INDEX

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STD. (LECTURE NOTE) DIV. 4th SEM ROLL NO.

6.1 Introduction to vibration and related terms (Amplitude, time period and frequency, cycle)

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6.3. Basic concept of Natural, forced and damped vibration.

6.4. Torsional and Longitudinal vibration.

6.5. Causes and remedies of vibration.

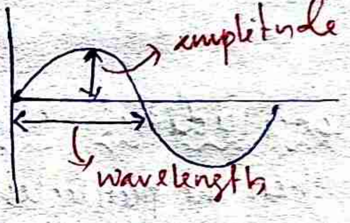
6.1: Introduction to vibration and related terms (Amplitude, timeperiod and frequency, cycle):

→ A type of motion in which a particle moves to and fro about a fixed point is called vibratory motion.

→ Any motion that repeat itself after a interval of time is called ossilation (or) ossilatory motion (or) vibratory motion.

Terminology:

(a) Amplitude:



→ It is the maximum displacement (or) distance moved by a body measured from its equilibrium position.

(b) Timeperiod:

→ It is the time required to complete one cycle by a harmonic motion, i.e the time interval after which the motion repeat itself.

→ It is generally in seconds. $\boxed{t_p = \frac{2\pi}{\omega_n}}$

t_p = Timeperiod of an ossilatory motion.

ω_n = Natural frequency of the system with which it ossilates.

(c) frequency:

→ It is the no. of cycles per unit time to cover by the harmonic motion.

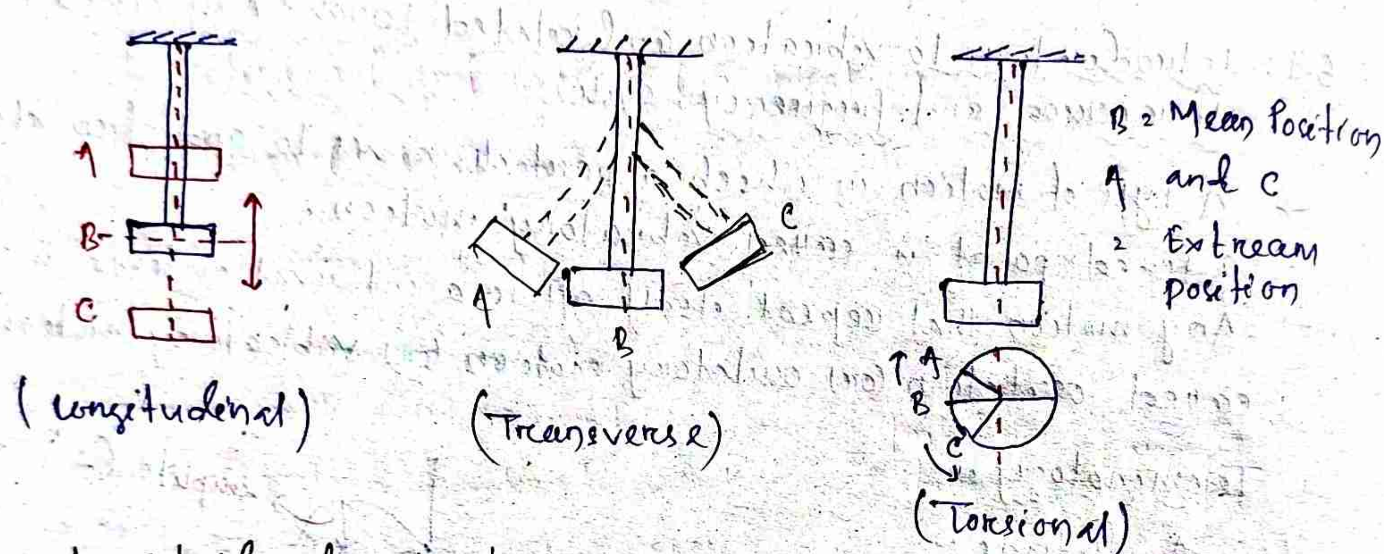
→ The unit of frequency is 'Hz' = 'Hertz' $\boxed{f_n = \frac{1}{t_p} = \frac{\omega_n}{2\pi}}$

cycle :- It is the motion completed during one time period.

6.2 classification of vibration :-

→ The following three types of free vibrations are important from the subject point of view.

1. Longitudinal vibrations
2. Transverse vibrations
3. Torsional vibration.



1. Longitudinal vibration :-

→ when the particles of the shaft or disc moves parallel to the axis of the shaft, the vibration is known as longitudinal vibration.

