Design Of Machine Elements

Chapter - 01

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INTRODUCTION

1.1 Introduction to Machine Design & Classify it

*** Machine Design :-**

→ Designing is the creation of new and better machines and improving the existing one.

→ A new or better machine is one which is more economical in the overall cost of production and operation.

→ From the study of existing ideas, a new idea has to be conceived.

→ Keeping the idea in the mind a shape and form is given in the form of drawings.

→ In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into the actual reality.

Classification of Design :-

The machine design may be classified as follows.

(1) ~~Ratio~~ Adaptive Design :- This is a process of modifying the existing technology into a better design.

→ In most cases, the designer's work is concerned with adaptation of existing designs.

This design needs no special knowledge or skill.

(2) Development Design :-

→ This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture.

→ Though the designing of new product is started from the existing ideas, but the final product may differ from the existing one.

(3) New Design :-

This type of design needs alot of research, technical ability and creative thinking.

(a) Rational Design :- This type of design depends upon mathematic formulae of principle of mechanics.

(b) Empirical design :- This type of design depends upon empirical formulae based on the practice and past experience.

(c) Industrial Design :- This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) Optimum design :- It is the best design for the given objective function under the specified constraints

→ It may be achieved by minimizing the undesirable effects.

(e) System design :- It is the design of any complex mechanical system like a motor car.

) Element design :- It is the design of any element of the mechanical system like piston, crankshaft, connecting rod.

g) Computer Aided Design :- This type of design depends upon the use of computer systems to assist in the creation, modification, analysis & optimisation of a design.

General Considerations in Machine Design :-

Following are the general considerations in designing a machine component.

(1) Types of Load and stresses caused by the load :-
The load, on a machine component, may act in several ways due to which the internal stresses are set up.

(2) Motion of the Parts or kinematics of the machine :-

The motion of the Parts may be :-

a- Rectilinear motion → Unidirectional
→ Reciprocating.

b- Curvilinear Motion — Rotary
— Oscillatory
— Simple Harmonic

c- Constant velocity

d- Constant or variable Acceleration

) Selection of Material :-

Mechanical Properties :- The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load.

- Mechanical properties of metals are :-

Strength
Stiffness
Elasticity
Plasticity
Ductility
Brittleness
Malleability
Toughness
Machinability
Resilience
Creep
Fatigue
Hardness

1. Strength :- It is the ability of a material to resist the externally applied forces without breaking or yielding.

or Max^m stress that can be resisted by a body.

2. Stiffness :-
It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

$\sigma \propto M.E$ $\quad$ $\sigma \propto \epsilon$

$\sigma = E\epsilon$

3. Elasticity :-
It is the property of a material to regains its original shape after deformation when the external forces are removed.

4. Plasticity :-
It is the property of a metal which retains the deformation produced under load permanently.

5. Ductility :- It is the property of a material enabling it to be drawn into wire. with the application of a tensile force.

6. Brittleness :- It is the property of breaking of a material with little permanent distortion.

1.2 Different Mechanical Engineering Materials Used in design with their uses & Their Mechanical & Physical properties

1.2.a Engineering Materials & Their Uses :-

(i) Selection of materials in designing :- The materials should be selected based on the properties of materials, which is suitable for operation.

→ Must consider the process of manufacturing.

Classification of Engineering Material :-

- Engineering Material
  - Metals & Alloys
    e.g- Iron, Steel, Copper, Aluminium.
    - Ferrous - Metal
      Iron as Main Constituent
      eg - Cast iron, Wrought Iron Steel.
    - Non-Ferrous Metal
      other than iron as main constituent.
      eg - Copper, Aluminium, brass Tin, Zinc.
  - Non-metals
    e.g - Glass, Rubber, Plastic Ceramic.

Physical Properties of Metals

→ Density

→ Melting Point

→ Thermal properties
  - → Heat Capacity
  - → Thermal Conductivity
  - → Thermal Expansion.

→ Electrical Conductivity

→ Magnetic properties

Malleability :- It is the property of metal which permits materials to be rolled or hammered into thin sheets.

8. Toughness :- Property of a material to resist fracture due to high impact loads like hammer blows.

- It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture.

- This property is desirable in parts subjected to shock & impact loads.

9. Machinability :- It is the ~~ability~~ property of a material which refers to a relative ease with which a material can be cut.

- The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials.

10. Resilience :- It is the property of a material to absorb energy energy and to resist shock & impact loads.
- It is measured by the amount of energy absorbed per unit volume within elastic limit.
- This property is essential for spring materials.

11. Creep :- When a part is subjected to constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep.

→ This property is considered in designing internal combustion engines, boilers & turbines.

12. Fatigue :- When a material is subjected to repeated stress, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue.

3 Define Working Stress - Yield stress, Ultimate stress & factor of Safety

Working Stress :- When designing machine parts, it is desirable to keep the stress lower than the maximum (or) ultimate stress at which failure of the material takes place.

→ This stress is known as the working stress or design stress. Also known as safe or allowable stress.

→ For Brittle material the max^m stress is the ultimate stress

→ For ductile material the max^m stress is the yield stress.

Yield Stress :- It is the minimum stress at which a solid will undergo permanent deformation or plastic flow without a significant increase in the load or external force.

Ultimate Stress :- It is defined as the maximum stress that a material can withstand ~~before~~ breaking when a force is applied.

Factor of Safety :- It is defined as the ratio of the max^m stress to the working stress.

Range - 1.2 to 4
Steel - 1.15
Concrete = 1.5

$$\text{Factor of Safety (FOS)} = \frac{\text{Max}^m \text{ stress}}{\text{Working stress (or) Design Stress}}$$

→ For Ductile Material :- As the Yield point is the max^m allowable stress point so

$$(FOS)_{ductile} = \frac{\text{Yield Point Stress}}{\text{Working stress}}$$

→ For Brittle Material :- As the yield point is not clearly defined as for ductile material, so to FOS for brittle material is based on Ultimate stress.

FOS - 1 to 6

$$(FOS)_{Brittle} = \frac{\text{Ultimate Stress}}{\text{Working or design stress.}}$$

*NB:- The above relation for FOS are for static loading.

Selection of Factor of Safety :-

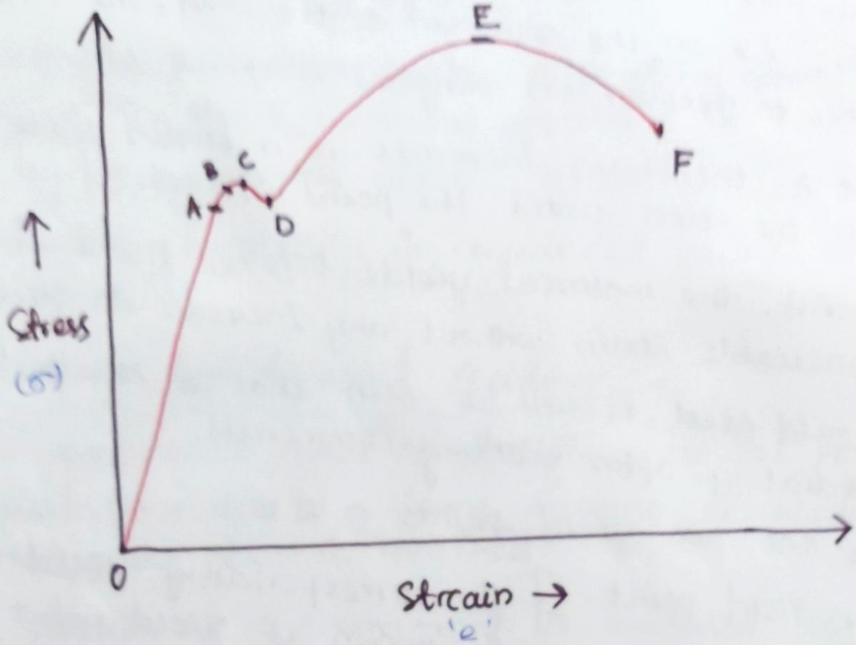
1. Reliability of the properties of the material and change of these properties during service.
2. The reliability of test results and accuracy of application of these results to actual machine parts.
3. The reliability of applied load.
4. The certainty as to exact mode of failure.
5. The extent of simplifying assumptions.
6. The extent of localised stresses.
7. The extent of loss of life if failure occurs.
8. · of property
9. The extent of initial stresses set up during manufacture.

FOS pressure vessel - 3.5 to 4

Aircrafts component - 1.5 to 2.5

Turbine " - 2 to 3

(1) Stress - Strain Curve for Mild Steel :-



From the above graph
OA - Propotional limit
AB - Elastic Region
C - Upper Yielding point
D - Lower Yielding Point
E - Ultimate tensile point
F - Breaking point.

(1) Propotional Limit :-

- From the graph we see that OA is a straight line which means that stress is directly propotional to strain.
- Hooke's Law holds good in this region and it is known as propotional limit.

(2) Elastic Limit :-

- Even if increasing the load from point A upto point B, the material will regain its shape & size after the removal of the load.
- This means that the material has elastic properties upto point B. This point is known as elastic limit.

(3) Yield Point :-

- If the material is stressed beyond point B, the plastic stage will reach, i.e on the removal of the load, the material will not be able to recover its original shape & size.
- Afte point B, the strain increases at a faster rate with any increase in the stress until the point C reached
- At this point, the material yields before the load and there is an appreciable strain without any increase in stress.
- In case of mild steel, it will be seen that a small load drops to D, immediately after yielding commences.
- Hence there are two yield points C & D.

C - Upper Yield point } The corresponding yield point stress
D - Lower " " } is known as Yield point stress.

Modes of Failure

(a) Modes of Failure By Elastic Deflection :-

In applications like transmission shafts supporting gears, the maximum force acting on the shaft, without affecting its performance is limited by the permissible elastic deflection.

- Lateral or torsional rigidity is considered as the criterion of design in such cases.

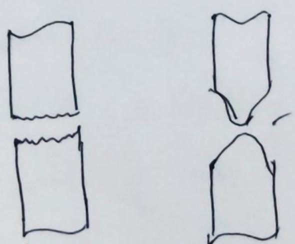
(b) Modes of Failure By General Yielding :-

A mechanical component made of ductile material loses its engineering usefulness due to a large amount of plastic deformation after the yield point stress is reached.

- Considerable portion of the component is subjected to plastic deformation, called general yielding.

(c) By Fracture :-

Components made of brittle material cease to function satisfactorily because of the sudden fracture without any plastic deformation.



Ductile Failure.

Fracture

Describe Design Procedure :-

- The general procedure to solve a design problem is as follows :-

(1) **Recognition of need** :- First of all, we have to identify the problem ~~and~~ by indicating the need, aim or purpose of designing the elements (machine).

(2) **Selecting the possible Mechanism** :- (or) (Synthesis)
Select the possible mechanism or group of mechanisms which will give the desired motion.

(3) **Analysis of Forces** :- Find the forces acting on each member of the machine and the energy transmitted by each member.

(4) **Material Selection** :- Select the material best suited for each member of the machine.

(5) **Design of Elements** :- Calculate the size of each member of the machine by considering the forces acting on the member and the permissible stresses for the material used.
- ~~It so should be~~ The deformation of the member should be in permissible limit.

(6) **Modification** :- Modify the size of the member to agree with the past experience and ~~a~~ judgement to facilitate manufacture.

Chapter - 02
Design of Fastening Elements

2.1 Joints and Their Classification

Joint is a section of a machine which is used to connect one or more mechanical parts to another.

Joints
- Temporary
- Permanent

Temporary: The temporary feastening which can be disassembled without destroying the connecting component.

e.g → Keys, cotters, pins splined

Permanent: The permanent fastening are those which can not be dis-assembled without destroying the connecting components.

e.g ⇒ Solder, Braze, Welded Riveted.

2.2 State type of welded Joints :-

* A welded joint is a permanent joint which is obtained by fusion of the edges of the two parts to be joined together with or without the application of pressure & a filler rod.

Welded Joint
- Lap Joint
- Butt joint

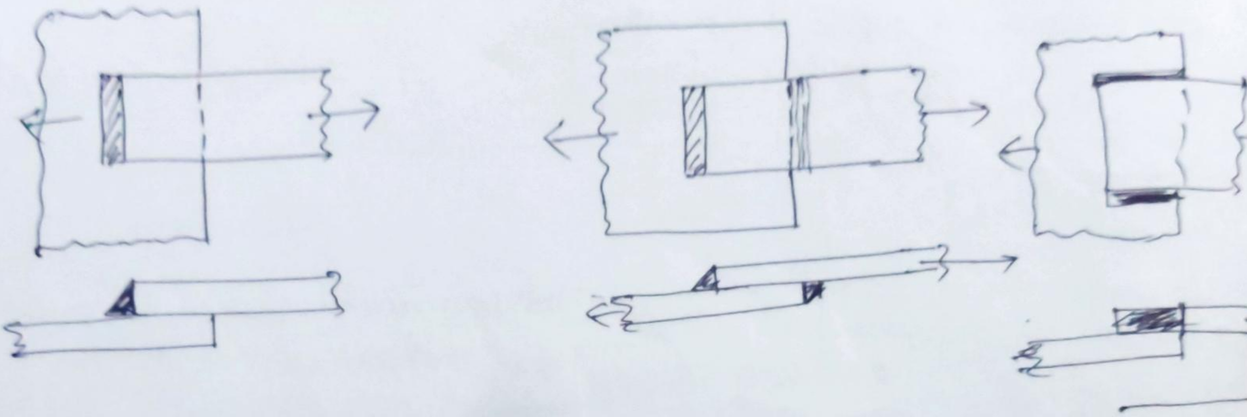
Lap Joint :- The lap joint or the fillet joint is obtained by overlapping the plates & then welding the edges of the plate

Lap Joint

Single ~~Trans~~ Transverse fillet

Double transverse

Parallel fillet



Butt Joint :-

the butt joint is obtained by placing the edges to edge

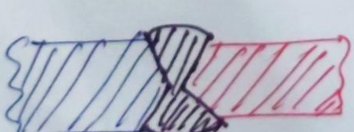
• In butt joint (welding) the plate edges do not ~~rep~~ require bevelling if thickness of plate is less than 5mm.

On the other hand, if the plate thickness is 5mm to 12.5mm the edges should be bevelled to V or U groove on both side.

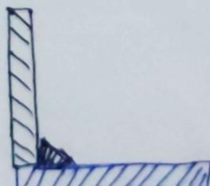
Square butt

Single V butt

Single U butt

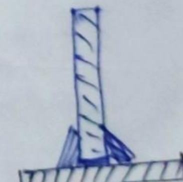


Double V butt



Corner Joint

Edge Joint



T-Joint

2.3 Advantages of welded Joint

- Welding establishes strong, durable and permanent Joint.
- The technique, when used with filler metal, produces a stronger weld than the base metal.
- It is an economical & affordable process.

Advantages of welded Joint over Riveted Joint

① The riveted structures are generally heavy in weight as compared to welded structure.

② welded joint always provide high-efficiency.

③ The addition & alteration can be easily made in welding structures as compared to riveted structure.

④ Welding Joint structures are ~~smoot~~ smoot than .

⑤ The welded joint are stronger than riveted joint.

(6) welded joints are durable.

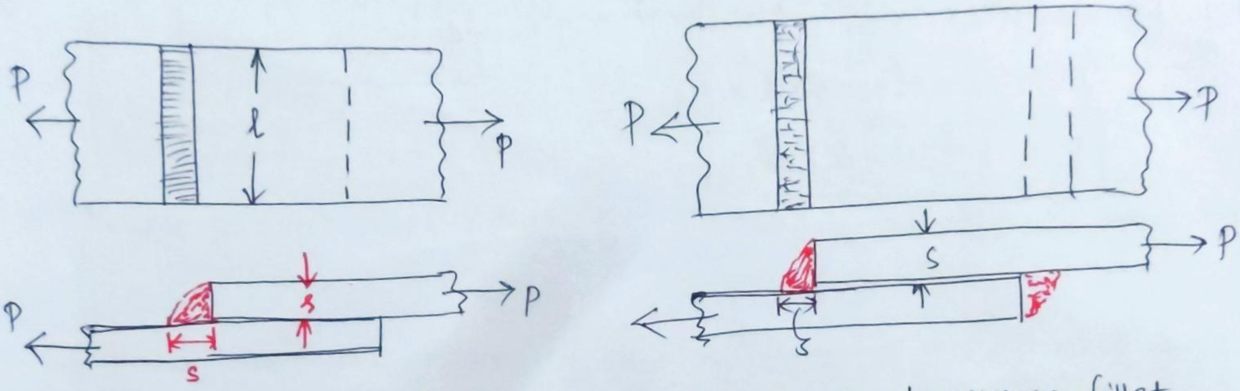
Advantages of welded Joint over Bolted Joint

- The strength of welded joint is more as compared to the bolting joint.
- The welded joints are better for fatigue load, impact load & vibration.
- As there is no deduction in the area because of no holes, thus the cross-section is effectively carried loads

Strength of Transverse Fillet Welded Joints

- Fillet or Lap joint is obtained by overlapping the plates and then welding the edges of the plates.

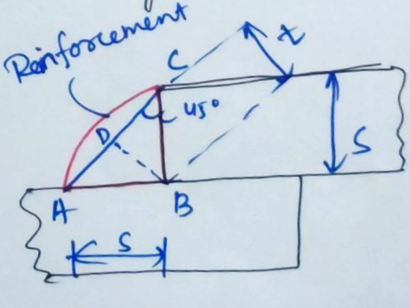
→ The transverse fillet welds are designed for tensile strength.



(a) Single transverse fillet weld.

(b) Double transverse fillet weld.

- In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angle triangle ABC with hypotenuse AC making equal angles with other two sides AB & BC.



(c)

- Let t = throat thickness (BD)

  s = Leg or size of weld
  = Thickness of plate.

  l = length of weld.

From fig 'c' throat thickness, $t = s \times \sin 45° = 0.707\,s$

∴ Minimum Area of the weld or throat area, A = Throat thickness × Length of weld

$$= t \times l$$

$$\boxed{A = 0.707\,s \times l}$$

* If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

P = throat area x Allowable tensile stress

$$\boxed{P = 0.7075 \times l \times \sigma_t}$$

For Double fillet weld, the tensile strength of the joint.

