivalent circuit [Fig. 9.11(*b*)] as  Fig. 9.11(*b*)
(*d*) Current in the capacitor is given by $i(t) = C\frac{dv(t)}{dt}$	$I(s) = C[V(s)s - V(0^+)] = I_C(s) - I_C(0^+)$ which gives the equivalent circuit [Fig. 9.11(*c*)] as  Fig. 9.11 (*c*)
8. Mutual inductance between two coils being *M*, currents being i_1 and i_2, self-inductances being L_1 and L_2, the voltage drops in each the linked coil is given by $v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$ and $v_2(t) = L_2\frac{di_2(t)}{dt} + M\frac{di_1(t)}{dt}$	$V_1(s) = L_1 sI_1(s) - Li_1(0^+) + MsI_2(s) - Mi_2(0^+)$ ...(*a*) $V_2(s) = L_2 sI_2(s) - Li_2(0^+) + MsI_1(s) - Mi_1(0^+)$ ...(*b*) The circuit configuration corresponding to equation (*a*),  Fig. 9.12 (*a*) and corresponding to equation (*b*),  Fig. 9.12 (*b*)

Step response of R-L series circuit :-



Assuming that the inductance of the circuit is initially deenergised, the governing equation when S is closed is given by

$$E\,u(t) = R\,i(t) + L\frac{di}{dt}$$

where, $E\,u(t)$ = Unit step function Voltage as E exists only when S is closed.

By taking Laplace transform

$$\frac{E}{S} = R\,I(S) + L[S\,I(S) - i(0)]$$

where, $i(0)$ is the initial value of current. Here it is zero as the circuit is assumed to be in the de-energised condition.

$$\frac{E}{S} = R I(S) + L S I(S)$$

$$= I(S)(R+SL)$$

$$\Rightarrow I(S) = \frac{E}{S(R+SL)} = \frac{E/L}{S\left(\frac{R}{L}+\frac{SL}{L}\right)} = \frac{E/L}{S\left(S+\frac{R}{L}\right)}$$

Expressing in Partial fraction form, $I(S) = \frac{A}{S} + \frac{B}{S+\frac{R}{L}}$

where, $A = \left[\frac{E/L}{S+\frac{R}{L}}\right]_{S=0} = \frac{E/L}{R/L} = \frac{E}{R}$

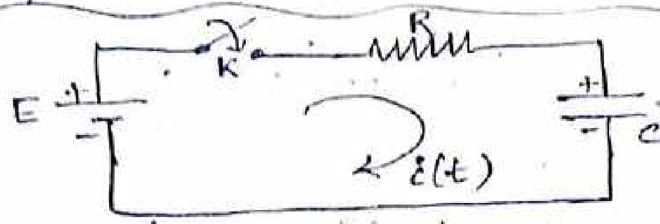
$$B = \left[\frac{E/L}{S}\right]_{S=-\frac{R}{L}} = \frac{E/L}{-R/L} = -\frac{E}{R}$$

Thus $I(S) = \frac{E/R}{S} + \frac{(-E/R)}{S+\frac{R}{L}}$

By taking inverse Laplace transform,

$$i(t) = \frac{E}{R} - \frac{E}{R}e^{-\frac{R}{L}t}$$

Step Response of series R-C circuit -



Assuming that the capacitance of the circuit is initially having a charge Q_0 owing to the flow of some previously applied current, the governing equation of the circuit when the switch is closed is given by

$$E\,u(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

Where; $E\,u(t)$ = step response voltage with magnitude E (d.c.).

By taking Laplace transform,

$$\frac{E}{s} = R\,I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{i(0)}{s}\right]$$

Where; $i(0)$ is the current in the circuit in initial condition when $i(0) = \int_{-\infty}^{0} i\,dt = Q_0$

$$\Rightarrow \frac{E}{s} = R\,I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{Q_0}{s}\right]$$

$$\Rightarrow \frac{E}{s} - \frac{Q_0}{Cs} = R\,I(s) + \frac{I(s)}{Cs}$$

$$\Rightarrow \frac{1}{s}\left(E - \frac{Q_0}{C}\right) = I(s)\left[R + \frac{1}{Cs}\right]$$

$$\Rightarrow I(s) = \frac{\frac{1}{s}\left(E - \frac{Q_0}{C}\right)}{R + \frac{1}{Cs}} = \frac{E - \frac{Q_0}{C}}{s\left(R + \frac{1}{Cs}\right)} = \frac{E - \frac{Q_0}{C}}{Rs + \frac{1}{C}}$$

$$= \frac{E - \frac{Q_0}{C}}{R\left(s + \frac{1}{RC}\right)} = \frac{E - \frac{Q_0}{C}}{R} \times \frac{1}{s + \frac{1}{RC}} = \left(\frac{E}{R} - \frac{Q_0}{RC}\right)\left(\frac{1}{s + \frac{1}{RC}}\right)$$

By taking inverse Laplace transform,

$$i(t) = \left(\frac{E}{R} - \frac{Q_0}{RC}\right) e^{-\frac{1}{RC}t}$$

Since, V_{in} is the initial voltage stored in the capacitor given by (Q_0/C)

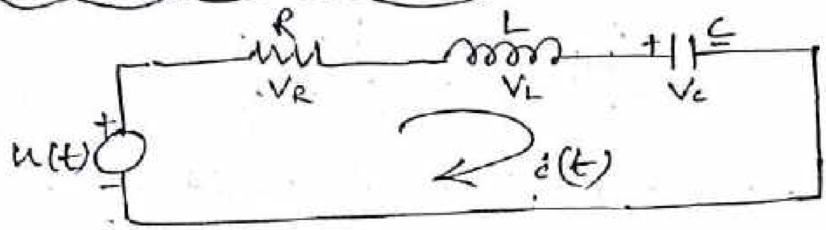
$$V_c(s) = \frac{V_{in}}{s} + \frac{I(s)}{Cs} = \frac{V_{in}}{s} + \frac{E - V_{in}}{s\left(R + \frac{1}{Cs}\right)Cs}$$

$$\Rightarrow V_C(S) = \frac{V_{in}}{S} + \frac{E - V_{in}}{S^2 C\left(R + \frac{1}{SC}\right)}$$

$$\therefore V_C(t) = V_{in} + E - V_{in} - (E - V_{in}) e^{-t/RC}$$

$$= E - (E - V_{in}) e^{-t/RC} \text{ volt.}$$

Step Response of series R-L-C circuit



A unit step voltage u(t) is applied across an R-L-C series circuit having zero initial conditions [$V_C(0^-) = 0$ and $I_L(0^-) = 0$]. The governing equation is given by

$$u(t) = R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt$$

By taking Laplace transform,

$$\frac{1}{S} = R\,I(S) + LS\,I(S) + \frac{1}{CS} I(S)$$

$$\Rightarrow \frac{1}{S} = I(S)\left(R + LS + \frac{1}{CS}\right) \quad \left[\because Z(S) = R + LS + \frac{1}{CS}\right]$$

$$\Rightarrow I(S) = \frac{1/S}{R + LS + \frac{1}{CS}} = \frac{1}{RS + LS^2 + \frac{1}{C}}$$

$$= \frac{1/L}{S^2 + \frac{R}{L} S + \frac{1}{LC}}$$

$$= \frac{1/L}{(S+\alpha)(S+\beta)}$$

$$\text{Where } \alpha, \beta = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Case-I

Let's assume α and β are real and not equal

$$\left[\frac{R}{2L} > \frac{1}{\sqrt{LC}}\right]$$

As $I(s) = \dfrac{1/L}{(s+\alpha)(s+\beta)} = \dfrac{K_1}{s+\alpha} + \dfrac{K_2}{s+\beta}$

Where, $K_1 = \left[\dfrac{1/L}{s+\beta}\right]_{s=-\alpha} = \dfrac{1}{L(\beta-\alpha)}$

$K_2 = \left[\dfrac{1/L}{s+\alpha}\right]_{s=-\beta} = \dfrac{1}{L(\alpha-\beta)}$

Thus, $I(s) = \dfrac{1/L(\beta-\alpha)}{s+\alpha} + \dfrac{1/L(\alpha-\beta)}{s+\beta}$

By taking inverse laplace transform,

$$i(t) = \left(\frac{1}{L(\beta-\alpha)} e^{-\alpha t} + \frac{1}{L(\alpha-\beta)} e^{-\beta t}\right) \text{ amp.}$$

Case-II

α and β are equal $\left[\dfrac{R}{2L} = \dfrac{1}{\sqrt{LC}}\right]$

Case-III

α and β are complex and not equal

$\left[\dfrac{R}{2L} < \dfrac{1}{\sqrt{LC}}\right]$

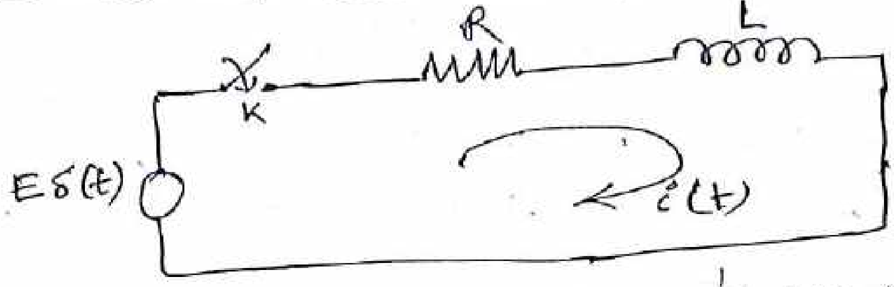
The case II and III can also be solved in the same way.

→ when $\alpha = \beta = \gamma$ i.e. $\dfrac{R}{2L} = \dfrac{1}{\sqrt{LC}}$

$$I(s) = \frac{1/L}{(s+\gamma)^2}$$

$$i(t) = \frac{1}{L} t e^{-\gamma t} \text{ Amp.}$$

Impulse Response of series R-L network



Assuming the circuit is initially de-energised, the governing equation is given by

$$E\delta(t) = Ri(t) + L\frac{di(t)}{dt}$$

The series R-L circuit is excited by an impulse function $E\delta(t)$.