2. Transverse vibration :-

→ when the particles of the shaft or disc moves approximately perpendicular to the axis of the shaft then it is called transverse vibration.

3. Torsional vibration :-

→ when the particle of the shaft (or) disc move in a circle about the axis of the shaft, then that is called Torsional vibration.

6.3. Basic concept of Natural, forced and damped vibration

(a) Free (or) Natural vibration:-

→ when no external force acts on a body, after giving it an initial displacement, then the body is said to be free (or) Natural vibration.

→ The frequency of the free vibrations is called free (or) Natural frequency.

(b) forced vibration:-

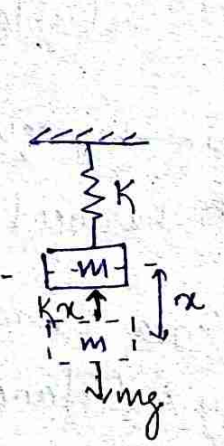
→ when the body vibrates under the influence of external force then the body is said to be under forced vibration.

→ when the frequency of the external force is same as that of the natural frequency then that is called Resonance.

(c) Damped vibration:-

→ when there is a reduction in amplitude over every cycle of vibration the motion is said to be damped vibration.

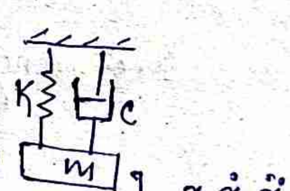
→ This happens due to the use of damper.



* Natural frequency of free vibration:

$$\boxed{\omega_n = \sqrt{\frac{K}{m}}}$$

* forced vibration :- $\boxed{K = \frac{f}{\delta}}$ N/m



* Damping coefficient:- It is the resistance against velocity

$$\boxed{c = \frac{f}{v}} \quad \frac{N\,sec}{m}$$

Critical damping coefficient :- $\boxed{C_c = 2m\omega_n = 2\sqrt{Km}}$

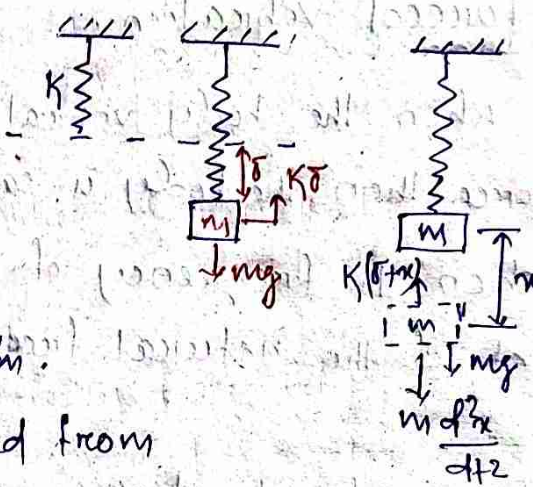
Damping factor (or) Damping Ratio :-

→ The ratio of the actual damping coefficient (c) to the critical damping coefficient (C_c) is known as damping factor (or) damping ratio.

$$\boxed{\text{Damping factor} = \frac{c}{C_c} = \frac{c}{2m\omega_n}}$$

6.4. Longitudinal and Torsional vibration :-

Longitudinal vibration :-



let K = stiffness of the spring

* It is the force require to produce unit displacement in the direction of vibration.

→ It is usually expressed in N/m.

m = mass of the body suspended from the constraint in kg.

W = Weight of the body in newton = mg

δ_{st} = static deflection of the spring in meters due to weight 'W' in newton.

x = Displacement given to the body by the external force in 'm'.

→ In the equilibrium position the gravitational pull W = mg is balanced by the spring force $W = K\delta_{st}$.

→ Since the mass is displaced from equilibrium position by a distance 'x' and then released, therefore after time 't'

Restoring force $= W - K(\delta_{st} + x) = W - K\delta_{st} - Kx$

$= -Kx$ (Taking upward force -ve)

→ Inertia force = mass × acceleration

$= m \times \dfrac{d^2x}{dt^2}$

→ Equating the equation of motion of the body

$$m\frac{d^2x}{dt^2} = -Kx$$

$$\Rightarrow Kx + m\frac{d^2x}{dt^2} = 0 \Rightarrow \frac{d^2x}{dt^2} + (K/m)\,x = 0$$