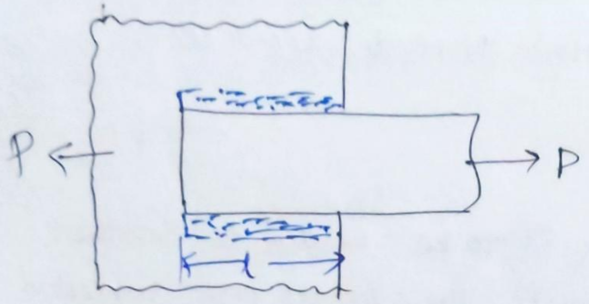
$$P = 2 \times 0.7075 \times l \times \sigma_t$$

$$\boxed{P = 1.4145 \times l \times \sigma_t}$$

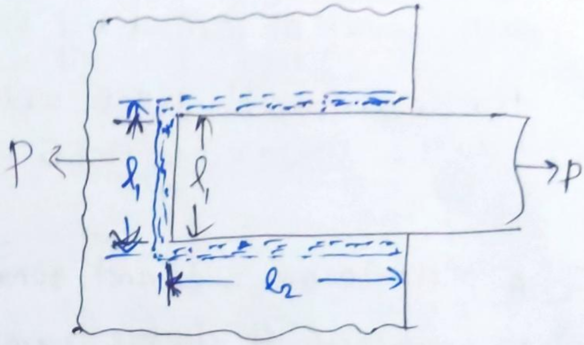
St

Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength



(a) Double parallel fillet weld



(b) Combination of transverse & parallel fillet weld

We know that the minimum area of weld or the throat area.

$$\boxed{A = 0.707 s \times l}$$

*If τ = allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld.

P = Throat Area × Allowable shear stress

$$\boxed{P = 0.707 s \times l \times \tau}$$

↳ Shear strength

• Shear strength of the joint for Double parallel fillet weld

$$P = 2 \times 0.707 s \times l \times \tau$$

$$\boxed{P = 1.414 s \times l \times \tau}$$

From fig. (b) For Combination of Transverse & Parallel fillet weld

The strength of the weld is calculated by the combine effect or strength of tensile & shear.

∴ The sum of strength of single & double parallel fillet welds

$$\boxed{P = 0.707 s \times l_1 \times \sigma_t + 1.414 s \times l_2 \tau}$$

- In order to allow for starting & stopping of the bead, 12.5mm should be added to the length of each weld obtained by the above expression.

- For reinforced fillet welds, the throat dimension may be taken as 0.85 t.

Q. A plate 100mm wide and 10mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.

Given width = 100 mm

Thickness = 10mm

$P = 80kN = 80 \times 10^3 N.$

$\tau = 55 MPa = 55 N/mm^2$

Let length of weld $= l$

s = size of weld = plate thickness = 10mm

∴ we know that max^m Load which the plates can carry for double parallel fillet weld (P)

$$P = 2 \times 0.707\, s \times l \times \tau$$

$$\Rightarrow 80 \times 10^3 = 2 \times 0.707 \times 10 \times l \times 55$$

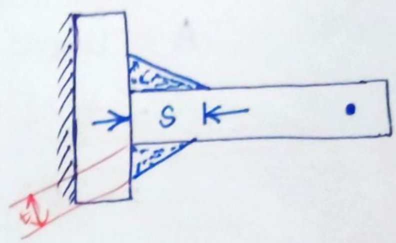
$$\Rightarrow l = \frac{80 \times 10^3}{2 \times 0.707 \times 10 \times 55}$$

$$\therefore l = 103 \text{ mm}$$

Adding 12.5mm for starting & stopping of weld run we have $l = 103 + 12.5 = 115.5$ mm (Ans)

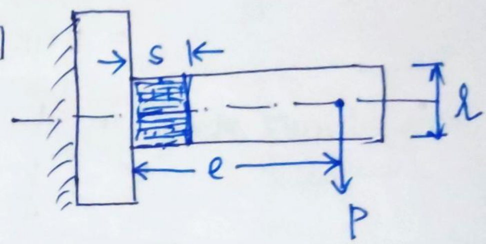
Design of Welded Joints for Eccentric Loads

When an eccentric load is imposed on welded joints → Shear & Bending stresses are simultaneously present in a joint



∴ Max^m Normal stress

$$\sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$



∴ Max^m Shear Stress

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

where σ_b = Bending stress
τ = Shear stress

Consider a T-Joint fixed at one end and subjected to an eccentric load P at a distance 'e' as shown in figure.

Let s = size of weld
l = length of weld
t = Throat thickness

The joint will be subjected to the following two types of stresses.

(1) Direct Shear stress due to shear force P acting at the welds

(2) Bending stress due to the bending moment. "P × e"

We know that area at the throat

A = Throat thickness × Length of weld

$= t \times l \times 2$ [∵ For Double fillet weld]

$= 2t \times l$

$= 2 \times 0.707s \times l$ [∵ $t = s\sin 45°$]

$= 1.414s \times l$

∴ Shear stress in the weld (Assuming uniformly Distributed)

$\tau = \frac{P}{A}$

$$\boxed{\tau = \frac{P}{1.414s \times l}}$$

Section Modulus of the weld metal through the throat.

[for both sides weld]

$Z = \frac{t \times l^2}{6} \times 2$

$= \frac{0.707s \times l^2}{6} \times 2$

$$\boxed{Z = \frac{s \times l^2}{4.242}}$$

Section Modulus $Z = \frac{I}{y}$

Direct measure of strength of Beam. Larger Z value ⇒ Increase the Load Bearing capacity

$\frac{\sigma}{y} = \frac{M}{I}$

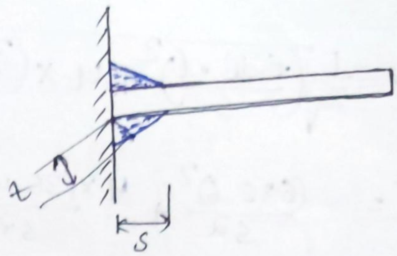
$M = \frac{\sigma}{y} \cdot I = \frac{I}{y} \cdot \sigma = Z \cdot \sigma$

Bending Moment, $M = P \times e$

Bending stress, $\sigma_b = \frac{M}{Z} = \frac{P \times e}{\left(\frac{s \times l^2}{4.242}\right)} = \frac{P \times e \times 4.242}{s \times l^2}$

$$\boxed{\therefore \sigma_b = \frac{4.242\,Pe}{s \times l^2}}$$

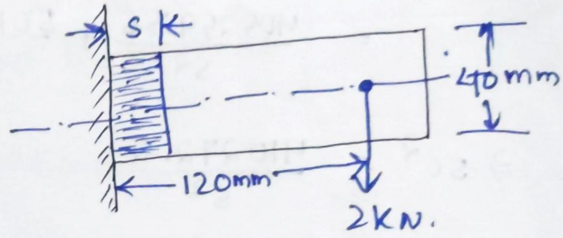
A welded joint as shown in figure is subjected to an eccentric load of 2KN. find the size of weld, if the max^m shear stress in the weld is 25 MPa.



Given $P = 2KN = 2000N$ $e = 120\ mm$

$l = 40\ mm$, $\tau_{max} = 25\ MPa$

$= 25\ N/mm^2$



Let s = size of weld

t = throat thickness

The joint will be subjected to direct shear stress due to shear force $P = 2000N$ & Bending stress due to the Bending moment of $P \times e$

$\therefore$ Throat Area, $A = 2t \times l$

or Area at the Throat $= 2 \times 0.707s \times l$

$= 1.414s \times l$

$A = 1.414s \times 40 = 56.56 \times s\ mm^2$

Shear Stress, $\tau = \frac{P}{A}$

$$\tau = \frac{2000}{56.56 \times s} = \frac{35.4}{s}\ N/mm^2$$

Bending Moment, $M = P \times e = 2000 \times 120 = 240 \times 10^3\ N\text{-}mm$

Section of modulus of the weld through the throat

$$Z = \frac{s \times l^2}{4.242} = \frac{s(40)^2}{4.242} = 377 \times s\ mm^3$$

$\therefore$ Bending Stress, $\sigma_b = \frac{M}{Z} = \frac{240 \times 10^3}{377 \times s} = \frac{636.6}{s}\ N/mm^2$

$\tau_{(max)} = \frac{1}{2}.$

We know that Max^m shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\Rightarrow 25 = \frac{1}{2}\sqrt{\left(\frac{636.6}{s}\right)^2 + 4 \times \left(\frac{35.4}{s}\right)^2}$$

$$\Rightarrow (50)^2 = \frac{(636.6)^2}{s^2} + \frac{4 \times (35.4)^2}{s^2}$$

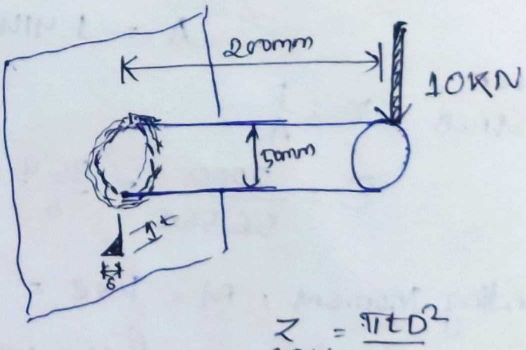
$$= \frac{405259.56}{s^2} + \frac{5012.64}{s^2}$$

$$\Rightarrow 50^2 = \frac{410272.2}{s^2}$$

$$\Rightarrow 50 = \frac{640.524}{s}$$

$$\Rightarrow \boxed{s = 12.81 \text{ mm}}$$ (Ans)

Q. A 50 mm diameter solid shaft is welded to a flat plate if the size of weld is 15mm. Find Max^m Normal & shear stress.



$$Z_{circl} = \frac{\pi t D^2}{4}$$

$$A = \pi D \times t$$

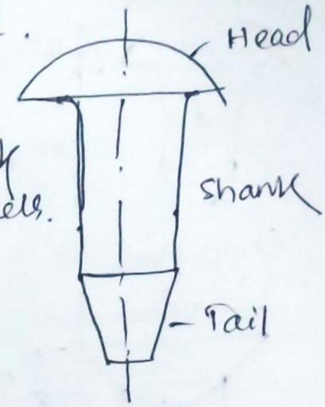
$\sigma_{max} = 94.6$ MPa 0.614

$\tau_{max} = 48.4$ MPa

State Types of Riveted Joints & Types of Rivets

Rivet :- A rivet is a short cylindrical bar with a head integral to it.

- The cylindrical portion of the rivet is called Shank or Body and the lower portion of shank is known as tail.



- Rivets are used to make permanent fastening between the plates such as in structural work Ship building, bridges, tanks and boiler shells.
- Riveted joints are used for joining light metal.

Fastening

- **Permanent** — e.g - Soldered, Brazed welded, Riveted
- **Temporary** — e.g Screwed, Key, Cotters Pins & splined joints.

Rivet

- **Cold** — Used in structural Joints
- **Hot** — Make leak proof joints.

Material of Rivets

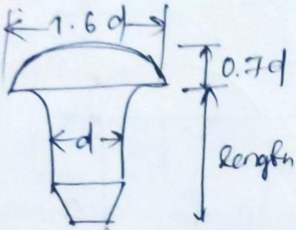
- Tough & Ductile
- Steel → Low carbon. Brass Copper
  → Nickle steel Aluminium
- For strength & a fluid tight joint → Steel Rivets are used.

Qualities of Rivets :-
- Must have Tensile strength $\geq 40\,N/mm^2$
- Elongation not less than 26%.

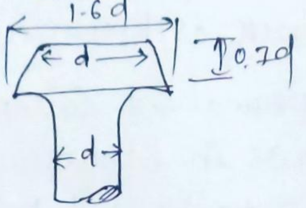
Manufacturing of Rivets → Cold Hedding
→ Hot Forging

Rivet :—

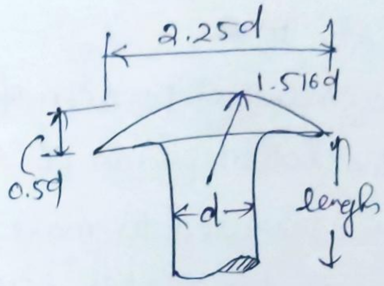
Rivet Heads



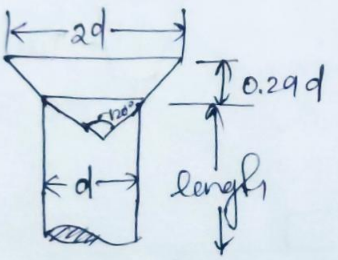
Snap Head



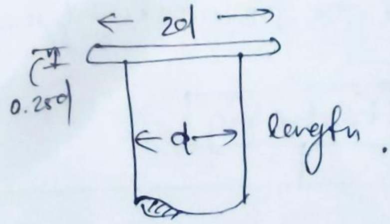
Pan Head



Mushroom Head.



Counter sunk Head.
120°.



[Rivet Heads for General Purposes (below 12 mm dia)]

* Snap Heads are used for structural work & machine riveting.
- Counter sunk head are mainly used for ship building
- Conical Heads → Hand Hammering.
- Pan Heads have Max^m strength

* The function of rivets in a joint is to make a connection that has strength & Tightness.

↳ It is necessary to prevent failure of the joint.

- Tightness. " ~~to~~ in order to contribute strength & to prevent leakage as in boiler or ship hull.

Types of Riveted Joints

2 types

- **Lap Joint**
- **Butt Joint**

A lap joint is that in which one plate overlaps the other and the two plates are then Riveted together.

- A butt joint is that in which the main plates are kept in alignment butting or touching each other & a cover plate is placed either on one side or both sides of the main plates.
- The cover plate is then riveted together with main plates.

Riveted Joints

- Lap Joint
- Butt Joint
  - Single ~~Riveted~~ Strap Butt Joint
  - Double strap Butt

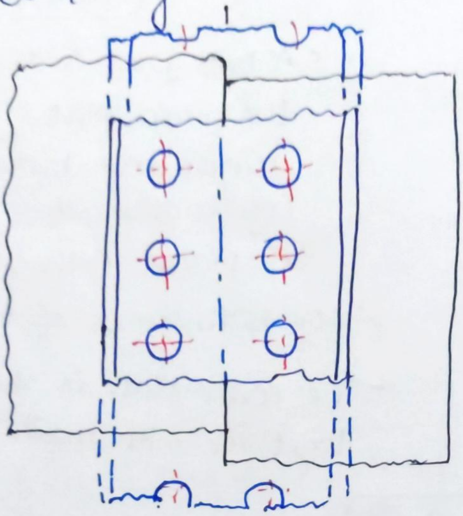
Riveted Joints

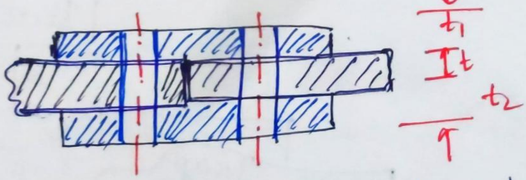
According to No. of Rows of the Rivets

- Single Riveted
- Double Riveted

Single Strap Butt Joint :-

The edges of the main plates butt against each other & only one cover plate is placed on one side of the main plates & then riveted together.

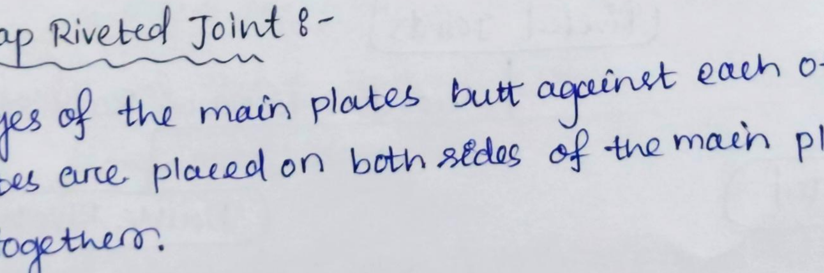




[Single Riveted Double strap butt joint]

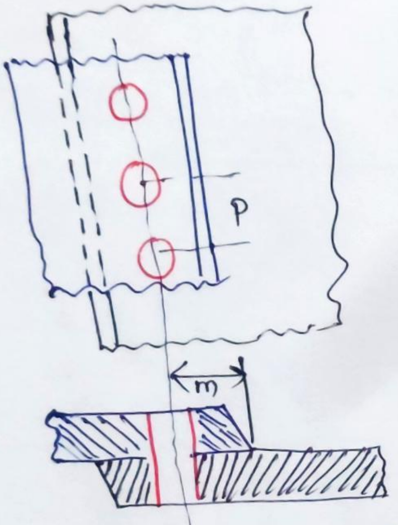
Double Strap Riveted Joint :-

The edges of the main plates butt against each other & two cover plates are placed on both sides of the main plates and then riveted together.

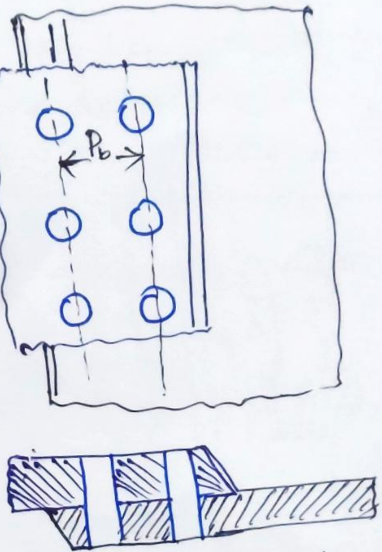


(Double Riveted Double strap Butt Joint)
(Equal)

Single Riveted Joint :- is that in which there is a single row of rivets in a lap joint. or Butt joint.

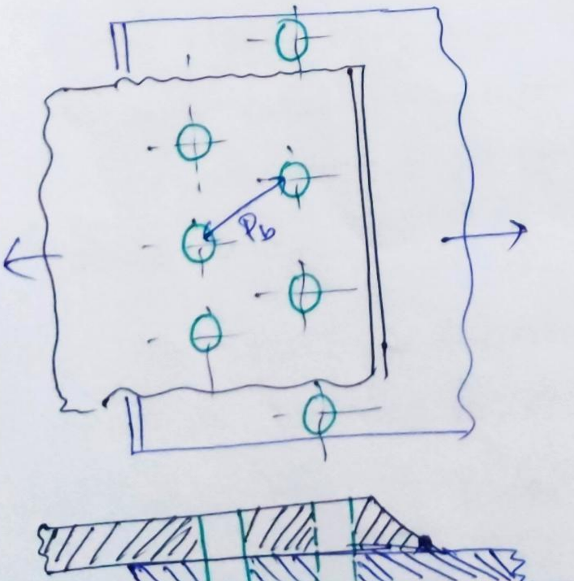


Single Riveted Lap Joint



Double Riveted Lap Joint
(Chain Riveting)

Double Riveted Joint :- is that in which there are two rows of rivets in a lap joint.



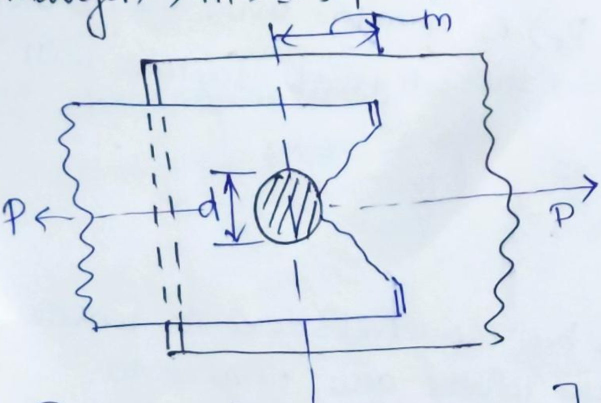
(Double Riveted Lap Joint)

.6 Describe Failure of Riveted Joints

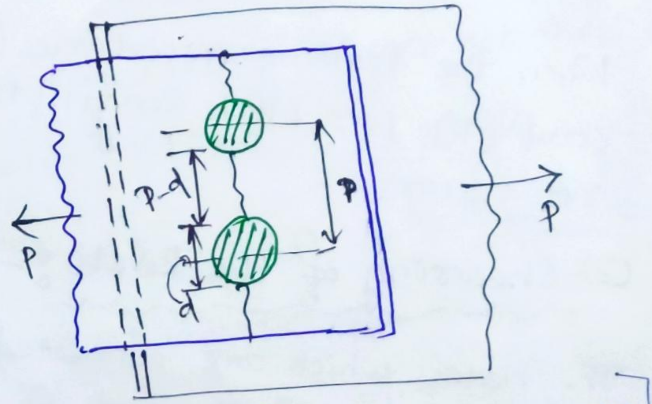
A riveted joint may fail in the following ways.