By taking Laplace transform,

$$E = R.I(s) + L[sI(s) - i(0^+)]$$

As $i(0^+) = 0$; $E = RI(s) + LsI(s)$

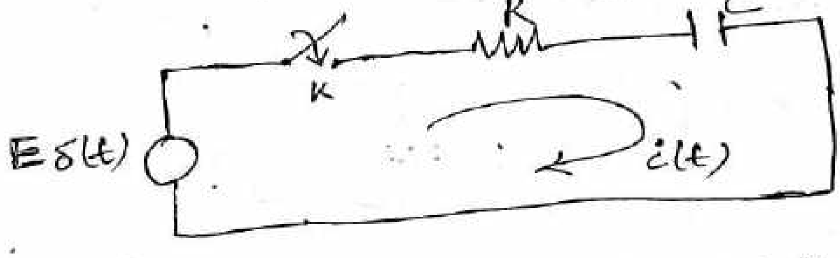
$$= I(s)[R+Ls]$$

$$\Rightarrow I(s) = \frac{E}{R+Ls} = \frac{E}{L(s+\frac{R}{L})} = \frac{E}{L}\cdot\frac{1}{(s+\frac{R}{L})}$$

By taking inverse laplace transform,

$$i(t) = \frac{E}{L}e^{-\frac{R}{L}t}$$

Impulse Response of series R-C network



The series R-C circuit is excited at $t=0$ by an impulse function of magnitude E. The governing equation is given by

$$E\delta(t) = Ri(t) + \frac{1}{C}\int i(t)\,dt$$

By taking laplace transform,

$$E = R\,I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{Q_0}{s}\right]$$

Assuming that the capacitor is initially discharged i.e. $Q_0 = 0$, the equation becomes

$$E = R\,I(s) + \frac{I(s)}{Cs}$$

$$= I(s)\left(R + \frac{1}{Cs}\right) = I(s)\cdot\frac{R}{s}\left(s + \frac{1}{RC}\right)$$

$$\Rightarrow I(s) = \frac{E}{\frac{R}{s}\left(s+\frac{1}{RC}\right)} = \frac{E}{R}\,\frac{s}{\left(s+\frac{1}{RC}\right)}$$

$$\Rightarrow I(s) = \frac{E}{R}\left[1 - \frac{1/RC}{\left(s+\frac{1}{RC}\right)}\right]$$

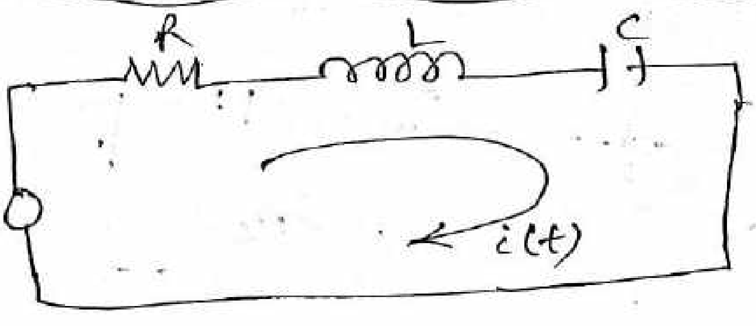
$$\Rightarrow I(s) = \frac{E}{R}\left[1 - \frac{1/T}{s+\frac{1}{T}}\right]$$

Where, T = time constant = RC.

By taking the inverse Laplace transform,

$$i(t) = \frac{E}{R}\,\delta(t) - \frac{E}{R.T}\,e^{-\frac{1}{T}t} \text{ Amp}.$$

Impulse response of series R-L-C circuit



The series R-L-C circuit is excited by an unit impulse function. The governing equation is given by

$$v(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

Assuming the circuit having zero initial condition i.e. $V_C(0^-) = 0$ and $I_L(0^-) = 0$, the equation can be written as in Laplace domain –

$$1 = R\,I(s) + Ls\,I(s) + \frac{1}{Cs}I(s)$$

$$= I(s)\left[R + Ls + \frac{1}{Cs}\right] \quad \left[\because Z(s) = R + Ls + \frac{1}{Cs}\right]$$

$$\Rightarrow I(s) = \frac{1}{R + Ls + \frac{1}{Cs}}$$

$$= \frac{s}{s^2L + Rs + \frac{1}{C}}$$

$$= \frac{s}{(s+\alpha)(s+\beta)}$$

Where $\alpha, \beta = \dfrac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L}$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Assuming α and β are real and unequal, i.e. $\dfrac{R}{2L} > \dfrac{1}{\sqrt{LC}}$

As. $I(s) = \dfrac{s}{(s+\alpha)(s+\beta)} = \dfrac{A}{s+\alpha} + \dfrac{B}{s+\beta}$

where, $A = \left[\dfrac{s}{s+\beta}\right]_{s=-\alpha} = -\dfrac{\alpha}{\beta - \alpha}$

$$B = \left[\frac{s}{s+\alpha}\right]_{s=-\beta} = -\frac{\beta}{\alpha - \beta}$$

$$I(s) = \frac{-\alpha/\beta-\alpha}{s+\alpha} + \frac{-\beta/\alpha-\beta}{s+\beta}$$

By taking inverse Laplace transform;

$$i(t) = -\frac{\alpha}{\beta-\alpha}e^{-\alpha t} - \frac{\beta}{\alpha-\beta}e^{-\beta t} \text{ amp}.$$

CHAPTER - 6
TWO PORT NETWORK ANALYSIS

6.1 Network elements -

When a number of impedances are connected together to form a system that consists of set of interconnected circuits comprising of electric elements like resistance, inductance, capacitance etc to perform specific functions, it is called a network. A network element is a component of an electric network or circuit.

Types of Network elements -

(i) Linear and Non-linear elements

A linear element is a component which shows linear characteristics of voltage vs current. Exa- resistor, inductor, capacitor etc.

A non-linear element is a component through which current does not change linearly with the linear change in applied voltage at a particular frequency. Exa - semiconductor devices like diode, transistor, FET, MOSFET, IC etc.

(ii) Active and passive elements -

Active element is a component which can deliver power or enhance the energy level of a signal passing through it. Exa- diode, transistor, op-amp etc.

Passive element is a component which can absorb the power and convert it into heat or store energy in electric or magnetic field. Exa- R, L, C.

(iii) Unilateral and Bilateral elements -

Unilateral element is a component through which magnitude of the current is affected with the change in polarity of the applied voltage. It allows unidirectional current flow. Exa- diode, transistor etc.

Bilateral element is a component through which magnitude of current remains the same even if the applied emf's polarity is changed. Exa- R, L, C.

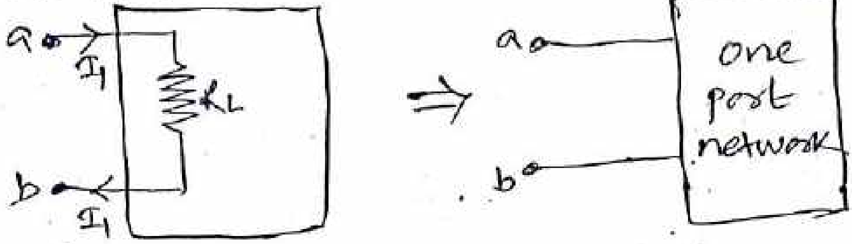
Types of Network -

(i) Linear and Non-Linear network
(ii) Active and passive network
(iii) Unilateral and Bilateral network
(iv) Lumped and Distributed network
(v) Recurrent and Non-recurrent network

* Linear network is the circuit which contains only linear elements. Non-linear network is the circuit which contains non-linear elements.
* Active network is a network which contains one or more than one sources of emf. Exa- battery, transistor. Passive network is a network which does not contain any source of emf. Exa- R, L, C.
* Unilateral network is the circuit which contains unilateral elements. Bilateral network is the circuit which contains bilateral elements.
* Lumped network is the circuit which contains physically separate network elements like R, L or C. A transmission line or cable is an example of distributed parameter network as throughout the line they are not physically separate.
* Recurrent network is called cascaded network in which similar networks are connected one after another. A single network is called non-recurrent network.

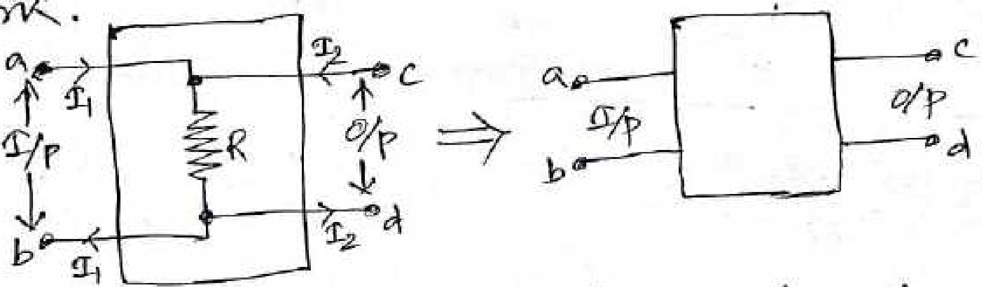
Posts in network -

(i) <u>one post network</u> — Any active or passive network having only two terminals can be represented by an one post network.



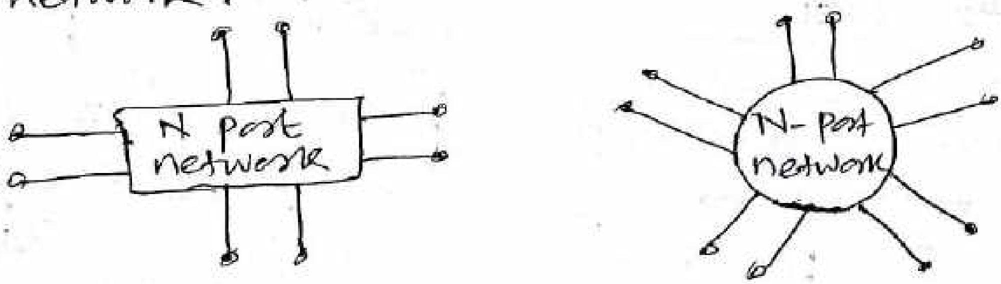
(Schematic representation of one post network)

(ii) <u>Two post network</u> - If a network consists of two pairs of terminals (i.e. four terminals) where one pair terminals can be designated as input and the other pair being output, it is called a two post network.



(Schematic representation of two post network)

(iii) <u>n-post network</u> - If any network contains 'n' number of pairs of terminals, it is called a n-post network.

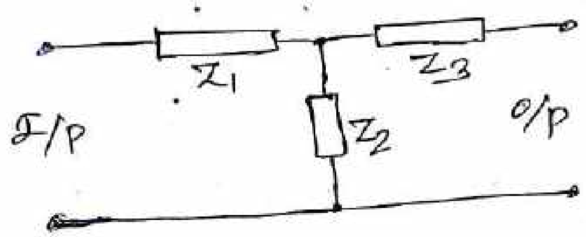


(Diagrammatic representation of n-post network)

6.2 Network configurations (T & π)

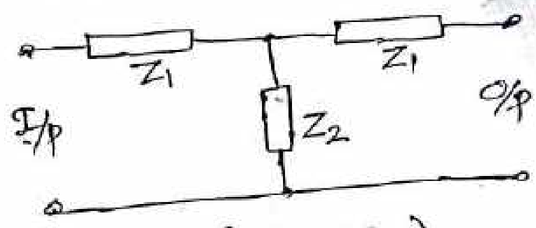
Depending on the configuration of impedance, a network can be specified as follows :-

(i) T section – when a network section looks like a 'T', it is called as T section.