We know that from the fundamental eqn of simple harmonic motion

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

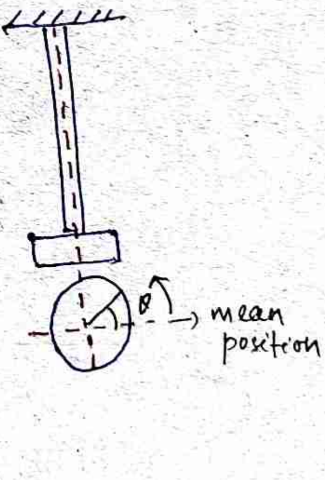
∴ comparing equations $\omega^2 = \sqrt{\dfrac{K}{m}} \Rightarrow \boxed{\omega^2 = K/m}$

∴ Time period $= T = \dfrac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\sqrt{\dfrac{m}{K}}}$

∴ frequency $= \boxed{\nu = \dfrac{1}{tp}}$

Torsional vibration :

→ Consider a shaft negligible mass whose one end is fixed and other end carrying a disc



let θ = Angular displacement of the shaft from mean position after time 't' in radians.

m = mass of disc in kg

I = mass moment of inertia of disc in kg-m² = mK^2

K = Radius of gyration in meter

q = Torsional stiffness of the shaft in N-m.

∴ Restoring force = $q\theta$ — (1)

and accelerating force = $I \times \dfrac{d^2\theta}{dt^2}$ — (2)

∴ Equating eqn (1) and (2) the eqn of motion is

$\Rightarrow I \times \frac{d^2\theta}{dt^2} = -q\theta$

$\Rightarrow I \times \frac{d^2\theta}{dt^2} + q\theta = 0$ —— (3)

$\therefore$ The fundamental equation of S.H.M is

$\frac{d^2\theta}{dt^2} + \omega^2 x = 0$ —— (4)

$\therefore$ comparing eqⁿ (3) and (4)

$\omega = \sqrt{\frac{q}{I}}$

$\therefore$ Time period $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{q}}$

$\therefore$ frequency $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

NOTE. Torsional stiffness 'q' may be obtained from the torsion eqⁿ.

$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow \frac{T}{\theta} = \frac{GJ}{L}$

$\therefore \boxed{q = \frac{GJ}{L}}$

G = modulus of rigidity

J = Polar moment of inertia of the shaft cross-section.

$= \pi/32\, d^4$

L = length of the shaft

BALANCING OF MACHINE

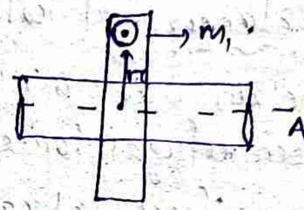
5.1. Concept of static and dynamic balancing:

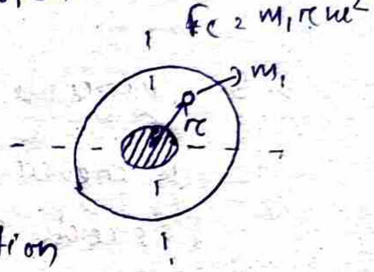
* Unbalanced system:

→ If the centre of gravity of the mass does not lie on the axis of rotation, then the system is called unbalanced system.

Dynamic force (centrifugal force)

$F_c = m_1 r \omega^2$





m_1 = Eccentrical mass.
r = Distance of the mass from the axis of rotation.
ω = Angular velocity.

Static Balancing:

→ If the C.G of the system lies on the axis of the rotation and the system is zero then the summation of dynamic forces acting on the system this is called static balancing.

→ Σ Dynamic forces acting on the system = 0.

Dynamic Balancing:

→ For dynamic balancing following two conditions must be satisfied.

(i) Summation of dynamic forces acting on the system is zero. i.e it should be statically balanced.

(ii) Summation of couple due to dynamic forces should be zero.

→ If the above two conditions are satisfied then the system is called dynamically balanced.

5.2. Static balancing of rotating parts :-

→ A system of rotating mass is said to be in static balance if the combine mass centre of the system lies on the axis of rotation.

→ For static balancing if there is a unbalanced mass in any direction then to balance the system we have to attach a mass (or) reaction mass of the same magnitude of the unbalanced mass, just in the opposite direction, so that the net effect of unbalance force due to gravity can be nullified.

5.3. Principle of Balancing of rotating parts :-

→ Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it.

→ In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such position so as to balance the effect of the centrifugal force of the first mass.

→ This is done in such a way that the centrifugal forces of both the masses are made to be equal and opposite.