1. Tearing of the Plate at an edge :-

A joint may fail due to tearing of the plate at an edge as shown in figure. This can be avoided by keeping the margin, $m = 1.5d$, where d = diameter of the rivet hole.



[Tearing of the plate at an edge]



(Tearing of the plate across the rows of rivets)

2. Tearing of the plate across a row of rivets

Due to tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown. In such cases, we consider only one pitch length of the plate since every rivet is responsible for that much length of the plate only.

The resistance offered by the plate against tearing is known as "Tearing Resistance" or "tearing strength" of plate

p = pitch of the rivets

d = Diameter of the rivet hole

t = Thickness of the plate.

σ_t = Permissible tensile stress for the plate mate

We know that tearing area per pitch length

$$\boxed{A_t = (p-d)\,t}$$

∴ Tearing resistance or pull required to tear off the plate per pitch length

$$P_t = A_t \times \sigma_t$$

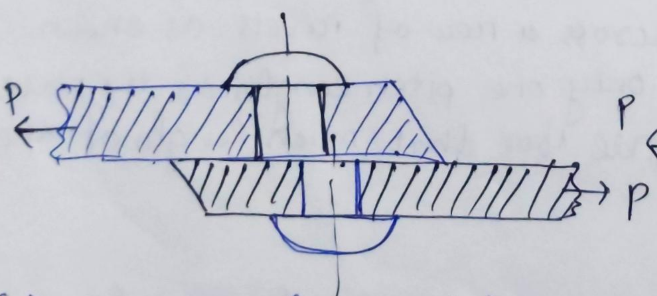
$$\boxed{P_t = (p-d)\,t\,\sigma_t}$$

When the tearing resistance (P_t) is greater than the applied load (P) per pitch length then this type of failure will not occur.

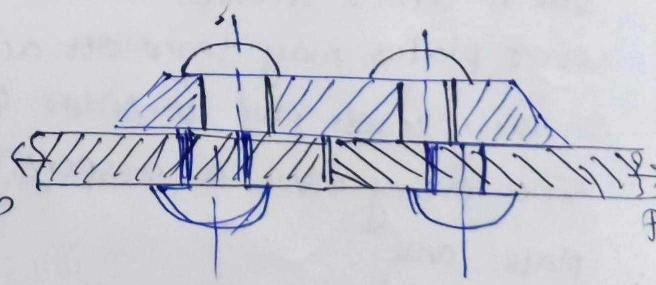
(3) Shearing of the Rivets :-

The plates which are connected by the rivets exert tensile stress on the rivets and if the rivets are unable to resist the stress, they are sheared off as shown in figure.

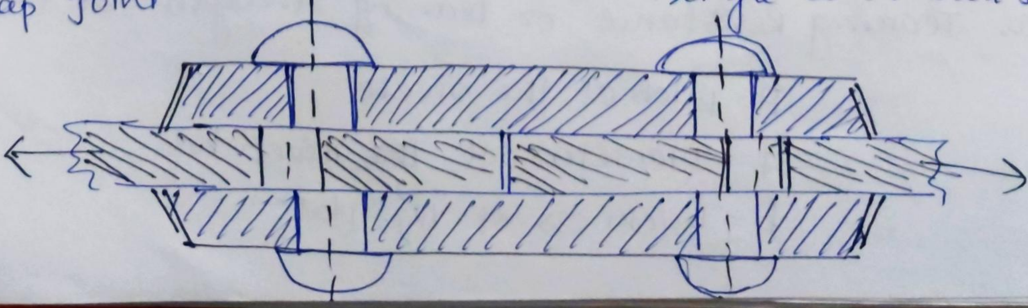
* It may be noted that the rivets are in "single shear in a lap joint & in a single cover butt joint,"



(a) Shearing off a rivet in a lap joint



(b) Shearing off a rivet in a single cover butt joint



d = Diameter of the rivet hole

τ = safe permissible shear stress for the rivet material

n = no. of rivets per pitch length.

∴ We know that shearing area, $A_s = \frac{\pi}{4} d^2$ [For single shear]

$= 2 \times \frac{\pi}{4} d^2$ [For double shear]

$= 1.875 \times \frac{\pi}{4} d^2$

∴ Shearing resistance or pull required to shear off the rivet per pitch length, $P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$ [For single shear]

$P_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau$. [in double shear]

$P_s = n \times 1.875 \times \frac{\pi}{4} d^2 \times \tau$ [in double shear for Indian Boiler Regulation]

* When the shearing resistance (P_s) is greater than applied load (P) per pitch length, then this type of failure will occur.

* Rivets are in double shear in a double cover butt joint as shown in figure.
* The Resistance offered by a rivet to be sheared off is known as 'Shearing Resistance or shearing strength"

4. Crushing of the Plate :-

- Some times, the rivets do not actually shear off under the tensile stress, but crushed.
- Due to this, the Rivet hole becomes of an oval shape and hence the joint becomes loose.
- The failure of rivets in such manner is known as 'Bearing failure'.

The resistance offered by a rivet to be crushed is known as crushing resistance or crushing strength.

d = Diameter of the rivet hole

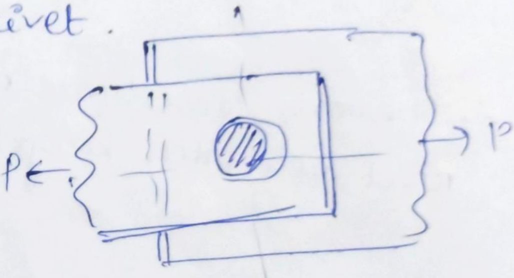
t = thickness of the plate.

σ_c = safe permissible crushing stress for the rivet plate material.

n = no. of rivets per pitch length under crushing

We know that crushing area per rivet.

$A_c = d \times t$



$\therefore$ Total crushing area $= ndt$

Crushing resistance on pull required to crush the rivet per pitch length $\boxed{P_c = ndt \times \sigma_c}$

* When the crushing resistance $P_c >$ applied Load 'P' per pitch length then this type of failure will occur.

* The number of rivets under shear shall be equal to the number of rivets under crushing.

Strength of a Riveted Joint :-

⇒ The strength of a Rivet joint may be defined as the max^m force, which can transmit, without causing it to failure.
We see that P_t - Tearing strength (P_t), Shearing strength (P_s) and P_c - Crushing strength are the pulls required to tear off the plate, shear off, & crushing off the rivet.

⇒ A little consideration will show that if we go on increasing the pull on a riveted joint, it will fail when the least of these three pulls is reached, because a higher value of the other pulls will never reach since the joint has failed, either by tearing off the plate, shearing off the rivet or crushing off the rivet.

- If the joint is continuous as in case of boilers, the strength is calculated per pitch length.
- But if the joint is small, the strength is calculated for the whole length of the plate.

Efficiency of a Riveted Joint :-

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

∴ Strength of the Riveted Joint = Least of P_t, P_s, P_c

& Strength of un-Riveted ~~joint~~ or solid plate per pitch length

$$P = p \times t \times \sigma_t$$

∴ Efficiency of the riveted joint

$$\eta = \frac{\text{Least of } P_t, P_s, P_c}{p \times t \times \sigma_t}$$

p = pitch of the rivets

t = thickness of the plate

σ_t = permissible tensile stress of the plate material.

Find the efficiency of the following riveted joints.

(a) Single Riveted lap joint of 6 mm plates with 20 mm diameter rivets having pitch of 50 mm.

(b) Double Riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm.

Permissible tensile stress in plate – 120 MPa

" shear stress in rivets – 90 MPa

" crushing " in rivets = 180 MPa

Ans Given t = thickness of plate = 6 mm

d = diameter of rivet = 20 mm

σ_t = 120 MPa

τ = 90 MPa

σ_c = 180 MPa.

(a) Efficiency of single Riveted Lap joint. of pitch, P = 50 mm

(i) Tearing Resistance of the Plate :-

We know that the tearing resistance of the plate per pitch length, $P_t = (p-d)\,t \times \sigma_t$

$= (50-20) \times 6 \times 120 = 21600$ N

(ii) Shearing Resistance of the Rivet :-

Since the joint is a single riveted lap joint,

∴ the strength of one rivet in single shear

We know that shearing resistance of one rivet.

$$P_s = \frac{\pi}{4} d^2 \times \tau = \frac{\pi}{4} \times (20)^2 \times 90 = 28278 \text{ N}$$

(iii) Crushing Resistance of the Rivet :-

Since the joint is a single riveted,

∴ Strength of one rivet is taken

We know that crushing resistance of one rivet

$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180 = 21600 N$.

∴ Strength of the joint = Least of P_t, P_s & P_c

= 21,600 N

We know that strength of the un-riveted or solid plate.

$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36000 N$.

∴ Efficiency of the joint $\eta = \frac{\text{Least of } P_t, P_s, P_c}{P} = \frac{21600}{36000}$

$\boxed{\eta = 60\%}$

(2) Efficiency of the second joint - Butt

pitch, $p = 65mm$

(i) Tearing Resistance of the Plate,

We know that the tearing resistance of the plate per pitch length, $P_t = (p-d) t \times \sigma_t$

$= (65 - 20) 6 \times 120$

$= 32400 N$

(ii) Shearing Resistance of the rivets :-

Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken.

∴ We know that shearing resistance of the rivets.

$P_s = n \times \frac{\pi}{4} d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2 \times 90 = 56554.8 N$.

(iii) Crushing Resistance of the Rivet :-

Since the joint is double riveted, therefore strength of two rivets is taken.

We know that ~~shearing~~ crushing resistance of the rivets

$$P_c = n \times d \times t \times \sigma_c$$

$$= 2 \times 20 \times 6 \times 180$$

$$= 43200 \text{ N.}$$

$\therefore$ Strength of the joint = Least of P_t, P_s & P_c

$$= 32400 \text{ N.}$$

We know that the strength of the unriveted or solid plate

$$P = p \times t \times \sigma_t$$

$$= 65 \times 6 \times 120$$

$$= 46800 \text{ N}$$

$\therefore$ Efficiency of the joint, $\eta = \dfrac{\text{Least of } P_t, P_s, P_c}{P}$

$$= \frac{32,400}{46,800} = 69.2\%$$

Q-2 A double riveted double cover butt joint in plates 20mm thickness is made with 25mm diameter rivets at 100mm pitch. The permissible stress are $\sigma_t = 120$ MPa, $\tau = 100$ MPa, $\sigma_c = 150$ MPa. Find the efficiency of Joint taking strength of rivet in double shear as twice than that of single shear.

Given:- $t = 20$ mm, $d = 25$ mm, $p = 100$ mm

$\sigma_t = 120$ MPa, $\tau = 100$ MPa, $\sigma_c = 150$ MPa

(i) Tearing Resistance of the plate per pitch length:

$$P_t = (p-d)t \times \sigma_t$$

$$= (100-25) \times 20 \times 120$$

$$= 180\,000 \text{ N}$$

(ii) Shearing Resistance of the Rivets

Since the joint is double riveted butt joint, therefore the strength of two rivets in double shear is taken

We know that shearing Resistance of the Rivets

$$P_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau$$

$$= 2 \times 2 \times \frac{\pi}{4} \times (25)^2 \times 100 = 196375\,N$$

(iii) Crushing Resistance of the Rivets

Since the joint is double Riveted, therefore the strength of two rivets is taken.

∴ Crushing Resistance of the Rivets

$$P_c = n \times d \times t \times \sigma_c$$

$$= 2 \times 25 \times 20 \times 150$$

$$= 150{,}000\,N.$$

∴ Strength of the Joint

$$= \text{Least of } P_t, P_s \;\&\; P_c = 150{,}000\,N.$$

(iv) Efficiency of the Joint :-

The strength of the Un-riveted or Solid Plate

$$P = p \times t \times \sigma_t$$

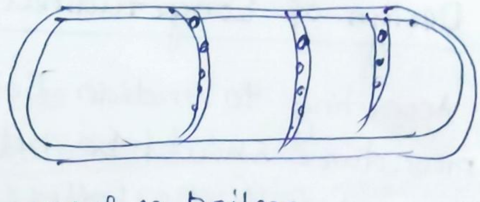
$$= 100 \times 20 \times 120 = 240{,}000\,N$$

$$\therefore \eta = \frac{\text{Least of } P_t, P_s, P_c}{P} = \frac{150{,}000}{240{,}000} = 0.625$$

$$= 62.5\%\ \text{(Ans)}$$

2.8 Design Riveted Joints for Pressure Vessel.

The pressure vessel or boiler has a longitudinal joint as well as circumferential joint.



- The longitudinal joint is used to join the ends of the plate to get the required diameter of a pressure vessel or boiler.

→ For this purpose, a butt joint with two cover plates is used.

* The circumferential joint is used to get the required length of the boiler. For this purpose, a lap joint with one ring overlapping the other alternately is used.

* Since a boiler is made up of number of rings, therefore the longitudinal joints are ~~are~~ staggered for convenience of connecting rings at places where both longitudinal & circumferential joints occur.

Assumptions in Designing Pressure Vessel Joints.

(1) The load on the joint is equally shared by all the rivets. The assumption implies that the shell & plate are rigid & that all the deformation of the joint takes place in the rivets themselves.

(2) The tensile stress is equally distributed over the section of metal between the rivets.

(3) The shearing stress in all the rivets is uniform.

(4) The crushing stress is uniform.

(5) There is no bending stress is the rivets.

(6) The holes into which the rivets are driven do not weaken the member.

(7) The rivet fills the hole after it is driven.

(8) The friction between the surfaces of the plate is neglected.

Design of Longitudinal Butt Joint for a Pressure Vessel.

According to Indian Boiler Regulation (I.B.R) the following procedure should be adopted for the design of logitudinal butt joint for a boiler.

1. Thickness of Boiler shell :- First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula.

$$\boxed{t = \frac{P \cdot D}{2 \times \sigma_t \times \eta_l} + 1\,mm}$$ as corrosion allowance.

Where t = Thickness of the boiler shell

P = Steam pressure in boiler

D = Internal diameter of boiler shell.

σ_t = Permissible tensile stress

η_l = Efficiency of the longitudinal joint.

- The thickness of the boiler shell should not be less than 7mm

(2) Diameter of Rivets :-

After finding out the thickness of the vessel (t), diameter of the rivet hole (d) may be determined by using Unwin's emperical formula $\boxed{d = 6\sqrt{t}}$ (t is greater than 8mm)

* But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance.

* In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing.

(3) Pitch of Rivets :-

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

$P_t = (P-d) t \times \sigma_t$ $\quad$ $P_s = n \times \frac{\pi}{4} d^2 \times \tau$

(a) The pitch of the rivets should not be less than $2d$, which is necessary for the formation of head. $P > 2d$

(b) The max^m value of the pitch of rivets for a longitudinal joint of a boiler as per IBR as

$$P_{max} = C \times t + 41.28 \text{ mm}$$

t = Thickness of the shell plate in mm

C = constant.

4. Distance between the Rows of Rivets :-

The distance between the rows of rivets as specified by Indian Boiler Regulations is as follows.

(a) For equal number of rives in more than one row for Lap joint or butt joint., the distance between the rows of rivets (P_b) should not be less than.

$P_b = 0.33P + 0.67d$ for zig-zig riveting.

$P_b = 2d$ for chain riverting.

(b) For joints in which the number of rivets in outer row is half the number of rivets in inner rows and if the inner rows are chain riveted, the distance between the outer rows & the next rows should not be less than $0.33P + 0.67$ or $2d$, whichever'' greater

The distance between the rows in which there are full number of rivets shall not be less than $2d$.

(c) For joint in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are zig-zag riveted, the distance between the outer rows and the next rows shall not be less than $0.2P + 1.15d$

The distance between the rows in which there are full number of rivets (zig-zag) shall not be less than $0.165p + 0.67d$.

(5) Thickness of Butt strap :-

(a) The thickness of butt strap in no case, shall be less than 10 mm.

(b) $t_1 = 1.125t$ for ordinary single butt strap.

$t_1 = 1.125t \left(\frac{p-d}{p-2d}\right)$, for single butt strap, every alternate rivet in outer rows being omitted.

$t_1 = 0.625t$ for double butt strap of equal width having ordinary riveting (chain riveting)

$t_1 = 0.625t \left(\frac{p-d}{p-2d}\right)$, for double butt straps of equal width having every alternative rivet in the outer rows being omitted.

(c) For unequal width of butt straps, the thickness of butt strap are $t_1 = 0.75t$ for wide strap on the inside.
$t_2 = 0.625t$, for narrow strap on the outside.

(6) Margin :- The margin (m) is taken as 1.5 d.

Design of Circumferential Lap Joint for Pressure Vessel:-

(1) Thickness of the shell and diameter of rivets:-

The thickness of the boiler shell and the diameter of the rivet will be same as for longitudinal joint.

$$t = \frac{P \cdot D}{2\sigma_t \times \eta_l} + 1\,mm \text{ as corrosion allowances}$$

t = thickness of the boiler shell.

P = Steam pressure in boiler.

D = Internal diameter of boiler shell

σ_t = permissible tensile stress.

η_l = Efficiency of the longitudinal joint.

(2) Number of Rivets:-

Since it is a lap joint, therefore the rivets will be in single shear, ∴ Shearing resistance of the rivets, $\boxed{P_s = n \times \frac{\pi}{4} \times d^2 \times \tau}$ —(1)

n = total no. of rivets

Knowing the inner diameter of the boiler shell (D), & the pressure of steam (P), the total shearing load acting on the circumferential joint.

$$W_s = \frac{\pi}{4} \times D^2 \times P \quad \text{—(2)}$$

equating eqn (1) & (2) ∴ $n \times \cancel{\frac{\pi}{4}} \times d^2 \times \tau = \cancel{\frac{\pi}{4}} \times D^2 \times P$

$$\Rightarrow \boxed{n = \left(\frac{D}{d}\right)^2 \frac{P}{\tau}}$$

(3) Pitch of Rivets :-

Efficiency of circumferential lap joint, $\eta_c = \dfrac{P_1 - d}{P_1}$ where P_1 = pitch of rivets for lap joint.

