INDEX

NAME:________ STD.: ________ SEC.: ________ ROLL NO.: ________ SUB.: ________

S. No.	Date	Title	Page No.	Teacher's Sign / Remarks
		Sidharth Sekhar Mallick (Lect. AE&I)		
		Sub- Digital Signal processing		
		Sem - 6th		

Digital Signal processing

Syllabus

1. Introduction.
   - 1.1 Discuss Signals, Systems & Signal processing.
     - 1.1.1 Explain basic element of a digital signal processing System.
     - 1.1.2 compare the advantages of digital Signal processing over analog Signal processing. ~~classify Signals.~~
   - 1.2 classify Signals
     - 1.2.1 multi channel & Multi dimensional Signals.
     - 1.2.2 continuous time Verses Discrete - time signals.
     - 1.2.3 continuous valued Verses Discrete - valued Signals.
   - 1.3 Discuss the concept of frequency in continuous time & discrete time Signals.
     - 1.3.1 continuous - time sinusoidal Signals
     - 1.3.2 Discrete - time sinusoidal Signals
     - 1.3.3 Harmonically related complex exponential.
   - 1.4 Discuss Analog to Digital & Digital to Analog conversion & explain the following.
     - 1.4.1 Sampling of Analog Signal.
     - 1.4.2 The Sampling theorem.
     - 1.4.3 Quantization of continuous amplitude signals.
     - 1.4.4 coding of quantized sample.
     - 1.4.5 Digital to analog conversion.
     - 1.4.6 Analysis of digital Systems Signals Vs discrete time Signals.
2. Discrete Time Signals & System.
   - 2.1 State and explain discrete time Signals.
     - 2.1.1 Discuss some elementary discrete time signals
     - 2.1.2 classify discrete time Signal.
     - 2.1.3 Discuss simple manipulation of discrete time Signal.
   - 2.2 Discuss discrete time System.
     - 2.2.1 Describe input - output System.
     - 2.2.2 Draw block diagram of discrete time ~~Signal~~ System.
     - 2.2.3 classify discrete time System
     - 2.2.4 Discuss inter connection of discrete - time System.
   - 2.3 Discuss discrete time time - invariant System.
     - 2.3.1 Discuss different technique for the analysis of linear System.
     - 2.3.2 Discuss the resolution of a discrete time Signal in to impulse.
     - 2.3.3 Discuss the response of LTI System to arbitary I/Ps using convolution theorem.

2.3.4 Explain the properties of convolution & interconnection of LTI system.

2.3.5 Study system with finite duration and infinite duration impulse response.

2.4 Discuss discrete time system described by difference system.

2.4.1 Explain recursive & non-recursive discrete time system.

2.4.2 Determine the impulse response of linear time invariant recursive system.

3. The Z-transform & it's application to the analysis of LTI system.

3.1 Discuss Z-transform & it's application to LTI system.

3.2.1 State & explain direct Z-transform.

3.1.2 State & explain inverse Z-transform

3.2 Discuss various properties of Z-transform.

3.3 Discuss rational Z-transform.

3.3.1 Explain poles & zeros.

3.3.2 Determine pole location time domain behaviour for casual signals.

3.3.3 Describe the system function of a linear time invariant system.

3.4 Discuss inverse-Z transform.

3.4.1 Determine inverse Z-transform by partial fraction expansion.

4. Discuss fourier transform: it's application properties.

4.1 Discuss discrete fourier transform.

4.2 Determine frequency domain sampling & reconstruction of discrete time signals.

4.3 State & explain Discrete time fourier transform (DTFT)

4.4 State & explain Discrete fourier transform (DFT)

4.5 compute DFT as a linear transformation.

4.6 Relate DFT to other transform.

4.7 Discuss the property of the DFT.

4.8 Explain multiplication of two DFT & circular convolution.

5. Fast fourier Transform Algorithm & Digital filters.

5.1 Compute DFT & FFT algorithm.

5.3 Discuss the radix-2 algorithm. (Small problems)

5.2 Explain direct computation of DFT.

5.4 Introduction to digital filters (FIR filters)

5.5 Introduction to DSP architecture, familarsation of different types of processor.

Text book

1. Digital Signal processing principles algorithm & application by J. G. Proakis & Dimities G. Monolakis, Pearson.
2. Digital Signal processing by Ramesh babu

Chapter - 1

1. Introduction

1.1 a) Signal

A Signal is defind as any physical quantity varies with time, space or any other independent variable and variables.

mathematically, we describe a signal as a function of one or more independent variables. for example the function

$S_1(t) = 5t$

$S_2(t) = 20t^2$

t = independent variable of two signal.

Ex- two independent variable x & y.

$S(x, y) = 3x + 2xy + 10y^2$

b) System :- ~~Apply~~ it is defind as physical device that perform an operation on a signal. ~~for~~

Ex- a filter used to reduce the noise & interference corrupting a desired information.

c) Signal processing

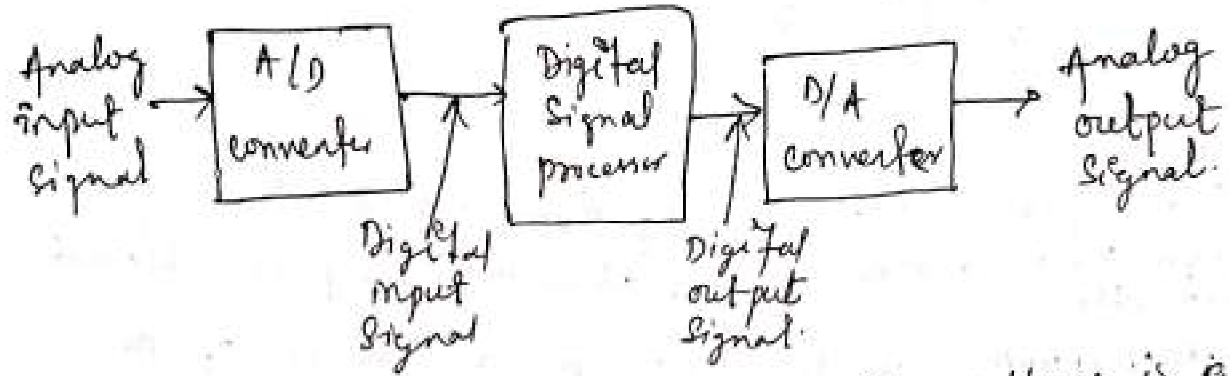
When we pass a signal through a system as in filtering we say that we have processed the signal.

1.1.1 Explain basic elements of a digital signal processing system.

→ most of the signals encountered in science & engineering are analog in nature. This is the signal are functions of a contineous variable, such as time or space, & usually take on values in a contineous range.

Such signals may be processed directly by appropriate analog

Digital Signal processing provides an alternative method for processing the analog signal.



→ To perform the processing digitally, there is a need for an interface betn the analog signal and the digital processor. This interface is called an analog-to-digital (A/D) converter.

→ The digital signal processor may be a large programmable digital computer or a small microprocessor programmed to perform the desired operation on the input signal.

→ where the digital output from the digital signal processor is to be given to the user in analog form ~~the~~ such as in speech communication, we must provide another interface from the digital domain to the analog domain. It is called digital to analog (D/A) converter.

1.1.2 Compare the advantage of digital signal processing over analog signal processing.

→ Digital programmable system allows flexibility in reconfiguring the digital signal processing operation simple by changing the program.

→ Reconfiguration of an analog system usually implies a redesign of the hardware followed by testing & verification to see that it operates properly.

→ Accuracy considerations also play imortant role in determining the form of the signal processor.

→ Tolerance in analog circuit components makes extremely difficult for the system designer to control accuracy of an analog signal processing system.

→ Digital provide much better accuracy of an requirement.

→ Digital signal are easily stored on magnetic media (tape or disc) without deterioration or loss of signal fidelity beyond the A/D conversion.

→ Digital signal processing method also allows for the implementation of more sophisticated signal processing algorithm.

→ Digital signal processing is cheaper than analog.

→ Speed of Digital signal processing higher than analog.

→ ~~Fast response~~ Digital signal processing fast response with large bandwidth.

1.2 Classification of Signals

1.2.1 multichannel & multidimensional signals

→ A signal is described by a function of one or more independent variable. the value of the function can real - valued scalar quantity, a complex valued quantity, or perhaps a vector.

→ vector $S_3(t)$

$$S_3(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \\ S_3(t) \end{bmatrix}$$

we refer to such a vector of signals as a multi channel signal. ECG have 3-leads means 3-channel signal & ECG have 12 - leads mean 12 channel. that is called multichannel.

↳ Electrocardiography

→ Multidimensional → if the signal is a function of a single independent variable, the signal is called a one - dimensional signal, if is M no. of independent variable is called M- dimensional signal.

$$I(x, y, t)] = \begin{bmatrix} I_r(x, y, t) \\ I_g(x, y, t) \\ I_b(x, y, t) \end{bmatrix}$$

This is function of color TV·picture.
Channel are (red, green, blue)
Dimension (x, y, t)

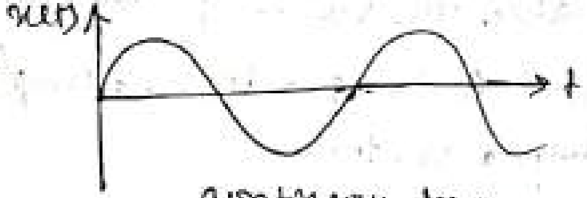
1.2.2 Continuous - Time Versus Discrete-Time Signals

Continuous-time Signals

The Signals that are defined for every instant of time are known as continuous-time signal. They are denoted by x(t).

Discrete-time Signals

The Signals that are defined at discrete instants of time are known as discrete-time Signals. The discrete-time Signals are continuous in amplitude and discrete in time. They are denoted by x(n) & define at certain specific values of time.

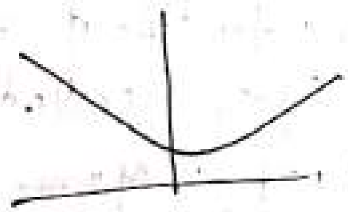


continuous time

1.2.3 Continuous-valued Versus Discrete valued Signals

→ if a Signal takes on all possible values on a finite or an infinite range, it is said to be a continuous-valued Signals.

→ if the Signal takes on values from a finite set of possible value it is said to be a discrete valued Signal.



continuous value

Discrete value

Deterministic Versus Random Signal

Deterministic Signal:- A Signal ~~is~~ exhibiting no uncertainty of value at any instant of time. if instaneous value can be accurately predicted by mathematical equation. One such Signal

$$x_1(n) = \sin(0.1\pi n)$$

Random Signal :-

A Random Signal is a Signal characterized by the uncertainty before it's actual occurance. Ex Noise.

1.3 The concept of frequency in continuous-time & Discrete-Time Signal

1.3.1 Continuous-Time Sinusoidal Signals

A simple harmonic oscillation is mathematically described by the following continous time Sinusoidal Signal.

$$x_a(t) = A\cos(\omega t+\theta), \quad -\infty < t < \infty$$

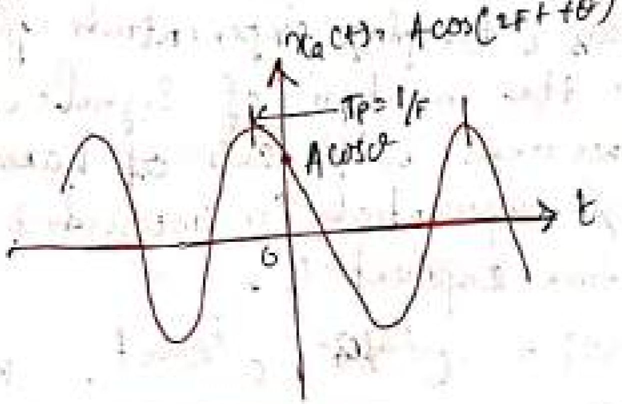
A = amplitude.

ω = frequency radian per second.

θ = phase radian.

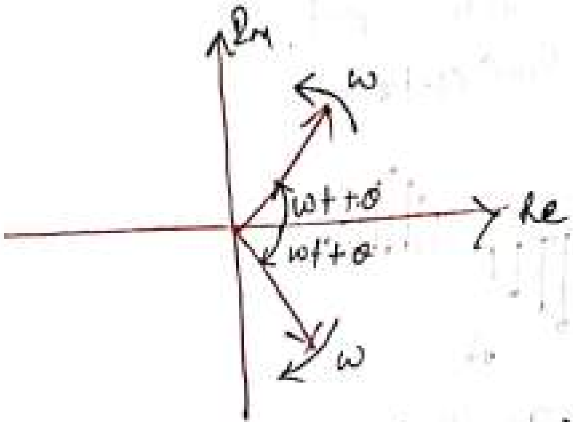
$\omega = 2\pi F$

$$x_a(t) = A\cos(2\pi F t + \theta), \quad -\infty < t < \infty$$



T_p = Fundamenta time periode.

$F = \frac{1}{T_p}$ $\boxed{T_p = \frac{2\pi}{\omega}}$



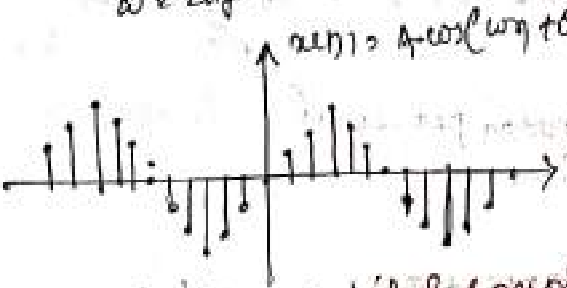
A positive frequency corresponds to counter clock wise uniform angular motion, a negative frequency simply correspond to clock wise angular motion.

1.3.2 Discrete-Time Sinusoidal Signals

A discrete-time sinusoidal signal may be expressed as

$$x(n) = A\cos(\omega n + \theta), \quad -\infty < n < \infty$$

A = amplitude
ω = frequency rad/sec
θ = phase.
$\omega = 2\pi f$



1.3.3 Harmonically Related complex exponentials

→ Sinusoidal signals & complex exponentials play a major role in the analysis of signals and system. In some cases we deal with sets of harmonically related complex exponentials (or sinusoids).

continuous-time exponential –

$$s_k(t) = e^{jk\omega_0 t}. \quad k = 0, \pm 1, \pm 2$$

Discrete - time exponential

$$S_k(n) = e^{jk\pi f_0 n} \quad k = 0, \pm 1, \pm 2$$

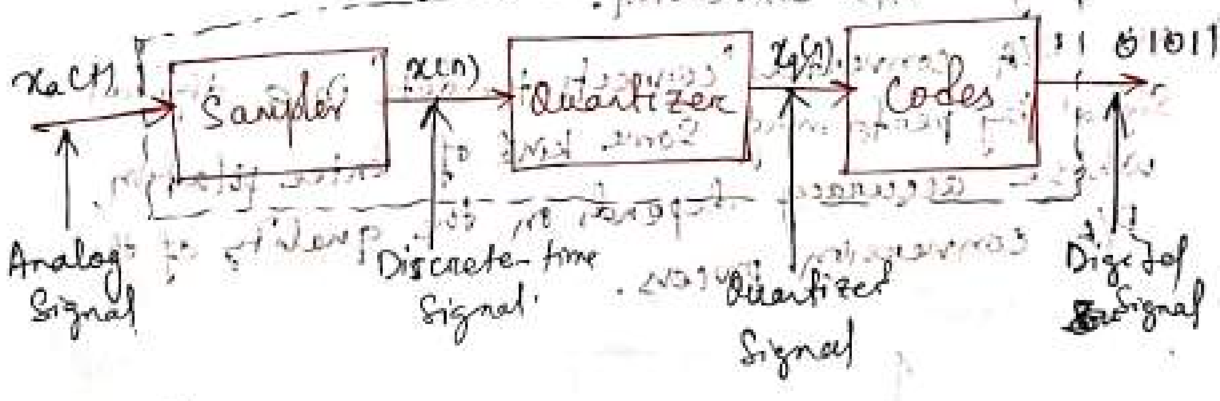
1.4 Analog - to - Digital & Digital - to Analog Conversion

→ Most signals of practical interest, such as speech, biological signals, seismic signals, radar signals, sonar signals & various communication signals such as audio & video signals are analog

→ To process analog signals by digital means, it is first necessary to convert them in to digital form; that is to convert them to a sequence of number having finite precision.

→ This procedure is called analog to digital (A/D) conversion & the corresponding devices are called A/D converters (ADCs).

→ We view A/D conversion as a three step process



1. Sampling → This is the conversion of a continuous time signal in to a discrete signal obtained by taking "samples" of the continuous - time signal at discrete time instants. Thus if $x_a(t)$ is the input to the sampler, the output is $x_a(nT) \equiv x(n)$ where T is called the sampling interval.

2. Quantization :- This is the conversion of a discrete-time continuous - valued signal in to a discrete - time, discrete valued (digital) signal.

→ The value of each signal sample is represented by a value selected from a finite set of possible value. The difference betn the quantized sample x(n) & the quantized output xq(n) is called the quantization error..

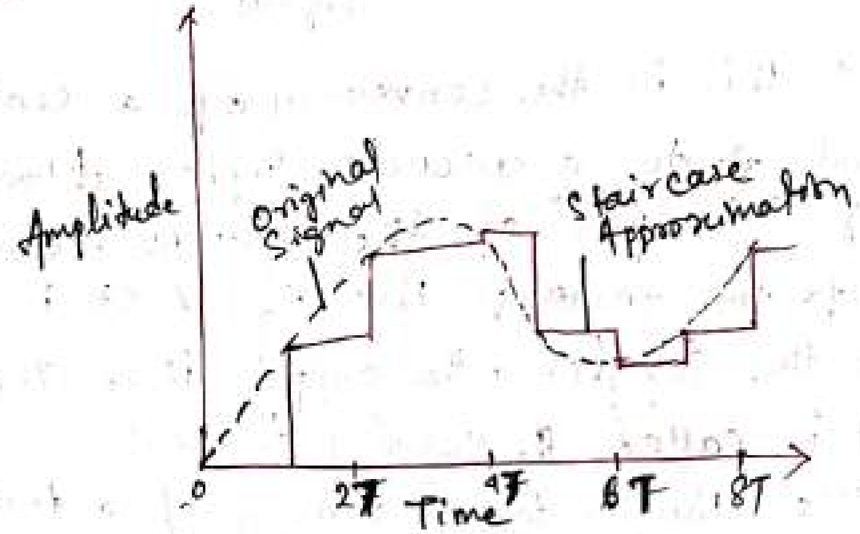
3. coding :- In the coding process each discrete value xq(n) is represented by a b-bit binary sequence.

→ Although we model the A/D converter as a sampler followed by a quantizer and coder, in practice the A/D conversion is performed by a single device that takes xa(t) & produces a binary_coded number.

→ the operation of sampling & quantization can be performed in either order but in practice, sampling is always performed before quantization.

→ The process of converting a digital signal into an analog signal is known as digital to analog (D/A) conversion.

→ All D/A converters "connects the dots" in a digital signal by performing some kind of interpolation, whose accuracy depends on the quality of the D/A conversion process.



→ A Simple form of D/A conversion is called a Zero-order hold or a staircase approximation.

→ Sampling and quantization are treated as we demonstrate that sampling doesnot result in a loss of information nor does it introduce distortion in the signal if the signal bandwidth is finite.