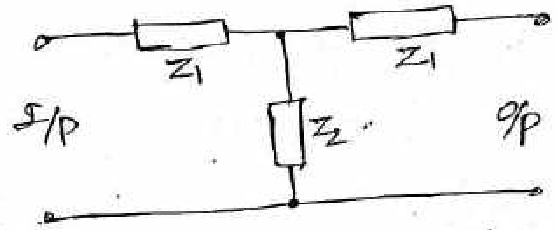
$(Z_1 \neq Z_3)$

(Unsymmetrical T-section)

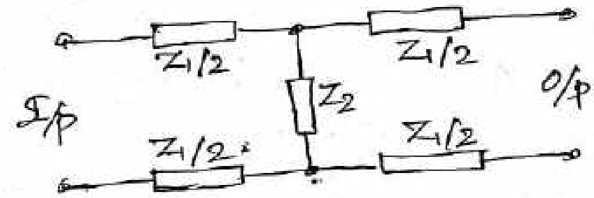


$(Z_1 = Z_3)$

(Symmetrical T-section)

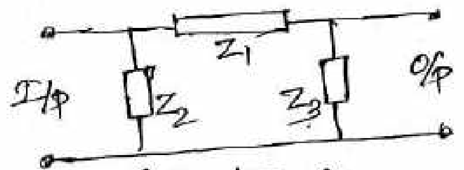


(Unbalanced symmetrical T-section)



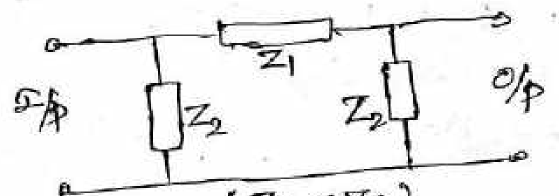
(Balanced symmetrical T-section or H-section)

(ii) π section – when a network section looks like a 'π', it is called as π-section.



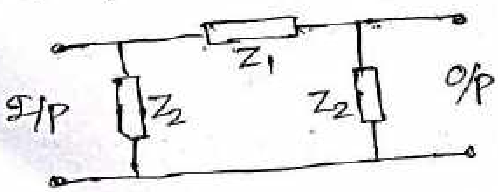
$(Z_2 \neq Z_3)$

(Asymmetrical π section)

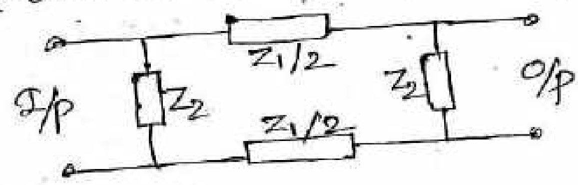


$(Z_2 = Z_3)$

(Symmetrical π section)



(Unbalanced symmetrical π-section)

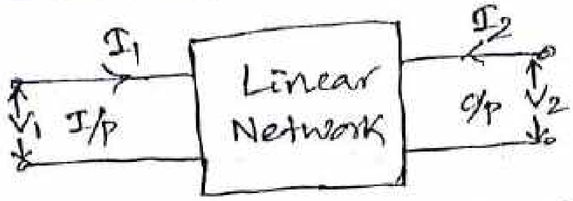


(Balanced symmetrical π-section or O section)

6.3 Parameter representation -

Different types of two port parameters are -
(i) Impedance (called Z parameters)
(ii) Admittance (called Y parameters)
(iii) Hybrid (called h-parameters)
(iv) Transmission (called ABCD parameters)

Z parameters (open circuit Impedance parameters)



In this two port network representation,
V_1 is the function of I_1 and I_2
V_2 is the function of I_1 and I_2
So, V_1 and V_2 are dependent parameters, I_1 and I_2 are independent parameters.

* The I/p and o/p voltages V_1 and V_2 can be expressed in terms of I/p and o/p currents I_1 and I_2 respectively as -

$$[V] = [Z][I]$$

where $[Z]$ is the impedance matrix.

In matrix form,
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{-----} (1)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{-----} (2)$$

Assuming the o/p of the two port network to be open circuited, $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1} \quad \text{and} \quad Z_{21} = \frac{V_2}{I_1}$$

Assuming the I/p of the two port network to be open circuited, $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2} \quad \text{and} \quad Z_{22} = \frac{V_2}{I_2}$$

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ called I/p driving point impedance.

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$ called o/p driving point impedance.

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ called reverse transfer impedance.

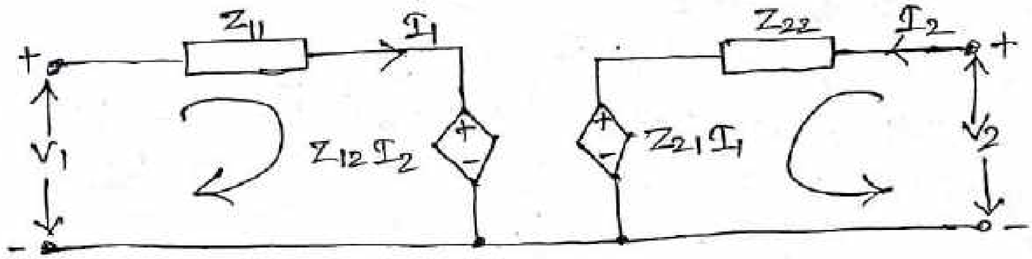
$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ called forward transfer impedance.

$Z_{11}, Z_{12}, Z_{21}, Z_{22}$ are called impedance parameters or open circuit parameters of the two port network.

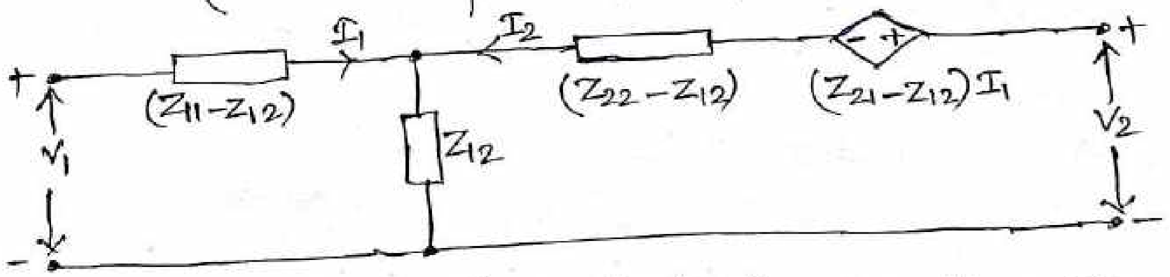
Equation ① & ② can be represented as

$$V_1 = (Z_{11} - Z_{12}) I_1 + Z_{12} (I_1 + I_2)$$

and $V_2 = (Z_{21} - Z_{12}) I_1 + (Z_{22} - Z_{12}) I_2 + Z_{12} (I_1 + I_2)$

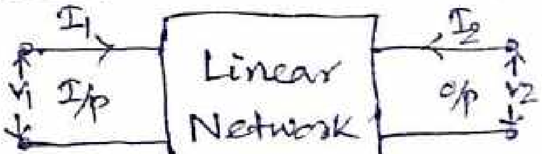


(Basic Z parameter equivalent circuit)



(Another form of equivalent circuit with Z parameters)

Y parameters (short circuit Admittance parameters)



In this two post network representation,

I_1 is the function of V_1 and V_2

I_2 is the function of V_1 and V_2

V_1 and V_2 are independent parameters, I_1 and I_2 are dependent parameters.

* The I/p and o/p currents I_1 and I_2 can be expressed in terms of I/p and o/p voltages V_1 and V_2 respectively as –

$$[I] = [Y][V]$$

where, $[Y]$ is the admittance matrix.

In matrix form, $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{------} \quad (1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{------} \quad (2)$$

Assuming the o/p of the two post network to be short-circuited, $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1} \quad \text{and} \quad Y_{21} = \frac{I_2}{V_1}$$

Assuming the I/p of the two post network to be short circuited, $V_1 = 0$

$$Y_{12} = \frac{I_1}{V_2} \quad \text{and} \quad Y_{22} = \frac{I_2}{V_2}$$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ called I/p driving point admittance.

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$ called o/p driving point admittance

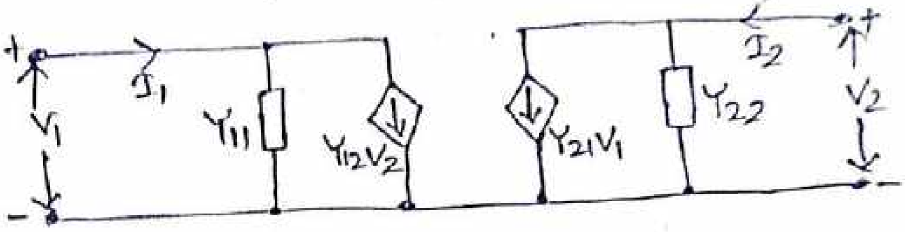
$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ called reverse transfer admittance

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ called forward transfer admittance.

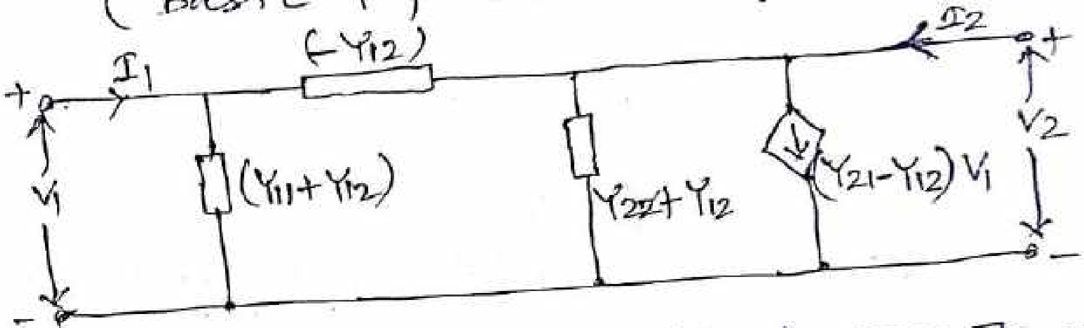
Equation (1) & (2) can be represented as

$$I_1 = (Y_{11} + Y_{12})V_1 - Y_{12}(V_1 - V_2)$$

and $$I_2 = (Y_{21} - Y_{12})V_1 + (Y_{22} + Y_{12})V_2 - Y_{12}(V_2 - V_1)$$



(Basic Y parameter equivalent circuit)



(Another form of equivalent circuit with Y parameters)

Z parameters in terms of Y-parameters (From Y to Z)

$$[Z] = [Y]^{-1}$$

or $$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

Thus $$Z_{11} = \frac{Y_{22}}{\Delta Y}, \quad Z_{12} = \frac{Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{Y_{21}}{\Delta Y} \text{ and } Z_{22} = \frac{Y_{11}}{\Delta Y}$$

where, $$\Delta Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = Y_{11}Y_{22} - Y_{12}Y_{21}$$

Y parameters in terms of Z parameters (From Z to Y)

$$[Y] = [Z]^{-1}$$

or

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

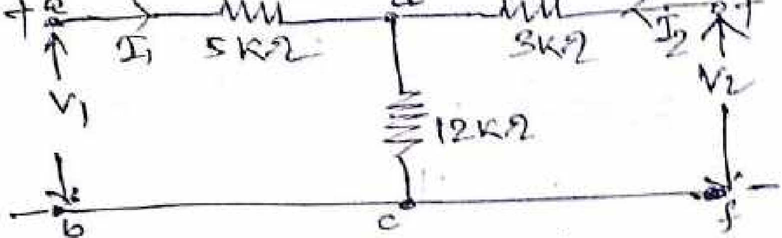
Thus $Y_{11} = \frac{Z_{22}}{\Delta Z}$, $Y_{12} = -\frac{Z_{12}}{\Delta Z}$

$Y_{21} = -\frac{Z_{21}}{\Delta Z}$ and $Y_{22} = \frac{Z_{11}}{\Delta Z}$

Where, $\Delta Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = Z_{11}Z_{22} - Z_{12}Z_{21}$

Problem 1

Find the Z parameters for the network shown in figure.