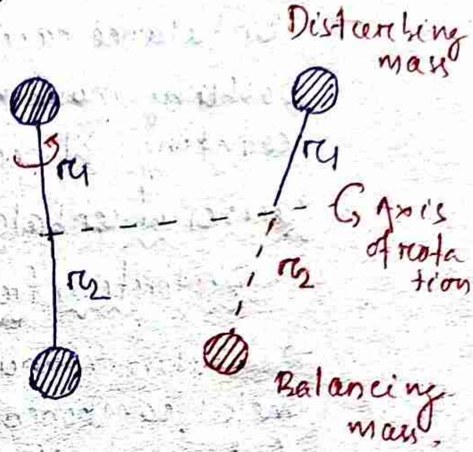
→ The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of the rotating masses.

→ The following cases are important from the subject point of view.

1. Balancing of a single rotating mass by a single mass rotating in same plane.
2. Balancing of single rotating mass by two masses rotating in different planes.
3. Balancing of different masses rotating in the same plane.
4. Balancing of different masses rotating in different planes.

1. Balancing of a single rotating mass by a single mass rotating in the same plane :

→ Consider a disturbing mass 'm_1' attached to a shaft rotating at 'ω' rad/sec. Let r_1 be the radius of rotation of mass 'm_1' (i.e distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1)



→ We know that the centrifugal force exerted by mass 'm_1' on the shaft

$$F_{c1} = m_1 \omega^2 r_1 \quad \text{---(1)}$$

→ This centrifugal force acts radially outward and thus produces bending moment on the shaft.

→ In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of rotating mass/disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.

r_2 = Radius of rotation of the balancing mass m_2 (i.e distance between the axis of rotation of the shaft and the centre of gravity of mass m_2)

Centrifugal force due to 'm_2'

$$F_{c2} = m_2 r_2 \omega^2 \quad \text{--- (2)}$$

∴ Equating both the equations

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2$$

$$\Rightarrow \boxed{m_1 r_1 = m_2 r_2}$$

→ From this we can find out the magnitude of the balancing mass (or) distance from the axis of rotation.

5.4. Causes and effect of unbalance :

→ Rotating unbalance is the uneven distribution of mass around an axis of rotation. A rotating mass, or rotor is said to be out of balance when its centre of mass is out of alignment with the centre of rotation.

→ Unbalance causes a moment which gives the rotor a wobbling movement characteristic of vibration of rotating structures.

Causes of unbalance :

(i) Distortion from stress :

→ Routine manufacturing processes can cause stress on metal components, without stress relief, the rotor will distort itself to adjust

(ii) Thermal distortion :

Thermal distortion often occurs with parts exposed to increased temperatures. Metals are able to expand when in contact with heat, so exposure to warmer temperatures can cause either to expand, or just certain parts, causing distortion.

(iii) Buildups and deposits :

→ Rotating parts involved in material handling almost always accumulate build up.

→ More over when exposed to oil, these parts can be very easily distorted. Without adhering to a maintenance routine or implementing an inspection process, oil can seep into the parts, causing unbalance.

Effect of unbalance :

* It creates vibration and noise.
* It decreases the life of bearings associated with the rotating parts.
* It creates unsafe work condition.
* It reduces the machine life.
* It increases the maintenance.

5.5. Difference between the static and dynamic balancing:

Static Balancing	Dynamic Balancing
1. Static balancing is done where the centre of gravity is on the axis of rotation of the body. Thus the body has no tendency to rotate under the influence of gravity.	1. It is generally referred to a case where the body is rotating about the axis due to external force (or) by change in the centre of gravity of the body.

chapter-4:

GOVERNERS AND FLYWHEEL

4.1- function of governor
4.2- Classification of governor.
4.3- working of watt, porter, proel and Hartnell governors
4.4- Conceptual explanation of sensitivity, stability and isochronisms.
4.5- function of flywheel.
4.6- Comparison between flywheel and governor.
4.7- fluctuation of energy and coefficient of fluctuation of speed.

4.1 function of Governor :-

→ The function of a governor is to regulate the mean speed of an engine, when there are variations in the load i.e when the load on an engine increases, its speed decreases therefore it becomes necessary to increase the supply of working fluid.

→ The Governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

4.2 classification of governor :-

→ The governors may broadly classified as
(a) centrifugal governor (b) Inertia governors

→ Centrifugal governors may be classified as follows.

Centrifugal governors
- Pendulum type
  - Watt governor
- Loaded type
  - Dead weight governor
    - Porter governor
    - Proell governor
  - Spring controlled governor
    - Hartnell governor
    - Hartung governor
    - Wilson-Hartnell governor
    - Pickering governor

4.3 Working principle of centrifugal governor :-

→ The centrifugal governors are based on balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the controlling force. It consists of two balls of equal mass, which are attached to the arms. These balls are known as governor balls (or) fly balls.