(4) Number of Rows :-

The number of rows of rivets for the circumferential joint may be obtained from.

$$\text{No of rows} = \frac{\text{Total no. of rivets}}{\text{No. of rivets in one row}}$$

$$\text{no. of rivets in one row} = \frac{\pi(D+t)}{P_1}$$

D = inner diameter of shell.

(5) After finding out the number of rows, the type of the joint (i.e single riveted or double riveted) may be decided. Then the number of rivets in a row and pitch may be re-adjusted.

(6) The distance between the rows of rives

- Pitch of rives = 2d or > 2d
- $P_{max} = C \times t + 41.28$ mm   t = thickness of the shell plate.
  C = constant.

(7) The overlap of the Plate may be fixed by

$$\text{Overlap} = (\text{No. of rows of rivets} - 1) P_b + m$$

m = margin

Q. A double riveted lap joint with zigzag riveting is to be designed for 13 mm thick plate. Assume $\sigma_t = 80$ MPa, $\tau = 60$ MPa & $\sigma_c = 120$ MPa

State how the joint will fail & find the efficiency of the joint.

Given $t = 13$ mm, $\sigma_t = 80$ MPa, $\tau = 60$ MPa, $\sigma_c = 120$ MPa

(i) Diameter of Rivet :-

As thickness of plate is 13 mm > 8 mm,

$\therefore$ diameter of Rivet Hole, $d = 6\sqrt{t}$

$= 6\sqrt{13}$

$= 21.6$ mm ≈ 22 mm

(As per standard)

(ii) Pitch of Rivets, 'p'

Since the joint is a double riveted lap joint with zig-zag riveting ⇒ There are two rivets per pitch length "n = 2"

- In lap joint, the rivets are in single shear

$\therefore$ Tearing Resistance of the Plate $P_t = (p-d)t \times \sigma_t$

$= (p-23)\,13 \times 80$

$= (p-23)\,1040$ N

shearing resistance of the rivets

$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$

$= 2 \times \frac{\pi}{4}(23)^2 \times 60$

$= 49864$ N.

∴ Equating P_t & P_s

$$(P-23) \times 1040 = 49864$$

$$\Rightarrow P = 48 + 23 = 71 \text{ mm}$$

[2 rivets per pitch length
$C = 2.62$]

The mx^m pitch, $P_{max} = C \times t + 41.28$

$$= 2.62 \times 13 + 41.28$$

$$= 75.28 \text{ mm}.$$

Since $P_{max} > P$ ∴ we shall adopt $P = 71$ mm (Ans)

(iii) Distance between the rows of Rivets :-

Distance between the rows of rivets [For zig-zag riveting].

$$P_b = 0.33P + 0.67d$$

$$= 0.33 \times 71 + 0.67 \times 23$$

$$= 38.8 \simeq 40 \text{ mm}$$

(iv) Margin :- 'm' $= 1.5 \times d = 1.5 \times 23 = 34.5 \simeq 35$ (Say)

• Failure of The Joint :-

Tearing Resistance of the Plate, $P_t = (p-d) \times t \times \sigma_t$

$$= (71-23) \times 13 \times 80$$

$$= 49920 \text{ N}.$$

Shearing Resistance of Rivets

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} \times (23)^2 \times 60$$

$$= 49864 \text{ N}.$$

Crushing Resistance of the rivets, $P_c = n \times d \times t \times \sigma_c$

$$= 2 \times 23 \times 13 \times 120 = 71760 \text{ N}$$

The least of P_t, P_s & P_c is $P_s = 49864\,N$.

Efficiency of the joint :-

Strength of the Un-reiveted

$$P = p \times t \times \sigma_t$$

$$= 71 \times 13 \times 80$$

$$= 73840\,N.$$

$$\therefore \eta = \frac{P_s}{P} = \frac{49864}{73840} = 0.675 = 67.5\%.$$

Q. Two plates of 10mm thickness each are to be joined by means of a single reiveted double strap butt jonit. Determine the reivet diameter, reivet pitch, strap thickness and efficiency of the joint! Take the working stress in tension & shear as 80 MPa & 60 MPa

Given data $t = 10\,mm$, $\sigma_t = 80\,MPa$, $\tau = 60\,MPa$, $\sigma_c = 120\,MPa$.

① Diameter of Rivet :-

Since $t > 8\,mm$ $\therefore d = 6\sqrt{t} = 6 \times \sqrt{10} = 18.97\,mm \approx 19\,mm$.

d = diameter of reivet hole $= 19\,mm$, 18mm reivet dia.

② Pitch of Rivets

Let p = pitch of reivets.

Since the joint is single Riveted Double Strap.

$\Rightarrow$ One Rivet per pitch length $= n = 1$.

& Rivets are in double shear.

We know Tearing Resistance, $P_t = (p-d)t \times \sigma_t$
of the plate
$$= (p-19) \times 10 \times 80 \quad \text{--- (1)}$$

Shearing Resistance of the Rivets, $P_s = n \times \overset{1.875}{\cancel{2}} \times \frac{\pi}{4} \times d^2 \times \tau$ (As per IBR)

$$= 1 \times 1.875 \times \frac{\pi}{4} \times (19) \times 60$$

$$= \cancel{10}\ 31900\,N.$$

Equating Eqn (1) & (2):

$$800(p-19) = 31900$$

$$\Rightarrow p = 58.87 \approx \cancel{60}$$

$$= 59\,mm$$

But According to IBR, Pitch max^m, $P_{max} = C(\cancel{t} + 41.28$

$$= 1.75 \times 10 + 41.28$$

$$= 58.78\,mm$$

$$= 59\,mm$$

(3) Thickness of cover platees.

$$t_1 = 0.625 \times t = \cancel{0}.0.625 \times 10$$

$$= 6.25\,mm$$

(4) Efficiency of the joint :-

Tearing Resistance of Plates, $P_t = (p-d)t \times \sigma_t$

$$= \cancel{0}(59-19) \times 10 \times 80$$

$$= \cancel{8}\ 32000\,N.$$

Shearing Resistance Rivets, $P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau$

$$= 1 \times 1.875 \times \frac{\pi}{4} \times (19)^2 \times 60$$

$$= 31896.98 \approx 32000\,N.$$

Crushing Resistance, $P_c = n \times d \times t \times \sigma_c$

$= 1 \times 19 \times 10 \times 120$

$= \cancel{19800}\ 22800\ N.$

Least of $P_t, P_s \,\&\, P_c = 22800N.$

Strength of un riveted plates per pitch length

$P = p \times t \times \sigma_t$

$= 59 \times 10 \times 80$

$= \cancel{480000}\ 47200\ N.$

$$\therefore \eta = \frac{\text{Least of } P_t, P_s, P_c}{P}$$

$$= \frac{22800}{47200} = .483 = 48.3\ \%$$

Design a double riveted butt joint with two cover plates for the longitudinal steam of boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm2. Assume joint efficiency as 75%, σ_t = 90MPa, compressive stress 140MPa, τ = 56MPa

Given:- D = 1.5m = 1500mm

P = 0.95 N/mm²

internal/steam pressure

longitudinal efficiency = 75% = 0.75

σ_t = 90MPa $\quad \tau$ = 56MPa

σ_c = 140 MPa

1) Thickness of the boiler shell plate:-

we know that thickness of boiler shell plate, $t = \dfrac{P.D}{2\sigma_t \times \eta_l} + 1\,mm$

$$= \frac{0.95 \times 1500}{2 \times 90 \times 0.75} + 1$$

$= 11.6 \approx 12mm$

2) Diameter of Rivet :-

As thickness of plate $t > 8mm$ $\therefore$ dia. of rivet hole, $d = 6\sqrt{t}$

$d = 6\sqrt{12} = 20.8mm$

The standard diameter of the reivet hole is 21mm and cor
diameter of the reivet is 20mm.

③ Pitch of reivets "p"

- The pitch of the reivets is obtained by equating the tearing resistance of the plate to the shearing resistan of the reivets.

$\therefore$ Tearing Resistance of the Plate, $P_t = (p-d)\,t \times \sigma_t$

$$= (p-21) \times 12 \times 90 \quad \text{—①}$$

$\therefore$ Shearing Resistane of Rivets $\Rightarrow P_s = n \times \frac{\pi}{4} d^2 \times \tau$

$$P_s = 2 \times \frac{\pi}{4} (21)^2 \times 56$$

[$\because$ As the Joint is Double reiveted bouble strap. Butt joint]

$\Rightarrow$ Double shearing of Rivets.

& There are two nos. of reivets per pitch length.

Assuming that the rivets in double shear are 1.875 times stronger than in single shear.

$\therefore$ Shearing strength of the rivets.

$$P_s = n \times 1.875 \times \frac{\pi}{4} d^2 \times \tau$$

$$= 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56$$

$$= 72745\,N. \quad \text{—②}$$

Equating eqn ① & ②

$$\therefore (p-21) \times 12 \times 190 = 72745$$

$$\Rightarrow p = 88.35 \approx 90\,mm$$

According to IBR, Max^m Pitch of reivets for longitudinal joint $= P_{max} = C \times t + 41.28\,mm$

$$= 3.50 \times 12 + 41.28 = 83.28 \approx 84\,mm$$

Since the value of p is more than P_{max} $p > P_{max}$

∴ We shall adopt pitch of the rivets, $p = P_{max} = 84$ mm

④ <u>Distance between rows of Rivets</u>

Assuming zig-zag riveting, the distance between the rows of the rivets, $P_b = 0.33p + 0.67d = 0.33 \times 84 + 0.67 \times 21 = 41.28 \approx 42mm$

⑤ <u>Thickness of cover plates</u> :- "t_1" $= 0.625t$ [if cover plates are of equal ~~length~~ width]

~~According~~ 1

$= 0.625 \times 12$

$= 7.5mm$

⑥ <u>Margin</u>, $m = 1.5d = 1.5 \times 21 = 31.5 \approx 32mm$

⑦ <u>Efficiency of Joint</u> :-

Tearing resistance, $P_t = (p-d) \times t \times \sigma_t = (84-21) \times 12 \times 90$
$= 68040N$

Shearing resistance, $P_s = n \times 1.875 \times \frac{\pi}{4} d^2 \times \tau$
$= 2 \times 1.875 \times \pi/4 (21)^2 \times 56 = 72745N.$

Crushing resistance, $P_c = n \times d \times t \times \sigma_c$
$= 2 \times 21 \times 12 \times 140 = 70560N$

∴ The strength of riveted joint is the least of P_t, P_s or P_c

$(P_t = 68040N)$

∴ The strength of un-riveted portion, or solid plate

$P = p \times t \times \sigma_t = 84 \times 12 \times 90 = 90720N.$

∴ Efficiency of joint, $\eta = \frac{P_t}{P} = \frac{68040}{90720} = 0.75$

$\approx 75\%$.

∴ Since the efficiency of joint (designed) is equal to the given efficiency $= 75\%$.

Chapter - 03
Shafts & Keys

3.1 State function of shafts:-

Shaft:- A shaft is a rotating machine element, usually circular in cross section, which is used to transmit power from one part to another. (or) from a m/c which produce power to a m/c which absorbs power.

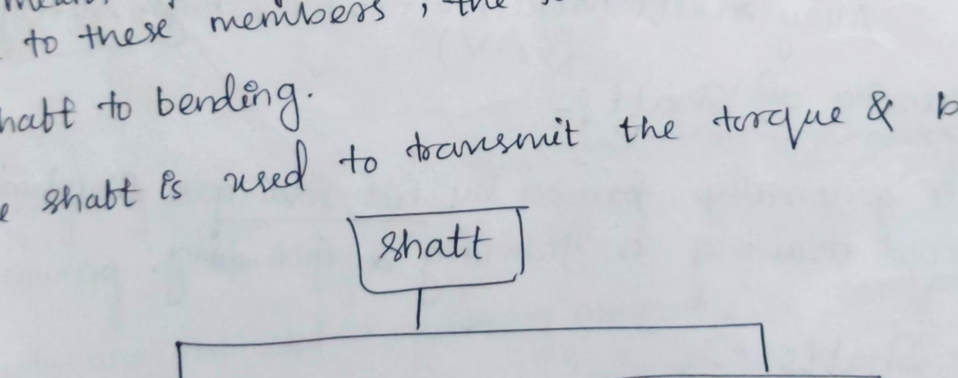
→ The power is delivered to the shaft by some tangential force & the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.

→ The various members such as gears, pulley & sprockets are mounted on shaft in order to transmit powers. by means of keys.

→ Due to these members, the forces exerted upon them causes the shaft to bending.

→ The shaft is used to transmit the torque & bending moment

→



- "Axle":- An axle is a stationary machines element & is used for the transmission of bending moment.
e.g - : wheels of car supported by axle.

Spindle :- is a short shaft that imparts motion either to a cutting tool or to a workpiece.

(e.g - drill press spindle) (eg - lathe spindle)

Materials Used for shafts

- Properties of shaft Materials

→ High strength

→ Good Machinability

→ Low notch sensitivity factor (less stress concentration)

→ Good Heat treatment

→ wear resistance ↑

Materials :- Mild steel For Ordinary shaft

Alloy steel (Nickel, Ni-Cr (Cr-V)) For High strength

Manufacturing of Shaft :-

Shafts are generally formed by hot rolling & finished to size by cold drawing or turning & grinding.

Types of Shafts :-

Transmission shaft :- These shafts are used to transmit power between the source and the machine absorbing power.

Examples :- The counter shaft

Line shaft

Over head shafts

(2) Machine shaft :- These are the general part of the machine itself.
e.g. - Crank shaft, Axle shaft,

3.3 Design of shafts (Transmission)

① Stresses in shafts :

→ Shear stresses due to the transmission of torque (Torsional load)

→ Bending stresses (tensile or compressive) due to the forces acting upon machine elements (gears, pulleys)

→ Stresses due to combined torsional & bending loads.

Design of shafts → Strength
Design of shafts → Rigidity

* In designing shafts on the basis of strength, the following cases may considered.

ⓐ Shafts are subjected to twisting moment or torque only.

ⓑ Shafts are subjected to bending moment

ⓒ Shafts are subjected to combined twisting & bending moments

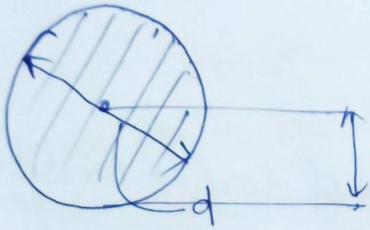
(a) Shafts Subjected to Twisting Moment

(1) Diameter of the shaft - may be obtained by using

$$\boxed{\frac{T}{J} = \frac{\tau}{r}} \quad \text{--- (1)}$$

T = Twisting moment acting upon the shaft

J = Polar moment of inertia of shaft about axis of rotation. $= \int_A r^2 dA$



τ = Torsional shear stress.

r = Distance from neutral axis to the outermost fiber

$= d/2$

* Polar moment of inertia of a solid shaft, $J = \frac{\pi}{32} \times d^4$

∴ Equ (1) became, $\frac{T}{J} = \frac{\tau}{r}$

$$\Rightarrow \frac{T}{(\pi/32\, d^4)} = \frac{\tau}{(d/2)}$$

$$\Rightarrow \boxed{T = \frac{\pi}{16} \times \tau \times d^3} \leftarrow \text{For solid shaft}$$

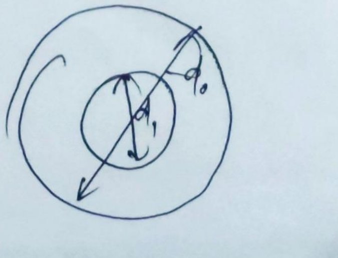
For Hollow Shaft

Diameter of hollow shaft :- $d_o - d_i$

∴ Polar moment of inertia of hollow shaft = $J_o = \frac{\pi}{32}(d_o^4 - d_i^4)$

∴ From equ (1) $\frac{T}{J} = \frac{\tau}{r}$

$$\Rightarrow \left(\frac{T}{\frac{\pi}{32}(d_o^4 - d_i^4)}\right) = \frac{\tau}{\left(\frac{d_o}{2}\right)}$$



$$\therefore T = \frac{\pi}{16} \times \tau \times \left(\frac{d_o^4 - d_i^4}{d_o}\right)$$

If $K = \frac{d_i}{d_o}$. $\boxed{T = \frac{\pi}{16} \times \tau \times d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]} \rightarrow$ Torque

$$\therefore T = \frac{\pi}{16} \times \tau \times \left(\frac{d_o}{d_o}\right)^4 \left\{1 - \left(\frac{d_i}{d_o}\right)^4\right\}$$

↳ Outer dia & inside dia of hollow shaft ~~can be~~ may be calculated.

* When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both shafts must be same.

∴ For the same material of both the shafts.

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\Rightarrow \frac{d_o^4 - d_i^4}{d_o} = d^3$$

$$\Rightarrow (d_o)^3 (1 - k^4) = d^3$$

(2) Twisting moment (T) :- maybe obtained from the relation $P = \frac{2\pi N T}{60}$ (watt)

P = power
N = speed in rpm of shaft.

(3) In case of Belt drives, (T)

$$T = (T_1 - T_2) R$$

T_1 = Tension in the tight side
T_2 = " in the slack side. } of belt.

R = Radius of the pulley.

Q. A line shaft rotating at 200rpm is to transmit 20kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft neglecting the bending moment on the shaft.

Given data :- Speed $N = 200rpm$

power, $P = 20kW$ $\quad$ d = dia of shaft.

$\tau = 42MPa$

Let T = torque.

Torque transmitted by the shaft, $T = \frac{P \times 60}{2\pi N}$

$= \frac{20 \times 10^3 \times 60}{2 \times \pi \times 200}$

$= 955 \ N\text{-}m$

$= 955 \times 10^3 \ N\text{-}mm$

As the shaft is subjected to twisting moment only.

$\therefore T = \frac{\pi}{16} \times \tau \times d^3$

$\Rightarrow 955 \times 10^3 = \frac{\pi}{16} \times 42 \times d^3$

$\Rightarrow d^3 = \frac{955 \times 10^3}{8.25} = 115733$

$\Rightarrow d = 48.7 \ mm \approx 50mm$ (standard.)

Find the diameter of a solid steel shaft to transmit 20kw at 200 rpm. The ultimate shear stress for the steel maybe taken as 360 MPa & a factor of safety as 8.

If a hollow shaft is to be used in place of the solid shaft, find the inside & outside diameter when the ratio of inside to outside diameter is 0.5.