Sol^n – Using KVL for loop abcd

$$V_1 = 5\times10^3 I_1 + 12\times10^3 (I_1 + I_2)$$
$$= 17\times10^3 I_1 + 12\times10^3 I_2$$

Using KVL for Loop cdef

$$V_2 = 3\times10^3 I_2 + 12\times10^3 (I_1 + I_2)$$
$$= 12\times10^3 I_1 + 15\times10^3 I_2$$

Z_{11} = I/p driving point impedance

$$= \frac{V_1}{I_1}\Big|_{I_2=0} = 17\times10^3 = 17\,K\Omega$$

Z_{12} = Reverse transfer impedance

$$= \frac{V_1}{I_2}\Big|_{I_1=0} = 12\times10^3 = 12\,K\Omega$$

Z_{21} = Forward transfer impedance

$$= \frac{V_2}{I_1}\Big|_{I_2=0} = 12\times10^3 = 12\,K\Omega.$$

Z_{22} = O/p driving point impedance

$$= \frac{V_2}{I_2}\Big|_{I_1=0} = 15\times10^3 = 15\,K\Omega$$

Shortcut method

$Z_{11} = Z_1 + Z_3$

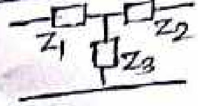
$Z_{12} = Z_3$

$Z_{21} = Z_3$

$Z_{22} = Z_2 + Z_3$

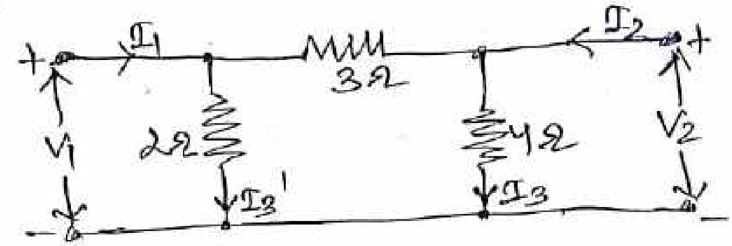
for T-section only

(Imp)



Problem 2

Find the Z parameters for the π-network shown in fig



Sol^n

Assuming the o/p port being open-circuited,

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{I_1\times[2\,||\,(3+4)]}{I_1}$$

$$= 2\,||\,7 = \frac{2\times7}{2+7} = \frac{14}{9} = 1.555\Omega$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{I_3\times4}{I_1} \qquad [\because V_2 = I_3\times4\Omega]$$

$$= \frac{2}{2+3+4}\times I_1\times\frac{4}{I_1} \qquad \left[\because I_3 = \frac{2}{2+3+4}I_1\right]$$

$$= \frac{8}{9} = 0.888\,\Omega$$

Assuming the I/p port being open circuited,

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{I_3'\times2}{I_2} \qquad [\because V_1 = I_3'\times2\Omega]$$

$$= \frac{4}{4+3+2}\times I_2\times\frac{2}{I_2} \qquad \left[\because I_3' = \frac{4}{4+3+2}I_2\right]$$

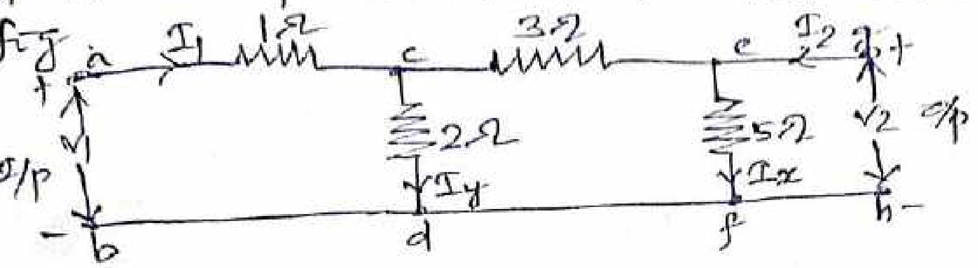
$$= \frac{8}{9} = 0.888\,\Omega$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{[4\,||\,(2+3)]\,I_2}{I_2} = 4\,||\,5 = \frac{4\times5}{4+5} = \frac{20}{9}$$

$$= 2.222\,\Omega \quad \text{(Ans)}$$

Problem 3

Find the Z parameters for the circuit shown in fig



Sol^n

Assuming the o/p being open circuited,

$I_2 = 0$

$R_{equivalent\,1} = [(3+5) \parallel 2] + 1 = (8 \parallel 2) + 1$

$= \frac{8\times 2}{8+2} + 1 = \frac{16}{10} + 1 = \frac{26}{10} = 2.6\,\Omega$

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{I_1 \times R_{eq1}}{I_1} = R_{eq1} = 2.6\,\Omega$

Let current through $5\,\Omega$ resistance $= I_x$

$I_x = \frac{2}{2+3+5} \times I_1 = \frac{I_1}{5}$ A [o/p being open]

$V_2 = \frac{I_1}{5} \times 5 = I_1$ volt

$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{I_1}{I_1} = 1\,\Omega$

Assuming the I/p being open circuited,

$I_1 = 0$

$R_{equivalent\,2} = (2+3) \parallel 5 = \frac{5\times 5}{5+5} = \frac{25}{10} = 2.5\,\Omega$

$V_2 = I_2 R_{eq2} = 2.5\, I_2$ volt.

$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{2.5\, I_2}{I_2} = 2.5\,\Omega$

Let current through $2\,\Omega$ resistance $= I_y$

$I_y = \frac{5}{2+3+5} \times I_2 = \frac{I_2}{2}$ A [I/p being open.

$V_1 = V_{a-b} = V_{c-d} = I_y \times 2 = \frac{I_2}{2} \times 2 = I_2$ volts

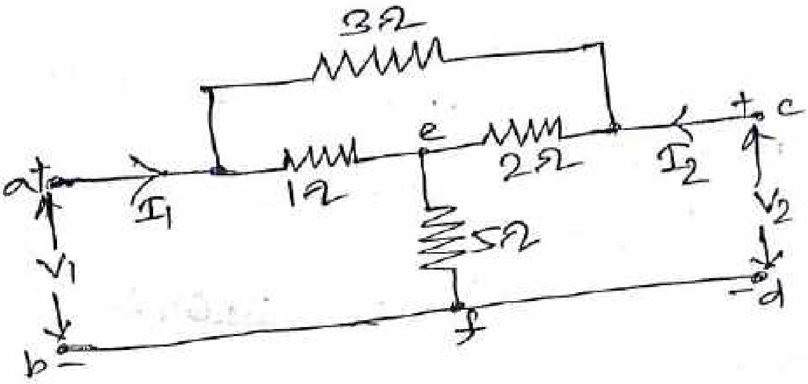
Thus $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{I_2}{I_2} = 1\,\Omega$

$Z_{11} = 2.6\,\Omega$, $Z_{12} = 1\,\Omega$

$Z_{21} = 1\,\Omega$; $Z_{22} = 2.5\,\Omega$ (Ans)

Problem 4

Find out the open circuit parameters and Loop quantities of the network shown in fig:



Sol Assuming the o/p being open-circuited,

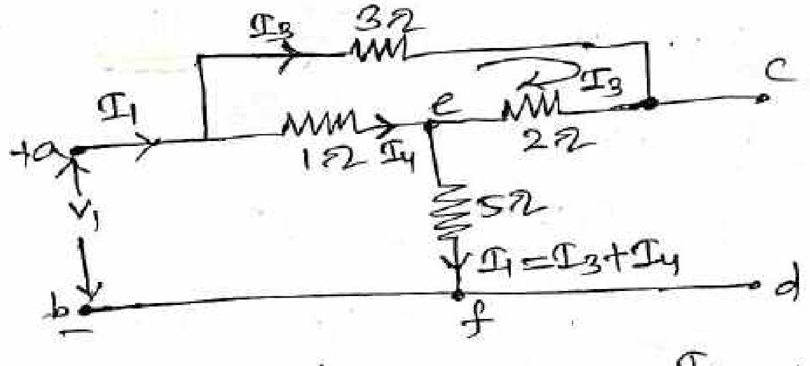
$I_2 = 0$

$R_{eq} = [((3+2) \| 1] + 5 = \frac{5\times1}{5+1} + 5 = \frac{5}{6} + 5$

$= \frac{5+30}{6} = \frac{35}{6}\,\Omega$

$V_1 = I_1 \times R_{eq} = \frac{35}{6} I_1$ volts.

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{\frac{35}{6} I_1}{I_1} = \frac{35}{6} = 5.833\,\Omega$



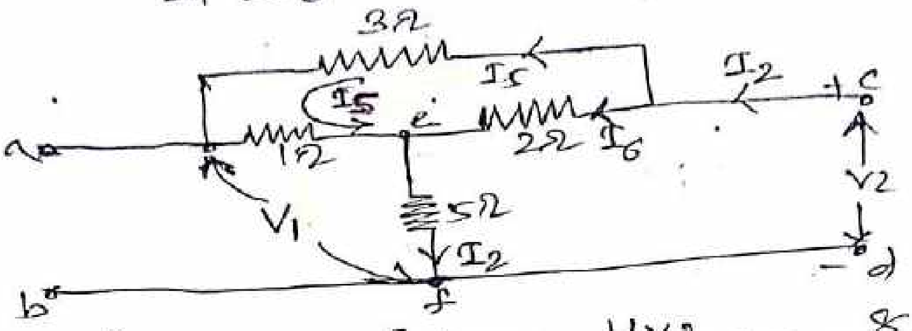
$I_3 = \frac{1}{1+3+2} \times I_1 = \frac{I_1}{6}$ A .