→ The balls revolve with a spindle, which is driven by the engine through the bevel gears. The upper ends of the arms are pivoted to the spindle so that the balls may rise or fall down as they revolve about the vertical axis.

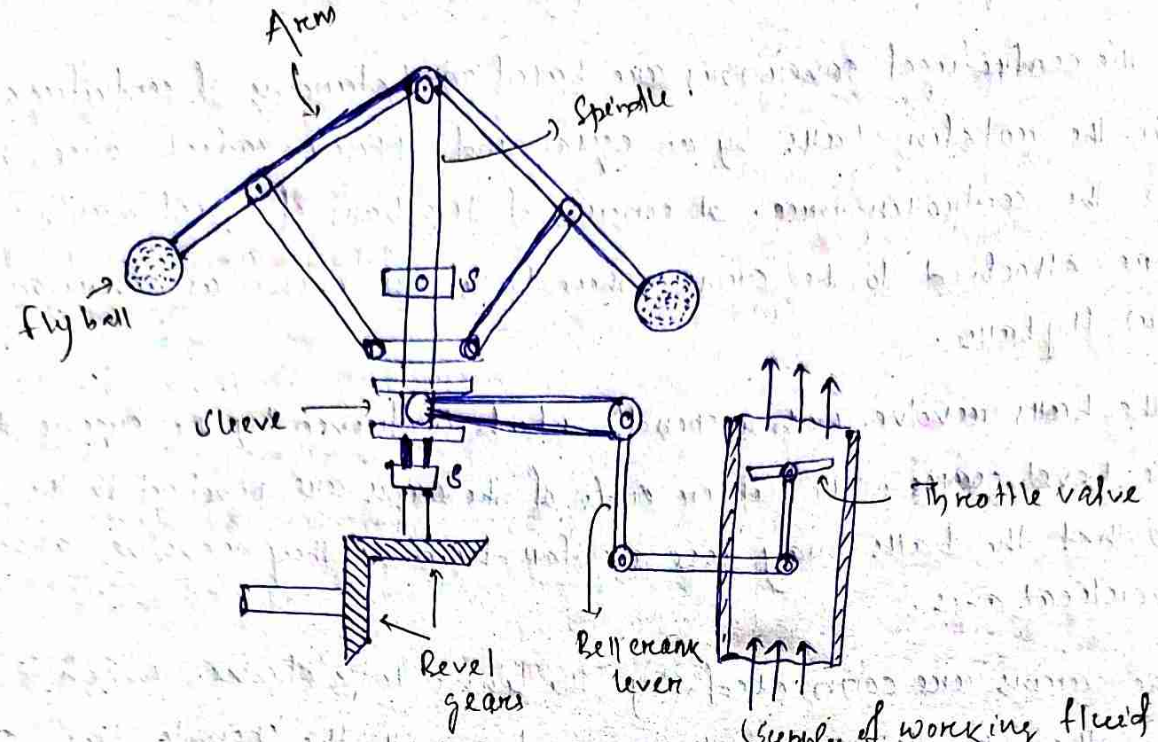
→ The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolve with the spindle, but can slide up and down. The balls and the sleeve rises when the spindle speed increases and falls when the speed decreases.

→ In order to limit the travel of the sleeve in upward and downward directions, two stops S, S are provided on the spindle. The sleeve is connected by bell crank lever to a throttle valve. The supply of working fluid decreases when the sleeve rises and increases when it falls.

→ When the load on the engine increases, the engine and governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve move downwards.

→ The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased.

→ When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreases.



TERMS USED IN GOVERNORS :-

1. Height of a governor :- It is the vertical distance from the centre of the ball to a point where the axes of arms intersect on the spindle axis. It is denoted by 'h'.

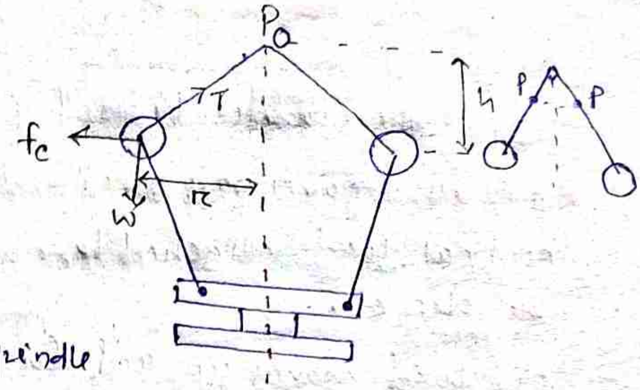
2. Equilibrium speed :- It is the speed at which the governor balls, arms, etc are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. Mean equilibrium speed :- It is the speed at the mean position of the balls (or) the sleeve.