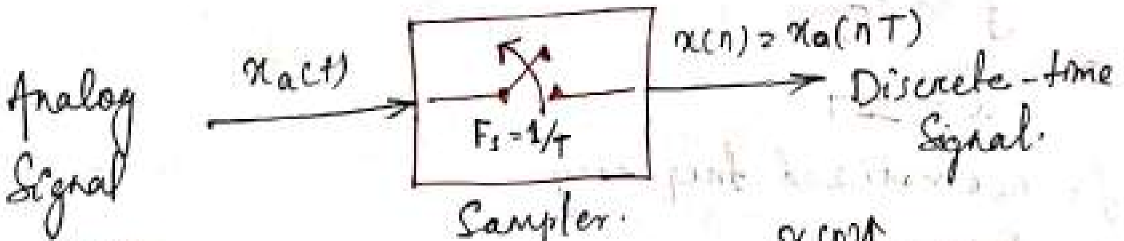
→ In principle, the analog signal can be reconstructed from the samples provided that the sampling rate is sufficiently high to avoid the problem commonly called aliasing.

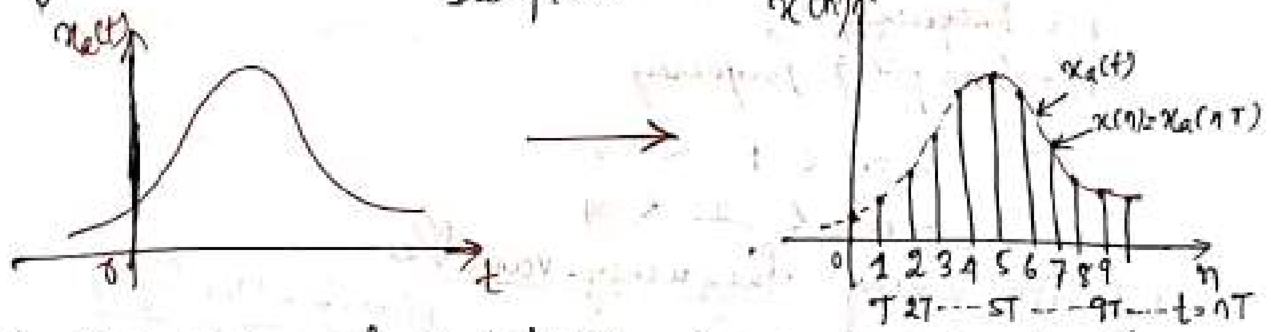
1.4.1 Sampling of Analog Signals

→ There are many ways to Sample an analog Signal. we limit our discussion to periodic or uniform Sampling,

$$x(n) = x_a(nT), \quad -\infty < n < \infty$$

When $x(n)$ is discrete-time Signal by taking of analog Signal $x_a(t)$ every T Seconds.





→ The time interval T between successive samples is called the Sampling period or Sample interval & it's reciprocal $1/T = F_s$ is called the sampling rate (Samples per Second) or the Sampling frequency (HZ).

→ Periodic Sampling establishes a relationship betn the time variables t and n of continuous-time and discrete-time signals respectively. these variables are linearly related through the Sampling period T or, equivalently through the Sampling rate $F_s = 1/T$

$$t = nT = \frac{n}{F_s}$$

there exists a relationship betn the frequency variable F (or Ω) for analog signals & the frequency variable f (or ω) for discrete-time signals.

$$x_a(t) = A\cos(2\pi Ft+\theta) \quad -①$$

which when sampled periodically at a rate $F_s = 1/T$ samples per second.

$$x_a(nT) \equiv x(n) \equiv A\cos(2\pi FnT+\theta) \quad -②$$

$$= A\cos\left(\frac{2\pi nF}{F_s}+\theta\right)$$

If we compare 2 & $x(n) = A\cos(2\pi fn+\theta)$, $-\infty < n < \infty$

$$f = \frac{F}{F_s}$$

or

$$\omega = \Omega T$$

f = normalized frequency

F = frequency

F_s = Sampling frequency

$$-\infty < F < \infty$$

$$-\infty < \Omega < \infty$$

Relation among frequency variable

continuous-time signal

$\Omega = 2\pi F$

$\frac{\text{radians}}{\text{sec}}$ Hz

$\xrightarrow{\omega = \Omega T,\ f = F/F_s}$

Discrete-time signal

$\omega = 2\pi f$

$\frac{\text{radians}}{\text{sample}}$: $\frac{\text{cycle}}{\text{sample}}$

$-\pi \le \omega \le \pi$

$-1/2 \le f \le 1/2$

$-\infty < \Omega < \infty$ $\xleftarrow{}$ $\Omega = \omega/T$, $F = f \cdot F_s$ $\quad -\pi/T \leq \Omega \leq \pi/T$

$-\infty < F < \infty$ $\quad -F_s/2 \leq F \leq F_s/2$

Discrete-time signal is $\omega = \pi$ or $f = 1/2$ with a sampling rate F_s the corresponding highest values of F & Ω

$$\boxed{\begin{aligned} F_{max} &= \frac{F_s}{2} = \frac{1}{2T} \\ \Omega_{max} &= \pi F_s = \frac{\pi}{T} \end{aligned}}$$

Ex analog signal

$$x_a(t) = 3\cos 100\pi t$$

a) Determine the min^m sampling rate required to avoid aliasing.

b) Suppose that the signal is sampled at the rate $F_s = 200Hz$ what is the ~~dec~~ discrete-time signal obtained after sampling?

Ans

a) analog frequency $F = 50Hz$.

min^m sampling rate freq^n $= F_{min} = \frac{F_s}{2}$

$= F_s = 2 \times 50 = 100Hz$

b) Signal is sampled at $F_s = 200Hz$

$x(n) = 3\cos\frac{100\pi}{200} = 3\cos\frac{\pi}{2}n$

1.4.2 The Sampling Theorem

~~Let $x_a(t) = \sum_{i=1}^{N} A_i \cos(2\pi F_i t + \theta_i)$~~

A continuous time signal can be completely represented in it's samples & recovered back, if the sampling frequency f_s is greater than or equal to the twice of the highest frequency component of the message signal (f_m) & $F_{max} = 3000Hz$ for the class of speech signals &

$F_{max} = 5MHz$ for television signal.

from our knowledge of F_{max} we can select the appropriate sampling rate $F_s = 1/T$ is $F_s/2$.

any frequency above $F_s/2$ or below $-F_s/2$ results in samples that are identical with a corresponding. Any frequency range $\boxed{-F_s/2 \leq F \leq F_s/2}$, to avoid corresponding aliasing. we must select the sampling rate to be sufficiently high. that is we must

$$\boxed{F_s/2 > F_{max}}$$

$$\boxed{F_s \geq 2F_{max}}$$

→ if the highest frequency contained in an analog signal $x_a(t)$ is $F_{max} = B$ and the signal is sampled at a rate $F_s > 2F_{max} \equiv 2B$, then $x_a(t)$ can be exactly recovered from it's sample values using the interpolation function.

$$g(t) = \frac{\sin 2\pi Bt}{2\pi Bt}$$

Thus $x_a(t)$ may be expressed as the sampling rate $\boxed{F_N = 2B = 2F_{max}}$ is called Nyquist rate

1.4.3 Quantization of Continuous - Amplitude Signals:-

→ As we have seen, a digital is a sequence of numbers (samples) in which each number is represented by a finite number of digits (finite precision).

→ The process of converting a discrete - time continuous amplitude signal into a digital signal by expressing each sample value as a finite (instead of an infinite) number of digits is called quantization.

→ the error introduced in representing the continuous-valued signal by a finite set of discrete value levels is called quantization error or quantization noise.

Samples $x(n)$ as $Q[x(n)]$ & let $x_q(n)$ denote the sequence of quantized samples at the output of the quantizer.

$$x_q(n) = Q[x(n)]$$

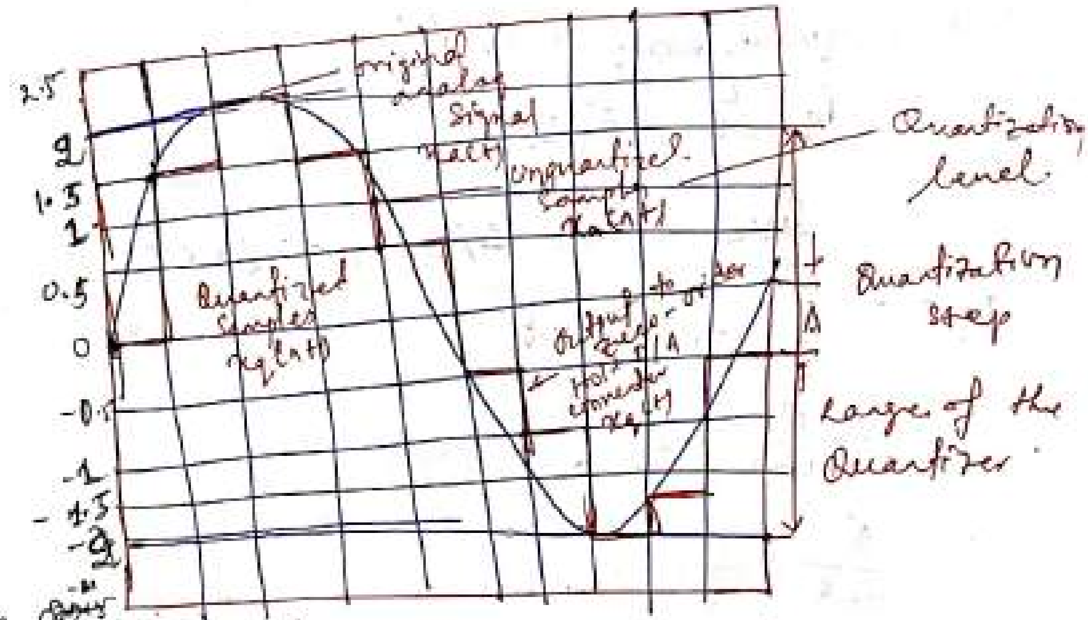
the quantization error is a sequence $e_q(n)$ defined as the difference betn the quantized value & the actual sample

$$e_q(n) = x_q(n) - x(n)$$

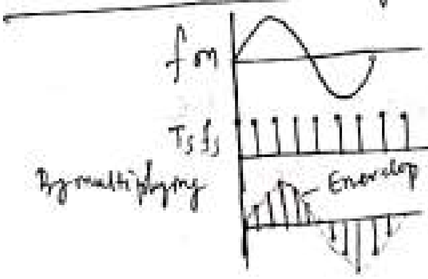
$$L = 2^n$$

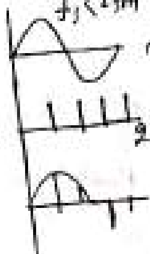
$$\Delta = \frac{x_{max} - x_{min}}{L - \cancel{1}}$$

if x_{min} & x_{max} represent the minm & maxm values of $x(n)$ & L is the number of quantization level
n = No. of Bits



continued from sampling theorem





$f_s < 2f_m$ message signal

$x(t) = \sin 2\pi t + \sin 3\pi t + \sin 4\pi t$

$w_1 = 2\pi$, $w_2 = 3\pi$, $w_3 = 4\pi$

$f_2 = 1.5$ $f_3 = 2$

$f_m = \max(f_1, f_2, f_3) = 2$

we can't reconstruct if $f_s < 2f_m$ because take more time period $T_s \uparrow$

1.4.4 Quantization of Sinusoidal Signals

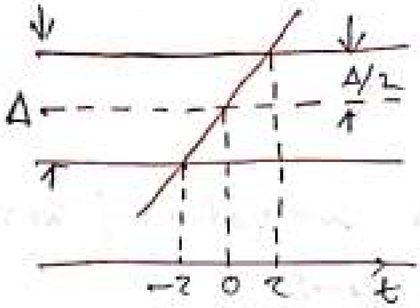
→ The Sampling & quantization of an analog Sinusoidal Signal $x_a(t) = A\cos\Omega_0 t$ using a rectangular grid.

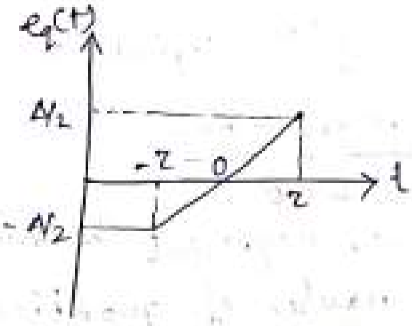
→ Horizontal lines within the range of the quantizer indicate the allowed levels of quantization.

→ vertical lines indicates the Sampling time

Thus, orginal analog Signal $x_a(t)$ we obtain a discrete-time Signal $x(n) = x_a(nT)$ by sampling & a discrete-time discrete-amplitude Signal $x_q(nT)$ after quantization.

$$x(n) = x_a(nT)$$





The quantization error $e_q(t) = x_a(t) - x_q(t)$

mean-Square error power P_q is

$$P_q = \frac{1}{2\tau}\int_{-\tau}^{\tau} e_q^2(t)\,dt$$

Since $e_q(t) = (\Delta/2\tau)t, \quad -\tau \le t \le \tau$

$$P_q = \frac{1}{\tau}\int_0^{\tau}\left(\frac{\Delta}{2\tau}\right)^2 t^2\,dt$$

$$= \frac{1}{\tau}\times\frac{\Delta^2}{4\tau^2}\left[\frac{t^3}{3}\right]_0^{\tau}$$

$$= \frac{\Delta^2}{4\cancel{\tau^3}}\times\frac{\cancel{\tau^3}}{3}$$

$$= \frac{\Delta^2}{12} \quad \text{— ①}$$

If the quantizer has b bits of accuracy & the quantizer covers the entire range 2A, the quantization step is $\Delta = 2A/2^b$.

$$P_q = \frac{A^2/3}{2^{2b}}$$

The average power of the signal $x_a(t)$

$$P_x = \frac{1}{T_p}\int_0^{T_p}(A \cdot \cos \Omega_0 t)^2 dt = \frac{A^2}{2}.$$

→ the quality of the output of the A/D converter is usually measured by the signal to quantization noise ratio (SQNR)

$$SQNR = \frac{P_x}{P_q} = \frac{3}{2}\, 2^{2b}$$

Expressed in (dB)

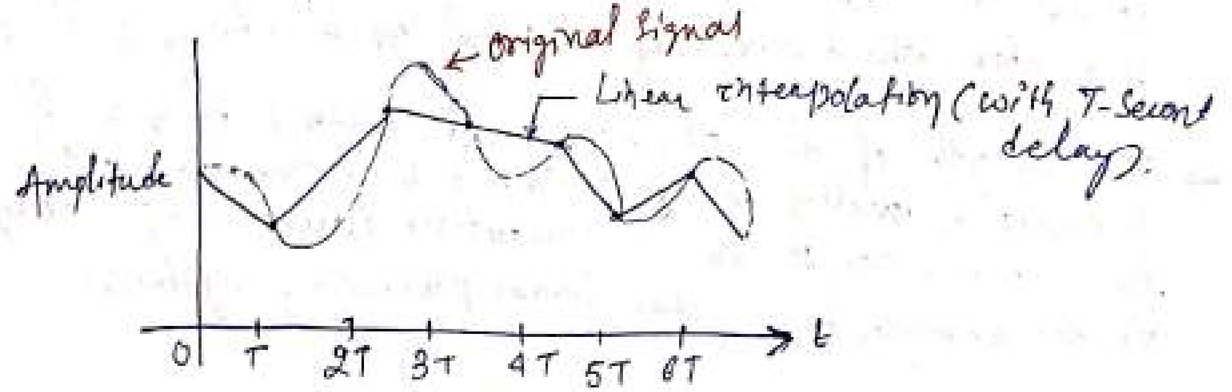
$$\boxed{SQNR\ (dB) = 10 \log_{10} SQNR = 1.76 + 6.02b}$$

1.4.5 Coding of Quantized Samples

→ The coding process in an A/D converter assigns a unique binary number to each quantization level. If we have L levels we need at least L different binary numbers.

→ With a word length of b bits we can create 2^b different binary number. Hence we have $2^b \geq L$, or equivalently, $b \geq \log_2 L$. Thus the number of bits required in the coder is the smallest integer greater than or equal to $\log_2 L$.

Ex If can easily be seen that we need a coder with b = 4 bits, commercially available A/D converters may be obtain with finite precision of b = 16 or less



1.4.6 Digital - to - Analog Conversion

→ To convert a digital signal in to an analog signal we can use a digital - to - analog (D/A) converter. As stated previously the task of a D/A converter is to interpolate betn samples.

→ The Sampling theorem specifies the optimum interpolation for a bandlimited signal practical viewpoint the simplest D/A converter is the zero-order hold.

→ Suboptimum interpolation technique results in passing frequencies above the folding frequency. Such frequency components are undesirable and are usually removed by passing the output of the interpolator through a proper analog filter which is called a postfilter or smoothing filter.

1.4.7 Analysis of Digital Signals and Systems Versus Discrete-Time Signals & Systems.

→ we have seen that a digital signal is defined as a function of an integer independent variable & it's values are taken from a finite set of possible values.

→ The usefulness of such signal is a consequence of the possibilities offered by digital computer. computers operate on number which are represented by a string of 0's & 1's.

→ The length of this string (word length) is fixed & finite & usually it is 8, 12, 16 or 32 bits. The effect of finite word length in computation cause complication in the analysis of digital signal processing systems.

CHAPTER-2

DISCRETE TIME SIGNALS & SYSTEMS

2.1 state and explain discrete time signals

→ A Discrete-time signal x(n) is a function of an independent variable that is an integer.

→ it is not defined at instants betn two successive samples

→ x(n) was obtained from sampling an analog signal $x_a(t)$, then $x(n) \equiv x_a(nT)$, where T is the sampling period (i.e time betn successive samples).

→ Some alternative representation that are often more convenient to use. These are:

All are Infinite duration

1. functional representation, such as

$$x(n) = \begin{cases} 1, & \text{for } n = 1,3 \\ 4, & \text{for } n = 2 \\ 0, & \text{else where.} \end{cases}$$

2. Tabular representation, such as

n	....	-2	-1	0	1	2	3	4	5	...
x(n)	....	0	0	0	1	4	1	0	0	...

3. Sequence representation

An infinite-duration signal or sequence with the time origin (n=0) indicated by the symbol ↑ is represented as

$$x(n) = \{ \ldots\ldots 0, \underset{\uparrow}{0}, 1, 4, 1, 0, 0, \ldots\ldots \}$$

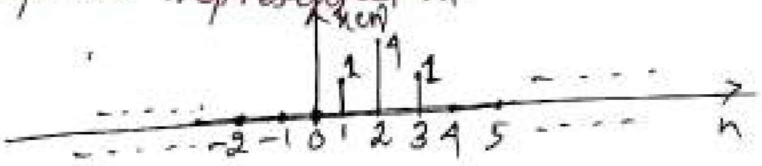
A sequence x(n), which is zero for n<0

$$x(n) = 0 \quad n<0$$

can be represented as

$$x(n) = \{ \underset{\uparrow}{0}, 1, 4, 1, 0, 0 \ldots\ldots \}$$

4. Graphical representation.



→ for Finite-duration Sequence can be represented as

$$x(n) = \{3, -1, \underset{\uparrow}{-2}, 5, 0, 4, -1\}$$ — Both side signal.

finite-duration sequence that satisfies the condition $x(n) = 0$ for $n < 0$

$$x(n) = \{\underset{\uparrow}{-2}, 5, 0, 4, -1\}$$

is called Right sided signal.

$x(n) = 0$ for $n > 0$

$$x(n) = \{3, -1, \underset{\uparrow}{-2}\}$$

is called left side signal.