∴ By applying KVL, drop across terminal c-d (V_2) is given by

$$V_2 = I_6 \times 2 + I_1 \times 5$$
$$= \frac{I_1}{6} \times 2 + 5I_1 = \frac{I_1}{3} + 5I_1 = I_1\left(\frac{1}{3}+5\right)$$
$$= \frac{16}{3} I_1 \text{ volts}$$

$$Z_{21} = \frac{V_2}{I_1}\Big]_{I_2=0} = \frac{\frac{16}{3}I_1}{I_1} = \frac{16}{3}\,\Omega = 5.333\,\Omega$$

Assuming the I/p being open circuited,
$$I_1 = 0$$



$$R_{eq2} = \left[(3+1)\,||\,2\right] + 5 = \frac{4\times 2}{4+2} + 5 = \frac{8}{6} + 5$$
$$= \frac{38}{6} = \frac{19}{3}\,\Omega$$

$$V_2 = I_2 \times R_{eq2} = \frac{19}{3} I_2 \text{ volts}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{\frac{19}{3}I_2}{I_2} = \frac{19}{3}\,\Omega = 6.333\,\Omega$$

$$I_5 = \frac{2}{2+3+1} I_2 = \frac{I_2}{3} \text{ A}$$

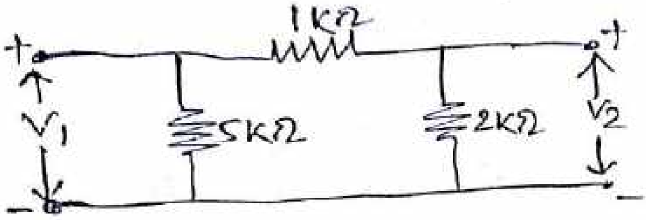
$$V_1 = I_5 \times 1 + 5I_2 = \frac{I_2}{3} \times 1 + 5I_2$$
$$= I_2\left(\frac{1}{3}+5\right) = \frac{16}{3} I_2$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{\frac{16}{3}I_2}{I_2} = \frac{16}{3} = 5.333\,\Omega$$

$$Z_{11} = 5.833\,\Omega, \quad Z_{12} = 5.333\,\Omega$$
$$Z_{21} = 5.333\,\Omega, \quad Z_{22} = 6.333\,\Omega \text{ (Ans)}$$

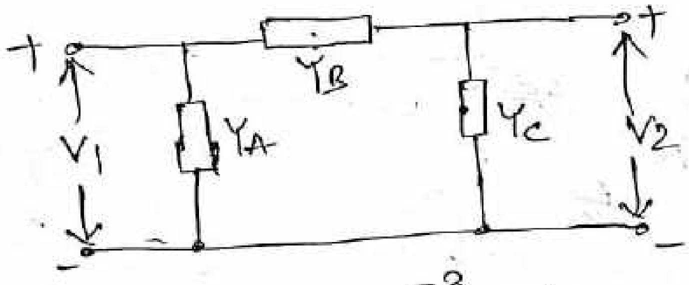
Problem 5 -

Find out the short circuit parameters and draw the equivalent Y parameter circuit.



Sol

The circuit can be redrawn in the form of admittances as



$$Y_A = \frac{1}{5\times10^3} = 0.2\times10^{-3} \text{ mho}$$

$$Y_B = \frac{1}{1\times10^3} = 10^{-3} \text{ mho}$$

$$Y_C = \frac{1}{2\times10^3} = 0.5\times10^{-3} \text{ mho}$$

The Y parameters are -

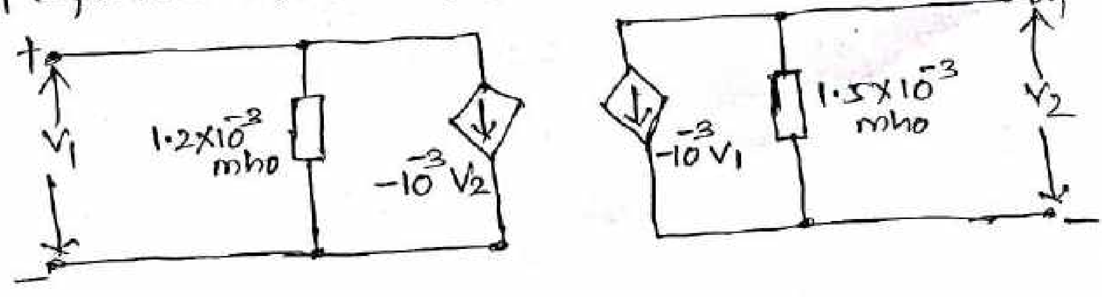
$$Y_{11} = Y_A + Y_B = 1.2\times10^{-3} \text{ mho}$$

$$Y_{12} = -Y_B = -10^{-3} \text{ mho}$$

$$Y_{21} = -Y_B = -10^{-3} \text{ mho}$$

$$Y_{22} = Y_B + Y_C = 1.5\times10^{-3} \text{ mho}$$

The Y-parameter equivalent ckt is as follows:



6.4 hybrid parameter (h-parameter)

h-parameter representation is widely used in modelling of electronic components and circuits, particularly transistors. As both short circuit and open circuit terminal conditions are utilized, this parameter representation is called hybrid parameters.

* In this form of representation, the voltage of the I/p port (V_1) and the current of the o/p port (I_2) can be expressed in terms of the current of I/p port (I_1) and the voltage of the o/p port (V_2).

Hence,
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

or,
$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Assuming the o/p of the two port network to be short circuited, $V_2 = 0$.

$$h_{11} = \frac{V_1}{I_1} \text{ and } h_{21} = \frac{I_2}{I_1}$$

Assuming the I/p of the two port network to be open-circuited, $I_1 = 0$

$$h_{12} = \frac{V_1}{V_2} \text{ and } h_{22} = \frac{I_2}{V_2}$$

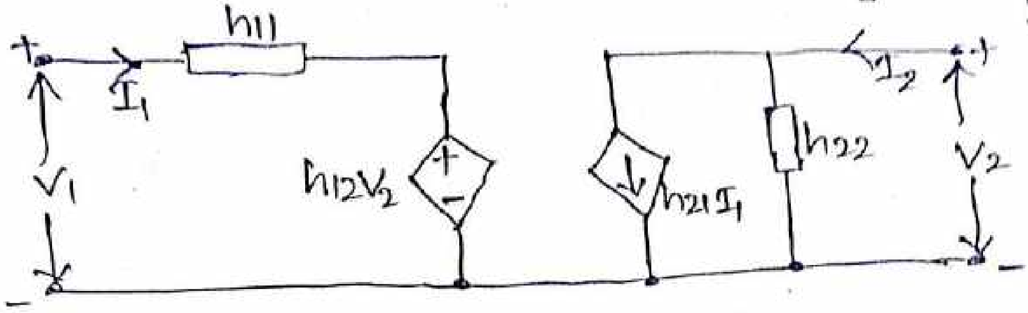
$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ called Input impedance

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ called Reverse voltage gain

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ called Forward current gain

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ called output admittance

Unit of h_{11} is Ω, Unit of h_{22} is mho($\mho$), h_{12} and h_{21} have got no unit.
h_{11}, h_{12}, h_{21} and h_{22} are called hybrid parameters.



(h-parameter equivalent circuit)

$h_{12}V_2$ is the controlled voltage source while $h_{21}I_1$ is the controlled current source.

Z parameters in terms of h-parameters –

In Z parameter representation,

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

In h-parameter representation,

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{--- (1)}$$
$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{--- (2)}$$

From eq^n (2), $V_2 = \dfrac{I_2 - h_{21}I_1}{h_{22}} = \dfrac{1}{h_{22}}I_2 - \dfrac{h_{21}}{h_{22}}I_1$

From eq^n (1), $V_1 = h_{11}I_1 + h_{12}\left(\dfrac{I_2}{h_{22}} - \dfrac{h_{21}}{h_{22}}I_1\right)$

$$= h_{11}I_1 + \frac{h_{12}}{h_{22}}I_2 - \frac{h_{12}h_{21}}{h_{22}}I_1$$
$$= I_1\left(\frac{h_{11}.h_{22} - h_{12}h_{21}}{h_{22}}\right) + \frac{h_{12}}{h_{22}}I_2$$
$$= \frac{\Delta h}{h_{22}}I_1 + \frac{h_{12}}{h_{22}}I_2 \quad [\text{Where, } \Delta h = h_{11}h_{22} - h_{12}h_{21}]$$

Hence, $Z_{11} = \dfrac{\Delta h}{h_{22}}$, $Z_{12} = \dfrac{h_{12}}{h_{22}}$

$Z_{21} = -\dfrac{h_{21}}{h_{22}}$, $Z_{22} = \dfrac{1}{h_{22}}$

h-parameters in terms of Z parameters –

In h parameter representation,

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

In Z parameter representation,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{---- (1)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{---- (2)}$$

From eq^n (2), $I_2 = \dfrac{V_2 - Z_{21} I_1}{Z_{22}} = \left(-\dfrac{Z_{21}}{Z_{22}}\right) I_1 + \left(\dfrac{1}{Z_{22}}\right) V_2$

From eq^n (1), $V_1 = Z_{11} I_1 + Z_{12}\left[-\dfrac{Z_{21}}{Z_{22}} I_1 + \dfrac{V_2}{Z_{22}}\right]$

$$= Z_{11} I_1 - \frac{Z_{12} Z_{21}}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$= I_1 \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}}\right] + \frac{Z_{12}}{Z_{22}} V_2$$

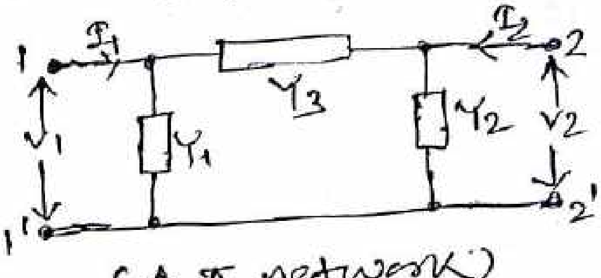
$$= \frac{\Delta Z}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2 \quad \left[\text{where, } \Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}\right]$$

Hence, $h_{11} = \dfrac{\Delta Z}{Z_{22}}$, $h_{12} = \dfrac{Z_{12}}{Z_{22}}$

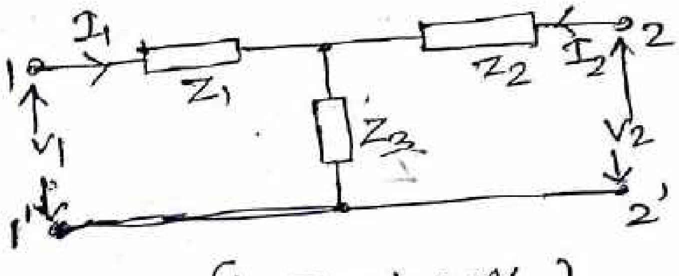
$h_{21} = -\dfrac{Z_{21}}{Z_{22}}$, $h_{22} = \dfrac{1}{Z_{22}}$

6.5 T and π network

These are two networks which are used frequently to represent the equivalence of transmission lines, filter etc. In electric circuits, T is referred as star (Y) connection and π is referred as delta (Δ) connection.



(A π network)



(A T network)

* If Y parameters of a network are known, the equivalence π-network can be constructed.

For π network, o/p port being short circuited

$$I_1 = V_1Y_1 + V_1Y_3 = V_1(Y_1 + Y_3)$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = Y_1 + Y_3$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

Here, $-I_2 = V_1Y_3$

$$\therefore Y_{21} = \frac{-V_1Y_3}{V_1} = -Y_3$$

Assuming I/p port being short circuited

$$I_2 = V_2Y_2 + V_2Y_3 = V_2(Y_2+Y_3)$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = Y_2 + Y_3$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

Here, $-I_1 = V_2Y_3$

$$\therefore Y_{12} = \frac{-V_2Y_3}{V_2} = -Y_3$$

$Y_{11} = Y_1 + Y_3$, $Y_{12} = -Y_3$

$Y_{21} = -Y_3$, $Y_{22} = Y_2 + Y_3$.