4. Maximum and minimum equilibrium speed :-
The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

5. Sleeve lift :- It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Watt Governor :



→ The simplest form of a centrifugal governor is a watt governor.

1. The pivot 'P' may be on the spindle axis.
2. The pivot P, may be offset from the spindle axis and the arms when produced intersect at 'O'.
3. The pivot 'P' may be offset, but the arms cross the axis at 'O'.

m = mass of the ball in kg
w = weight of the ball in newtons = mg
T = Tension in the arm in newtons.
ω = Angular velocity of the arms and balls about the spindle axis in rad/sec.
r = Radius of the path of rotation of the ball i.e Horizontal distance from the centre of the ball to the spindle axis in 'm'.
f_c = Centrifugal force acting on the ball in newton
$= m r \omega^2$
h = Height of the governor in 'm'.

→ It is assumed that the weight of the arms, links and the sleeves are negligible as compared to the balls. Now the ball is in equilibrium under the action of (1) Centrifugal force acting on the ball (2) tension (T) in the arm (3) The weight of the ball.

→ Taking moment about the point 'O' we have

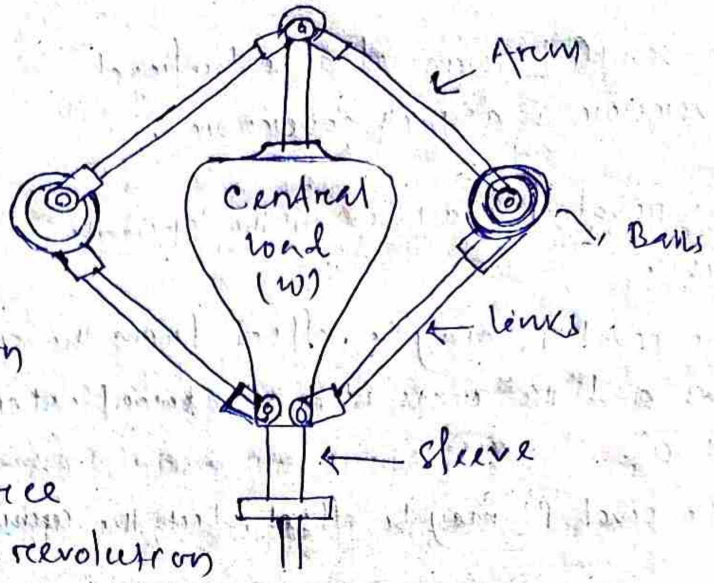
$$f_c \times h = w \times r = mgr$$

$$\Rightarrow m\omega^2 r h = mgr$$

$$\Rightarrow h = \frac{g}{\omega^2}$$

As $\omega = \frac{2\pi N}{60}$ (N is the speed in rpm) $\Rightarrow \boxed{h = \frac{895}{N^2}}$ meter.

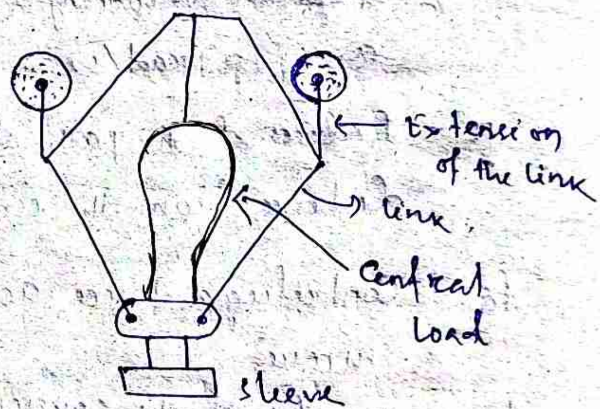
PORTER GOVERNOR :-



→ It is the modification of a watt's governor ; with central load attached to the sleeve.