2.1.1 Some elementary Discrete-Time Signals

1. unit Sample Sequence (Impulse Signal)

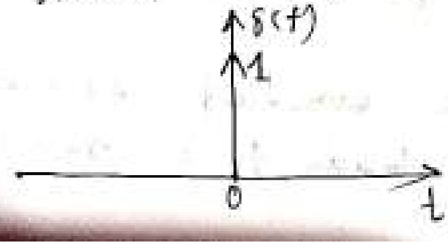
Continuous Time

→ Impulse signal is one of the most important signal and play an important role in the analysis. unit impulse can be regarded as a rectangular pulse with a width that has become infinitly large and and overall area remain unit. Hence unit impulse signal is a signal with zero amplitude everywhere except at t=0 at t=0 the amplitude is infinite such that the area under the curve is equal to one.

$$* \quad \delta(t) = \begin{cases} 0, & \text{for } t \neq 0 \\ 1, & \text{for } t = 0 \end{cases}$$

$$* \int_{-\infty} \delta(t) \cdot dt = 1$$ Area under $\delta(t) = 1$

Delayed unit impulse and impulse signal is as shown in fig



Discrete time

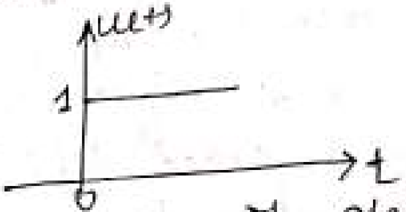
$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n\neq 0 \end{cases}$$

$$\boxed{\delta[n] = u(n) - u(n-1)}$$

② Unit - Step Function

The continuous time version eq^n the unit step functions is defined by .

$$u(t) = \begin{cases} 1, & t\geq 0 \\ 0, & t<0 \end{cases}$$



The Discrete time version eq^n the unit step function is defined by

$$u[n] = \begin{cases} 1, & n\geq 0 \\ 0, & n<0 \end{cases}$$

A step function is said to exhibit a discontinuity at $t=0$ since the value of $u(t)$ change in instantaneously from 0 to 1 at $t=0$.

The simple example is a DC source applied at $t=0$ by closing a switch.

③ Ramp Signal

The continuous time version the ramp signal or function is

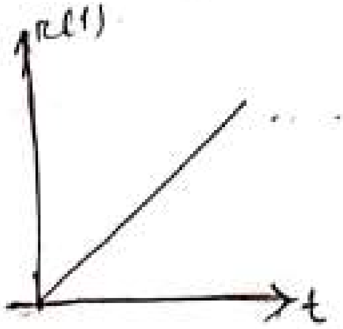
$$r(t) = \begin{cases} t, & t\geq 0 \\ 0, & t<0 \end{cases}$$

$$r(t) = t \cdot u(t)$$

the impulse function $\delta(t)$ is the derivative of the step function $u(t)$ w.r.t time. the integral of the step function $u(t)$ is the ramp function of the unit slop.

The physical example is a constant current flowing through a capacitor leads to a ramp

Voltage across Capacitor.



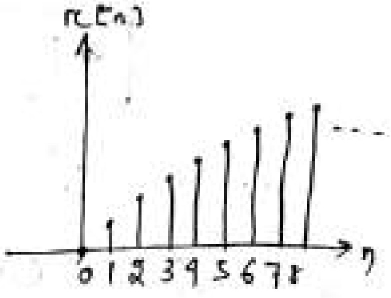
$$V_c(t) = \frac{1}{c}\int i \cdot dt$$

$$= \frac{I}{c}\int dt$$

$$= \frac{I}{c} t .$$

$$V_c(t) \propto t$$

The Discrete time version of the ramp function is defined by



$$r[n] = \begin{cases} n, & n>0 \\ 0, & n<0 \end{cases}$$

or $r[n] = n.u[n]$

④ Exponential Signals :

A continuous time real exponential signal is given by,

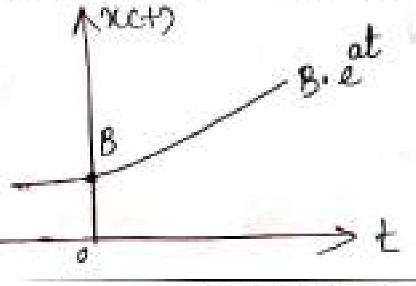
$$\boxed{x(t) = B \cdot e^{at}}$$

where B & a are real parameter.

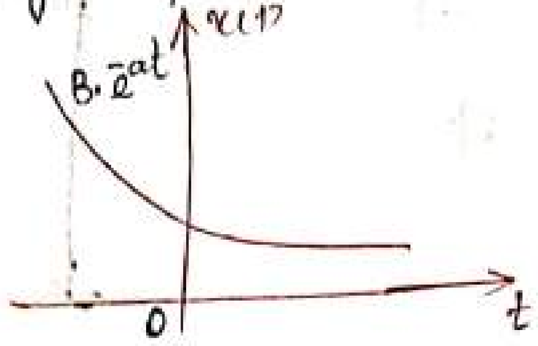
The parameter 'B' is the amplitude of the exponential signal measured at t=0. The behaviour of the signal is ~~defined~~ depends on the parameter 'a' and is of two types.

1) Growing exponential and 2) Decaying exponential.

* If 'a' is +ve i.e. a>0 then the signal x(t) is called as growing exponential. This form is used in describing many physical process including chain reaction in atomic explosions and complex chemical reaction.



* if a is -ve i.e $a<0$ then the signal $x(t)$ is called Decaying exponential.



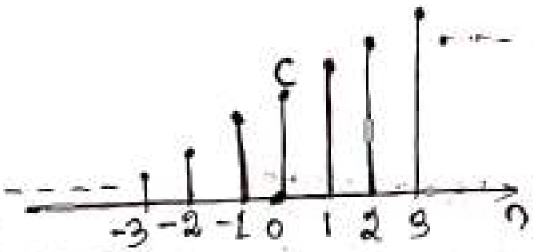
* In case of Discrete time signal

$$\boxed{x[n] = c\,a^n}$$

where 'c' and a are real constants.

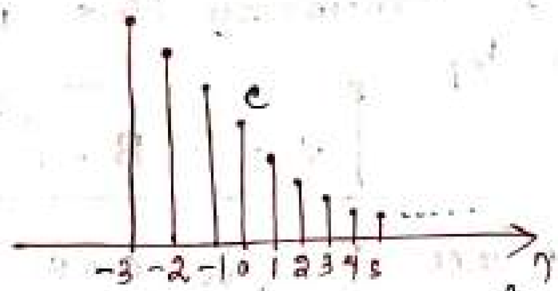
For $x[n] = c \cdot a^n$, if

1) $a>1$



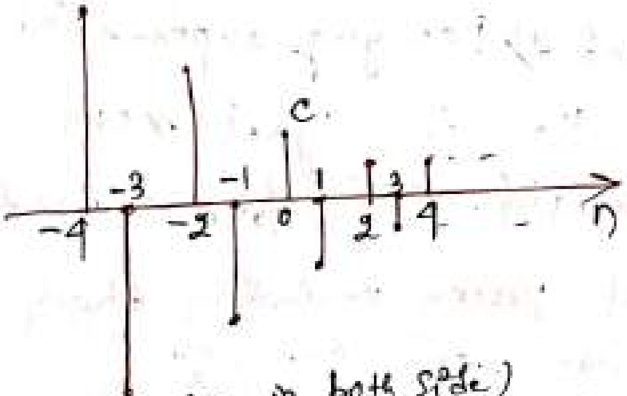
(growing signal)

2) $0 \leq a < 1$



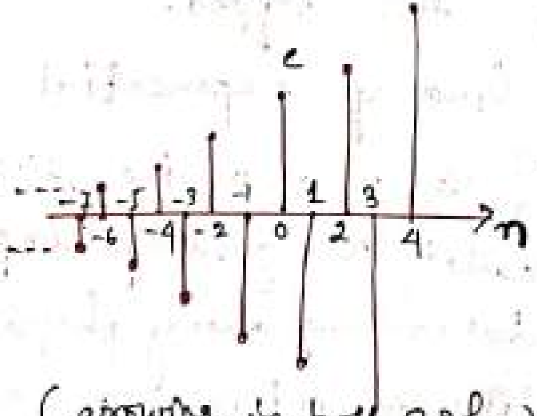
Decaying signal.

3) $-1 < a < 0$



(Decaying in both side)

4) $a < -1$



(growing in both side)

5) Sinusoidal Signals

* the contineous time version of sinusoidal signal in it's most general from may be written as

$$x(t) = A\cos(\Omega_0 t + \phi)$$

$$x(t) = A\sin(\Omega_0 t + \phi)$$

A = amplitude , ω → Angular frequency in radian/sec

φ = phase angle in radian.

∴ A sinusoidal signal is example of periodic signal with $\boxed{T = \frac{2\pi}{\Omega}}$ i.e $\boxed{x(t) = x(t+T)}$

Angular frequency $\Omega = \frac{2\pi}{T}$ radian/sec

Proof: $x(t+T) = A\cos[\Omega(t+T)+\phi]$

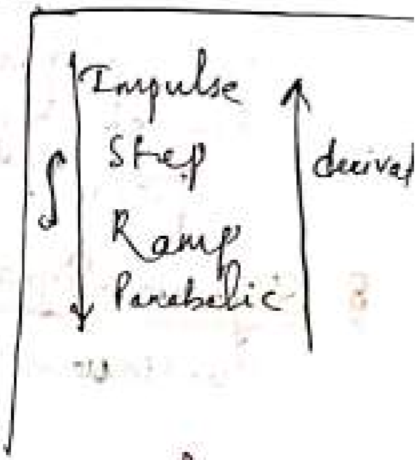
$= A\cos(\Omega t + \Omega T + \phi)$

$= A\cos(\Omega t + \Omega\frac{2\pi}{\Omega} + \phi)$

$= A\cos(\Omega t + 2\pi + \phi)$

$= A\cos(\Omega t + \phi)$

$x(t+T) = x(t)$ Proved.



* Discrete time version of a sinusoidal signal:

The period of periodic discrete time signal is measured in samples - Thus x[n] is said to be periodic with a period of N samples.

$$x[n] = A\cos[\omega n + \phi]$$

for periodic ⇒ $x(n+N) = x[n]$

∴ $x(n) = A\cos(\omega(n+N) + \phi)$

only if ~~cos~~ $\omega N = 2\pi m$ radians

i.e $\boxed{N = \frac{2\pi m}{\omega}}$ samples, where m, N are integers

As in contineous time signals, this is not periodic for any ~~ab~~ arbitary value of Ω. For the discrete

) time signals to be periodic. the angular frequency must be integer multiple of 2π.

6) Complex Exponential Signal:-

The complex exponential signal is defined by

$$\boxed{x(t) = A\, e^{j\Omega_0 t}}$$

$A \rightarrow$ amplitude
Ω_0 = angular frequency
$\Omega_0 = 2\pi f_0 = \frac{2\pi}{T}$

complex exponential function can be resolved in to real and imaginary part i.e

$$x(t) = A \cdot e^{j\Omega_0 t}$$
$$= A\left[\cos\Omega_0 t + j\sin\Omega_0 t\right]$$

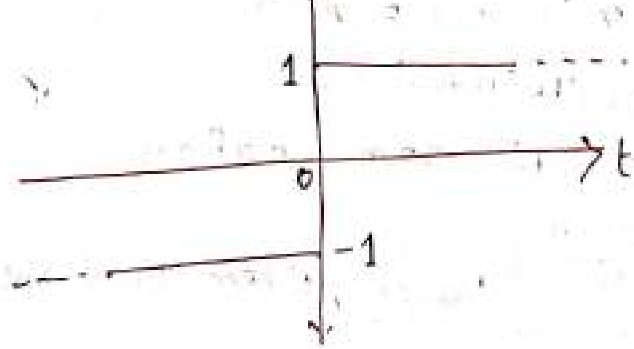
. The exponential Discrete time signal is defined by

$$\boxed{e(n) = a^n \cdot x(n)}$$

8) Signum Signal

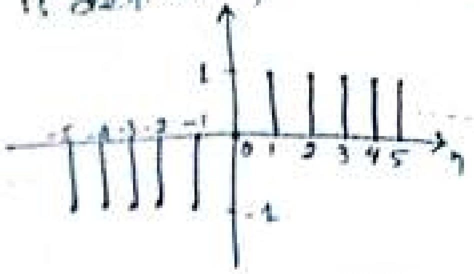
Continuous time signum signal is defined as

$$Sgn(t) = \begin{cases} 1 & : t>0 \\ 0 & : t=0 \\ -1 & : t<0 \end{cases}$$



Discrete-time Signum Signal is defined as

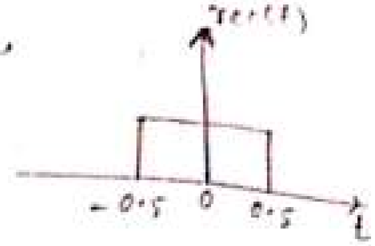
$$\mathrm{Sgn}(n) = \begin{cases} 1 : & n > 0 \\ 0 : & n = 0 \\ -1 : & n < 0 \end{cases}$$



⑨ Rectangular function

The rectangular function is defined as

$$\begin{matrix} \mathrm{rec}(t) \\ \text{or} \\ \pi(t) \end{matrix} = \begin{cases} 1 : & |t| < 1/2 \\ 1/2 : & |t| = 1/2 \\ 0 : & |t| > 1/2 \end{cases}$$



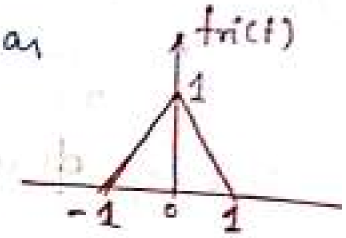
Discrete rectangular function is

$$\mathrm{rect}\, N_0(n) / \pi[n] = \begin{cases} 1 : & |n| < 4 \\ 1/2 : & |n| = 4 \\ 0 : & |n| > 4 \end{cases}$$

⑩ Unit triangular function

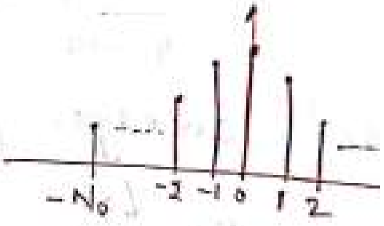
The triangular function is defined as

$$\mathrm{tri}(t) = \begin{cases} 1 - |t| & : |t| \leq 1 \\ 0 & : |t| > 1 \end{cases}$$



The discrete time triangular function is defined as

$$\mathrm{tri}(n) = \begin{cases} 1 - \frac{|n|}{N_0} & : |n| \leq N_0 \\ 0 & : |n| \geq N_0 \end{cases}$$



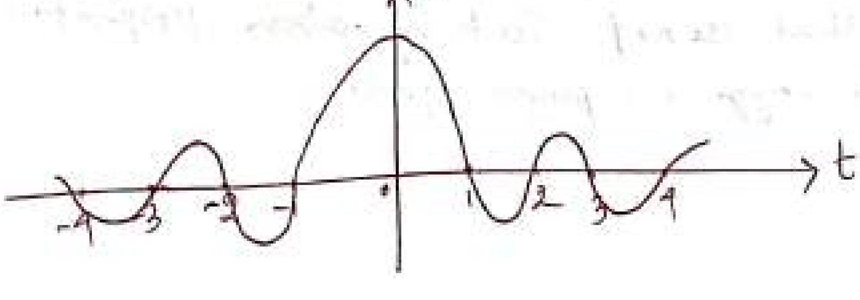
⑪ Sinc function

The sinc function is defined as

$$\mathrm{Sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

At $t = 0$:

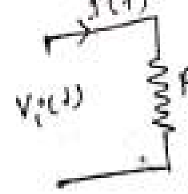
$$\mathrm{Sinc}(t) \Big|_{t=0} = 1$$



2.1.2 Classification of Discrete-Time Signals

In CT   Energy & Power Signal

→ In electrical sim, Signal may represents voltage or current.



consider voltage v(t) developed across the resistor R. producing urgent I(t). the instaneous power dissipated in resistor R is defined by.

$$\boxed{P(t) = \frac{V^2(t)}{R}} \quad — ①$$

equivalently $P(t) = R \cdot I^2(t)$

Energy of Signal

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\, dt \quad (or) \quad E = \int_{-\infty}^{\infty} |x^2(t)|^2 \cdot dt$$

↳ (~~Non-periodic signal~~) Infinite duration

= Finite duration ↳ ~~periodic~~ Signal

Average power as

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [x(t)]^2 \cdot dt \quad or \quad P = \frac{1}{T} \int_{-T/2}^{T/2} [x(t)]^2 \cdot dt$$

↳ (~~non-periodic~~ Infinite duration)   Finite duration

In case of discrete time Signal

$$E = \sum_{n=-\infty}^{\infty} [x(n)]^2$$

~~[illegible]~~

and average power is

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

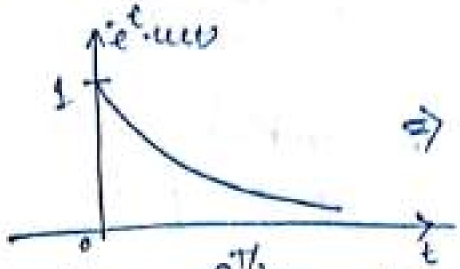
condition:

* A signal is an energy signal, if and only if the total energy of the signal is finite for power P=0
* The signal is said to be power signal if the average power of the signal is finite & Energy E=∞
* The signal that do not satisfy above properties are neither energy nor power signals.

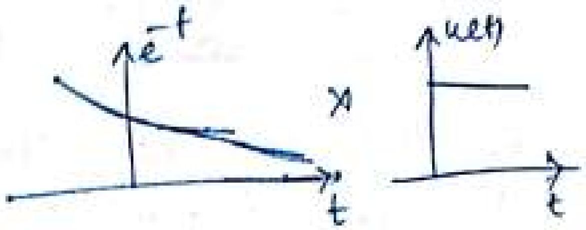
Problem

Q Calculate Energy and power for the following C.T.S & classify it as Energy or power signal.

1. $x(t) = e^{-t} . u(t)$.



$\Rightarrow$



$$E_{x(t)} = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t) . dt$$

$$= \int_{-\infty}^{\infty} |x^2(t)|^2 . dt$$

$$= \int_{0}^{\infty} (e^{-t} . u(t))^2 . dt$$

$$= \int_{0}^{\infty} e^{-2t} . 1 . dt$$

$$= \int_{0}^{\infty} e^{-2t} dt$$

$$= \left[\frac{e^{-2t}}{-2}\right]_0^{\infty}$$

$$= \left|\frac{e^{0} - e^{-\infty}}{-2}\right|$$

$$= 1/2$$

$$P_{av} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} |x(t)|^2 . dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T/2} e^{-2t} . dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-2t}}{-2}\right]_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[-\frac{e^{-2T}}{2} + \frac{1}{2}\right]$$

$$= \frac{1}{\infty} = 0$$

This is energy signal.

* Periodic Signals and aperiodic Signals :-

For CT

$$\boxed{x(t) = x(t+T)}$$

$\boxed{T = \frac{2\pi}{\Omega}}$ Time periode / Fundamental time periode

if the above condition is satisfies they the signal is periodic otherwise non periodic.

~~For DT~~

* a contineous time signal $x(t)$ is said to be periodic with period T, if there is a positive non-zero value of T for which . .

for all T $(-\infty$ to $\infty)$

* . $\boxed{x(t) \neq x(t+T)}$ is called non-periodic signal.

For DT

A Discrete time signal $x(n)$ is said to be periodic if it satisfies the .

$\boxed{x(n) = x(n+N)}$ for all N.

Where N is a +ve integer.

N = Samples.

$\boxed{x(n) \neq x(n+N)}$ is non-periodic.

$N = 2\pi\left(\frac{m}{\omega_0}\right)$ m = Smallest integer m & min value of m.

Q. Determine whether the following signals are periodic or not. if periodic find the fundamental time periode.

i) $x(t) = \cos(t + \pi/2)$

Ans - $x(t) = x(t+T)$

comparing with $x(t) = A\cos(\Omega_0 t + \phi)$

$\Omega_0 = 1$ $T = \frac{2\pi}{\Omega_0} = 2\pi$

$x(t+T) = \cos(t + 2\pi + \pi/2) \Rightarrow \cos(t + \pi/2)$

Periodic Signal.