Given:- Power, $P = 20KW$, Speed $N = 200rpm$

$\tau_{ult} = 360 MPa$, $FOS = 8$, $k = d_i/d_o = 0.5$

We know Allowable shear stress, $\tau = \frac{\tau_{ult}}{F.S} = \frac{360}{8}$

$\tau = 45 N/mm^2$

① Diameter of the Solid shaft :- "d"

Toreque transmitted by the shaft, $T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2 \times \pi \times 200}$

$= 955 N\text{-}m$

$= 955 \times 10^3 N\text{-}mm$

As the shaft is subjected to twisting moment only

$T = \frac{\pi}{16} \times \tau \times d^3$

$\Rightarrow 955 \times 10^3 = \frac{\pi}{16} \times 45 \times d^3$

$\Rightarrow d^3 = \frac{955000}{8.84} = 108032$

$\Rightarrow d = 47.6 \approx 50 mm$ (say)

② Diameter of Hollow shaft $(d_o - d_i)$

Torque transmitted by the hollow shaft $(T) = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$

$\Rightarrow 955000 = \frac{\pi}{16} \times 45 \times (d_o)^3 \{1 - (0.5)^4\}$

$\Rightarrow (d_o)^3 = \frac{955000}{8.3} = 48.6 \approx 50 mm = d_o$

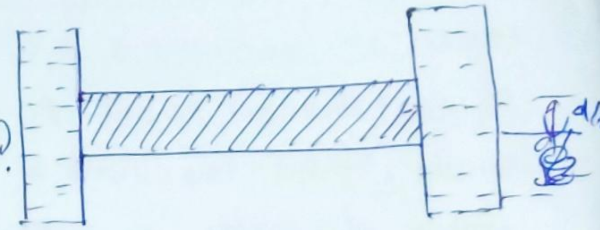
$\therefore k = \frac{d_i}{d_o} = 0.5$

$\Rightarrow d_i = 0.5 \times d_o = 0.5 \times 50 = 25 mm$

rotating at 200rpm is to

(b) Shafts subjected to Bending Moment Only :-

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compression) is given by the bending eqn.



$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y}} \quad \text{—(1)}$$

M = Bending Moment

I = moment of inertia of cross-sectional area of the shaft about the axis of rotation.

σ_b = Bending stress

y = Distance from neutral axis to the outer most fiber.

∴ Moment of inertia of solid shaft

$$I = \frac{\pi}{64} d^4 \quad \& \quad y = d/2$$

∴ Eqn (1) becomes.

$$\frac{M}{\left(\frac{\pi}{64}\right) d^4} = \frac{\sigma_b}{(d/2)}$$

$$\Rightarrow \boxed{M = \frac{\pi}{32} \times d^3 \times \sigma_b}$$

· Dia. of shaft may be calculated

For Hollow shaft :-

We know that, the moment of inertia of a hollow shaft

$$I_h = \frac{\pi}{64}\left[d_o^4 - d_i^4\right] = \frac{\pi}{64} \times (d_o)^4 (1-K^4)$$

$$\& \; y_h = d_o/2$$

∴ Eqn (1) becomes

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1-K^4)} = \frac{\sigma_b}{(d_o/2)}$$

$$\Rightarrow \boxed{M = \frac{\pi}{32} \times \sigma_b \times (d_o)^3 \cdot (1-K^4)}$$

Dia of Hollow shaft may be obtained.

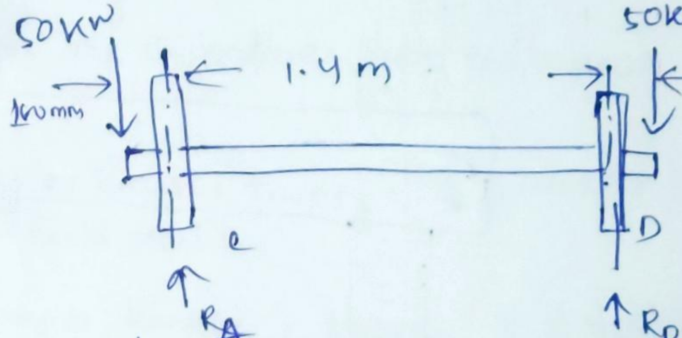
Q. A pair of wheels of a railway wagon carries a load of 50KN on each axle box, acting at a distance of 100mm outside the wheel base. The gauge of the rails is 1.4 m Find the diameter of the axle between the wheels, if the stress is not to exceed 100MPa.

Given $W = 50KN = 50 \times 10^3 N$

$L = 100mm$

$x = 1.4m$

$\sigma_b = 100 MPa$



From, figure, a little consideration will show that the maximum bending moment acts on the wheels at A & B

$\therefore$ Max^m Bending moment $M = W \cdot L$

$= (50 \times 10^3) \times 100$

$= 50 \times 10^5$ N-mm

Let d = dia of the axle.

We know that the max^m bending moment (M)

$M = \frac{\pi}{32} \times \sigma_b \times d^3$

$\Rightarrow 50 \times 10^5 = \frac{\pi}{32} \times 100 \times d^3$

$\Rightarrow d^3 = \frac{5 \times 10^6}{9.82} = 0.51 \times 10^6.$

$\Rightarrow d = 79.8mm \approx 80 mm$ (Ans)

Shafts Subjected to Combined Twisting Moment & Bending Moment

When the shaft is subjected to combined twisting moment & bending moment, then the shaft must be designed on the basis of the two moments simultaneously.

- The following two theories are important from the subject point of view.

① Max^m shear stress theory or Guest's theory. [It is used for ductile materials such as mild steel]

② Max^m normal stress theory or Rankine's theory [It is used for brittle materials such as cast iron]

* **Max^m Shear Stress Theory :-**

Let τ = Shear stress induced due to twisting moment

σ_b = Bending stress (tensile (or) compressive) induced due to bending moment.

According to max^m shear stress theory, The max^m shear stress in the shaft

$$\boxed{\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}} \quad —— ①$$

$\because \frac{M}{I} = \frac{\sigma_b}{y}$

$\Rightarrow \sigma_b = \left(\frac{My}{I}\right)$

where $y = d/2$

$I = \frac{\pi}{64}d^4$

$$\boxed{\sigma_b = \left(\frac{32M}{\pi d^3}\right)}$$

$\frac{T}{J} = \frac{\tau}{r}$

where $J = \frac{\pi}{32}d^4$

$r = d/2$

$\Rightarrow \tau = \frac{Tr}{J}$

$= \frac{T \times d/2}{\pi/32\, d^4}$

$$\boxed{\tau = \frac{16T}{\pi d^3}}$$

Substituting the values of σ_b & τ in equ^n ①

$$\therefore \tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\boxed{\Rightarrow \frac{\pi}{16} \times d^3 \times \tau_{max} = \sqrt{M^2 + T^2}} \quad —— ②$$

(b) Shafts subjected to Bending Moment Only :-

where $\sqrt{M^2+T^2} = T_e$ = Equivalent Twisting moment

Defined as that Twisting moment which when acting alone, produces the same Shear stress (τ) as the actual twisting moment.

* By limiting the max^m shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the eqn (2) may be written as

$$\boxed{T_e = \sqrt{M^2+T^2} = \frac{\pi}{16} \times \tau \times d^3} \quad \text{—— (3)}$$

From this eqn, the diameter of shaft (d) may be calculated.

* Max^m Normal stress Theory:-

According to max^m Normal stress theory, the max^m normal stress in the shaft,

$$\boxed{\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}} \quad \text{—— (4)}$$

We know $\frac{M}{I} = \frac{\sigma_b}{y}$ $\qquad$ $\frac{T}{J} = \frac{\tau}{r}$

$\Rightarrow \boxed{\sigma_b = \frac{32M}{\pi d^3}}$ $\qquad$ $\Rightarrow \boxed{\tau = \frac{16T}{\pi d^3}}$

Substituting the values of σ_b & τ in eqn (4)

$$\sigma_{b(max)} = \frac{1}{2}\left(\frac{32M}{\pi d^3}\right) + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\times\left(\frac{16\tau}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)\right]$$

$$\Rightarrow \boxed{\frac{\pi}{32} \times \sigma_{b(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right]} \quad \text{—— (5)}$$

Where . $\frac{1}{2}M + \sqrt{M^2+T^2} = M_e$ = Equivalent Bending moment

Defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.

- By limiting the max^m Normal stress $\sigma_{b(max)}$ equal to the allowable bending stress (σ_b), then the equation e ⑤ becomes

$$\boxed{M_e = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3} \quad — ⑥$$

From this equa^n, diameter of the shaft (d) may be calculated.

Note In case of a hollow shaft, the equations

$$T_e = \sqrt{M^2+T^2} = \frac{\pi}{16} \times \tau\, d_o^3\,(1-K^4)$$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2+T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1-K^4)$$

* Diameter of the shaft may be obtained by using both the theories & the larger of the two values is adopted.

Q- A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10000 N-m. The shaft is made of 45C8 steel having ultimate tensile stress of 700 MPa & a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Given $M = 3000$ N-m $= 3\times10^6$ N-mm F.S = 6

$T = 10\,000$ N-m $= 10\times10^6$ N-mm

$\sigma_{tu} = 700$ MPa $\tau_u = 500$ MPa

We know that allowable tensile stress $\sigma_t = \frac{\sigma_{tu}}{F.S} = \frac{700}{6} = 116.7\ N/mm^2$

" Shear stress, $\tau = \frac{\tau_u}{F.S} = \frac{500}{6} = 83.3\ N/mm^2$

Let d = diameter of the shaft

* According to max^m shear stress theory, equivalent twisting moment $T_e = \sqrt{M^2 + T^2}$

$= \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2}$

$= 10.44 \times 10^6$ N-mm

We also know that equivalent twisting moment, $T_e = \frac{\pi}{16} \times \tau \times d^3$

$\Rightarrow 10.44 \times 10^6 = \frac{\pi}{16} \times 83.3 \times d^3$

$\Rightarrow d^3 = \frac{10.44 \times 10^6}{16.36} = 0.636 \times 10^6$

$\Rightarrow \boxed{d = 86 \text{ mm}}$

* According to max^m normal stress theory, Equivalent bending moment. $M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$.

$= \frac{1}{2}(M + T_e)$

$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6)$

$M_e = 6.72 \times 10^6$ N-mm

also We know that equivalent bending moment $M_e = \frac{\pi}{32} \times \sigma_b \times d^3$

$\Rightarrow 6.72 \times 10^6 = \frac{\pi}{32} \times 116.7 \times d^3$

$\Rightarrow d^3 = \frac{6.72 \times 10^6}{11.46} = 0.586 \times 10^6$.

$\Rightarrow \boxed{d = 83.7 \text{mm}}$

$\therefore$ The diameter of shaft = max^m $\{86 \& 83.7\}$ = 86mm

$\approx$ 90mm

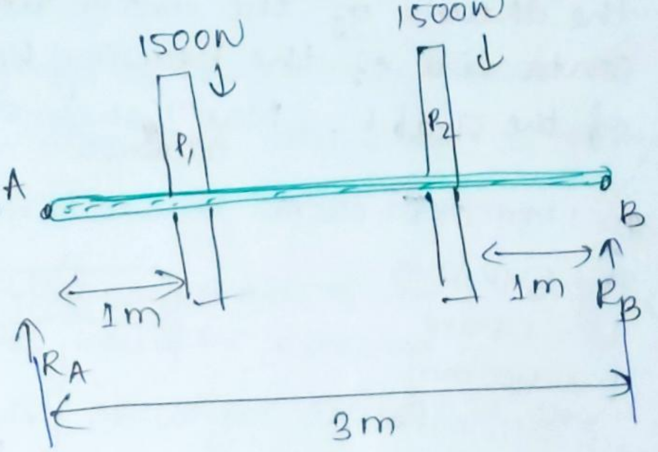
Q2 A shaft made of mild steel is required to transmit 100kW at 300 rpm. The supported length of the shaft is 3meters. It carries two pulleys each weighing 1500N supported at a distance of 1m from the ends respectively. Assuming the safe value of stress, determine the diameter of shaft

$P = 100\,kW$ (d) = dia of shaft

$N = 300$ rpm

$L = 3m$

$W = 1500N$



∴ Torque transmitted by the shaft

$$T = \frac{P \times 60}{2\pi N}$$

$= 3183$ N-m

Shaft carrying two pulleys is like a simply supported Beam

⇒ The reaction at each support = 1500N

$R_A = R_B = 1500N$.

A little consideration will show that the max^m Bending moment lies at P_1 & P_2.

Max^m Bending moment = $W \times 1m$

$= 1500 \times 1$

=

Equivalent twisting moment

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \times 10^3 \text{ N-mm}$$

Equivalent twisting moment $T_e = \pi/16 \times \tau \times d^3$

$\Rightarrow 3519 \times 10^3 = \pi/16 \times 60 \times d^3$

$\Rightarrow d = 66.8$ mm ≈ 70 mm.

Q. A line shaft is driven by means of a motor placed vertically below it. The pulley on the shaft is 1.5m in diameter and has belt tensions 5.4kN & 1.8kN. on the tight side & slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhanging from the shaft the distance of the centre line of the pulley from the centre line of the bearing being 400mm. Find the diameter of the shaft. Max^m shear stress of 42MPa allowable.

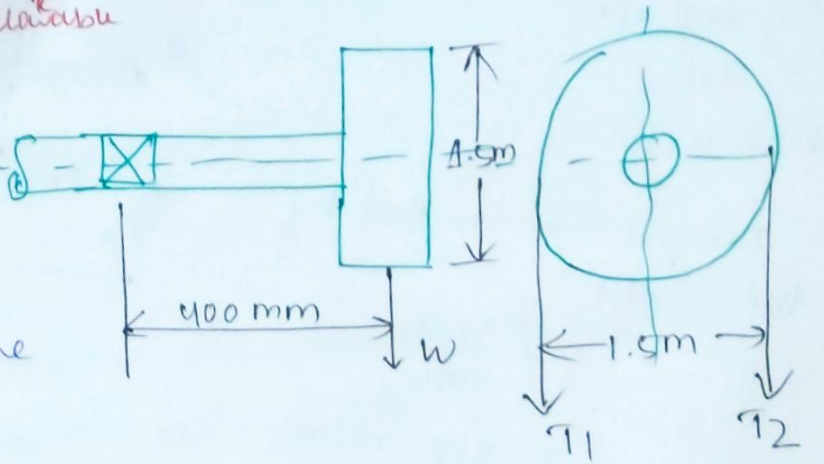
$D = 1.5m$ or $R = 0.75m$

$T_1 = 5.4kN$

$T_2 = 1.8kN$

$L = 400mm$

$\tau = 42MPa$



Torque transmitted by the shaft, $T = (T_1 - T_2)R$

$=$

$= 2700 N.m$

Neglecting the weight of shaft, total vertical load acting on the pulley, $W = T_1 + T_2 = 5400 + 1800 = 7200W$

$\therefore$ Bending moment, $M = W \times L = 7200 \times 400 = 2880 \times 10^3$ N-mm

Let d = dia of shaft.

$\therefore$ Equivalent twisting moment, $T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$

$= 3950 \times 10^3$ N-mm

$\therefore$ Equivalent twisting momet.

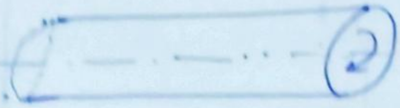
$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3$

$= \frac{\pi}{16} \times 42 \times d^3$

$\Rightarrow d = 78 \approx 80mm$

Design of Shaft on the Basis of Rigidity

(b) Rigidity



(i) Angle of Twist

Torsional Rigidity :- It is defined as how much an object of specified material resists twisting force, also known as torque.

- It depends upon the material as well as on its shape.
- T.R is important in the case of camshaft of an I.C engine where the timing of the valves would be effected.
- The permissible amount of twist should not exceed 0.25° per metre length of shaft.
- for Line shaft or Transmission shaft, deflections 2.5 to 3° per meter length may be used as limiting value.

The torsional deflection may be obtained by using torsion equation

$$\frac{T}{J} = \frac{G \cdot \theta}{L}$$

$$\Rightarrow \boxed{\theta = \frac{T \cdot L}{J \cdot G}}$$

T = Twisting moment or torque (N-m)

θ = Torsional deflection or Angle of twist in radians

J = Polar moment of inertia of the cross-sectional area about the axis of rotation. [m^4]

$J = \frac{\pi}{32} \times d^4$ (Solid shaft)

$= \frac{\pi}{32}(d_o^4 - d_i^4)$

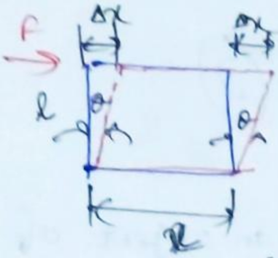
G = Modulus of Rigidity for the shaft (N/m^2)

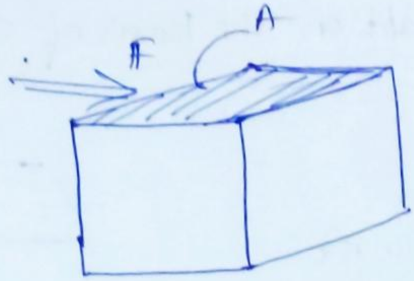
L = length of the shaft. (m)

* Torsional Rigidity depends on "G" & "J"

If $G\uparrow$ & $J\uparrow \Rightarrow T.R\uparrow$

Modulus of Rigidity :-





- Modulus of Rigidity or also called as shear modulus

we know that shear stress ∝ shear strain

$$\Rightarrow \tau \propto \gamma$$

$$\Rightarrow \tau = G \cdot \gamma$$

$$\Rightarrow \boxed{G = \frac{\tau}{\gamma}} \quad \text{or} \quad \boxed{G = \frac{(F/A)}{(\Delta x / l)}}$$

- Modulus of rigidity is the measure of the elastic shear stiffness of a material and is defined as the ratio of shear stress to shear strain.
- G is a measure of shearing stress which produce shear strain.

Modulus of Elasticity (E) = 2G(1+ μ)

From Torsion Equation $\frac{T}{J} = \frac{G \cdot \theta}{L}$

$$\Rightarrow \boxed{G = \frac{TL}{J\theta}}$$

* Rigidity

The property of a substance in which the shape of the substance does not change due to an external force.

- G tells us about the shear deformation undertaken by a body on exposure to shearing forces of different magnitude.

Q- A steel spindle transmits 4KW at 800 rpm. The angular deflection should not exceed 0.25° per meter of the spindle. If the modulus of rigidity for the material of the spindle is 84GPa, Find the diameter of the spindle & the shear stress included in the spindle?

Given $P = 4KW$, $N = 800 rpm$, $\theta = 0.25^\circ = 0.25 \times \frac{\pi}{180} = 0.0044$ rad.

$L = 1m = 1000mm$, $G = 84GPa = 84 \times 10^9 N/m^2 = 84 \times 10^3 N/mm^2$

Diameter of the spindle:-

let d = dia

we know that the torque transmitted by the spindle.

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \, N\text{-}m$$

we also know that $\frac{T}{J} = \frac{G \times \theta}{L}$

$$\Rightarrow J = \frac{T \times L}{G \times \theta}.$$

$$\Rightarrow \frac{\pi}{32} \times d^4 = \frac{47740 \times 1000}{84 \times 10^3 \times 0.0044} = 129167$$

$$\Rightarrow d^4 = 129167 \times 32/\pi$$

$$\Rightarrow d = 33.87 \approx 35 \text{ mm (Ans)}$$

Shear Stress induced in the spindle:-

τ = shear stress induced in the spindle

we know that the torque transmitted by the spindle "T"

$$47740 = \frac{\pi}{16} \times \tau \times d^3$$

$$= \frac{\pi}{16} \times \tau \times (35)^3$$

$$\Rightarrow \tau = \frac{47740}{8420} = 5.67 \, N/mm^2 = 5.67 \, MPa \text{ (Ans)}$$

Q-2 Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of shaft. The inside diameter of the hollow shaft being half the external diameter. Both the ~~side~~ shafts have the same material & length.