Thus, $Y_1 = Y_{11} + Y_{12}$

$Y_2 = Y_{22} + Y_{12}$

$Y_3 = -Y_{12}$

* If Z parameters of a network are known, the equivalence T-network can be constructed.

For a T network, o/p port being open circuited

$$V_1 = I_1(Z_1 + Z_3)$$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = Z_1 + Z_3$$

As $V_2 = I_1Z_3$ when o/p is open

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{I_1Z_3}{I_1} = Z_3$$

Assuming I/p port being open circuited,

$$V_2 = I_2(Z_2+Z_3)$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = \frac{I_2(Z_2+Z_3)}{I_2} = Z_2+Z_3$$

As, $V_1 = I_2 Z_3$ when o/p is open,

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{I_2 Z_3}{I_2} = Z_3$$

$Z_{11} = Z_1+Z_3$, $Z_{12} = Z_3$

$Z_{21} = Z_3$, $Z_{22} = Z_2+Z_3$

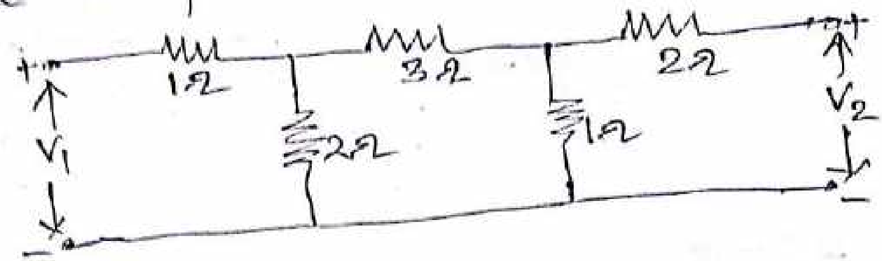
Thus, $Z_1 = Z_{11} - Z_{12}$

$Z_2 = Z_{22} - Z_{12}$

$Z_3 = Z_{12}$

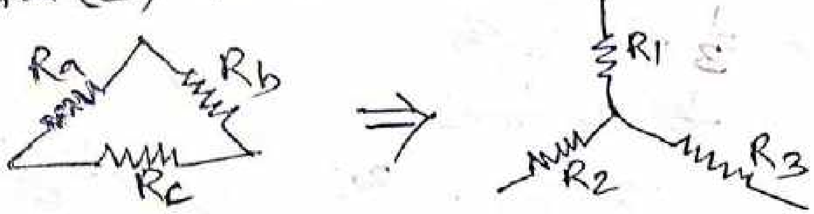
<u>Problem 5</u>

Find out the Z-parameters of the following network.



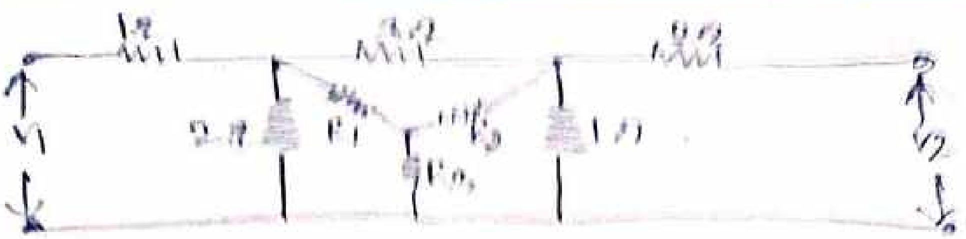
<u>Sol</u>^{n}

By converting from π to T network or delta (Δ) to star (Y) connection



$$R_1 = \frac{R_a R_b}{R_a+R_b+R_c} , \quad R_2 = \frac{R_c R_a}{R_a+R_b+R_c}$$

$$R_3 = \frac{R_b R_c}{R_a+R_b+R_c}$$

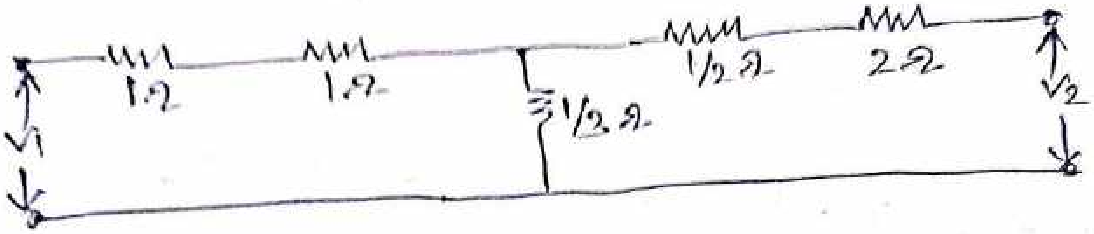


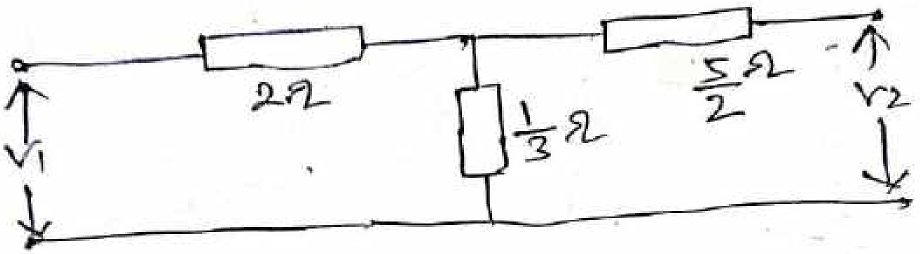
(Delta to star connection)

$$R_1 = \frac{2\times 3}{2+3+1} = \frac{6}{6} = 1\,\Omega$$

$$R_2 = \frac{3\times 1}{2+3+1} = \frac{3}{6} = \frac{1}{2}\,\Omega$$

$$R_3 = \frac{2\times 1}{2+3+1} = \frac{2}{6} = \frac{1}{3}\,\Omega$$





$$Z_{11} = Z_1 + Z_3 = 2 + \frac{1}{3} = \frac{7}{3}\,\Omega$$

$$Z_{12} = Z_3 = \frac{1}{3}\,\Omega$$

$$Z_{21} = Z_3 = \frac{1}{3}\,\Omega$$

$$Z_{22} = Z_2 + Z_3 = \frac{5}{2} + \frac{1}{3} = \frac{15+2}{6} = \frac{17}{6}\,\Omega \quad \text{(Ans)}$$

Port parameters conversion table

	[Z]	[Y]	[h]
[Z]	$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
[Y]	$\begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$	$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\Delta h = h_{11} h_{22} - h_{12} h_{21}$$

CHAPTER 7

FILTERS & ATTENUATORS

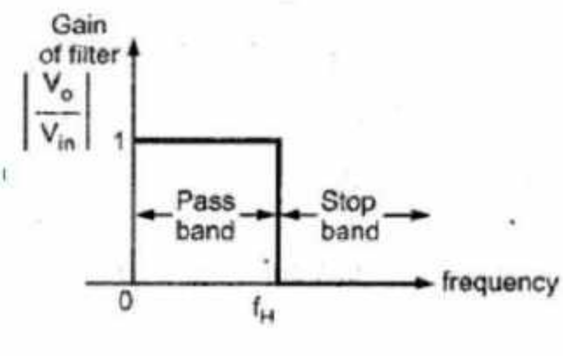
FILTERS

Filters are essential building blocks of any electronic and communication systems that alter the amplitude and/or phase characteristics of a signal with respect to frequency. Filter is basically linear circuit that helps to remove unwanted components such as noise, interference and distortion from the input signal.

In simple words, you can understand it as the circuit rejects certain band of frequencies and allows others to pass through.

7.1 Ideal & Practical filters and its applications, cut off frequency, pass band and stop band.

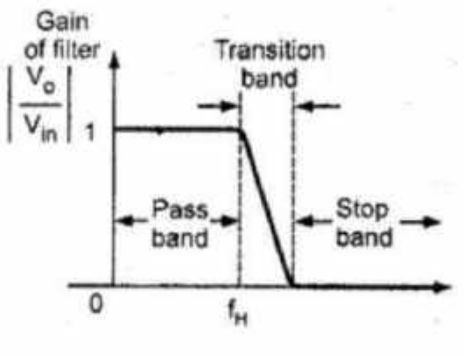
An ideal filter is the filter which would transmit signals under pass band frequencies without attenuation and completely suppress the signal with attenuation band of frequencies with a sharp cut off profile. Ideal low-pass filters should have a steep transition band and excellent gain flatness in the pass band.



Ideal Filter Characteristics

No filter can entirely get rid of all the unwanted frequencies.

Practical filter are the filters which do not ideally transmit the pass band signal unattenuated due to absorption, reflection or due to other loss. This results in loss of signal power. These filters do not completely suppress the signal in attenuation bands.



Practical Filter Characteristics

The response of some commonly used practical filters are shown in the colored lines. If the pass band gain is not flat or exhibits ripples, this response may scale the fundamental signal.

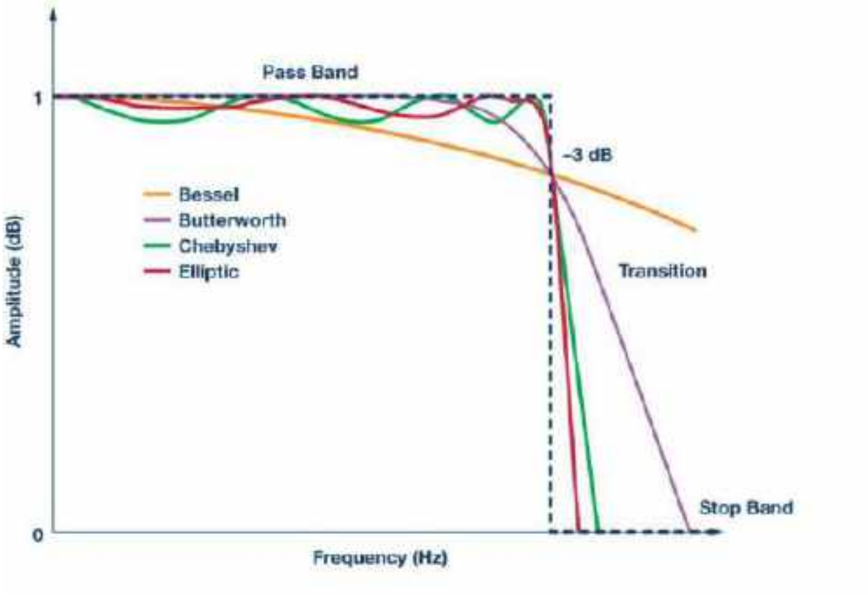


Figure 2. Ideal filter vs. practical filters amplitude response.