(ii) $x(t) = 3 \sin \pi/4\, t$

$x(t) = A \sin(\Omega_0 t + \phi)$

$A = 3 \quad \Omega_0 = \pi/4$

$T = \frac{2\pi}{\pi/4} = 8$ Sec.

This is periodic signals.

$x(t) = x(t+T)$

$x(t+T) = 3 \sin(\pi/4 (t+T))$

$= \cancel{3 \sin(\pi/4\, t + 8)}$

$= 3 \sin(\pi/4\, t + \frac{\pi}{4} \times 8^{2})$

$= 3 \sin(\pi/4\, t + 2\pi)$

$= \underline{3 \sin \pi/4\, t}$

~~Thus~~ $x(t) = x(t+T)$

This periodic Signal.

(iii) $x(t) = 4 \cos(4\pi/7\, t + \pi/6)$

$\Omega_0 = 4\pi/7 \quad T = \frac{2\pi}{4\pi/7}$

$= 7/2$ Sec.

$x(t)$ is periodic with 7/2 Sec.

(iv) $x(t) = 1 + \cos(3t + 6)$

$\Omega_0 = 3 \quad T = \frac{2\pi}{3}$ Sec

$x(t) = x(t+T)$

$x(t+T) = 1 + \cos(3(t + 2\pi/3) + 6)$

$= 1 + \cos(3t + 2\pi + 6)$

$= 1 + \cos(3t + 6)$

So, this is periodic Signal.

5)

$x(t) = \cos^2 \pi/8\, t$

$\cos^2\theta = \frac{1}{2}[1+\cos 2\theta)$

$= \frac{1}{2}[1+\cos 2\pi/8\, t),$

$T = \frac{2\pi}{\Omega_0}$ $\Omega_0 = 2 = \frac{1}{2}[1+\cos \pi/4\, t])$

$= \frac{2\pi}{\pi/4} = 8$ Sec

this is periodic Signal.

6) $x(t) = e^{j\pi t/10}$

Given x(t) is periodic complex exponential Signal comparing with

$x(t) = A e^{j\omega t}$

$\Omega_0 = \frac{2\pi}{10}$

$T = \frac{2\pi}{\Omega_0} = \frac{2\pi}{\frac{2\pi}{10}} = 10$ sec

This periodic Signal

7) $x(t) = e^{j\omega t}$

prove x(t) is periodic

Ans for the Signal to be periodic $x(t) = x(t+T)$

So replace t by t+T

$x(t+T) = e^{j\omega(t+T)}$

$= e^{j\omega t} \cdot e^{j\omega T}$

$= e^{j\omega t} \cdot e^{j\omega \frac{2\pi}{\omega}}$

$= e^{j\omega t} \cdot e^{j 2\pi}$

$\therefore e^{i\theta} = \cos\theta + i\sin\theta$

$e^{j2\pi} = \cos 2\pi + i\cancel{\sin 2\pi}^{0}$

$= 1$

$= e^{j\omega t} \cdot 1$

$x(t+T) = x(t)$

It is periodic

8) Determine whether the following Signals are periodic or not periodic if periodic find the period of the Signal.

a) $x(t) = \sin \pi/3\, t + \cos \pi/4\, t$

Ans $x(t) = x_1(t) + x_2(t)$

$x_1(t) = \sin \pi/3\, t$

$x_2(t) = \cos \pi/4\, t$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\pi/3} = 6$ sec

$T_1 = \frac{2\pi}{\omega_2} = 2\pi/\pi/4 = 8$ sec

consider

$\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4}$

rational number

x(t) is periodic.

$\frac{T_1}{T_2} = \frac{3}{4}$

$4T_1 = 3T_2 = T$

$4 \times 6 = 3 \times 8$

$T = 24$ sec

9) $x(t) = \cos\sqrt{2}t + \sin t$

$x(t) = x_1(t) + x_2(t)$

$x_1(t) = \cos\sqrt{2}t \quad T_1 = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

$x_2(t) = \sin t \quad T_2 = 2\pi$ sec.

$\frac{T_1}{T_2} = \frac{\sqrt{2}\pi}{2\pi} = \frac{1}{\sqrt{2}}$

if is rational number

x(t) is not periodic.

10) $x(n) = 5\sin 2n$

$w_0 = 2$

$N = \frac{2\pi}{w_0} m$

$= \frac{\cancel{2}\pi}{\cancel{2}}$

$= \pi\ m$

$N = \pi$

x(n) is a non-periodic.

11) $x(n) = 3\sin(\pi/3\, n + \pi/6)$

$w_0 = \pi/3$

$N = \frac{2\pi}{w_0} m$

$= \frac{2\pi}{\pi/3} = 6$

N=6 Sample.

12) $x(n) = 3\cos(4\pi/3\, n + \pi/6)$

$w_0 = \frac{4\pi}{3}$

$N = \frac{2\pi}{w_0} m$

$= \frac{2\pi}{4\pi/3} m$

$= \frac{6}{4} = \frac{3}{2} m$

$N = 3$ for $m = 2$

x(n) is periodic with a period N=3.

13 $x(n) = \cos 0.6\pi n + \sin \frac{3\pi}{8} n$

$x(n) = x_1(n) + x_2(n)$

$x_1(n) = \cos 0.6\pi n$

$N_1 = \frac{2\pi}{w_1} K = \frac{2\pi}{0.6\pi} K$

$= \frac{10}{3} K$

$N_1 = 10$ Sample. for K=3

$x_2(n) = \sin \frac{3\pi}{8} n$

$N_2 = \frac{2\pi}{\frac{3\pi}{8}} K$

$= \frac{16}{3} K$

$N_2 = 16$ for K=3

$\frac{N_1}{N_2} = \frac{10}{16} = \frac{5}{8}$

rational number

x(n) is periodic

⑭ $x(t) = \cos\pi/3\, t + \cos\pi/6\, t + \cos\pi/9\, t + 1$

As $x(t) = x_1(t) + x_2(t) + x_3(t)$

$x_1(t) = \cos\pi/3\, t$

$w_1 = \pi/3$

$T_1 = \frac{2\pi}{w_1} = \frac{2\pi\times 3}{\pi}$

$= 6$

$w_2 = \pi/6$

$T_2 = \frac{2\pi/6}{\pi} = 12$

$w_3 = \pi/9$

$T_3 = \frac{2\pi}{w_3} = \frac{2\pi\times 9}{\pi} = 18$

$\frac{T_1}{T_2} = \frac{6}{12} = 1/2$ rational

$\frac{T_1}{T_3} = \frac{6}{18} = \frac{1}{3}$ rational

x(t) is periodic

Lcm (2, 3) = 6

$\frac{T_1}{T_2} = \frac{1}{2}\times\frac{3}{3} = \frac{3}{6}$

$\frac{T_1}{T_3} = \frac{1}{3}\times\frac{2}{2} = \frac{2}{6}$

$T = 6T_1$ (or) $3T_2$ (or) $2T_3$

$= 6 \times 6 = \underline{36}$

⑮ $x(n) = \cos(3\pi/4)n + \cos(4\pi/5)n + \cos\frac{5\pi}{6}n.$

$x(n) = x_1(n) + x_2(n) + x_3(n)$

$x_1(n) = \cos\left(\frac{3\pi}{4}\right)n$

$N_1 = \frac{2\pi}{\Omega_1}K$

$= \frac{2\pi}{3\pi/4}$

$= \frac{8}{3}K$

$= \frac{8}{3} \times 3 \quad K=3$

$\boxed{N_1 = 8 \text{ Samples}}$

$x_2(n) = \cos(4\pi/5)n$

$N_2 = \frac{2\pi}{\Omega_2}K$

$= \frac{2\pi}{4\pi/5}$

$= \frac{5}{2}K$

$\boxed{N_2 = 5 \text{ Samples}}$

$K=2$

$x_3(n) = \cos\left(\frac{5\pi}{6}\right)n$

$N_3 = \frac{2\pi}{5\pi/6}K = \frac{12}{5}K$

$\boxed{N_3 = 12} \quad K=5$

consider

$\frac{N_1}{N_2} = \frac{8}{5}$ → rational number

$\frac{N_1}{N_3} = \frac{8}{12} = \frac{2}{3}$ rational number.

x(n) is a periodic signal.

To find fundamental time period

LCM (5, 3) = 15

$\frac{N_1}{N_2} = \frac{8}{5} \times \frac{3}{3} = \frac{24}{15}$

$\frac{N_1}{N_3} = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$

$N = 15N_1 = 24N_2 = 10N_3$

$= 15(8) = 120 \underline{\text{ Samples}}$

Q calculate Energy & power for the following C.T.S and classify it as energy or power signals.

Ans

i) $x(t) = 5\cos 5\omega t$

The Energy of the C.T.S is

First find out signal is periodic or aperiodic.

Ex $\Omega_0 = 5\omega$

$T = \frac{2\pi}{5\omega}$ not finite not periodic.

~~$x(t+T) = 5\cos\left(5\omega\left(t+\frac{2\pi}{5\omega}\right)\right)$~~

~~$= 5\cos(5\omega t + 2\pi)$~~

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \cdot dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} (5\cos 5\omega t)^2 \cdot dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} 25\cos^2 5\omega t \cdot dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} 25\left[\frac{1+\cos 10\omega t}{2}\right] dt$$

$$= \lim_{T\to\infty} \frac{25}{2} \int_{-T}^{T} (1+\cos 10\omega t) \cdot dt$$

$$= \lim_{T\to\infty} \frac{25}{2} \left[t + \frac{\sin 10\omega t}{10\omega}\right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{25}{2} \left[T+T + \frac{\sin 10\omega T}{10\omega} + \frac{\sin 10\omega T}{10\omega}\right]$$

$$= \infty$$

The power P of a C.T.S is

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} (5\cos 5\omega t)^2 \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 25 \cos^2 5wt \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 25 \left(\frac{1 + \cos 10wt}{2} \right) \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \cancel{\int_{-T}^{T}} \frac{25}{2} \int_{-T}^{T} (1 + \cos 10wt)\, dt$$

$$= \lim_{T\to\infty} \frac{25}{4T} \left[t + \frac{\sin 10wt}{10w} \right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{25}{4T} \left[T + T + \frac{\sin 10wT}{10w} + \frac{\sin 10wT}{10w} \right]$$

$$= \lim_{T\to\infty} \frac{25}{4T} \left[2T + 2 \cancel{\frac{\sin 10wT}{10w}}^{\,0} \right]$$

$$= \lim_{T\to\infty} \frac{25}{\cancel{4T}_{2}} \left[\cancel{2T} \right)$$

$$= \underline{\underline{12.50}}$$

<u>Short cut</u>

For sinusoidal signal is always a power signal.

$x(t) = A \cos(\omega t + \theta)$

$\quad\;\; = A \sin(\omega t + \theta)$

$$\boxed{P = \frac{A^2}{2}}$$

Q. $x(t) = t\, u(t)$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt$$

$$= \int_{0}^{\infty} t^2\, dt$$

$$= \left[\frac{t^3}{3} \right]_0^{\infty} = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |t\, u(t)|^2 \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} (t^2)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{t^3}{3} \right]_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T} \times \frac{T^3}{24} = \infty$$

Niether power Nor energy Signal.

Q. $x(t) = e^{-5t} . u(t)$

find Energy & power.

So $T = \frac{2\pi}{-5}$ = irrational number non periodic

$$E = \lim_{T\to\infty} \int_{-\infty}^{\infty} |x(t)|^2 . dt$$

$$= \lim_{T\to\infty} \int_{-\infty}^{\infty} |e^{-5t} u(t)|^2$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-10t} . dt$$

$$= \lim_{T\to\infty} \left[\frac{e^{-10t}}{-10}\right]_0^T$$

$$= \lim_{T\to\infty} \left[\frac{e^{-10T}}{-10} + \frac{1}{10}\right]$$

$$= \lim_{T\to\infty} \left[\frac{1}{10} - \frac{e^{-10T}}{10}\right]$$

$$= \frac{1}{10} - \frac{0}{10} = \frac{1}{10} \text{ J}$$

This is energy Signal.

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [e^{-5t} u(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} e^{-10t} . dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-10t}}{-10}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-10T}}{-10} + \frac{1}{10}\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{1}{10} - \frac{0}{10}\right]$$

$$= 0 \text{ W}$$

Q. $x[n] = a^n u[n]$ check whether Energy Signal or power Signal.

Ans

case 1

for $|a| < 1$

So $x[n] = (1/2)^n u[n]$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} |(1/2)^n|^2$$

formula

$$\boxed{\text{for } a<1 \quad \therefore \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}}$$

$$= \sum_{n=0}^{\infty} \left(1/4\right)^n$$

$$= 1 + \frac{1}{4} + \frac{1}{16} + \cdots\cdots \cdots + \infty$$

$$= \frac{1}{1 - 1/4} = 4/3$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left[\left(1/2\right)^n\right]^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[1 + \frac{1}{4} + \frac{1}{16} \cdots\cdots \left(\frac{1}{4}\right)^N\right]$$

$= \underline{0}$

This energy signal when $\underline{a<1}$.

Case-2

$a = 1$

Eg

$x(n) = (1)^n u[n]$

$= u[n]$

$$E = \sum_{n=0}^{\infty} |1|^2$$

$$= \sum_{n=0}^{\infty} 1$$

$= ~~1+2+~~ \cdots$

$= \cancel{\infty}\ 1\cdot(\infty - 0 + 1)$

$= \infty$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1$$

$$\therefore \sum_{n=-N}^{N} b = b(N-(-N)+1)$$

$$= \underline{b(2N+1)}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot N+1$$

$$= \underline{\underline{1/2}}$$

This is power signal.

Case-3

$a \ngtr -1$

$$\boxed{\therefore \sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}}$$

$x(n) = (-2)^n u(n)$

$E = \sum_{n=\infty}^{\infty} |(-2)^n u(n)|^2$

$= \sum_{n=0}^{\infty} |(-2)^n|^2$

$= \sum_{n=0}^{\infty} 4^n$

$= 1 + 4 + 16 + \cdots\cdots 4^{\infty}$

$= \infty$

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N-1} |x(n)|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N-1} |(-2)^n u(n)|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N-1} |(-2)^n|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N-1} 4^n$

$=$

$$\boxed{\therefore \sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a} \quad \text{for } a \neq 1}$$

$= \lim_{N\to\infty} \frac{1}{2N+1} \times \frac{1-4^N}{1-4} =$

power signal

Case-4

$|a| > 1$

$x(n) = 3^n u(n)$

$E = \sum_{n=0}^{\infty} 9^n$

$= \infty$

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 9^n$

$= \infty$

(Niether power Nor energy signal).

$x(n) = e^{j(\pi/2 n + \pi/4)}$

$E = \sum_{n=-\infty}^{\infty} |e^{j(\pi/2 n + \pi/4)}|^2$

$|e^{j(\omega + \theta)}| = 1$

$= \sum_{n=-\infty}^{\infty} 1 = \infty$

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |e^{j(\pi/2 n + \pi/4)}|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$

$= \lim_{N\to\infty} \frac{2N+1}{2N+1}$

$= \lim_{N\to\infty} \frac{N(2+1/N)}{N(2+1/N)} = 1$

Q. $x(n) = \sin(\pi/4\, n)$

$E = \sum_{n=-\infty}^{\infty} |\sin^2(\pi/4\, n)|$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos 2\theta = 1 - 2\sin^2\theta$

$= \sum_{n=-\infty}^{\infty} \left|\frac{1-\cos(\pi/2\, n)}{2}\right|$

$= \infty$

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos(\pi/2\, n)}{2}$

$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$

$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \times 2N+1$

$= \frac{1}{2} \lim_{N\to\infty} \frac{\cancel{N}(2+1/N)}{\cancel{N}(2+1/N)}$

$= \boxed{1/2}$ ←

Power Signal.

Q. $x(n) = u(n)$

$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$= \sum_{n=0}^{\infty} 1$

$= \infty$

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} u^2(n)$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1$

$= \lim_{N\to\infty} \frac{1}{2N+1} N+1$

$= \lim_{N\to\infty} \frac{\cancel{N}(1+1/N)}{\cancel{N}(2+1/N)}$

$= 1/2$

Power Signal

* Even Signal & odd Signal

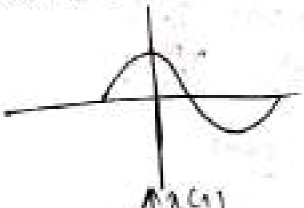
[illegible]

A Signal $x(t)$, or $x(n)$ is Said to be even Signal if it is identical to it's time reversal counter part or with reflection about the origin or Symmetrical about vertical axis.

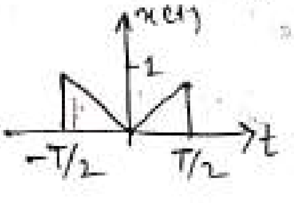
i.e. $x(t)$ is even $x(-t) = x(t)$ $\forall t$

$x(n)$ is even $x(-n) = x(n)$ $\forall n$

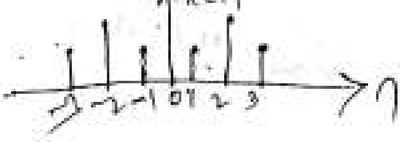
Eg 1) $x(t) = \cos t$



②



③

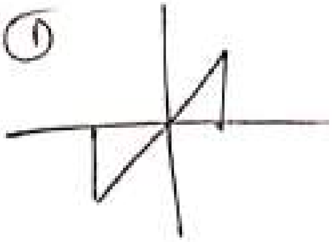


* $x(t)$ is odd

$x(-t) = -x(t)$,
for all value of t

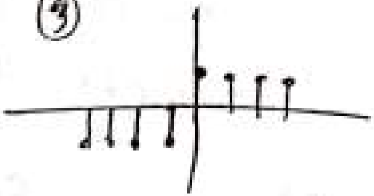
$x(-n) = -x(-n)$ ∀ n

Ex ①



② $x(t) = \underline{\sin \omega t}$

③



Decomposition of a Signal

- Any arbitary signal $x(t)$ can be decomposed in to an even & odd signal by applying the corresponding :

$$x(t) = x_e(t) + x_o(t) \quad \text{——} ①$$

$x_e(t)$ = even component
$x_o(t)$ = odd component

$$\boxed{\begin{aligned} x_e(t) &= \frac{x(t)+x(-t)}{2} \\ x_o(t) &= \frac{x(t)-x(-t)}{2} \end{aligned}}$$

DT

$$\boxed{\begin{aligned} x_e(n) &= \frac{x(n)+x(-n)}{2} \\ x_o(n) &= \frac{x(n)-x(-n)}{2} \end{aligned}}$$

→ if the signal is complex-valued signal then a complex valued signal $x(t)$ is said to be conjugate symmetric

if $x(-t) = x^*(t)$

Let $x(t) = \underset{\text{Real}}{\underset{\uparrow}{a(t)}} + j\underset{\text{Imaginary}}{\underset{\uparrow}{b(t)}}$

$$\underline{x^*(t) = a(t) - jb(t)}$$

Q Find the even and odd components of the following signal.

a) $x(t) = 1+t+3t^2+5t^3+9t^4$

$x_e(t) = 1+3t^2+9t^4$ $\quad x_o(t) = t+5t^3$

b) $x(t) = 1+t\cos t + t^2\sin t + t^3\sin t \cdot \cos t$

$x_e(t) = 1+t^3\sin t \cdot \cos t$

$x_o(t) = t\cos t + t^2\sin t$

c) $x(n) = \sin \pi/_3 n + \cos 4\pi/_3 n$

$x_e(n) = \cos 4\pi/_3 n$

$x_o(n) = \sin \pi/_3 n$

Properties

- sum of two even signal are even signal
- Sum of two odd signal are odd signal
- Sum of even signal & an odd signal is neither even or nor odd.
- product of two even signal is even signal
- product of two odd signal is ~~odd signal~~ even signal
- product of even & odd signal is odd signal.
- $\frac{\text{even signal}}{\text{odd signal}}$ = odd signal

* Causal & Non Causal Signal

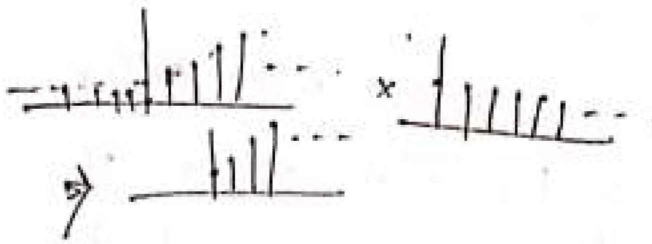
→ A signal $x(n)$ is said to be causal if it value is zero for $n<0$

$$\boxed{x(n) = 0 \quad , \; n<0}$$

other wise the signal is non causal

Ex

$x(n) = a^n u(n)$



Causal Signal.