Given $d_o = d$, $d_i = d_o/2$ or $k = d_i/d_o = 1/2 = 0.5$

Comparison of weight

Weight of hollow shaft, W_H = Area × Length × density — ①

$$= \frac{\pi}{4}(d_o^2 - d_i^2) \times L \times \rho$$

Weight of solid shaft, W_s = Area × Length × density — ②

$$= \frac{\pi}{4} d^2 \times L \times \rho$$

Since both the shafts have the same material, & length.

$$\frac{eq^n ①}{eq^n ②} = \frac{W_H}{W_S} = \frac{d_o^2 - d_i^2}{d^2} = 1 - \frac{d_i^2}{d_o^2} = 1 - k^2 = 1 - \frac{1}{4}$$

$$= 3/4 = 0.75 \text{ (Ans)}$$

Comparison of strength

Strength of the hollow shaft, $T_H = \frac{\pi}{16} \times \tau \times d_o^3 (1-k^4)$ — ③

Strength of the solid shaft, $T_S = \frac{\pi}{16} \times \tau \times d^3$ — ④

$$\frac{eq^n ③}{eq^n ④} = \frac{T_H}{T_S} = \frac{\pi/16 \times \tau \times d_o^3 (1-k^4)}{\pi/16 \times \tau \times d^3} = \frac{d_o^3}{d^3}(1-k^4) = 0.9375 \text{ (Ans)}$$

Comparison of stiffness

We know that stiffness ⇒ $\frac{T}{\theta} = \frac{G \times J}{L}$

Stiffness of the Hollow shaft, $S_H = \frac{G}{L} \times \frac{\pi}{32}(d_o^4 - d_i^4)$ — ⑤

Stiffness of the solid shaft, $S_S = \frac{G}{L} \times \frac{\pi}{32} d^4$ — ⑥

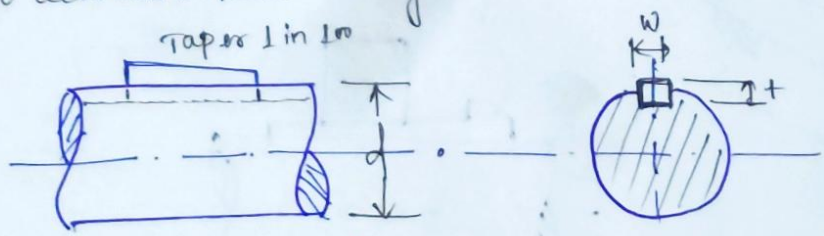
$$\frac{eq^n ⑤}{eq^n ⑥} = \frac{S_H}{S_S} = \frac{d_o^4 - d_i^4}{d^4} = 1 - \left(\frac{d_i}{d_o}\right)^2 = 1 - k^4 = 0.9375 \text{ (Ans)}$$

Key

3.5 State Function of Keys, Types of keys & material of keys

Key :- A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.

- It is always inserted parallel to the axis of the shaft.
- Keys are used as temporary fastening & are subjected to considerable crushing & shearing stresses.
- A keyway is a slot or recess in a shaft & hub of the pulley to accommodate a key.



(Rectangular Sunk key)

Types of Keys :-

(1) Sunk Keys
(2) Saddle Keys
(3) Tangent keys
(4) Round keys
(5) Splines

(1) Sunk keys :- The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley.

(a) Rectangular Sunk Key :-

width of key $w = \frac{d}{4}$, Thickness of key, $t = \frac{2w}{3} = d/6$

d = diameter of the shaft

or " of the hole in the hub

② Square Sunk Key :- width of the key = Thickness of the

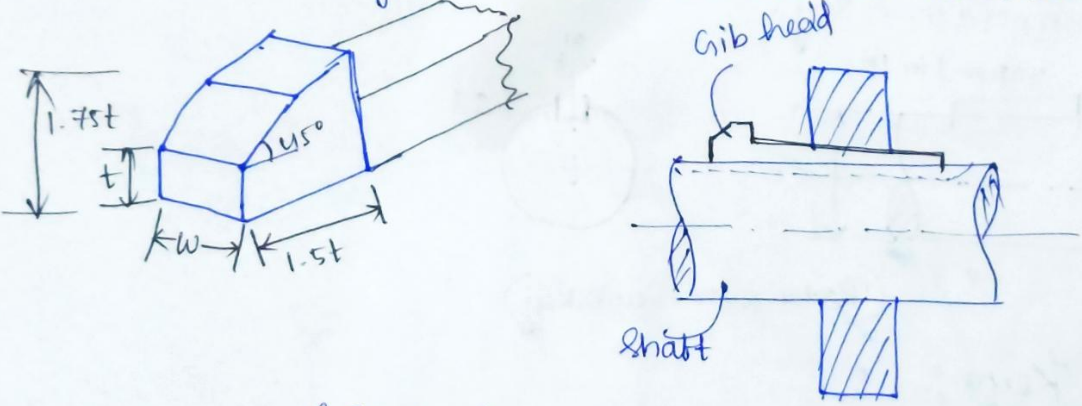
$= w = t = d/4$

③ Parallel Sunk Key :- The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout.

- It is taperless & is used where the pulley, gear or other mating piece is required to ~~slid~~ slide along the shaft.

④ Gib-head key :- It is a rectangular sunk key with a head at one end known as gib head.

- It is usually provided to facilitate the removal of key.



: Gib head key :-

The usual proportion of the gib head key are

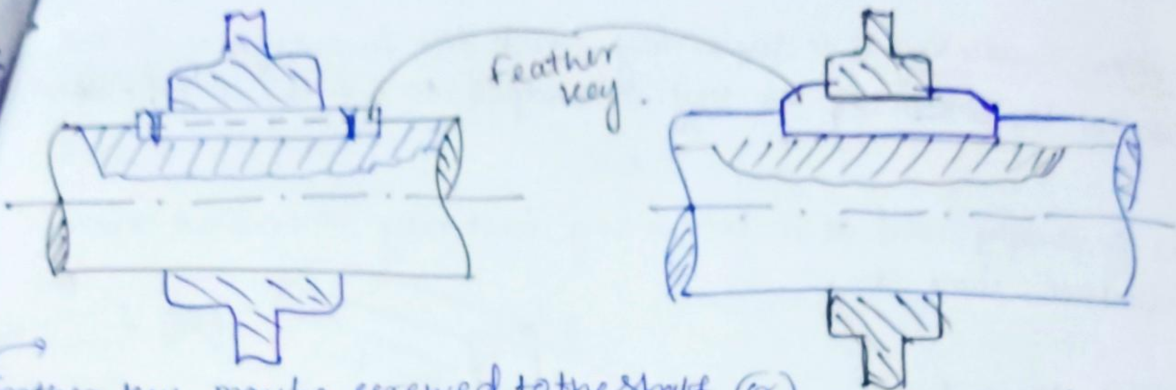
$w = d/4$

the thickness at large end, $t = 2w/3 = d/6$

⑤ Feather Key :- A key attached to one member of a pair and which permits relative axial movement is known as feather key.

- It is a special type of parallel key which transmits a turning moment & also permits axial movement.

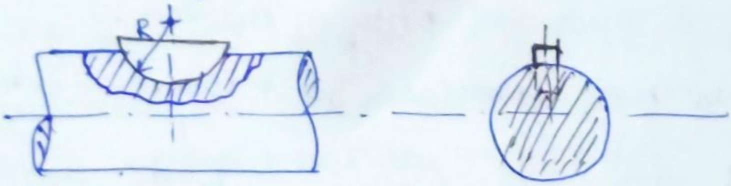
- It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



Feather key maybe screwed to the shaft (or) it may have double gib heads

6. Woodruff Key :-

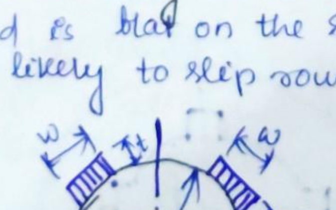
- The woodruff key is an easily adjustable key.
- It is a piece from a cylindrical disc having segmental cross section
- It is largely used in machine tools & automobile construction.



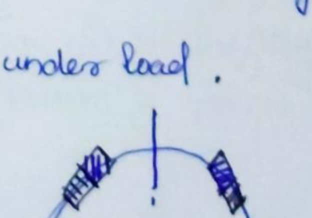
woodruff key.

Saddle Keys :- ① Flat saddle key ② Hollow saddle key.

A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft.
- It is likely to slip round the shaft under load.



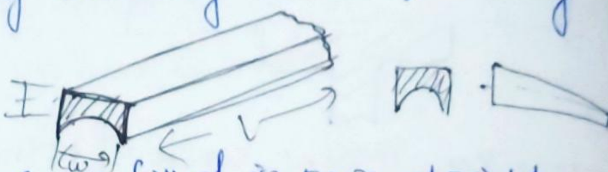
$t = w/3 = d/12$



Tangent key

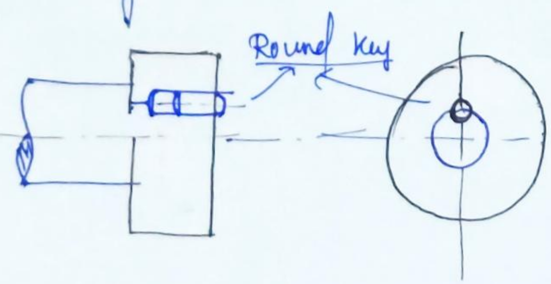
A hollow Saddle Key is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface.

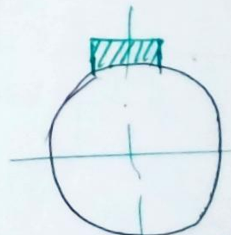
- It is usually used as a temporary fastening in fixing & setting eccentric, cams etc.



Tangent Keys :- The tangent keys are fitted in pair at right angles. Each key is to withstand torsion in one direction only.
- These are used in large heavy duty shafts.

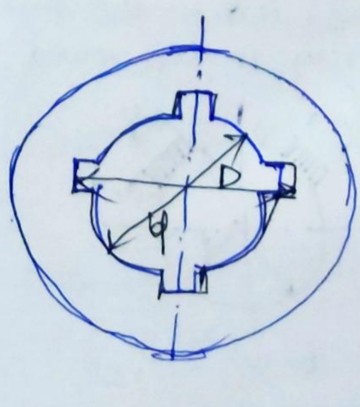
Round Keys :- The round keys are circular in section and fit into holes drilled partly in shaft and partly in the hub.
- Usually considered to be most appropriate for low power drives.





Hollow Saddle Key

Splines :- Keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as splined shafts.
- These shafts have 4, 6, 10 or 16 splines.
- The splined shafts are relatively stronger than shafts having a single keyway.



Materials of Keys :-

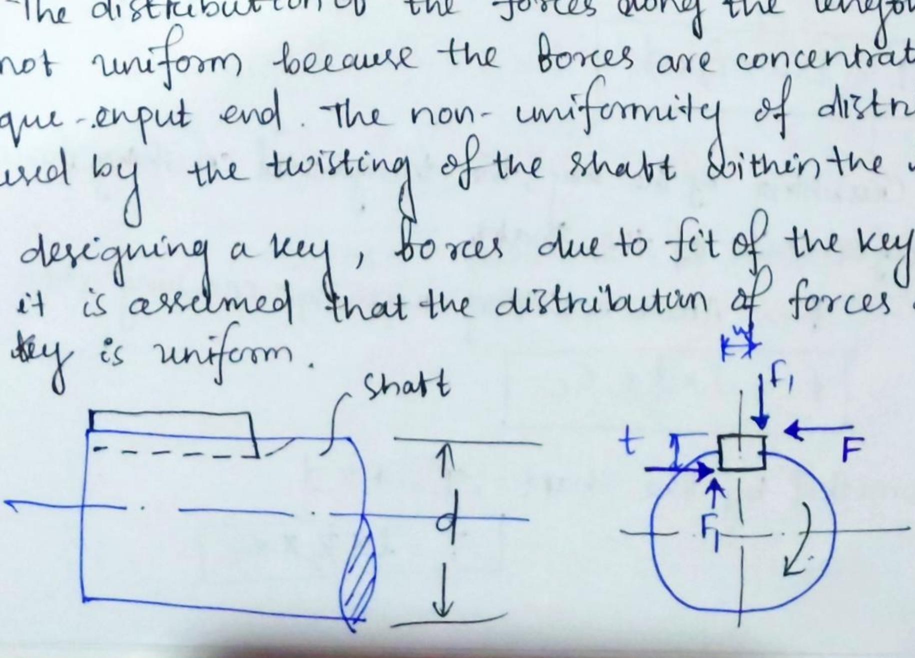
Failures of Key :-

Forces Acting on a Sunk Key :-

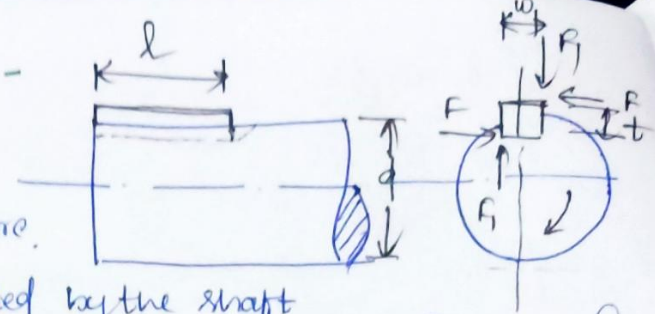
(1) Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

(2) Forces (F) due to the torque transmitted by the shaft; These forces produce shearing & compressive (or crushing) stress in the key.

* The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.
* In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.



Strength of a Sunk Key :-



A key connecting the shaft and hub is shown in figure.

Let T = Torque transmitted by the shaft

F = Tangential force acting at the circumference of the shaft

d = diameter of shaft

l = length of key

t = thickness of key.

w = width of key.

τ = shear stress

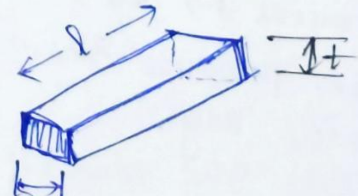
σ_c = crushing stress.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing (or) crushing

- Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft.

$$F = \text{Area resisting shearing} \times \text{shear stress}$$

$$\boxed{F = l \times w \times \tau}$$



∴ Torque transmitted by the shaft

$$T = F \times \frac{d}{2}$$

$$\boxed{T = (l \times w \times \tau) \times d/2}$$

- Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft.

$$F = \text{Area resisting crushing} \times \text{crushing stress}$$

$$\boxed{F = l \times \frac{t}{2} \times \sigma_c}$$

∴ Torque transmitted by the shaft, $T = F \times \frac{d}{2}$

$$\boxed{T = l \times t/2 \times \sigma_c}$$

The key is equally strong in shearing & crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$\Rightarrow \boxed{\frac{w}{t} = \frac{\sigma_c}{2\tau}} \quad \text{———}$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress.

From eqn $\quad$ if $w = t$ $\quad$ [square key]

then $\sigma_c = 2\tau$

i.e in square key, the shearing strength = crushing strength

Length of the key :- In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{———}$$

Torsional shear strength of the shaft

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \text{——} \quad [\tau_1 = \text{shear stress for the shaft material}]$$

Equating eqn ① &

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3 \qquad (w = d/4)$$

$$\Rightarrow l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{2\tau} = 1.571 d \times \frac{\tau_1}{\tau}$$

$\therefore \boxed{l = 1.571 d}$ $\left\{ \begin{array}{l} \text{when the key material is same as that} \\ \text{of the shaf. } \tau_1 = \tau \end{array} \right.$

Q Design the rectangular key for a shaft of 50mm diameter. The shearing and crushing stresses for the key material are 42MPa & 70MPa.

Given: $d = 50mm$, $\tau = 42MPa = 42 N/mm^2$

$\sigma_c = 70MPa$,

The rectangular key is designed as :-

From standard table, for a shaft of dia 50mm,

width of key, $w = 16mm$

thickness of key, $t = 10mm$.

The length of key is obtained by considering the key in shearing & crushing.

Let l = length of key.

Considering shearing of the key, we know that shearing strength or (torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2}$$

$$= l \times 16 \times 42 \times \frac{50}{2} = 16800\, l \text{ N-mm} \quad \text{— (1)}$$

& Torsional shearing strength of the shaft, $T = \frac{\pi}{16} \times \tau \times d^3$

$$= \frac{\pi}{16} \times 42 \times (50)^3$$

$$T = 1.03 \times 10^6 \text{ N-mm} \quad \text{— (2)}$$

From eqn (1) & (2)

$$l = \frac{1.03 \times 10^6}{16800} = 61.31 \text{ mm}$$

Now considering Crushing ~~strength~~ of the key. We know that ~~shearing~~ crushing strength (or torque transmitted) of key.

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$= l \times \frac{10}{2} \times 70 \times \frac{50}{2}$$

$$T = 8750\, l \text{ N-mm} \quad \text{— (3)}$$

From eqn (2) & (3) $\therefore$ $l = 1.03 \times 10^6 / 8750 = 117.7mm$

Taking larger of the two values, we have length of key

$l = 117 \approx 120mm$ (Ans)

A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide & 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use max^m shear stress theory & assume a factor of safety of 2.

Given, $d = 45\,mm$ $\quad w = 14\,mm$ $\quad \sigma_{yt_{key}} = 340\,MPa$ $\quad F.S = 2$

$\sigma_{yt_{shaft}} = 400\,MPa$ $\quad t = 9\,mm$

According to max^m shear stress theory, max^m shear stress for the shaft

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S} = \frac{400}{2 \times 2} = 100\,N/mm^2$$

& Max^m shear stress for the key $\tau_k = \frac{\sigma_{yt}}{2 \times F.S} = \frac{340}{2 \times 2} = 85\,N/mm^2$

We know that the max^m torque transmitted by the shaft & key.

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100 \times (45)^3 = \overset{1.789}{1.8} \times 10^6\,N\text{-}mm$$

Consider the failure of key due to ~~Shearing~~ crushing. We know that max^m torque transmitted (T) by the shaft & key

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{c_{key}} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17213\,l$$

$\boxed{\text{Taking } \sigma_{cu} = \frac{\sigma_{yt}}{F.S}} = 209.15\,mm$

$\Rightarrow l = 104.6\,mm$

Consider the failure of key due to shearing. The max^m toreque transmitted (T) $1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26775\,l$

$l = 67.2\,mm$

Taking the larger value $\{104.6, 67.2\} = 104.6 \approx 105\,mm$ (Ans)

* The shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension,

$$\boxed{\therefore \tau_{max} = \frac{\sigma_{yt}}{2 \times F.S}}$$

Effects of Key ways :-

- The keyway cut into the shaft reduces the load carrying capacity of the shaft.
- This is due to the stress concentration near the corners of the key way and reduction in the cross-sectional area of the shaft.
- The torsional strength of the shaft is reduced.
- The following relation for the weakening effect of the key way is based on the experimental results by H.F. Moore.

$$\boxed{e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)}$$

e = shaft strength factor $= \dfrac{\text{Strength of the shaft with keyway}}{\text{Strength of the same shaft without keyway}}$.

w = width of keyway

d = dia of shaft

h = depth of keyway $= \dfrac{\text{Thickness of key}(t)}{2}$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too ~~high~~ long & the key is sliding type, then the angle of twist is increased in the ratio K_θ

$$\boxed{K_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)}$$

K_θ = Reduction factor for angular twist.