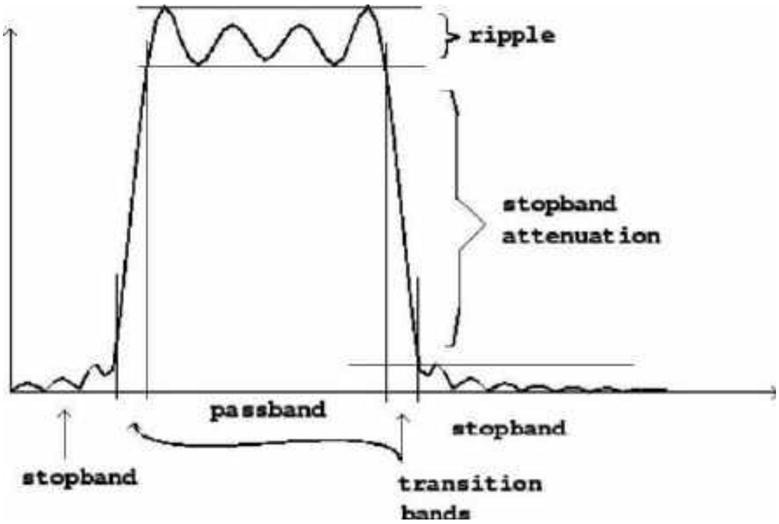
APPLICATIONS:-

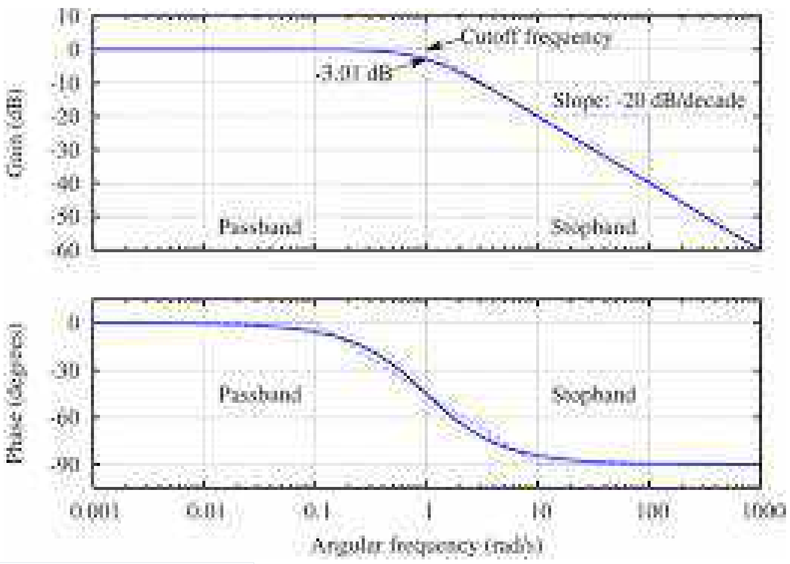
- Radio communications :Filters enable radio receivers to only "see" the desired signal while rejecting all other signals (assuming that the other signals have different frequency content).
- DC power supplies: Filters are used to eliminate undesired high frequencies (i.e., noise) that are present on AC input lines. Additionally, filters are used on a power supply's output to reduce ripple.
- Audio electronics: A crossover network is a network of filters used to channel low-frequency audio to woofers, mid-range frequencies to mid range speakers, and high-frequency sounds to tweeters.
- Analog to digital conversion: Filters are placed in front of an ADC input to minimize aliasing.

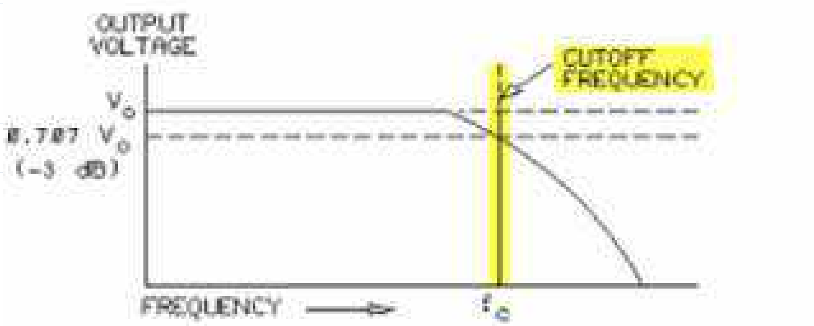
PASS BAND, STOP BAND & CUTOFFFREQUENCY

A filter is an electrical network that can transmit signals within a specified frequency range. This frequency range is called **pass band** and other frequency band where the signals are suppressed is called **attenuation band or stop band**. The frequency that separates the pass and attenuation bands is known as **cut-off frequency**. There may also be two cut off frequencies in the entire zone of operation of the filter (i.e. upper cut off frequency & lower cut off frequency)

Cut off frequency (also known as corner frequency, or break frequency) is defined as a boundary in a system's frequency response at which energy flowing through the system begins to be attenuated (reflected or reduced) rather than passing through. Most frequently this proportion is one-half the pass band power, also referred to as the 3 dB point since a fall of 3 dB corresponds approximately to half power. As a voltage ratio, this is a fall to approximately 0.707.







7.2 Classify filters- low pass, high pass, band pass, band stop filters & study their Characteristics.

Active Filters:

Filter Circuit which consists of active components like Transistors and Op-amps in addition to Resistors and Capacitors is called as ***Active Filter***.

Passive Filters:
Filter circuit which consists of passive components such as Resistors, Capacitors and Inductors is called as ***Passive Filter***.

The filter can be further categorized based on the operating frequency of a particular circuit. They are:

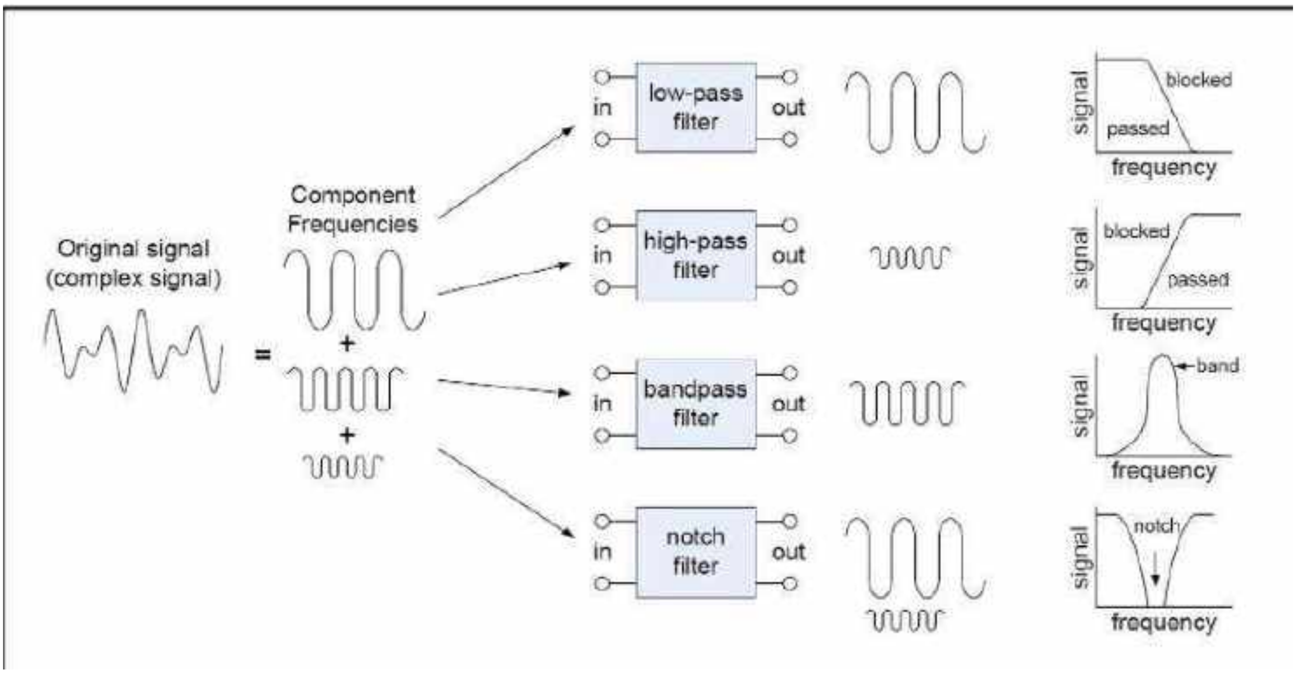
- Low Pass Filter
- High Pass Filter
- Band Pass Filter
- Band Stop or Band Reject or Notch Filter
- All Pass Filter

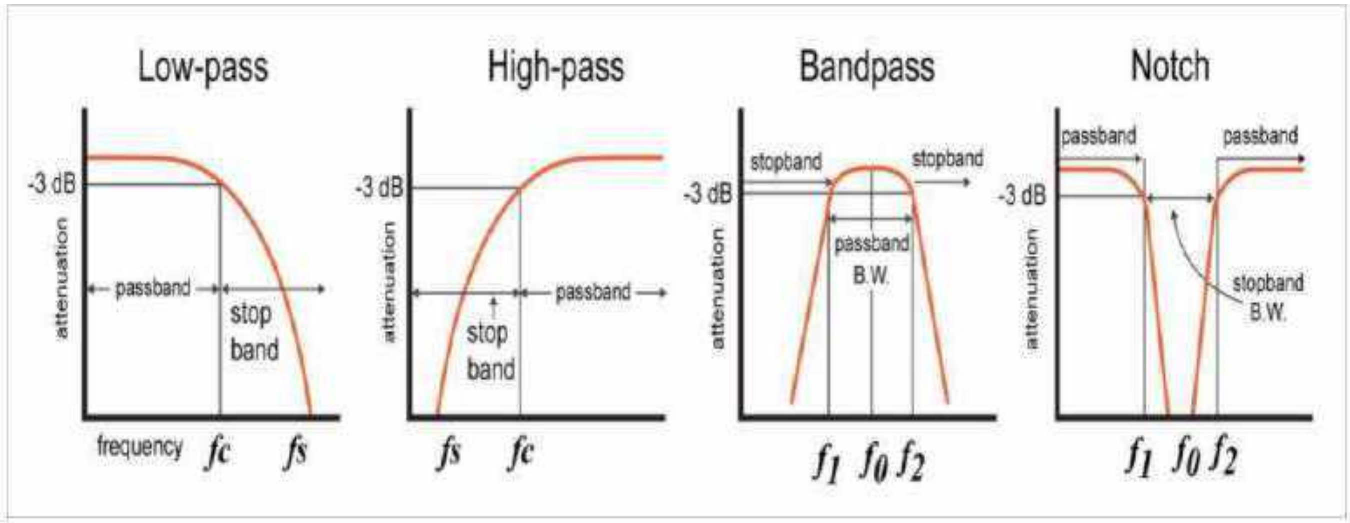
Low Pass Filters:
It is a type of Filter which attenuates all the frequencies above the cut-off frequencies. It provides a constant output (gain) from zero to cut-off frequency.

High Pass Filters:
It is a type of Filter which attenuates all the frequencies below the cut-off frequencies. It provides a constant output (gain) above the cut-off frequency.

Band Pass Filters:
It is a type of filter which allows specific band of frequencies to pass through and all other frequencies outside the band are attenuated.

Band Stop Filters:
It is a type of filter which rejects specific band of frequencies and allows passing of frequencies outside the band.