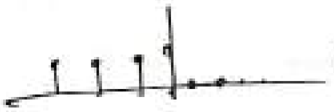
ex $x(n) = \{ \underset{\uparrow}{1}, 2, -3, -1, 2 \}$

Signal is causal.

Anti causal

A signal is zero for all $n \geq 0$

$\boxed{x(n) = 0 \quad \text{for all } n \geq 0}$



anti causal sig.

2.1.3 Simple manipulation of Discrete-Time Signals

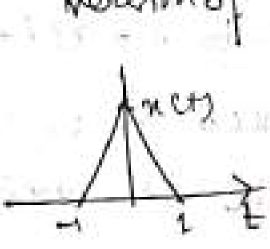
1. Time Scaling

If x(t) is a continuous time signal then signal y(t) is obtained by scaling the independent variable time 't' by a factor 'a' and is defind as

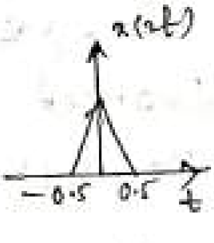
$\boxed{y(t) = x(at)}$

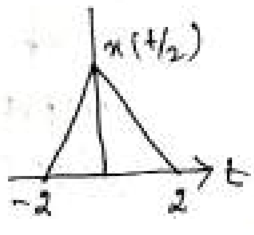
- if a>1 then the resultant signal y(t) is a compressed version of x(t).
- if a<1, then the resultant signal y(t) is expanded version of x(t)

Ex

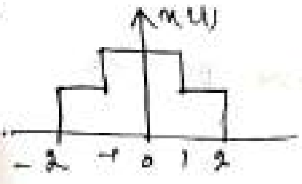


$x(2t)$, $x(t/2)$

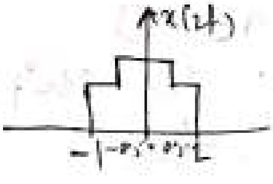


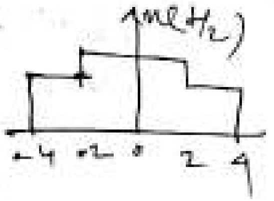


Ex



$x(2t)$, $x(t/2)$



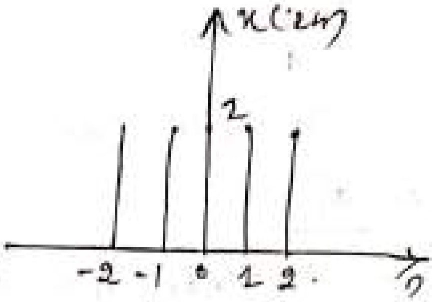


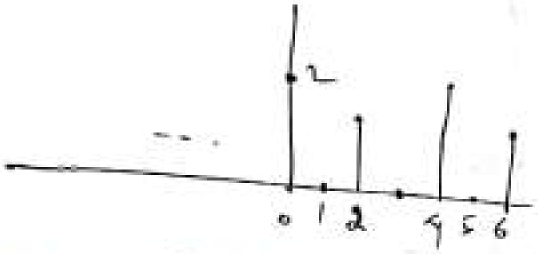
* In case of DTS

$y[n] = x[kn]$

when $k > 0$ and k can take only integer if $k > 1$ then some values of the discrete time signals is lost in $y[n]$

Ex





2) Time Shifting

CT let x(t) denotes a contineous time signal then the time shifted version of x(t) is defined as

$$\boxed{y(t) = x(t \pm t_0)}$$

where t_0 = time shift.

* if $t_0 > 0$ i.e +ve then $x(t - t_0)$ represents the delayed version of the x(t) i.e it is shifted to the right relative to the time axis.

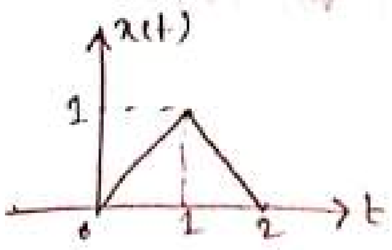
* if $t_0 < 0$ i.e -ve then $x[t - (-t_0)] = x(t + t_0)$ represents advanced version of x(t) i.e it is shifted to the left relative to the time axis.

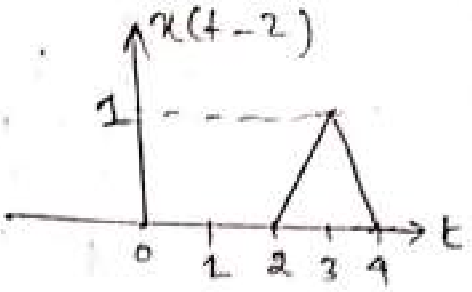
DT

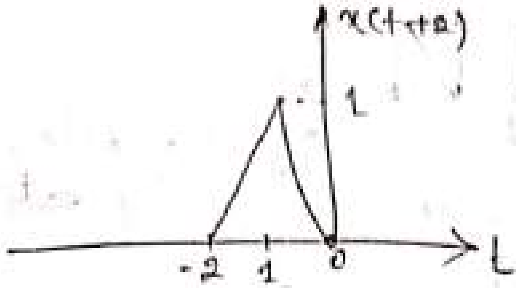
$y[n] = x[n \pm n_0]$

where n_0 is the shift either +ve or -ve integer value.

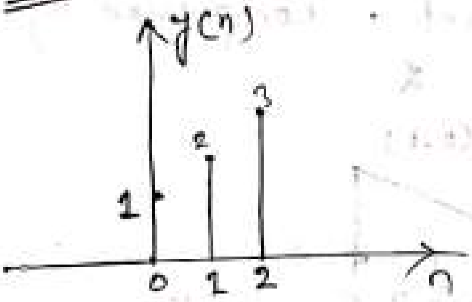
* Ex

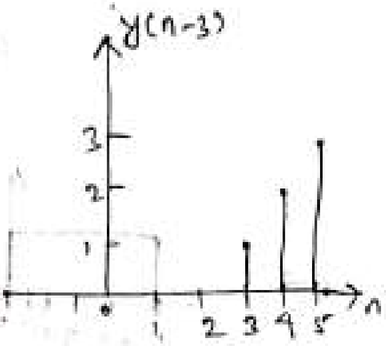


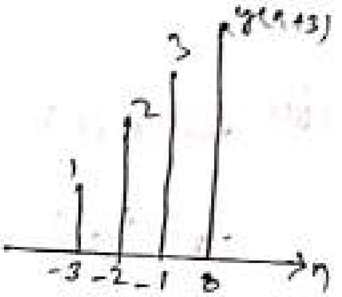




* Ex







③ Reflection or Time Reversal or time folding

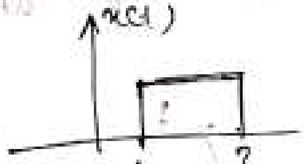
CTS ~~text y(t)~~ of x(t) is continuous time signal; then y(t) is the signal obtained by replacing time t by '-t'

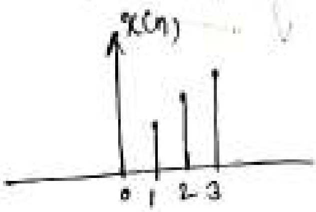
$$\boxed{y(t) = x(-t)}$$

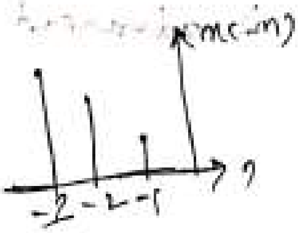
DTS

$y(n) = x[-n]$

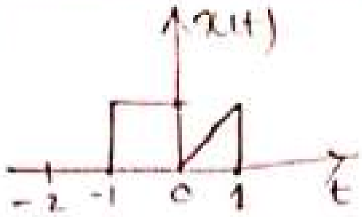
Ex







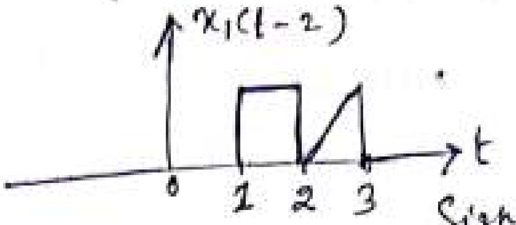
Q



find & sketch
i> $x_1(t-2)$, v) $x_5(t+3)$
ii) $x_2(2t)$
iii) $x_3(0.2t)$
(v) $x_4(-t)$

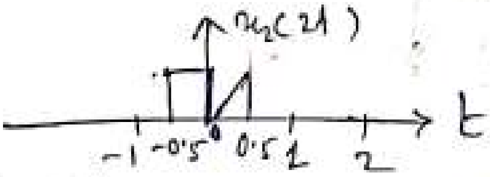
Ans

i) $x_1(t-2)$
$t=2$ →



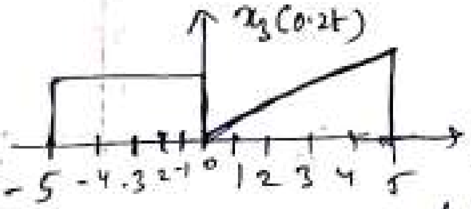
Signal is delay by 2[illegible]

(ii) $x_2(2t)$ →



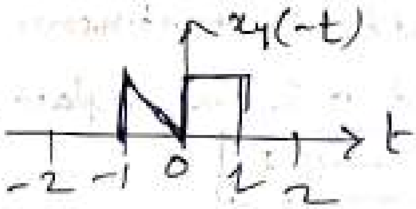
Signal is compressed by 2.

(iii) $x_3(0.2t)$
$x_3(t/5)$ →



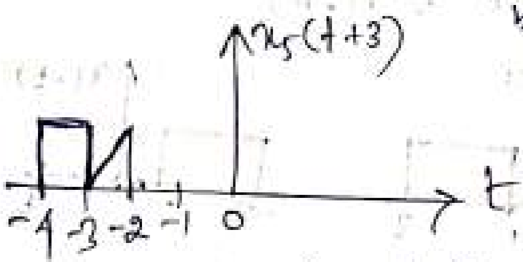
Signal is Expand by 1/5.

(iv) $x_4(-t)$ →



Signal is mirror image of original signal by vertical axis.

v) $x_5(t+3)$
$t=-3$ →



Signal is advanced by t+3.

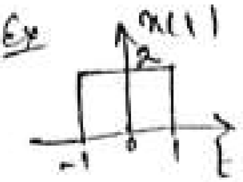
* Amplitude Scaling

CT

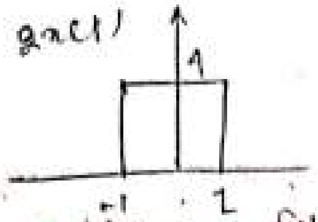
→ $x(t) = \underline{a\,x(t)}$ Same as in DTS

$\underline{x[n] = a\,x[n]}$

Ex



$\frac{1}{2}x(t) = ?$

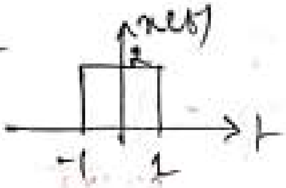


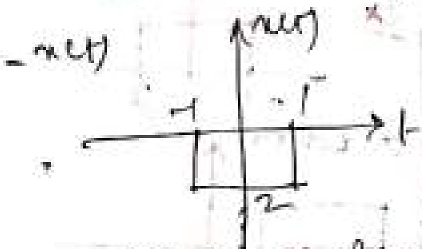
* Amplitude folding

$x(t) = -x(t)$ Same as in DTS

$x[n] = -x[n]$

Ex





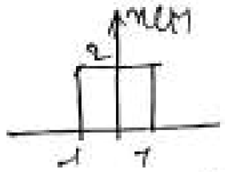
* Amplitude Shifting

CTS $x(t) = \pm K \pm x(t)$ DTS $x[n] = K \pm x[n]$

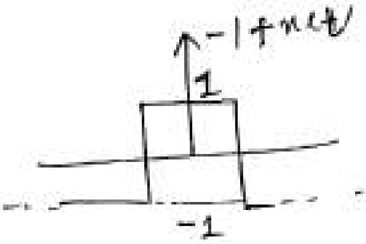
when
k<0

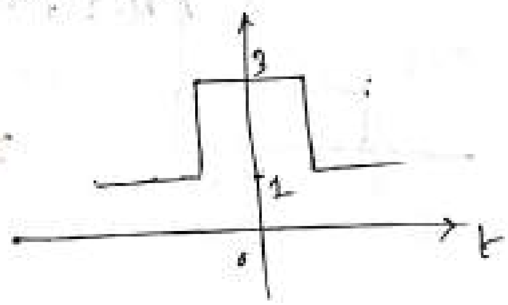
k = -1

Ex

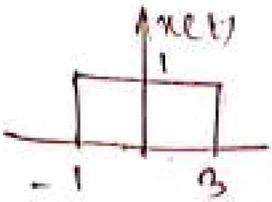


$-1 + x(t) = ?$ (ii) $1 + x(t) = ?$





Q



Draw Signal

i) x(2t+8) = ?

ii) x(-2t+4) = ?

Sol

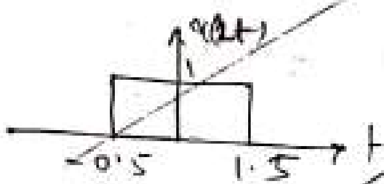
i) x(2t+8) = ?

x(2(t+4)) = ?

Step to be followed always remember

~~Scaling → Shifting → folding~~ | Shifting → Scaling → folding

First is Scaling

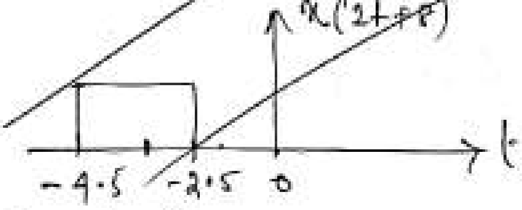


Then Shifting

t+4 = 0

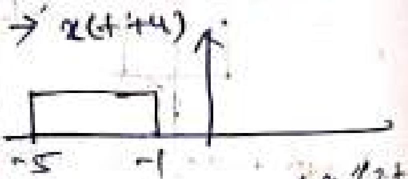
t = -4

shifting by -4

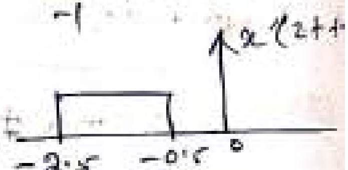


Ex

→ x(t+4)



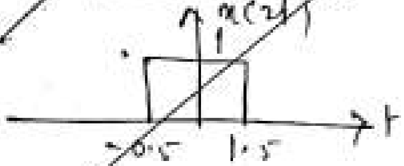
→



(ii) x(-2t+4) = ?

→ x(-2(t-2)) = ?

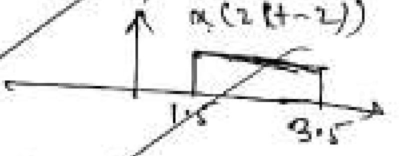
First Scaling



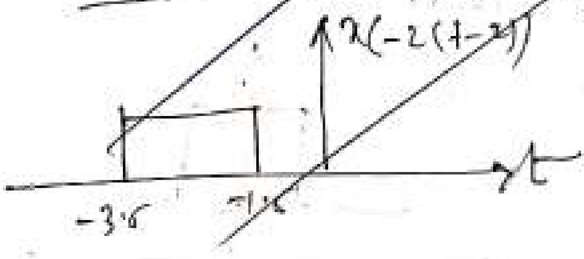
shifting

t-2 = 0

t = 2



folding



⑤ Addition of Signal

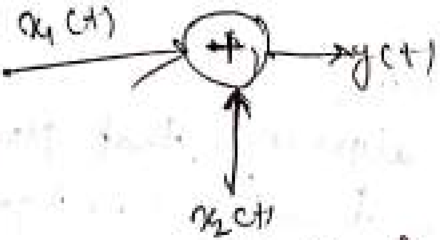
→ if $x_1(t)$ and $x_2(t)$ are the two CTS then

$$\boxed{y(t) = x_1(t) + x_2(t)}$$

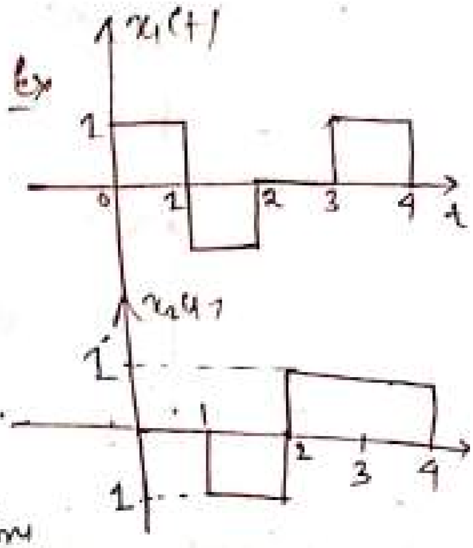
In case of DTS

$$\boxed{y(n) = x_1(n) + x_2(n)}$$

Addition is represented as

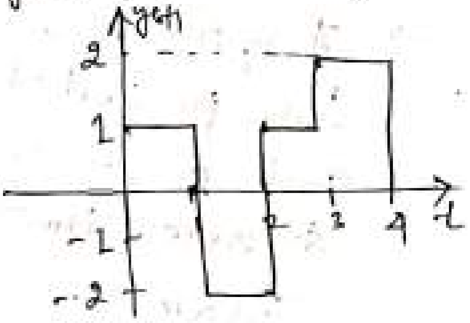


Ex



Ans

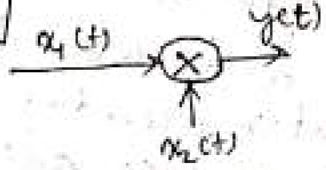
$y(t) = x_1(t) + x_2(t)$



⑥ multiplication of Signal

If $x_1(t)$ and $x_2(t)$ are the two CTS then y(t) resulting for the multiplication of the signals is

$$\boxed{y(t) = x_1(t) \cdot x_2(t)}$$

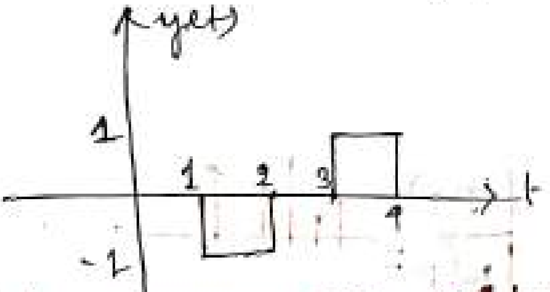


DTS

$$\boxed{y(n) = x_1(n) \cdot x_2(n)}$$

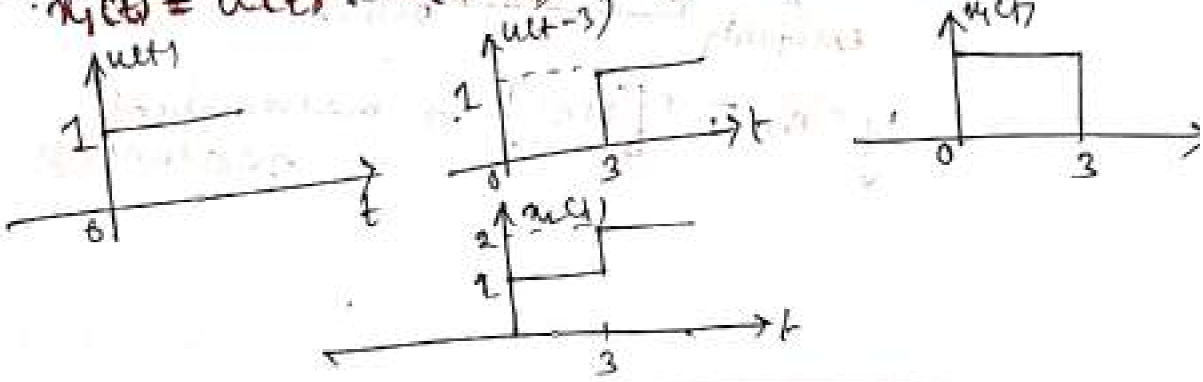
Ex we take above example

$y(t) = x_1(t) \times x_2(t)$

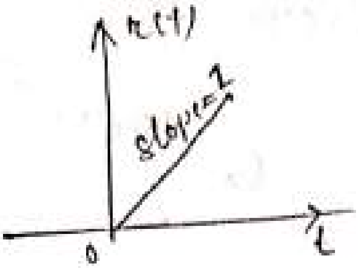


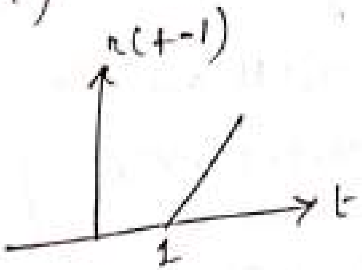
Q plot $x_1(t) = u(t) - u(t-3)$ & $x_2(t) = u(t) + u(t-3)$