Design of Couplings

4.1 Design of Shaft Coupling

* **Coupling:-** A coupling is a device (or mechanical component) used to connect two shafts together at their ends for the purpose of transmitting power.

* **Shaft Coupling:-** A shaft coupling is a mechanical component that connects the drive shaft & driven shaft of a motor in order to transmit power.

Purpose of shaft Coupling:-

(1) To provide for the connection of shafts units that are manufactured separately such as a motor & generator & to provide for disconnection for repairs or alteration.

(2) To provide for mis-alignment of the shaft or to introduce mechanical flexibility.

(3) To reduce the transmission of shocks loads from one shaft to another.

(4) To introduce protection against overloads.

(5) It should have no projecting parts.

Requirements of a Good Shaft Coupling:-

(1) It should be easily connect or disconnect.

(2) It should transmit the full power from one shaft to the other shaft without losses.

(3) It should hold the shafts in perfect alignment.

(4) It should reduce the transmission of shock loads from one shaft to another shaft.

(5) It should have no projectioning parts.

Types of Coupling

Coupling

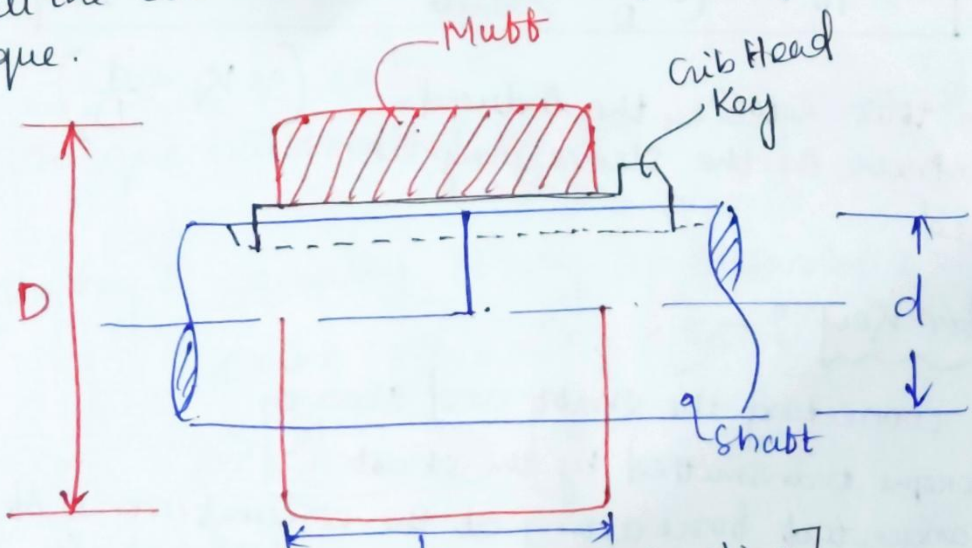
- Riged
  - Sleeve (or) Muff
  - Clamp or split muff or Compression
  - Flange Coupling
- Flexible
  - Bushed pin type
  - Universal coupling
  - Oldham coupling

* Riged Coupling is used to connect two shafts which are perfectly aligned.

* Flexible coupling is used to connect two shafts having both lateral & angular misalignment.

Design of Sleeve or Muff - Coupling :-

- Simplest type of coupling
- Material — cast Iron
- It consists of a hollow cylinder whose inner diameter is the same as that of the shaft.
- It is fitted over the ends of the two shafts by means of a gib head key.
- The power is transmitted from one shaft to another shaft by means of key & a sleeve.
- So all the elements must be strong enough to transmit the torque.



[Sleeve or Muff Coupling]

* The usual proportions of a cast iron sleeve coupling :-

Outer diameter of the sleeve, $D = 2d + 13$ mm

& Length of the sleeve, $L = 3.5d$

d = diameter of shaft.

(1) Design of sleeve :-

The sleeve is designed by considering it as hollow shaft

Let T = Torque to be transmitted by the coupling

τ_c = Permissible shear stress for the material of the sleeve (cast iron)

= 14 MPa (cast iron)

We know that torque transmitted by a hollow section

$$\boxed{T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - K^4)} \quad \text{(1)}$$

From this eqn, the induced shear stress in the sleeve may be checked. $\left(\because K = \frac{d}{D}\right)$

(2) Design for Key :-

A key is connecting the shaft and sleeve

Let T = Torque transmitted by the shaft

F = Tangential force acting at the circumference of the shaft

d = diameter of shaft

l = length of key

w = width of key

t = thickness of key.

τ & σ_c = Shear & crushing stress for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing & crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft.

F_s = Area resisting shearing × Shear stress

$F_s = l \times w \times \tau$

∴ Torque transmitted by the shaft, $T = F_s \times \frac{d}{2}$

$T = l \times w \times \tau \times \frac{d}{2}$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft. F = Area resisting crushing × crushing stress

$F_c = l \times \frac{t}{2} \times \sigma_c$

∴ Torque transmitted by the shaft, $T = F_c \times \frac{d}{2}$

$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$

If, the key is equally strong in shearing & crushing.

$\therefore \cancel{l} \times w \times \tau \times \frac{\cancel{d}}{2} = \cancel{l} \times \frac{t}{2} \times \sigma_c \times \frac{\cancel{d}}{2}$

$\Rightarrow \boxed{\frac{w}{t} = \frac{\sigma_c}{2\tau}}$ . . . .

Length of the key

In coupling, the length of the coupling key is atleast equal to the length of the sleeve (i.e 3.5d)

The coupling key is usually made into two parts. So that the length of the key in each shaft,

$l = \frac{L}{2} = \frac{3.5d}{2}$

After fixing the length of key in each shaft, the induced shearing & crushing stress may be checked,

Torque transmitted, $T = l \times w \times \tau \times \frac{d}{2}$ (Shearing of key)

$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$ (crushing of key)

Q. Design & make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40kW at 350 rpm. The material for the shafts and key is plain carbon steel for which allowable shear & crushing stresses may be taken as 40MPa & 80MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15MPa.

<u>Given data</u>

Power, $P = 40 kW$

$= 40 \times 10^3 W$

Speed, $N = 350$ rpm

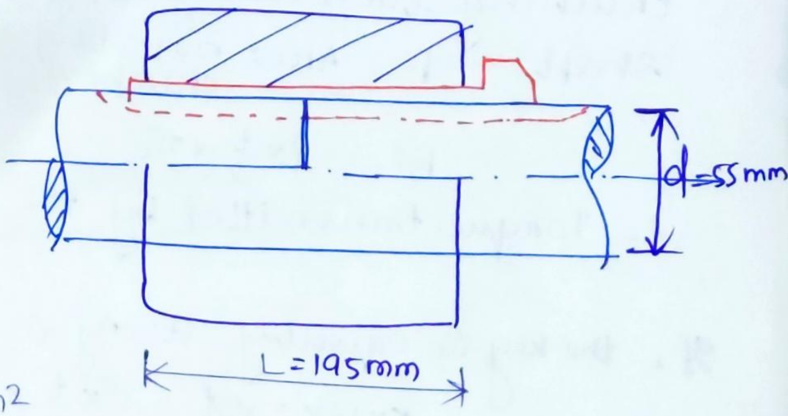
<u>Shaft material & Key material</u>

Shear stress, $\tau_s = 40 MPa$

$= 40 N/mm^2$

Crushing stress, $\sigma_c = 80 MPa$

$= 80 N/mm^2$



<u>Muff</u> :- Material - cast iron :-

Allowable shear stress, $\tau_{CI} = 15 MPa = 15 N/mm^2$

① Design of for Shaft :-

Let d = Diameter of the shaft

Torque transmitted by the shaft, $T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2 \times \pi \times 350}$

$T = 1100 N\text{-}m$

$= 1100 \times 10^3 N\text{-}mm$

Also we know that the torque transmitted,

$T = \frac{\pi}{16} \times \tau_s \times d^3$

$\Rightarrow 1100 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3$

$\Rightarrow d^3 = 1100 \times 10^3 / 7.86 \Rightarrow d = 52 \approx 55 mm$ (Ans)

Design for sleeve :-

Outer diameter of the muff, $D = 2d + 13$ mm

$= 2 \times 55 + 13 = 123$ mm $\simeq 125$ mm (Ans)

Length of the muff, $L = 3.5d = 3.5 \times 55 = 192.5 \simeq 195$ mm (Ans)

Checking of the Induced Shear Stress in the Muff :-

Let τ_I be the induced shear stress in the muff (cast iron)

Since the muff is considered to be a hollow shaft,

$\therefore$ The torque transmitted, $T = \frac{\pi}{16} \times \tau_I \left[\frac{D^4 - d^4}{D}\right]$

$$= \frac{\pi}{16} \times \tau_I \left[\frac{125^4 - 55^4}{125}\right]$$

$$\boxed{T = 370 \times 10^3 \tau_I}$$

$$\Rightarrow 1100 \times 10^3 = 370 \times 10^3 \tau_I$$

$$\Rightarrow \boxed{\tau_I = 2.97 \text{ N/mm}^2}$$

Induced shear stress

Since, τ_I (Induced shear stress of muff) is less than the allowable shear stress of muff i.e 15 MPa

Therefore the design is safe.

③ Design for Key :-

From the standard table for shaft, for a shaft of 55mm diameter.

Width of key, $w = 18$ mm

Since $\sigma_{crushing} = 2\tau_{shearing}$ stress of key material

Therefore a square key may be used.

$\therefore$ Thickness of key, $t = w = 18$ mm (Ans)

Length of key :- We know that length of key in each shaft

$$l = \frac{L}{2} = \frac{195}{2} = 97.5\text{ mm (Ans)}$$

Checking of induced shear & crushing stresses in the key

First of all considering ~~the~~ shearing of the key :-

We know that torque transmitted $\boxed{(T) = l \times w \times \tau_s \times \frac{d}{2}}$

$$\Rightarrow 1100 \times 10^3 = 97.5 \times 18 \times \tau_{s.I} \times \frac{55}{2}$$

$$\Rightarrow \tau_{s.I} = \frac{1100 \times 10^3}{48.2 \times 10^3} = 22.8\text{ N/mm}^2$$

Let induced shearing & crushing stress are $\tau_{s.I}$ & $\sigma_{c.I}$

Now considering crushing ~~stress~~ of the key :-

We know that the torque transmitted

$$T = l \times \frac{t}{2} \times \sigma_{c.I} \times \frac{d}{2}$$

$$\Rightarrow 1100 \times 10^3 = 97.5 \times \frac{18}{2} \times \sigma_{c.I} \times \frac{55}{2}$$

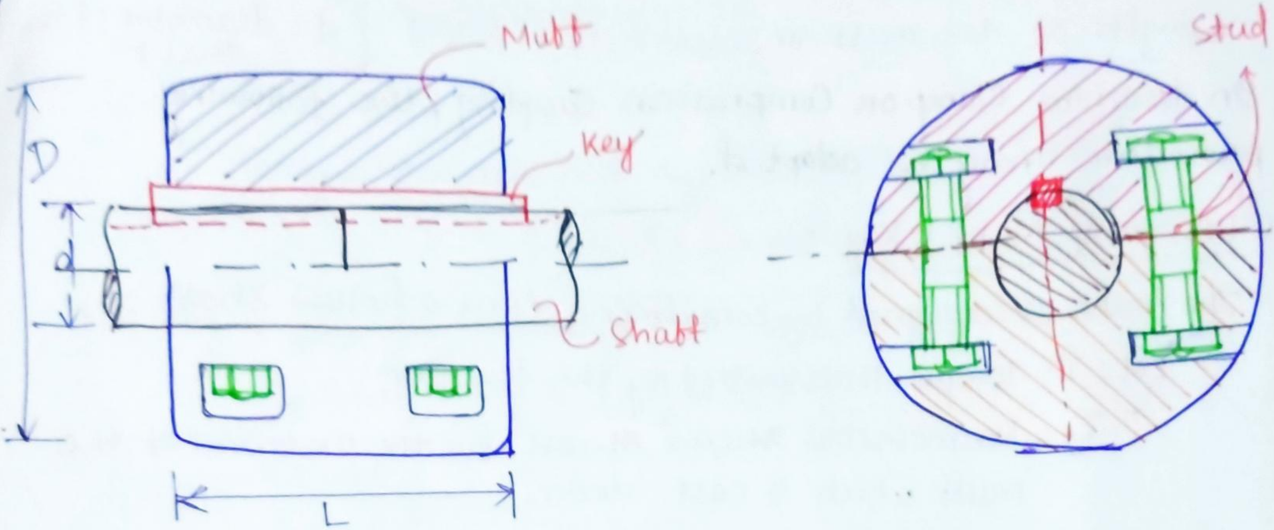
$$\Rightarrow \sigma_{c.I} = \frac{1100 \times 10^3}{24.1 \times 10^3} = 45.6\text{ N/mm}^2$$

Since induced $\tau_{s.I} = 22.8\text{ MPa} < 40\text{ MPa}$

& induced $\sigma_{c.I} = 45.6\text{ MPa} < 80\text{ MPa}$

Therefore the design of key is safe.

Design of Clamp or Compression Coupling



- It is also known as split muff coupling
- In this case, the muff or sleeve is made into two halves and are bolted together..
- The halves of the muff are made of cast iron.
- The shaft ends are made to abutt each other and a single key is fitted directly in the keyways of both the shafts
- One half of the muff is fixed from below & the other half is placed from above.
- Both the halves are held together by means of mild steel studs or bolts and nuts.
- The number of bolts may be two, four or six.
- This coupling may be used for heavy duty & moderate speeds.

Advantages of clamp or Compression Coupling :-

The position of the shafts need not be changed for assembling on disassembling of the clamping

* In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key & the friction between the muff and shaft

Design

Diameter of the muff or sleeve, $D = 2d + 13$ mm

Length of the muff or sleeve, $L = 3.5d$ [d = diameter of the shaft]

In designing Clamp or Compression Coupling, the following procedure may be adopted.

1. Design of Muff & Key :-

The muff is designed by considering it as a hollow shaft.

Let T = torque transmitted by the coupling

τ_c = Permissible shear stress for the material of the muff which is cast iron.

We know that torque transmitted by a hollow section,

$$\boxed{T = \frac{\pi}{16} \times \tau_c \cdot \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \times D^3 \left[1 - k^4\right]}$$

$$\left[\because k = d/D\right]$$

From this expression, the induced shear stress in the muff may be checked.

2. Design of Key :-

Let l = length of key, t = thickness of key

w = width of key, F = tangential force acting at the circumference of the shaft

τ = shear stress
σ_c = Crushing stress] of key material

Considering Shearing of the Key,

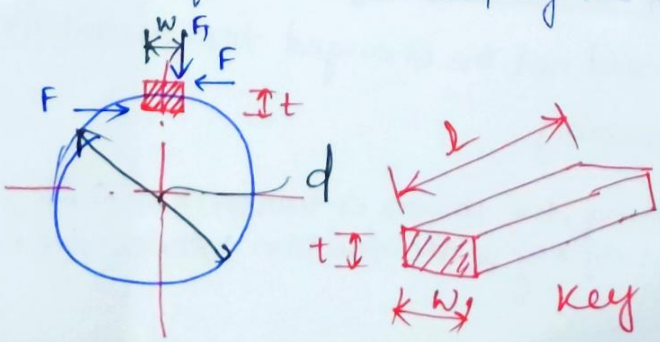
The tangential force acting at the circumference of the shaft,

F = Area resisting shearing × Shear stress

$F = (l \times w) \times \tau$

$\therefore$ Torque transmitted by the shaft, $T = F \times \frac{d}{2}$.

$$\boxed{T = l \times w \times \tau \times \frac{d}{2}} \quad \text{--- (1)}$$



Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft

F = Area resisting crushing × Crushing stress

$$F = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft, $T = F \times \frac{d}{2}$

$$\boxed{T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}} \quad \text{—— (2)}$$

The key is equally stronge in shearing & Crushing

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$\Rightarrow \boxed{\frac{w}{t} = \frac{\sigma_c}{2\tau}}$$

- Length of the key "l" = Total length of the muff. = L

2 Design of Clamping Bolts :-

Let T = Torque transmitted by the shaft

d = Diameter of shaft

d_b = Root or effective diameter of bolt

n = number of bolts

σ_t = Permissible tensile stress for bolt material

μ = coefficient of friction between the muff & shaft

L = Length of muff.

We know that the force exerted by each bolt. $= \frac{\pi}{4}(d_b)^2 \sigma_t$

∴ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface

$$p = \frac{\text{Force}}{\text{Projected Area}} = \frac{\left\{\frac{\pi}{4}(d_b)^2 \times \sigma_t \times \frac{n}{2}\right\}}{\left(\frac{1}{2} L \times d\right)}$$

∴ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area}$$

$$= \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \left\{ \frac{\frac{\pi}{4}(d_b)^2 \times \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \right\} \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2} \times \pi$$

$$\boxed{F = \mu \frac{\pi^2}{8} (d_b)^2 \times \sigma_t \times n}$$

The torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8} (d_b)^2 \times \sigma_t \times n \times \frac{d}{2}$$

$$\boxed{T = \mu \times \frac{\pi^2}{16} (d_b)^2 \times \sigma_t \times n \times d}$$

The root diameter of the bolt (d_b) may be evaluated.

Note :- The value of μ may be taken as 0.3.

Ch-05

Design a Closed Coil Helical Springs

A spring is defined as an elastic body, whose function is to distort when loaded & to recover its original shape when the load is removed.

Applications:-

→ To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers & vibration dampers.