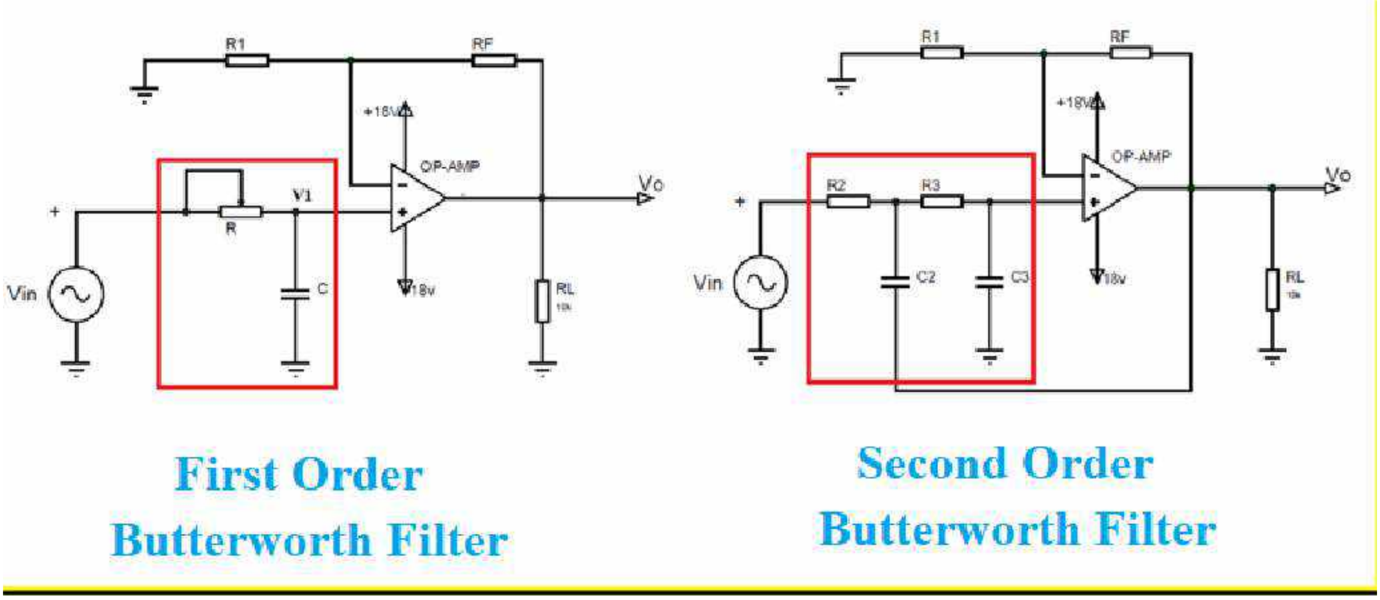


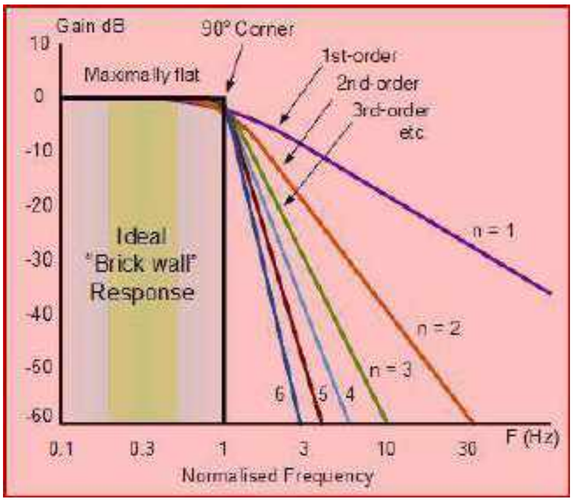


7.3 Butterworth Filter Design

Butterworth Active Filter

The Butterworth active filter is also named as flat filter. The implementation of the Butterworth active filter guarantees a flat response in the pass bandandanampleroll-off.Thisgroupoffiltersapproximatestheperfectfilterfitinthe pass band. Frequency response curves of different kinds of filters are shown. This filter includes essentially flat amplitude, frequency response up-to the cut-off frequency.





7.4 Attenuation and Gain, Bel, Decibel & Neper and their relations

Attenuation: It is the process that refers to any reduction in the strength of a signal. Attenuation occurs with any type of signal, whether digital or analog. Sometimes called *loss*, attenuation is a natural consequence of signal transmission over long distances. Attenuation is expressed either in decibels (dB) or in nepers. Attenuators weaken or attenuate the strong signal.

Attenuation (dB)= 10 X log(P_I/P_O)

Where P_I is input power and P_O is the output power. P_I is the power applied at one end of the cable, while P_O is the wattage at the end of the cable.

Gain: It is a measure of the ability of a two-port circuit (often an amplifier) to increase the power or amplitude of a signal from the input to the output port by adding energy converted from some power supply to the signal. It is the ratio of output signal power to the input signal power of an amplifier. Gain is expressed in db.

Gain (dB)= 10 X log(P_O/Pi)

Where P_I is input power and P_O is the output power.

Bel:

The bel is a unit for comparing two power levels, equal to the logarithm to the base ten of the ratio of the two powers.

$N_{dB}=\log_{10}(P_2/P_1)$

One bel = 10 decibels (dB)

Decibels:

The decibel is the base-10 logarithm ratio used to express an increase or decrease in power, voltage, or current in a circuit.

The deciBel formula or equation for power is given below:

$N_{dB}=10\log_{10}(P_2/P_1)$

Where:

N_{db} is the ratio of the two power expressed in deciBels, dB

P_2 is the output power level

P_1 is the input power level or reference power level

If the value of P_2 is greater than P_1, then the result is given as a gain, and expressed as a positive value, e.g. +10dB. Where there is a loss, the deciBel equation will return a negative value, e.g. -15dB. In this way a positive number of deciBels implies a gain, and where there is a negative sign, it implies a loss.

Although the deciBel is used primarily as comparison of power levels, deciBel current equations or deciBel voltage equations may also be used provided that the impedance levels are the same. In this way the voltage or current ratio can be related to the power level ratio.

$N_{dB}=10\log_{10}(V_2^2/V_1^2)$

Or $N_{dB}=20\log_{10}(V_2/V_1)$

Where:

N_{db} is the ratio of the two powers expressed in deciBels, dB
V_2 is the output voltage level
V_1 is the input voltage level

Neper:
The neper is a logarithmic scale based on natural logarithms to base e. Although decibels are the main unit for expressing the ratios of gain and loss for quantities like electrical power in electronic and other systems, the neper is also used.

One neper Np = 20 / (ln10) = 8.685889638 dB.

Attenuation in dB =8.686 / attenuation in nepers.

one neper = 8.686 dB

7.5 Attenuators & its applications. Classification-T-Type & PI–Type attenuators

Attenuators:

Attenuators are simple active or passive two-port electronic devices used for attenuation or to reduce the strength of signals without causing disturbance to its waveform**.** The passive type is often just a resistor divider but could also be followed by a buffer (a type of op amp). The active type of attenuator can be an inverting op amp attenuator or fully differential op amps.

They are used at the input of an electronic instrument to minimize a voltage or current to a value that can be handled by the device. An Attenuator is well the opposite of an amplifier, though the two work by different methods. In an amplifier, the gain is usually much higher than 1, while an attenuator provides loss or gain less than 1.

Applications:

- To control the volume of electronic equipment such as speakers, electric guitars, etc.
- In some digital or analog circuits, a surge in voltage may damage the courses, which can be avoided using attenuators to reduce high voltages.
- In microwave communication, radar, sonar etc.
- Used in medicine, physics, acoustics, fiber optics, nuclear power, materials science, biology, seismology, radiology, and many other disciplines.

Classification of Attenuators:

The attenuator should match source and load impedances in addition to the desired attenuation. There are commonly used attenuators: T attenuators and Pi attenuators. Besides T- and Pi attenuators, other types of fixed passive attenuators are laid out in L, H, and O configurations, too. Other attenuator types include continuously variable, programmable, DC passing, DC blocking, waveguide, and optical attenuators.

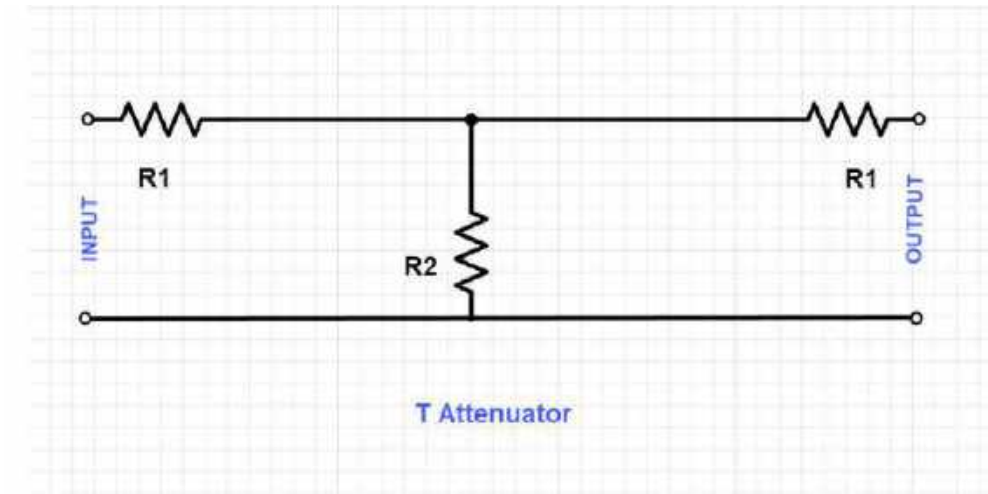


Figure 2: T-attenuator, a passive resistor divider circuit, looks like a “T”.

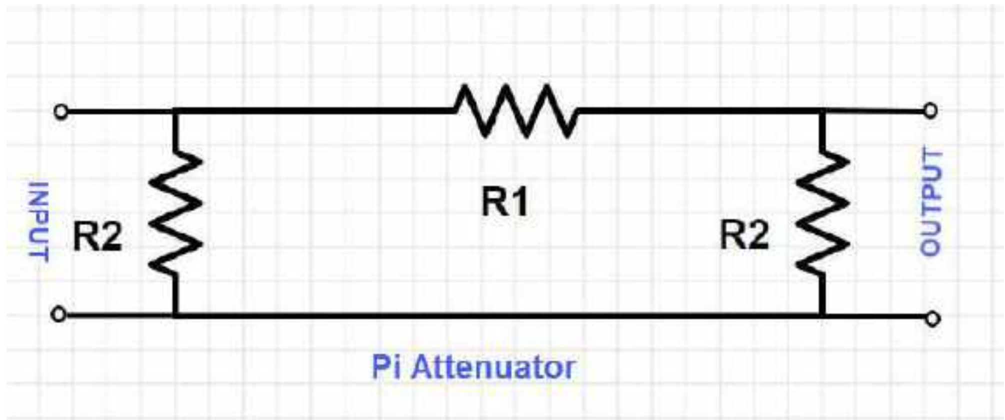


Figure 3: The Pi attenuator is also a passive resistor divider circuit. It looks like the Greek letter “Π”.