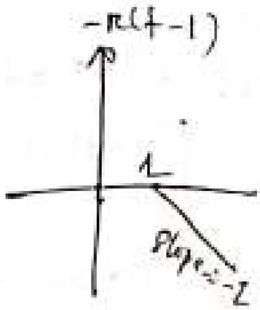
Ans

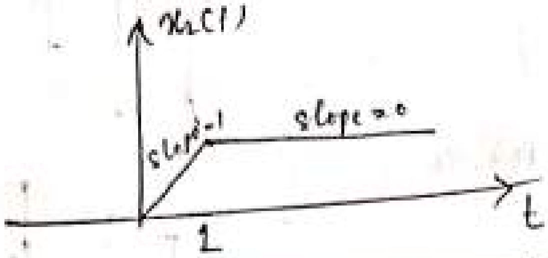


Q $x_2(t) = r(t) - r(t-1)$





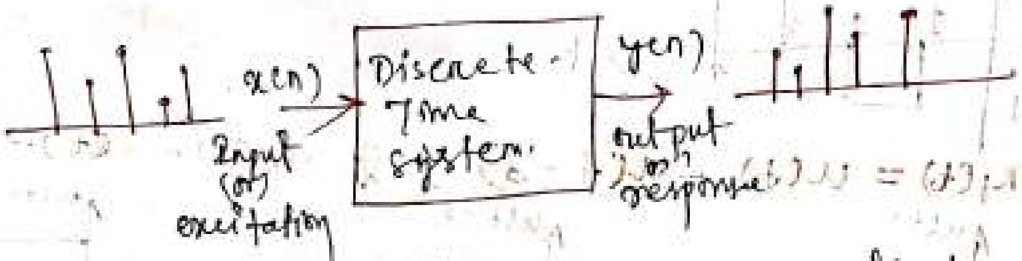




2.2 Discuss Discrete Time System :-

→ A system is a device or an algorithm that perform some prescribed operation on a discrete - time signal. Such a device or algorithm ~~that operates on a discrete - time signal called~~ is called Discrete - time system.

→ Discrete - time ~~signal~~ System is a device or algorithm that operates on a discrete - time signal called input or excitation, according to some well-defined rule to produce another discrete - time signal called the output or response of the system.



$y(n) \equiv T[x(n)]$ → mathematical relationship

2.2.1 Input - Output Description of System

→ the input - output description of a discrete - time system consists of a mathematical expression or a rule. defines input & output signal / relationship

$$x(n) \xrightarrow{T} y(n)$$

Ex determine the response of the following system to the input signal.

$$x(n) = \begin{cases} |n| , & -3 \leq n \leq 3 \\ 0 , & \text{otherwise} \end{cases}$$

An

i) $y(n) = x(n)$

$x(n) = \{ \ldots 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots \}$

(ii) $y(n) = x(n-1)$

$\{ \ldots 0, 3, 2, \underset{\uparrow}{1}, 0, 1, 2, 3, 0, \ldots \}$

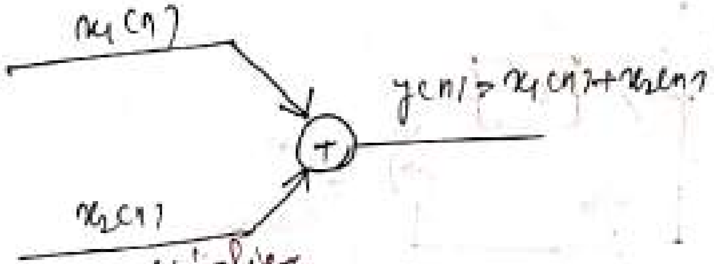
(iii) $y(n) = x(n+1)$

$\{ \ldots 0, 3, 2, 1, 0, \underset{\uparrow}{1}, 2, 3, 0, \ldots \}$

2.2.2 Block Diagram representation of DTS

- An adder

a system adder that perform the addition of two signal Sequence to form another (the Sum) Sequence, in other Sequence to form another (the sum) sequence, in other words, the addition operation is memoryless.

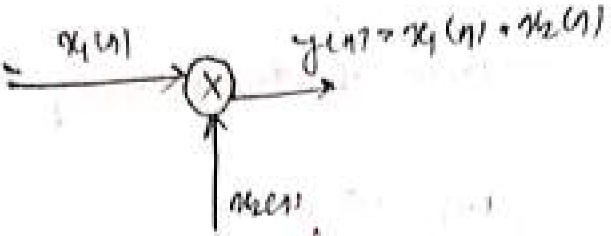


- A constant multiplier

Simple represent applying a Scale factor on the input x(n)

* A signal multiplier

product of two signal



* A unit delay element

→ the unit delay is a special system that simply delays the passing through it by one sample x(n) inputs the output x(n-1) from memory at time n to from

→ it is called y(n) = x(n-1), This basic building block requires memory.

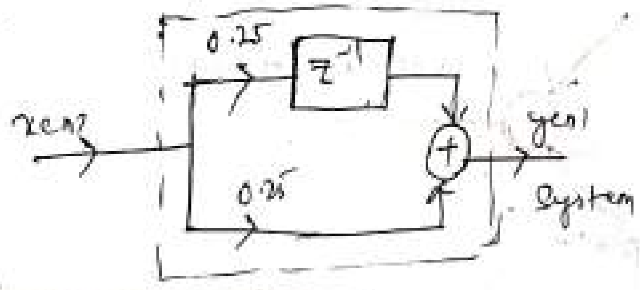
* A unit advance element

~~we can recall any sample of any t.~~

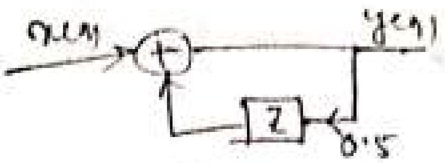
Such advance element is physical impossible in real time since infact of involved looking in to the future of the signal.

Ex Sketch the block diagram representation of the DTS $y(n) = \frac{1}{4}[x(n) + x(n-1)]$

An



Ex $y(n) = x(n) + \frac{1}{2} y(n+1)$



2.2.3 Classify Discrete time System

① Static and Dynamic System

→ A DTS is called static or memory less if it's output depends on at any instant 'n' depends on the inputs samples at the same time, but not on past or further samples of the input otherwise Dynamic system.

Ex (i) $y(t) = 2x(t)$

$t=0$

$y(0) = 2x(0)$

$y(1) = 2x(1)$

$y(2) = 2x(2)$

This signal is static.

(ii) $y(n) = x(n) + x(n-1) + x(n-2)$

$n=0$

$y(0) = x(0) + x(-1) + x(-2)$

$-1, -2$ are past value

~~So~~ This signal is Dynamic.

② Causal and Non Causal System

→ A System is said to be causal if the o/p of the System is independent of future values of i/p of the System is dependent or if the o/p of the system is dependent only on the present and past value of the i/p.

→ A System is said to be non-causal if the o/p at any instant of time depends on the future value of the i/p signal.

Ex i) $y(t) = x(t)$ → causal.

$t=0$

$y(0) = x(0)$

$y(1) = x(1)$

this is depends on present value is called causal.

(ii) $y(t) = x(t + x(t-1)$

$t=0$

$y(0) = x(0) + x(-1)$

$t=1$

$y(1) = x(1) + x(0)$

This is depends past value is called Causal

(iii) $y(t) = 2x(t) + x(t+1)$

$t=0$

$y(0) = 2x(0) + x(1)$

$y(1) = 2x(1) + x(2)$

this is depends on Future value so this Non-Causal system

* Anti-causal system

If the output of the system depends only on the future value of i/p. this is exactly opposite to causal system.

Ex. $y(t) = x(t+2)$

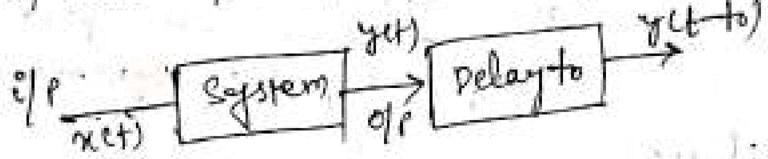
$y(0) = x(2)$

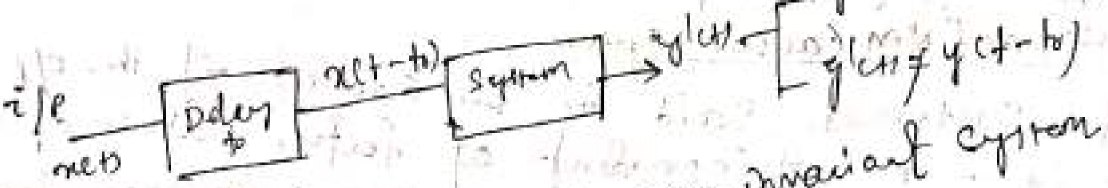
$y(1) = x(3)$

Depends on future value is called anticausal.

3) Time variant and Time invariant system

This property of systems is very important in LTI system.





i) if $y'(t) = y(t-t0)$ ⇒ Time invariant system

ii) if $y'(t) \neq y(t-t0)$ ⇒ Time variant system

Ex i) $y(t) = x(2t)$

Check whether time variant or time invariant

Sol

$$y(t) \xrightarrow{t_0} y(t-t_0) = x(2(t-t_0))$$
$$= x(2t-2t_0) \longrightarrow ①$$

$$x(t) \xrightarrow{t_0} x(t-t_0) \longrightarrow \boxed{\text{System}} \longrightarrow x(2(t)-t_0)$$

$$y(t) = x(2t-t_0) \longrightarrow ②$$

Comparing ① & ②

$y(t) \neq y(t-t_0)$

System Time variant.

Note: Always time Scaling → Time variant.

: Amplitude Scaling → Time invariant.

Ex $y(t) = 2+x(1)$

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t) = 2+x(1)$$

Check whether time variant or Time invariant.

Sol

$$y(t) \xrightarrow{t_0} y(t-t_0) = 2+x(t-t_0) \quad — ①$$

$$x(t) \xrightarrow{t_0} x(t-t_0) \longrightarrow \boxed{\text{S/m}} \rightarrow y(t) = 2+x(t-t_0) \quad — ②$$

Cmp: ① & ②

$y(t) = y(t-t_0)$

The given system is time invariant.

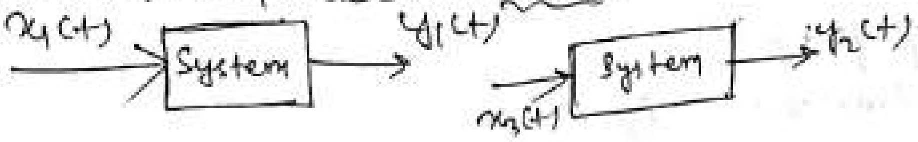
4) Linearity or Non-linearity

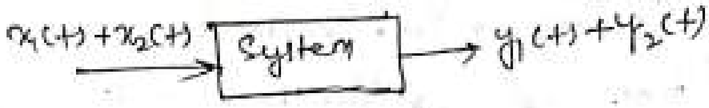
→ A system is said to be linear interms of the system input excitation x(t) and the system output response y(t) if it satisfies the following propert i.e Law of Superposition and law of homogenity

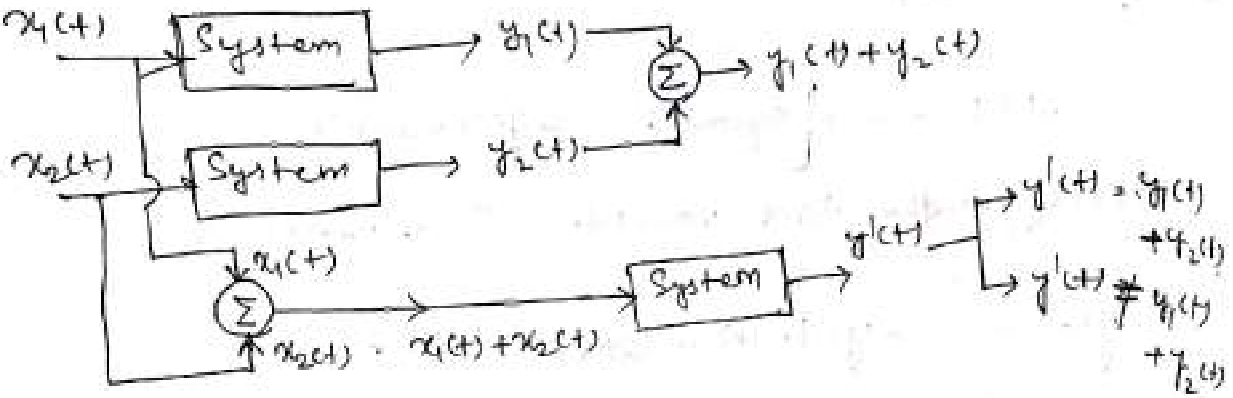
1) law of Superposition is also called as law of additivity [LOA)
2) Law of Homogenity is also called as law of multiplication (LOM) or Scalar multiplication.

→ A system which does not satisfies any of the above proper then it is called as non-Linear System

Law of Superposition [LOA]





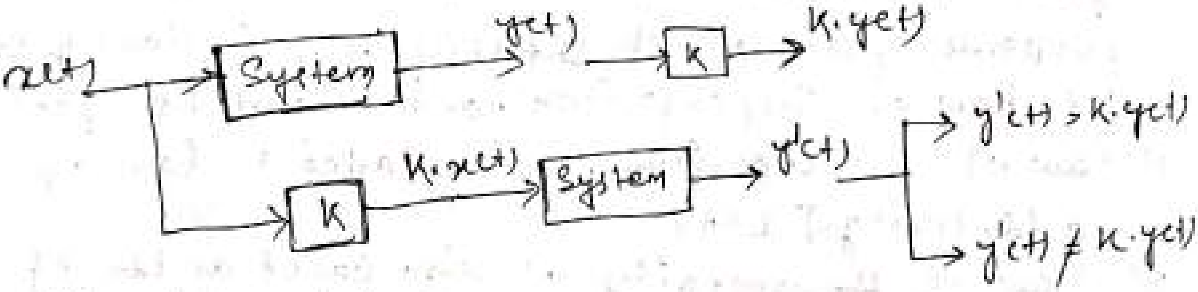


System is Same in all cases

if $y'(t) = y_1(t) + y_2(t) \Rightarrow$ system is following law of additivity.

$y'(t) \neq y(t) + y_2(t) \Rightarrow$ system is not following Law of additivity.

Law of Homogenity [LOH]



if $y'(t) = k\cdot y(t) \Rightarrow$ system is following law of Homogenity.

$y'(t) \neq K\cdot y(t) \Rightarrow$ System is not following Law of Homogenity.

Ex $y(t) = x(\sin t)$

1) Law of additivity

$$x(t) \longrightarrow \boxed{System} \longrightarrow y(t) = x(\sin t)$$

$y_1(t) = x_1(\sin t)$, $y_2(t) = x_2(\sin t)$

$y_1(t) + y_2(t) = x_1(\sin t) + x_2(\sin t) \rightarrow ①$

$x_1(t) + x_2(t) \longrightarrow \boxed{System} \longrightarrow y'(t) = x_1(\sin t) + x_2(\sin t)$. — ②

① = ② ⇒ LOA is followed.

2) LOH

$y(t) = x(\sin t)$

$k \cdot y(t) = k \cdot x(\sin t)$ — ①

$k \cdot x(t) \rightarrow \boxed{System} \longrightarrow y'(t) = k \cdot x(\sin t)$ — ②

① = ② ⇒ LOH is followed.

Both law of Superposition and law of Homogeneity are followed. Hence the given System is linear system.

⑤ Stability

A System is Said to be bounded input, bounded o/p [BIBO] Stable if and only if every bounded input results in bounded o/p. The output of such a System does not diverge if the input does not diverge.

i.e for input $|x(t)| \leq M_x < \infty$ for all.

for o/p $|y(t)| \leq M_y < \infty$ for all.

M_x & M_y represents to finit positive number

Ex- For bounded Signals are dc value, Sin t, cos t, u(t) constant -- -1 to 1, 1 to -1, 0 or 1.

Problem

* Find whether the following Signals are Static or Dynamic

1) $y(n) = \log\cdot[x(n)]$.

$n=0$

$y(0) = \log[x(0)]$

$n=1$

$y(1) = \log[x(1)]$

$n=2$

$y(2) = \log[x(2)]$

System is static.

2) $y(n) = x(n)\cdot x(n-1)$:

$n=0$

$y(0) = x(0)\cdot x(-1)$

$n=1$

$y(1) = x(1)\cdot x(0)$

$n=2$

$y(-1) = x(-1)\,x(-2)$

it shows past value this systems Dynamic

3) $y(n) = x^2(n) + x(n)$: Static

4) $y(n) = 4\,x(n)$: Static

5) $y(n) = \sum_{k=0}^{\infty} x(n-k)$: Dynamic

6) $y(n) = e^{x^2(n)}$: Static

* Test whether the following Systems are time invariant:

1) $y(n) = x(n) - x(n-1)$

$$x(n) \longrightarrow \boxed{S/m} \longrightarrow y(n) = x(n) - x(n) \longrightarrow \boxed{\text{Delay } n_0} \longrightarrow x(n-n_0) - x(n-1-n_0)\ ①$$

$$x(n) \longrightarrow \boxed{\text{Delay } n_0} \longrightarrow x(n-n_0) \longrightarrow \boxed{\text{System}} \longrightarrow x(n-n_0) - x(n-n_0-1)\ ②$$

① = ② System is time invariant

② $y(n) = n x(n)$

$$x(n) \longrightarrow \boxed{\text{System}} \longrightarrow y(n) \longrightarrow \boxed{\text{Delay } n_0} \longrightarrow y(n-n_0)$$

$$= (n-n_0)\, x(n-n_0) \rightarrow ①$$

$$x(n) \longrightarrow \boxed{\text{Delay } n_0} \longrightarrow x(n-n_0) \longrightarrow \boxed{\text{System}} \longrightarrow y(n) = n x(n-n_0) \rightarrow ②$$

① ≠ ②

The given system is time variant.