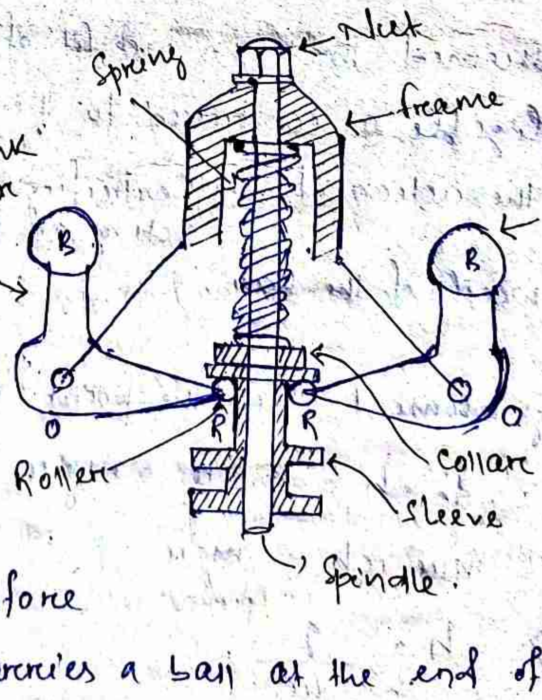
→ The load moves up and down the central spindle. This additional downward force increases the speed of the revolution required to enable the balls to rise to any predetermine level.

Proell Governor :-



→ The proell governor has the balls fixed to the extension of the links.

Hartnell Governor :-



→ It is a spring loaded governor. It consists of two bell crank levers pivoted at O, O to the frame.

→ The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up (or) down on the sleeve.

4.4 Sensitivity of Governor :-

→ It is the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

→ N_1 = Minimum equilibrium speed

N_2 = Maximum equilibrium speed.

N = Mean equilibrium speed $= \frac{N_1+N_2}{2}$

Sensitiveness of the governor $= \frac{N_2-N_1}{N} = \frac{2(N_2-N_1)}{N_1+N_2}$

$= \frac{2(\omega_2-\omega_1)}{(\omega_1+\omega_2)}$

→ Consider two governor A and B running at the same speed. When this speed increases (or) decreases by a certain amount the lift of the sleeve of the governor 'A' is greater than the lift of the sleeve of governor B, It is then said that the governor 'A' is more sensitive than governor B.

Stability of Governor :-

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor if the equilibrium speed increases, the radius of governor balls must also increase.

Isochronism :- A governor is said to be isochronous when the equilibrium speed is constant i.e (range of speed is zero) for all radii of rotation of the balls with-in the working range, neglecting friction.

→ Isochronism is the stage of infinite sensitivity.

4.5 Function of flywheel:-

→ Flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.

→ A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and release it during the period when the requirement of energy is more than the supply.

→ In I.C engines the energy is developed only during the expansion (or) power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of 4-stroke engines and during compression in case of two stroke engine.

→ The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crank shaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed.

→ When flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases.

4.6 Comparison between flywheel and Governor:

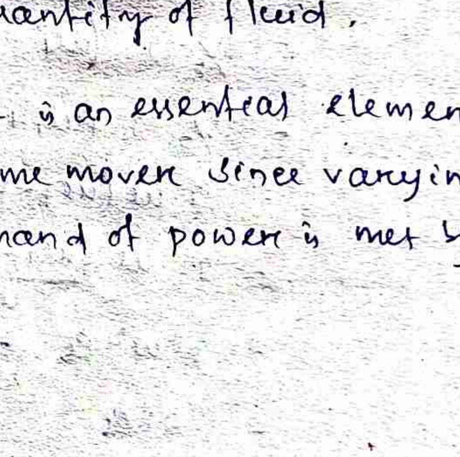
Flywheel	Governor
1. It reduces the fluctuation of speed during the thermodynamic cycles but it does not maintain a constant speed.	1. It is a device to control the speed variation caused by the varying load.

2. Its function to control the speed variations caused by the fluctuations of engine turning moment during a cycle.	2. Its function is to regulate the mean speed of engine within prescribed limit when there are variations of load.
3. Mathematically it controls $\frac{dN}{dt}$.	3. Mathematically it controls dN
4. Flywheel acts as a reservoir, it stores energy due to its mass moment of inertia and releases energy when required during a cycle.	4. A governor regulates the speed by regulating the quantity of charge/working fluid of prime mover.
5. It regulates speed in one cycle only.	5. It regulates speed over a period of time.
6. Flywheel has no control over supply of fluid/charge	6. Governor takes care of quantity of fluid.
7. It is not essential element of every prime mover. It is used when there are undesirable cyclic fluctuations.	7. It is an essential element of prime mover since varying demand of power is met by it.

4.7 Fluctuation of Energy :

Let m = mass of the flywheel in kg.

K = Radius of gyration of the flywheel in meter.



I = mass moment of inertia of the flywheel about its axis of rotation in kg-m²

$I = mK^2$

N_1 and N_2 = Maximum and minimum speeds during the cycles in r.p.m.