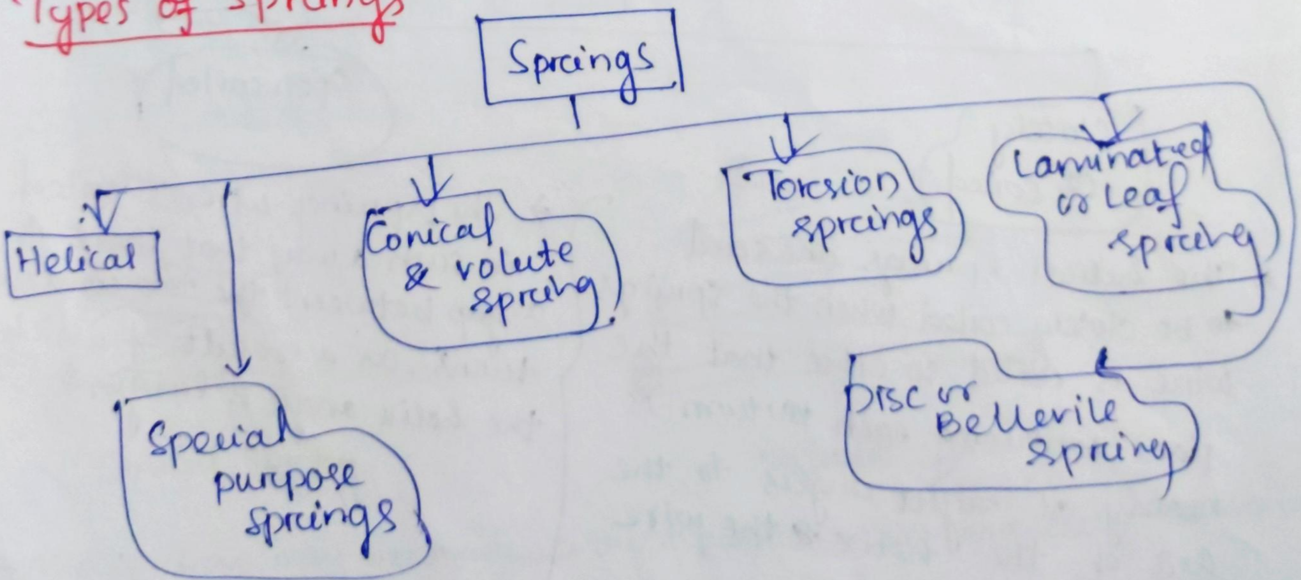
→ To apply forces as in brakes, clutches & spring loaded valves

→ To control motion by maintaining contact between two elements as in cam & followers.

→ To measure forces as in spring balances & engine indicators.

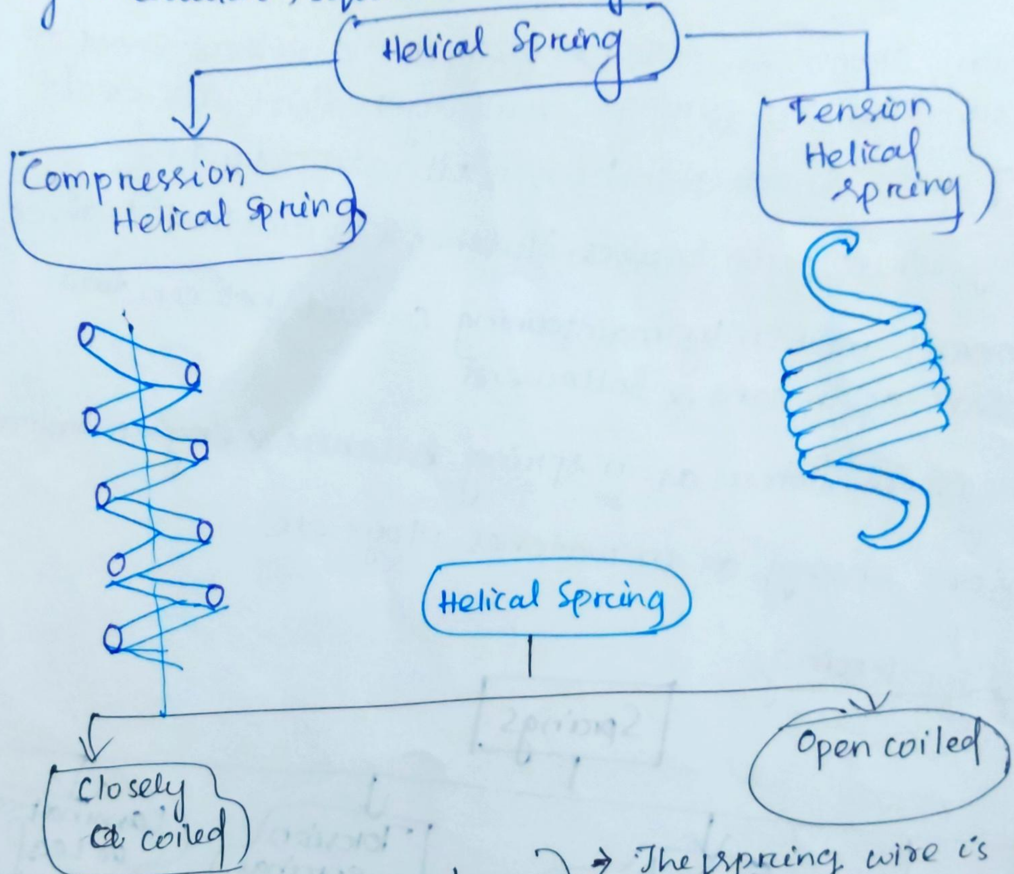
→ To store energy as in watches, toys etc.

Types of Springs



① Helical Spring

The helical springs are made up of a wire coiled in the form of a helix & is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular.



→ The helical springs are said to be closely coiled when the springs wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix & the wire is subjected to torsion

→ The spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large.

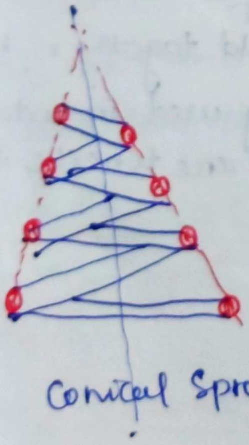
...antages of Helical spring

- These are easy to manufacture
- These are available in wide range
- These are reliable.
- These have constant spring rate.
- Their performance can be predicted more accurately
- Their characteristics can be varied by changing dimensions.

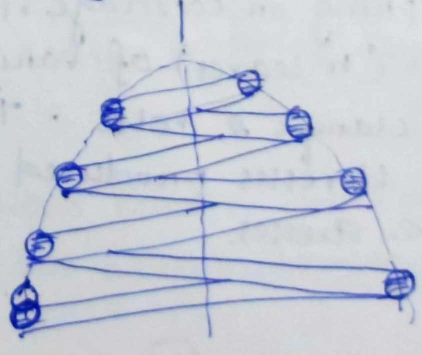
Conical & volute springs :-

The conical & volute springs are used in special applications where a telescoping spring or a spring rate that increases with the load is desired.

- The conical spring is wound with a uniform pitch where-as the volute springs are wound in the form of paraboloid with constant pitch. & lead angles.



Conical Spring

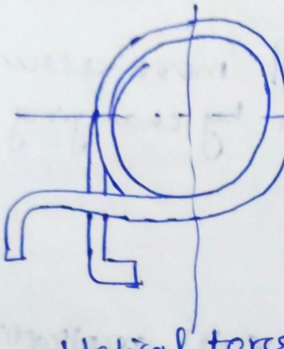


Volute Spring

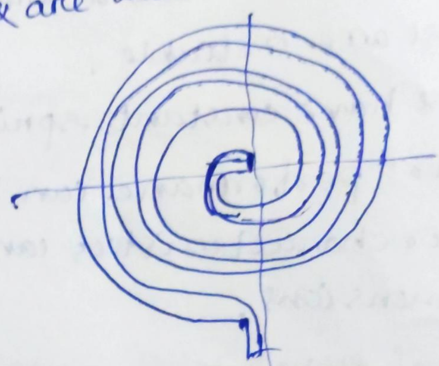
- The springs may be made either partially or completely telescoping. In either case, the number of active coils gradually decreases. The decreasing no. of coils results in an increasing spring rate.

Torsional Springs :-

The torsion springs may be of helical or spiral type. The helical type may be used only in applications where the load tends to wind up the spring & are used in various electrical mechanisms.



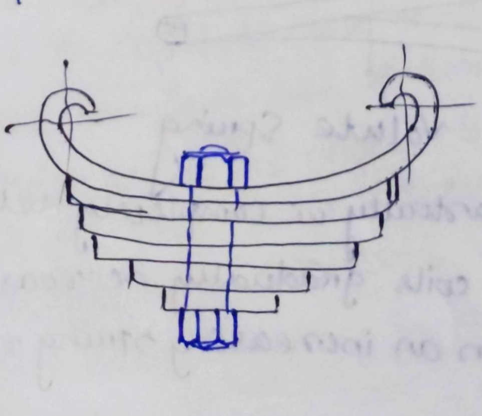
Helical torsion spring



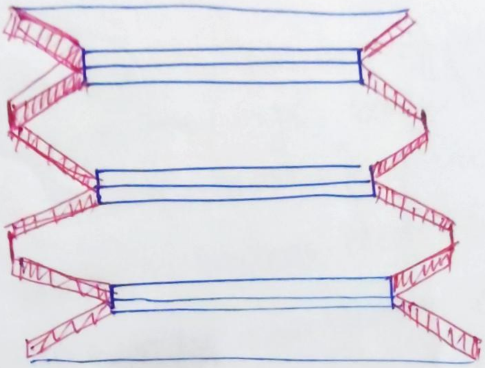
Spiral torsion spring

→ The spiral type is also used where the load tends to increase the number of coils & when made of flat strip are used in watches & clocks.

Laminated or Leaf Springs :- The laminated or leaf spring or flat spring or carriage spring consists of a number of flat plates (or leaves) of varying lengths held together by means of clamps & bolts. These are mostly used in automobiles. The major stresses produced in leaf springs are tensile & compressive stresses.



Cosc on Belleviel Springs :- These springs consist of a numbers of conical discs held together against slipping by a central bolt or tube. These springs are used in applications where high spring rates & compact spring units are required.



Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience & it should be creep resistant. It largely depends upon the service for which they are used i.e severe service, average service or light service.

Severe Service :- rapid continuous loading where the ratio of min^m to max^m load is $\frac{1}{2}$ or less as in automotive valve springs.

Average Service :- Includes the same stress range as in severe service but with only intermittent operations as in engine governor.

Materials for Helical Springs

Properties :- High fatigue strength
" ductility "
" resilience "
" creep resistance

- Selection of materials for spring ~~may~~ depend on the type of service

- Service type — 3 types

Severe

- Rapid continuous loading.
- $\frac{\text{Max}^m \text{ Load}}{\text{Min}^m \text{ Load}} \leq \frac{1}{2}$

e.g. - Automotive valve spring

Average

- Intermittent operation.

e.g. :- Engine governor spring

Automobile suspension spring

Light

spring subjected to loads (static) loading

e.g. - safety valve spring

- Material – oil-tempered carbon steel wires
0.6 to 0.7 % of Carbon
0.6 to 1 % manganese

(1) Solid length :- when the compression spring is compressed until the coils come in contact with each others, then the spring is said to be solid.

→ The solid length of the spring = total no. of coils (n') × diameter of wire (d)

$$\boxed{L_s = n' \times d}$$

(2) Free length :- The free length of a compression spring is the length of the spring in the free (or) unloaded condition.

Free length of spring, L_f = solid length + Max^m compression + clearance b/w adjacent coils (1 mm)

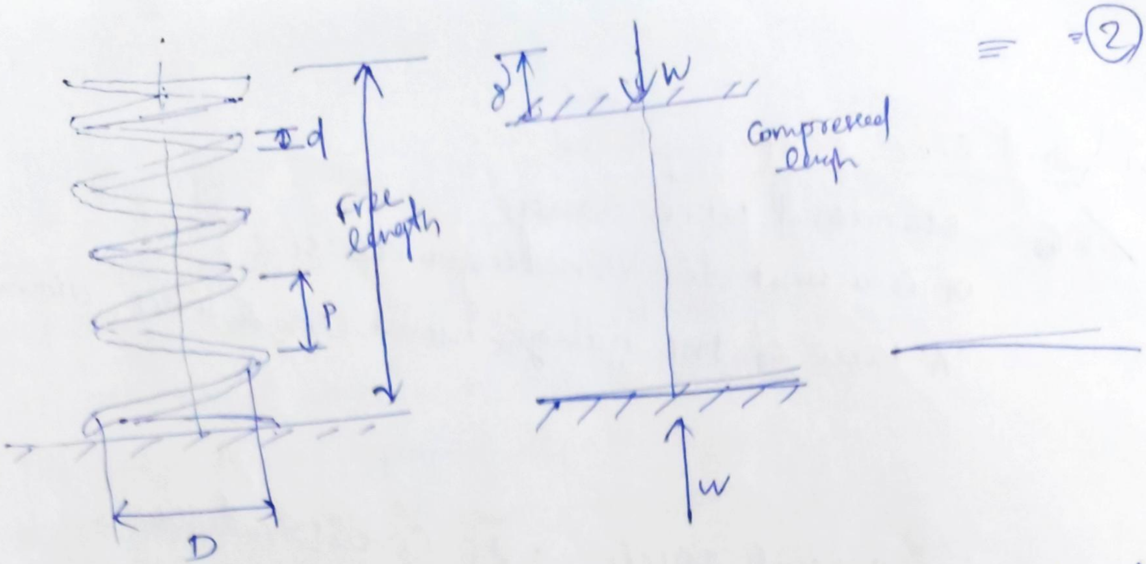
$$\boxed{L_f = n'd + \delta_{max} + 0.15\,\delta_{max}}$$

$$\boxed{L_f = n'd + \delta_{max} + (n'-1) \times 1\,mm}$$

(3) Spring index :-

$$\text{spring index} = \frac{\text{Mean dia. of the coil}}{\text{Dia. of the wire}}$$

$$C = \frac{D}{d}$$



(4) Spring rate :- 'K' or Stiffness or Spring constant is defined as the load required per unit deflection of the spring.

$$\text{spring rate } 'K' = \frac{W}{\delta} = \frac{\text{Load}}{\text{deflection of the spring}}$$

(5) Pitch :- The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state.

$$\text{pitch of the coil, } p = \frac{\text{Free length}}{n' - 1}$$

or

$$\text{pitch of the coil, } p = \frac{L_f - L_s}{n'} + d$$

* The Pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the Yield point stress in torsion.
* The spring should not close up before max^m service load is reached.

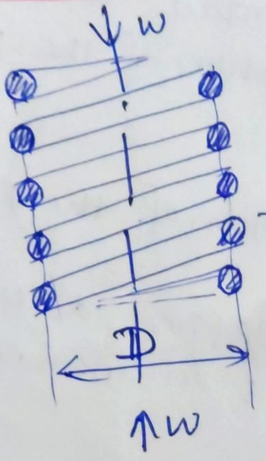
Standard size of spring wire

SWG :- standard wire Gauge

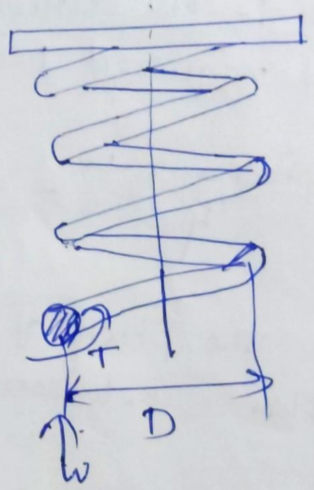
It is a unit for denoting wire size

A table of the gauge numbers & wire diameter

Stresses in Helical springs of circular wire :-



Axially loaded helical spring



Free body diagram showing that wire is subjected to torsional shear & direct shear

d = dia of spring wire

D = mean dia spring coil

n = no. of active coils

G = Modulus of rigidity of spring

W = Axial Load on the spring.

τ = Max^m shear stress induced in the wire.

C = spring ~~ratio~~ index = D/d

P = pitch of the coil

δ = deflection of the spring

from fig . 2

The Load W tends to rotate the wire due to the twisting moment (T) set up in the wire.

∴ torsional shear stress is induced in the wire

- The part of the spring is in equilibrium under the action of two forces 'W' & twisting moment 'T'

∴ Twisting moment , $T = W \times \frac{D}{2}$

$= \frac{\pi}{16} \times \tau_1 \times d^3$

$$\Rightarrow \boxed{\tau_1 = \frac{8WD}{\pi d^3}}$$

τ_1 = induced shear stress

* Two types of shear stresses act on the spring wire

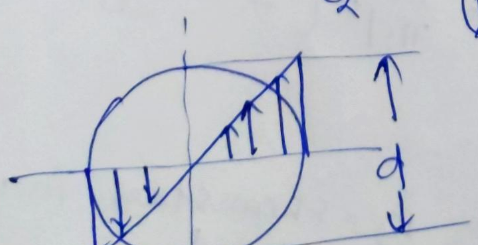
(1) Direct shear stress due to the Load W

(2) Stress due to curvature of wire.

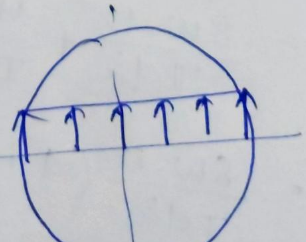
∴ Direct shear stress due to Load 'W'

$$\tau_2 = \frac{\text{Load}}{\text{cross-sectional area of the wire}}$$

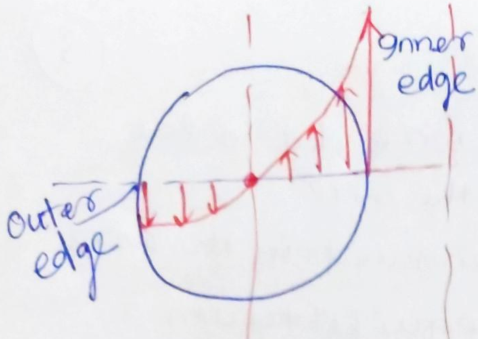
$$\tau_2 = \frac{W}{(\frac{\pi}{4} d^2)} = \frac{4W}{\pi d^2}$$



Torsional shear-stress

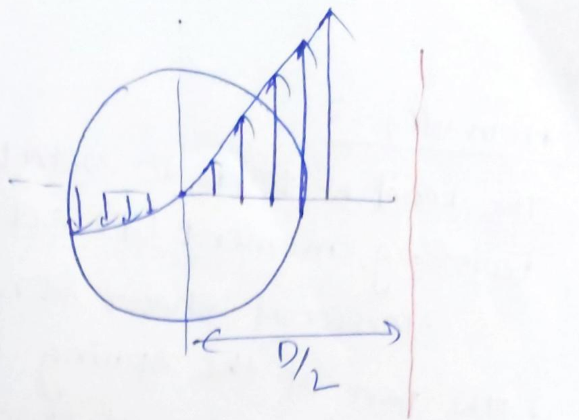


direct shear stress



Resultant torsional shear & direct shear stress

Axis of spring



Resultant torsional shear + Direct shear stress + Curvature shear stress.

∴ The Resultant shear stress induced in the wire.

$$\tau = \tau_1 \pm \tau_2$$

$$\boxed{\tau = \frac{8\,W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}}$$

+ve sign :- used for the inner edge of the wire.

-ve sign :- " Outer edge " "

Max^m stress is at the inner edge of wire.

Max^m shear stress induced in the wire.

∴ Max^m shear stress = Torsional shear stress + Direct shear stress

$$= \frac{8\,W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8WD}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8W.D}{\pi d^3}\left(1 + \frac{1}{2C}\right)$$

$$\boxed{\tau_{max} = K_s \times \frac{8\,W.D}{\pi d^3}}$$

K_s = shear stress factor $= 1 + \frac{1}{2C}$