\# Determine whether the following systems are linear or Non-linear

1) $y(n) = n \cdot x(n)$

$a x_1 \rightarrow y_1(n) = n a x_1(n)$

$b x_2 \rightarrow y_2(n) = b n x_2(n)$

$y_1(n) + y_2(n) = n a x_1(n) + n \cdot b x_2(n)$

$= n\{a x_1(n) + b x_2(n)\} \rightarrow ①$

$$a x_1(n) + b x_2(n) \longrightarrow \boxed{S/m} \longrightarrow y(n) = n\{x(n)\}$$

$= n\{a x_1(n) + b x_2(n)\} \rightarrow ②$

① = ②

The system is linear

② $y(n) = x(n^2)$

$a x_1(n) \longrightarrow y_1(n) = a x_1(n^2)$

$b x_2(n) \longrightarrow y_2(n) = b x_2(n^2)$

$y_1(n) + y_2(n) = a x_1(n^2) + b x_2(n^2) \rightarrow ①$

$x(n) = a x_1(n) + b x_2(n)$

$$x(n) \longrightarrow \boxed{S/m} \longrightarrow y(n) = x(n^2) = [a x_1(n^2) + b x_2(n^2)]^2 \rightarrow ②$$

① ≠ ② system is non linear

⑤ $y(n) = x^2(n)$

$a_1 x_1(n) \rightarrow y_1(n) = a x_1^2(n)$

$b_1 x_2(n) \rightarrow y_2 = b x_2^2(n)$

$y_1(n) + y_2(n) = a x_1^2(n) + b x_2^2(n)$

$$x(n) \rightarrow \boxed{S/m} \rightarrow y(n) = x^2(n) = [a_1 x_1(n) + b_1 x_2(n)]^2 \rightarrow \bigcirc$$

1 ≠ 2 system is non linear.

④ $y(n) = A x(n) + B$

$a x_1(n) \longrightarrow a A x_1(n) + B$

$b x_2(n) \longrightarrow b A x_2(n) + B$

$y_1(n) + y_2(n) = A a x_1(n) + B + A b x_2(n) + B$

$= A[a x_1(n) + b x_2(n)] + 2B \rightarrow$ ②

$x(n) = a x_1(n) + b x_2(n)$

$\xrightarrow{x(n)} \boxed{\text{System}} \longrightarrow y(n)$

$y(n) = A\{a x_1(n) + b x_2(n)\} + B \longrightarrow$ ②

① ≠ ② The system is Non lin

⑤ $y(n) = e^{x(n)}$

~~$a x(n) = a e^{x_1(n)}$~~

$a x_1(n) \rightarrow y_1(n) = a e^{x_1(n)}$

$b x_2(n) \rightarrow y_2(n) = b e^{x_2(n)}$

$y_1(n) + y_2(n) = a e^{x_1(n)} + b e^{x_2(n)}$ — ①

$x(n) = a x_1(n) + b x_2(n)$ —

$x(n) \longrightarrow \boxed{S/m} \longrightarrow y(n)$

$y(n) = e^{a x_1(n)} + e^{b x_2(n)}$ — ②

$1 \neq 2$

Non linear.

* Determine whether the following systems are causal or Non causal

---

1) $y(n) = x(n) + \frac{1}{x(n-1)}$: causal.

2) $y(n) = x(n^2)$: causal.

3) $y(n) = x(n) - x(n-1)$: causal.

4) $y(n) = \sum_{k=-\infty}^{\infty} x(k+n)$ = causal

⑤ $y(n) = A x(n)$ = causal

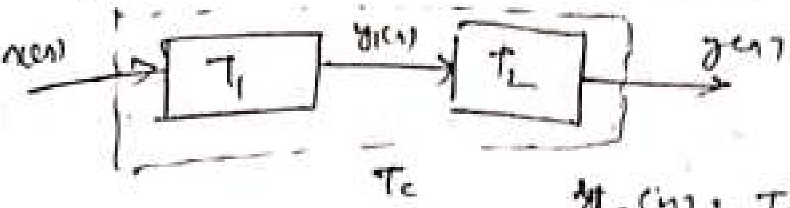
⑥ $y(n) = x(n) + 3(x+4)$ = Non causal.

⑦ $y(n) = x(2n)$. Non causal.

2.2.4 Interconnection of Discrete-Time System

Two types

Cascade (series) →



$y_n(n) = T_c[x(n)]$

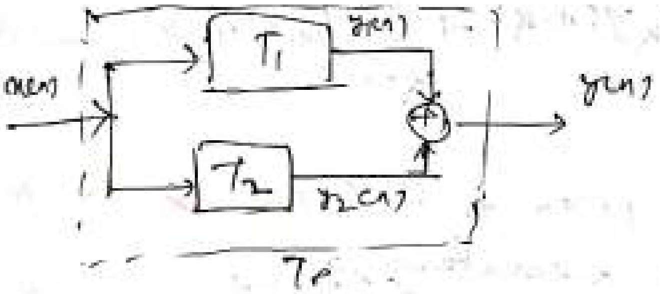
$y_1(n) = T_1[x(n)]$

$y(n) = T_2[y_1(n)]$

$= T_2[T_1[x(n)]]$

$\boxed{T_c \equiv T_1 T_2}$, But $\boxed{T_2 T_1 \neq T_1 T_2}$

Parallel



$y(n) = T_p[x(n)]$

$y_1(n) = T_1[x_1(n)]$

$y_2(n) = T_2[x_2(n)]$

$y(n) = y_1(n) + y_2(n)$

$= T_1[x_1(n)] + T_2[x_2(n)]$

$T_p[x(n)] = x(n)[T_1 + T_2]$

$\boxed{T_p \equiv T_1 + T_2}$

2.3 Discrete-Time Linear Time-invariant System

→ LTI - combination of both linear and time invariant Systems.

Linear - which satisfies both superposition & law of homogenity.

Time invariant - Any delay in input will reflect in output.

2.3.1 Techniques for the analysis of linear System

There are two basic method linear System input-output equation

First method →

$$y(n) = F[y(n-1), y(n-2), \dots y(n-N), x(n), x(n-1), x(n-2) \dots x(n-M)].$$

Specifically for an LTI system.

$$y(n) = -\sum_{K=1}^{N} a_K y(n-K) + \sum_{K=0}^{M} b_K x(n-K) \quad ①$$

Second method →

Linear system to a given to the system in the input Signal in to Sum of elementary Signal.

So, that the input signal x(n) is resolved in to weighted Sum of elementary Signal components $[x_K(n)]$. So that

$$x(n) = \sum_{K} c_K x_K(n)$$

c_K are set of amplitudes (weighting Co-efficients of the Signal x(n), now elementary signal component are $x_K(n)$ is $y_K(n)$

$$y_K(n) \equiv T[x_K(n)]$$

$$y(n) = T[x(n)] = T\left[\sum_K c_K x_K(n)\right]$$

$$= \sum_{K} C_k T[x_k(n)]$$

$$= \sum_{K} C_k y_k(n)$$

2.3.2 Resolution of a Discrete-Time Signal in to impulse

~~x(n)~~ we have an arbitrary signal x(n) that we wish to resolve in to a sum of unit sample sequence. we select the elementary signal $x_k(n)$

$$x_k(n) = \delta(n-k)$$

where K represent the delay of the unit sample sequence. To handle an arbitrary signal x(n).

~~resolution of a D~~

$\delta(n-k)$ is zero every where except $n=k$ where it is value unity. the result of this multiplication is another sequence that is zero every where except at $n=k$

$$x(n)\,\delta(n-k) = x(k)\,\delta(n-k)$$

the Sequence is

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)}$$

Q/ consider the special case of a finite-duration sequence given as

$$x(n) = \{2, 4, 0, 3\}$$

Resolve the sequence x(n) in to a sum of weighted impulse sequence.

Ans

$$n = -1, 0, 1, 2$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) = \sum_{k=-1}^{2} x(k)\,\delta(n-k)$$

$$= x(-1)\delta(n+1) + x(0)\delta(n+0) + x(1)\delta(n-1) + x(2)\delta(n-2)$$

$$= 2\delta(n+1) + 4\delta(n) + 0.\delta(n-1) + 3\delta(n-2)$$

2.3.3 Response of LTI system to arbitary input : the convolution Sum (Theorem)

→ arbitrary input Signal $x(n)$ in to a weighted Sum of impulse. now ~~we~~ ready to determine the respons of any relaxed lihan

we denote response $y(n, K)$ of the System to the input unit Sample Sequence $n = k$ by Special Symbol $h(n,k)$, $-\infty < k < \infty$

$$y(n,k) \equiv h(n,k) = T[\delta(n-k)]$$

n = time index

k = ~~[illegible]~~ location of the input impulse.

Expression of Sum weighted impulse

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right]$$

$$y(n) = \cancel{T\sum_{k=-\infty}} \sum_{k=-\infty}^{\infty} x(k)\,T[\delta(n-k)]$$

$\therefore C_k = x(k)$ - Scaled co-efficient

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)} \quad \text{——} ①$$

$\therefore h(n) \equiv T[\delta(n)]$

we Satisfies the Superposition property.

Now we have to Time invariant system

$$h(n) \equiv T[\delta(n)]$$

$$A(n-k) = T[\delta(n-k)$$

above eq^n ①

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)}$$

* The formula giv the respons y(n) of the LTI ~~sys~~ System, as input Signal x(n) & the unit ~~sa~~ impulse respon h(n) is called Convolution Sum.

Suppose that we wish to compute of the System Some time instant Say $n = n_0$

$$\boxed{y(n_0) = \sum_{k=-\infty}^{\infty} x(k).\, h(n_0-k)}$$

So we have to follow four steps

1. Folding :- Fold h(k) about k=0 to obtain h(-k)
2. Shifting :- shift h(-k) by n_0 to the right ~~(left)~~ if n_0 is positive, then if ~~no to k~~ shift h(-k) by ~~no of the~~ n_0 to the left, if n_0 is negative, to obtain $h(n_0-k)$.
3. multiplication :- multiply x(k) by $h(n_0-k)$ to obtain product of sequence.

$$\boxed{v_{n_0}(k) \equiv x(k)\, h(n_0-k)}$$

4. Summation :- Sum all the values of the product sequence $v_{n_0}(k)$ to ~~at~~ obtain the value of the output at time $n = n_0$. , $-\infty < n < \infty$.

Q The impulse response of a linear time-invariant System is

$$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$$

Determine the response of the System to the input signal

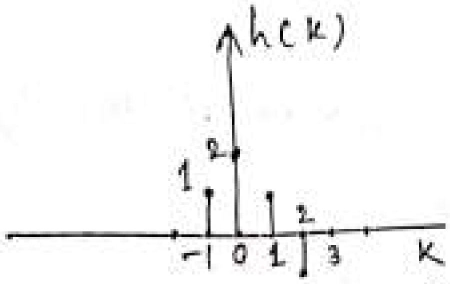
$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

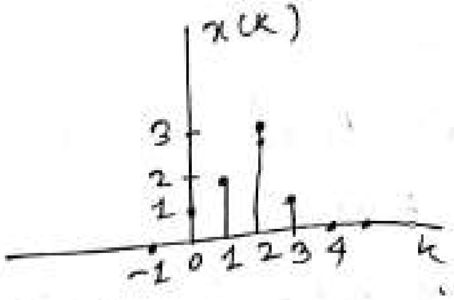
Ans

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

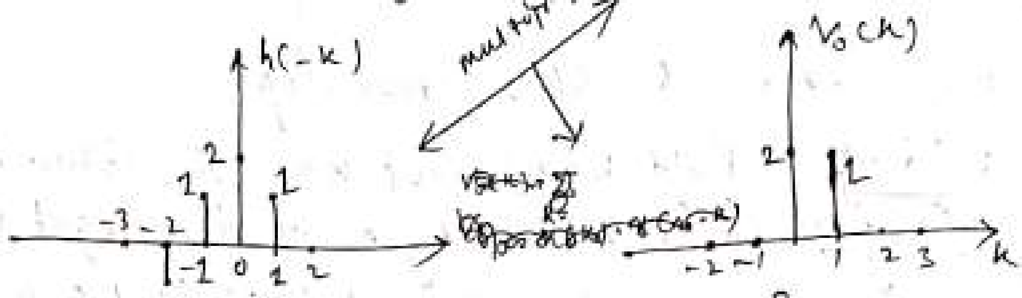
$n = 0$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(-k)$$





$\overset{n = n_0 = 0}{v_0 = x(k) \cdot h(0-k)}$



$V_0(k) = \{0, 0, \underset{\uparrow}{2}, 2, 0, 0\}$

Sum of all terms in the product sequence $= 4$.

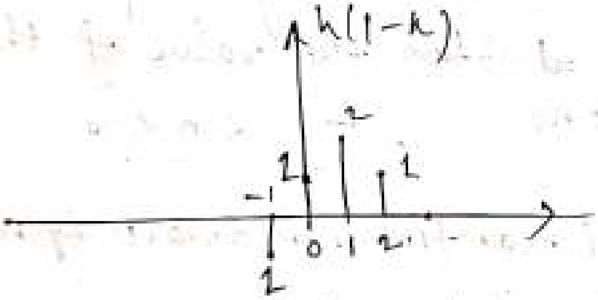
Then continue $n = 1$

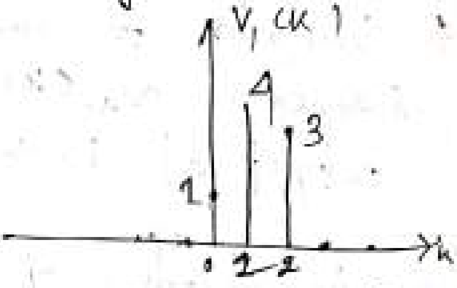
$$y(1) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(1-k)$$

~~Shift by n_0 ... $h(-(k-1))$~~

~~$= (\sum_{k=-\infty} x(k)$ ... shift~~

$n_0 = +1$ the $h(-k)$ Shift by 1 to right



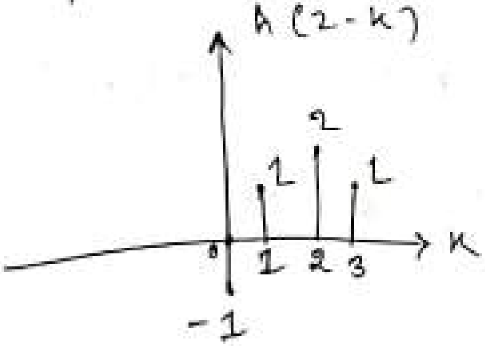


Then $V_1(k) = x(k) \cdot h(1-k)$

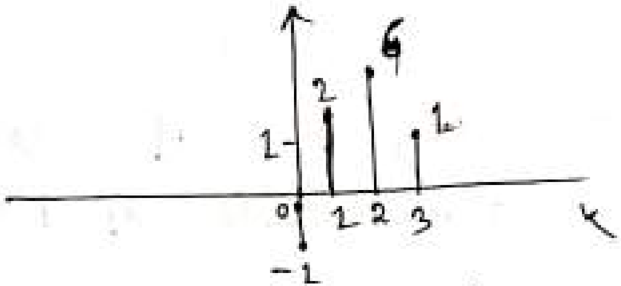
Sum of all terms in the product Sequen.

$= 8$

$n_0 = 2.$



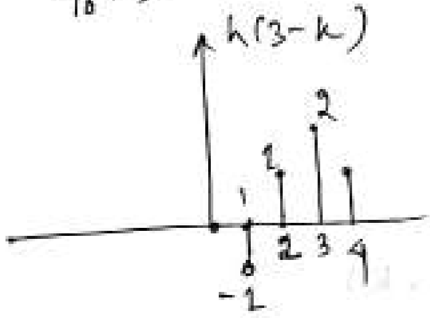
$V_2(k) = x(k) \cdot h(2-k)$



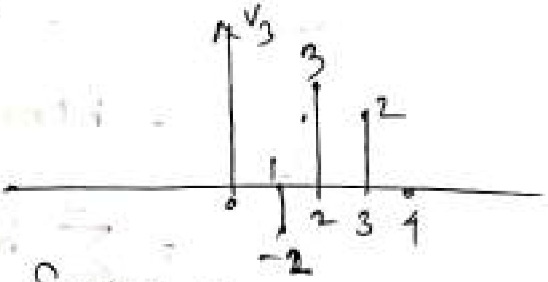
Sum of all terms in the product sequence

$= 9 - 1 = 8.$

$n_0 = 3$

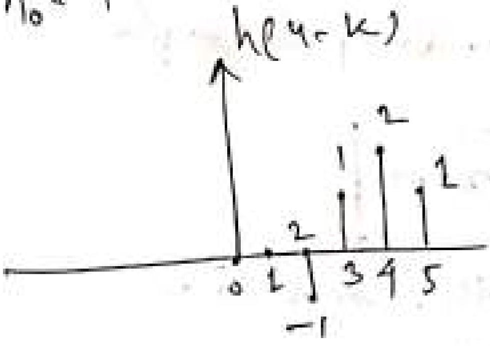


$V_3(k) = x(k) \cdot h(3-k)$

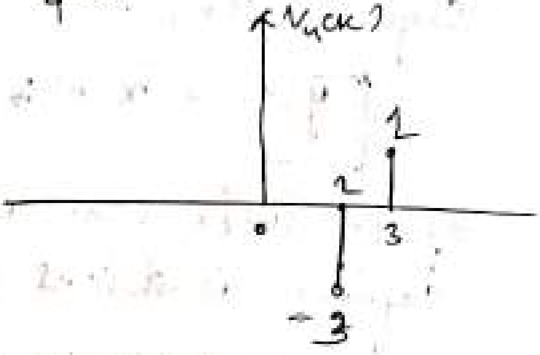


Sum = 3

$n_0 = 4$

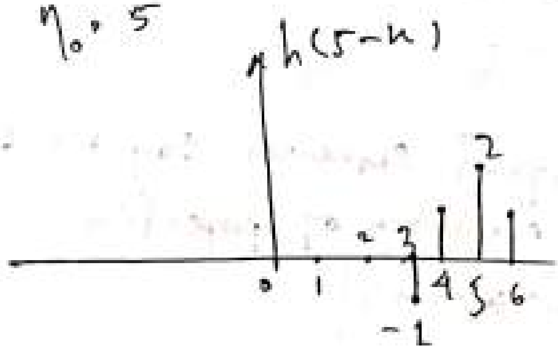


$V_4(k) = x(k) \cdot h(4-k)$

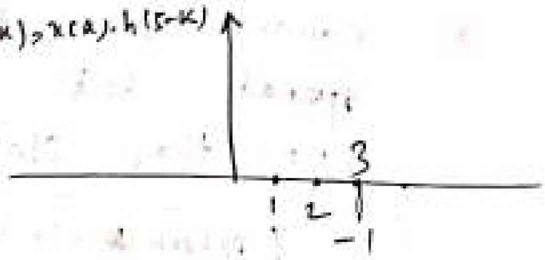


Sum = −2.

$n_0 = 5$

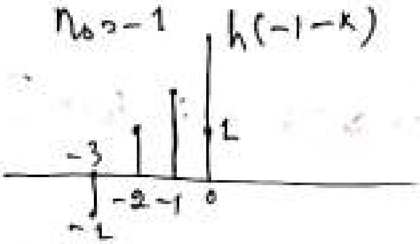


$V_5(k) = x(k).h(5-k)$

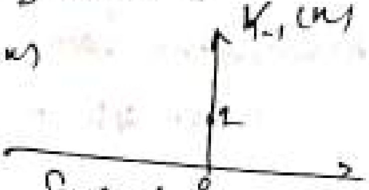


Sum = −1.

$n_0 = -1$



$V_{-1}(k) = x(k) \cdot h(-1-k)$



Sum = 1

Now we have the entire response.

$$y(n) = \{ \ldots 0, 0, 1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1, 0 \ldots \}$$

2.3.4 properties of convolution & the interconnection of LTI

$$\boxed{y(n) = x(n) * h(n) = \sum_{K=-\infty}^{\infty} x(K) \cdot h(n-K)}$$

$$y(n) = h(n) * x(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

* identity & shifting properties

we also note that the unit sample sequence $\delta(n)$.

$$\boxed{y(n) = x(n) * \delta(n) = x(n)}$$

if we shift $\delta(n)$ by K, the convolution sequence is shifted also by K.

$$\boxed{x(n) * \delta(n-K) = y(n-K) = x(n-K)}$$

* commutative law

operation betn two signal sequences say $x(n)$ & $h(n)$, that statisfies a number of properties.

$$\boxed{x(n) * h(n) = h(n) * x(n)}$$

* Associative law

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

Distributive law

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

2.3.7 System with finite Duration & infinite-Duration Impulse response

→ a linear time-invariant system in term of it's impulse response h(n). it is also convenient however to subdivide the class of linear time-invariant system in to two types finite-duration impulse response (FIR) & that have an infinite-duration impulse response (IIR).

For causal FIR system

$$h(n) ,\quad n<0 \ \& \ n \geq M$$

$$\boxed{y(n) = \sum_{k=0}^{M-1} h(k) \cdot x(n-k)}$$

for ~~Causal~~ IIR

$$\boxed{y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)}$$

2.4 Discrete-Time System Described by Difference Equations

we have treated as linear & time-invariant system unit sample response h(n) & to determine output y(n) of the system for any given input sequence x(n),

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)}$$

FIR System is readily implemented by convolution sum. & IIR however it's practical implementation by convolution sum.

2.9.1 Recursive & Non recursive Discrete-Time System

Suppose that we wish to compute the cumulative average of a signal $x(n)$ in the interval $0 \le k \le n$.

$$y(n) = \frac{1}{n+1} \sum_{k=0}^{n} x(k) \quad , n = 0, 1, \ldots$$

the computation of $y(n)$ requires the storage of all the input samples $x(k)$ for $0 \le k \le n$.

→ $y(n)$ can be computed more efficiently by utilizing the previous o/p value $y(n-1)$

$$(n+1)\, y(n) = \sum_{k=0}^{n-1} x(k) + x(n)$$

$$= n\, y(n-1) + x(n)$$

$$\boxed{y(n) = \frac{n}{n+1} y(n-1) + \frac{1}{n+1} x(n)}$$

2.9.9 The impulse response of a Linear-Time Invariant Recursive System

→ 1st order recursive system, the zero-state response given

$$y_{zs}(n) = \sum_{k=0}^{n} a^k x(n-k)$$

when $x(n) = \delta(n)$.

$$y_{zs}(n) = \sum_{k=0}^{n} a^k \delta(n-k)$$

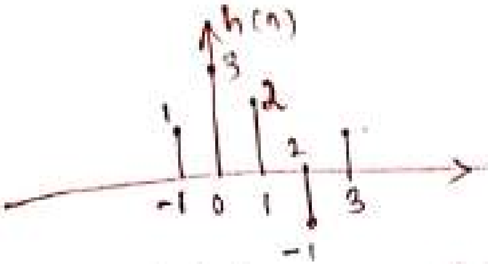
$$= a^n , \quad n \geq 0$$

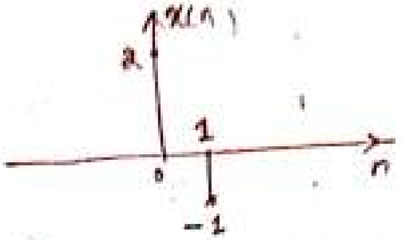
Hence $h(n) = a^n u(n)$

for Linear time invariant system

$$y_{zs}(n) = \sum_{k=0}^{n} h(k) \cdot x(n-k) \quad n \geq 0$$

Q $x(n) = 2\delta(n) - \delta(n-1)$, $h(n)$ is as shown in fig





Calculate convolution Sum.

Q.) Find the convolution of 2 sequence $x_1(n)$ & $x_2(n)$.

$x_1(n) = \{\underset{\uparrow}{1}, 2, 3\}$, $x_2(n) = \{\underset{\uparrow}{2}, 1, 4\}$

Sol

Sol^n length of $x_1(n) = L_1 = 3$

$x_2(n) = L_2 = 3$

length of $y(n) = L_1 + L_2 - 1 = 3+3-1 = 5$

$x_2(n)$ \ $x_1(n)$	↓ 1	2	3		
→ 2	2	4	6		
1	×	1	2	3	
4	×	×	4	8	12
	2 .	5	12	11	12

$y(n) = \{\underset{\uparrow}{2}, 5, 12, 11, 12\}$

Q $x_1(n) = \{\underset{\uparrow}{1}, 2, 3\}$

$x_2(n) = \{1, 2, \underset{\uparrow}{3}, 4\}$

Convolute the two sequence.

Soln

$y(n) = x_1(n) * x_2(n)$

$x_1(n)$ / $x_2(n)$	$\overset{\downarrow}{1}$	2	3			
1	1	2	3			
2	x	2	4	6		
→ 3	x	x	3	6	9	
4	x	x	x	4	8	12
	1	4	10	16	17	12

$y(n) = \{1, 4, \underset{\uparrow}{10}, 16, 17, 17\}$

Q Obtain the graphical convolution of a discrete Linear time invariant System for input $x(n)$ shown in fig. The System impulse response is $h(n)$ as shown in fig.

$x(n) = \{-1, 1, 0, 1, -1\}$, $h(n) = \{1, \underset{\uparrow}{2}, 3\}$

CHAPTER - 3

The Z - Transform & it's application to the analysis of LTI System.

3.1 The Z - Transform:-

⇒ The Z-transform play the same role in the analysis of discrete - time signal & LTI System. in the frequency domain.

⇒ In Z- domain, the convolution of 2-time domain signal is equal to the multiplication of their corresponding Z- transform.

$$\boxed{X(Z) = \sum_{-\infty}^{\infty} x(n)\, z^{-n}} \quad — ①$$

where Z is a complex variable

$$\boxed{Z = re^{j\omega}}$$

$$X(Z) = Z\{x(n)\}$$

relationship betn $x(n)$ & $X(Z)$:

$$x(n) \xleftrightarrow{Z} X(Z)$$

Region of Convergence:-

The region of convergence of $X(Z)$ is all the set of all values of Z for which $X(Z)$ attains a finite value.

$-\infty$ to ∞ = Two sided Z-Transform

Ex → $x(n) = \{-1, \underset{\uparrow}{2}, 1, 3, 4\}$

0 to ∞ = one Sided Z - Transform.

Ex → $x(n) = \{\underset{\uparrow}{1}, 0, 3, -1, 2\}$

Ex
Q Find of Causal Sequence in to Z- Transform.

$x(n) = \{\underset{\uparrow}{1}, 0, 3, -1, 2\}$ → (Right hand Side Signal)

Ans

$$X(Z) = \sum_{-\infty}^{\infty} x(n) \cdot z^{-n} = \sum_{0}^{4} x(n) \cdot z^{-n}$$

$$= x(0) z^{-0} + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + x(4) z^{-4}$$

$$= 1 + 0 \times z^{-1} + 3 \cdot z^{-2} + (-1) z^{-3} + 2z^{-4}$$

$$= 1 + 3z^{-2} - z^{-3} + 2z^{-4}$$

ROC: All values of z except zero, is the region of convergence.

Q) $x(n) = \{-3, -2, -1, 0, \underset{\uparrow}{1}\}$ → left hand Sided Signal

Ans Find Z-Transform.

$$X(z) = \sum_{-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{-\infty}^{0} x(n) \cdot z^{-n}$$

$$= x(0) \cdot z^{0} + x(-1) z^{1} + x(-2) z^{2} + x(-3) z^{3} + x(-4) z^{4}$$

$$= 1 + 0 \times z - 1 \cdot z^{2} - 2z^{3} - 3z^{4}$$

$$= 1 + -z^{2} - 2z^{3} - 3z^{4}$$

ROC: All value of z except $z = \infty$; is the region of convergence.

Q) $x(n) = \{2, -1, 3, 2, \underset{\uparrow}{1}, 0, 2, 3, -1\}$ → (Two Sided Signal)

Ans

$$X(z) = \sum_{-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{-4}^{4} x(n) z^{-n}$$

$$= x(-4) \cdot z^{4} + x(-3) z^{3} + x(-2) z^{2} + x(-1) z^{1} + x(0) z^{0}$$
$$+ x(1) z^{-1} + x(2) \cdot z^{-2} + x(3) z^{-3} + x(4) \cdot z^{-4}$$

$$= 2 \cdot z^{4} + (-1) \cdot z^{3} + 3z^{2} + 2z^{1} + 1 \cdot z^{0} + 0 \cdot z^{-1} + 2z^{-2} + 3z^{-3} + (-1) z^{-4}$$

$$= 2z^{4} - z^{3} + 3z^{2} + 2z + 1 + 2z^{-2} + 3z^{-3} - z^{-4}$$

ROC: All values of z except $z = 0$ & $z = \infty$

* Z-Transform of infinite duration causal sequence

Q) $x(n) = a^{n} u(n)$

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{-\infty}^{\infty} a^n u(n) z^{-n}$$

$$= \sum_{0}^{\infty} a^n z^{-n}$$

$$= \sum_{0}^{\infty} (az^{-1})^n$$

Let $\therefore az^{-1} = x$

$$= \frac{1}{1 - az^{-1}}, \text{ for } |x| < 1$$

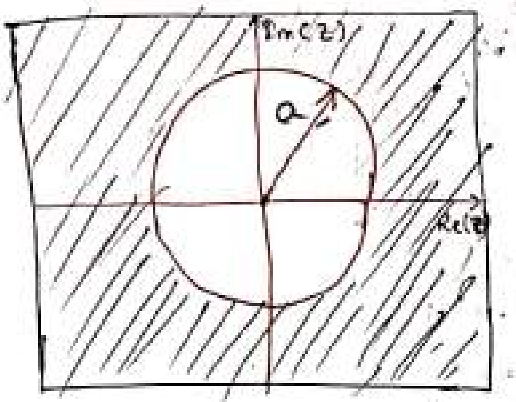
$$= \frac{z}{z-a}$$

$\therefore$ ROC: $|x| < 1$

$|az^{-1}| < 1$

$a < z$

$|z| > |a|$



⊛ $x(n) = -b^n u(-n-1) \to$ non causal Sequence or Anti causal Sequence.

Ans

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

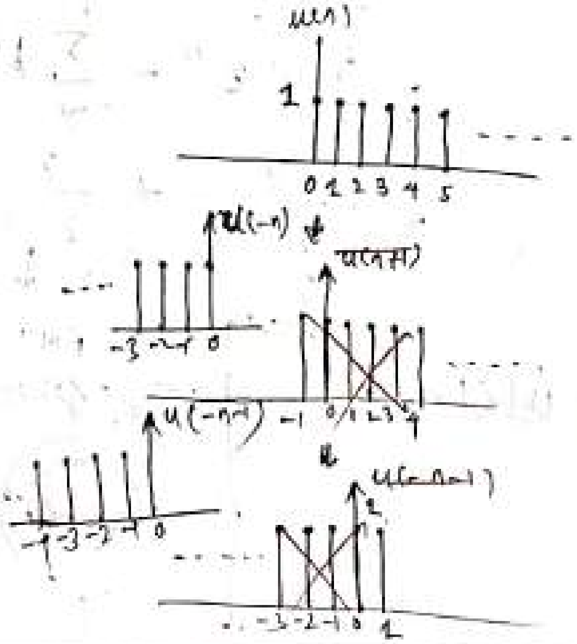
$$= \sum_{-\infty}^{\infty} -b^n u(-n-1) z^{-n}$$

$$= \sum_{-\infty}^{-1} -b^n z^{-n}$$

$$= -\sum_{-\infty}^{-1} (b^{-1} z^{-1})^n$$

$$= \frac{1}{1-bz} \to$$

$$= -\left[b^{-1} z^{+1} + b^{-2} z^{+2} + b^{+3} + b^{-\infty} z^{+\infty} \right]$$



$$= -\left[b^{-1} z^{1} + b^{-2} z^{2} + b^{-3} z^{3} \ldots\ldots + 0 \right]$$

$$= -\left[\sum_{0}^{-\infty} b^{n} z^{-n} - b^{0} z^{0} \right]$$

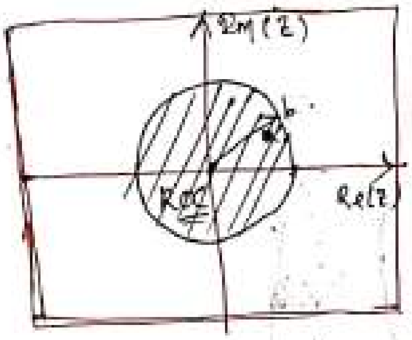
$$= -\left[\sum_{0}^{-\infty} (b^{-1} z)^{n} - 1 \right]$$

$$= -\left[\frac{1}{1-b^{-1}z} - 1 \right]$$

$$= \frac{z}{z-b}$$

∴ ROC of this sequence is $|b^{-1}z| < 1$

$|z| < |b|$



Q $x[n] = a^{n} u[n] - b^{n} u(-n-1)$

Ans $X(z) = \sum_{-\infty}^{\infty} x[n] \cdot z^{-n}$

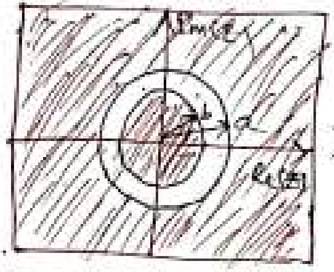
$$= \sum_{-\infty}^{\infty} \{ a^{n} u[n] - b^{n} u(-n-1) \} z^{-n}$$

$$= \sum_{0}^{\infty} a^{n} z^{-n} - \sum_{-\infty}^{-1} b^{n} z^{-n}$$

$$= \frac{z}{z-a} + \frac{z}{z-b}$$

↓ ROC: $|z| > |a|$ ↓ ROC: $|z| < |b|$

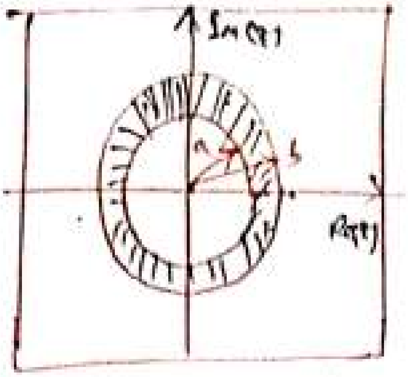
∴ If $|b| < |a|$



→ $|b| > |z| > |a|$

if $|a| < |b|$

ROC: $|a| < |z| < |b|$



5.1.2 The inverse Z-Transform

we have the Z-Transform X(Z) of a Signal & we must determine the Signal Sequence. The procedure for Transforming from the Z-domain to the time domain is called inverse Z-Transform.

we know

$$X(Z) = \sum_{K-\infty}^{\infty} x(n) \cdot Z^{-n}$$

∴ Suppose z^{n-1} multiply both sides & integrate both side.

$$\oint_C X(z) z^{n-1} dz = \oint_C \sum_{K=-\infty}^{\infty} x(K) \cdot z^{n-1-k} dz$$

$$\boxed{x(n) = \frac{1}{2\pi j} \oint X(Z) Z^{n-1} dz}$$

* **Stability & ROC**

→ let h(n) is an impulse function of a causal or non-causal linear time invariant system & H(Z) be the system function then the stability of the system can be found from ROC in the the following theorem.

A system is said to be BIBO stable.

if all the poles of the system function lies inside the unit circle or ROC contains in the in unit circle.

$$h(n) = 2^n u(n), \quad \sum_{-\infty}^{\infty} h(n) < \infty$$

The series cannot converge, so it is unstable

$$H(z) = \sum_{-\infty}^{\infty} 2^n u(n) \cdot z^{-n}$$

$$= \sum_{0}^{\infty} 2^n \cdot z^{-n}$$

$$= \sum_{0}^{\infty} (2z^{-1})^n$$

$$= \frac{z}{z-2}$$ ∴ ROC: $|z| > 2$ doesn't contain in the unit circle.

Properties of ROC:-

→ ROC is a rim or disc in the z plane centered at origin.

→ ROC must be a connected region.

→ ROC doesn't contain any pole.

→ if $x(n)$ is a causal sequence, then ROC is all the values of z except $z=0$.

→ if $x(n)$ is a non-causal sequence, then ROC is all the values of z except $z = \infty$.

→ ROC of any linear time invariant contains unit circle.

32 Properties of z-Transform :-

(1) **Linearity** :-

if $X(z) = Z[x(n)]$

$$Z[a_1 x_1(n) + a_2 x_2(n)] = a_1 X_1(z) + a_2 X_2(z).$$

→ $X_1(z) = Z[x_1(n)]$

$X_2(z) = Z[x_2(n)]$

Proof $Z[a_1 x_1(n) + a_2 x_2(n)]$

$$= \sum_{-\infty}^{\infty} [a_1 x_1(n) + a_2 x_2(n)] z^{-n}$$

$$= \sum_{-\infty}^{\infty} a_1 x_1(n) \cdot z^{-n} + \sum_{-\infty}^{\infty} a_2 x_2(n) \cdot z^{-n}$$

$$= a_1 X_1(z) + a_2 X_2(z)$$

(2) Time shift

if $X(z) = Z[x(n)]$

$$Z[x(n-m)] = z^{-m} X(z)$$

Proof

$Z[x(n-m)]$ $\quad \therefore l = n-m$, $n = l+m$

$$= \sum_{=-\infty}^{\infty} x(n-m) z^{-n}$$

$$= \sum_{l=-\infty}^{\infty} x(l) \cdot z^{-(l+m)}$$

$$= \sum_{l=-\infty}^{\infty} z^{-m} x(l) \cdot z^{-l}$$

$$= z^{-m} \sum_{l=-\infty}^{\infty} x(l) \cdot z^{-l}$$

$$= z^{-m} \cdot X(z)$$

(3) Time Reversed

if $X(z) = Z[x(n)]$

$$Z[x(-n)] = X(z^{-1})$$

Proof

$$Z[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n) z^{-n} \quad \therefore l = -n$$

$$= \sum_{l=-\infty}^{\infty} x(l) z^{l}$$

$$= \sum_{l=-\infty}^{\infty} x(l)(z^{-1})^{-l}$$

$$= X(z^{-1})$$

Q find the z-transform of $\delta(n)$..

$x(n) = \delta(n)$

Ans

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \delta(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} z^{-0}$$

$$= 1 \qquad \text{ROZ all } z$$

Q find the z-Transform of

$x(n) = u(n)$

Ans

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} z^{-n} \qquad \therefore \text{let } a = z^{-1}$$

$$= \sum_{n=0}^{\infty} a^{n}$$

$$= \frac{1}{1-a} \qquad \therefore \text{For } |a| < |1|$$

$$= \frac{1}{1-z^{-1}} \qquad \text{ROC} \therefore |z^{-1}| < |1|$$

$$|z| > 1$$