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad/sec

N = Mean speed during the cycle in r.p.m = $\frac{N_1 + N_2}{2}$

ω = Mean angular speed during the cycle in rad/s

$= \frac{\omega_1 + \omega_2}{2}$

C_s = coefficient of fluctuation of speed

$= \frac{N_1 - N_2}{N}$ (or) $\frac{\omega_1 - \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel.

$E = \frac{1}{2} I \omega^2 = \frac{1}{2} m.k^2 \omega^2$

As the speed of the flywheel changes from ω_1 ~~and~~ to ω_2 the maximum fluctuation of energy

ΔE = Maximum K.E − Minimum K.E

$= \frac{1}{2} I(\omega_1)^2 - \frac{1}{2} I(\omega_2)^2$

$= \frac{1}{2} I(\omega_1^2 - \omega_2^2)$

$= \frac{1}{2} I(\omega_1 + \omega_2)(\omega_1 - \omega_2)$ $\qquad \omega = \frac{\omega_1 - \omega_2}{2}$

$= I\omega(\omega_1 - \omega_2)$

$= I\omega^2 \frac{(\omega_1 - \omega_2)}{\omega}$

$= I\omega^2 C_s = mk^2\omega^2 C_s$

$= 2EC_s$

→ The radius of gyration may be taken as equal to the mean radius of the rim 'R'.

$\Delta E = mR^2\omega^2 C_s = mv^2 C_s$

v = mean linear velocity = ωR.

Coefficient of fluctuation of speed:

→ The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed.

→ The ratio of the maximum fluctuation of speed to the mean speed is called the coefficient of fluctuation of speed.

N_1 and N_2 = Maximum and minimum speeds in r.p.m during the cycle.

N = Mean speed in r.p.m = $\frac{N_1 + N_2}{2}$

coefficient of fluctuation of speed

$$C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

$$C_s = \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2}$$

POWER TRANSMISSION

3.1. Concept of power transmission

3.2. Type of drives, belt, gear and chain drive

3.3. Computation of velocity ratio, length of belts (open and cross) with and without slip.

3.4. Ratio of belt tensions, centrifugal tension and initial tension.

3.5. Power transmitted by the belt.

3.6. Determine belt thickness and width for given permissible stress for open and crossed belt considering centrifugal tension.

3.7. V belts and V belt pulleys.

3.8. Concept of crowning of pulleys.

3.9. Gear drives and its terminology.

3.10. Gear trains, working principle of simple, compound, reverted and epicyclic gear trains.

3.1 Concept of power transmission:

→ Power transmission is the movement of energy from its place of generation to the location where it is applied to perform useful work.

→ When the distance between the shafts is large belts (or) ropes are used and for intermediate distance chains can be used.

→ When the distance between the shafts is small then gears are used.

→ For belt drive the distance is maximum but it should not be more than ten meters.

3.2 TYPE OF DRIVES :-

→ There are mainly three types of driving system by which the power is transmitted.

(i) Belt drive (ii) chain drive (iii) Gear drive.

TYPES OF BELT DRIVE :-

The belt drives are mainly classified in to the following three groups

(1) Light Drive :- These are used to transmit small powers at belt speeds up to 10 m/s as in agricultural machines and small machine tools.

(2) Medium Drive :- These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s as in machine tools.

(3) Heavy drives :- These are used to transmit large powers at belt speeds above 22 m/s as in compressors and generators.

(ii) Chain Drives :-

→ In belt and rope drives slipping may occure at some point. In order to avoid slipping steel chains are used.

→ The chain are made up of rigid links which are hinged together in order to provide the necessary flexibility for wraping around the driving and the driven wheels.

→ The wheels have projecting teeth and fit into the corresponding racesses in the links of the chain.

→ The toothed wheels are known as sprocket wheels or simply sprockets.

(iii) Gear drives

The gear (or) toothed wheels may be classified as

1. According to the position of axes of the shafts :

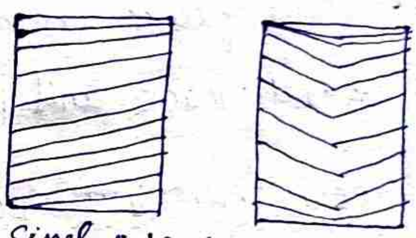
The axes of the two shafts between which the motion is to be transmitted, may be

(a) Parallel (b) Intersecting (c) Non intersecting and non parallel.

→ The two parallel and coplanar shafts connected by the gears is known as spur gears. Another name of spur gear is Helical gears.

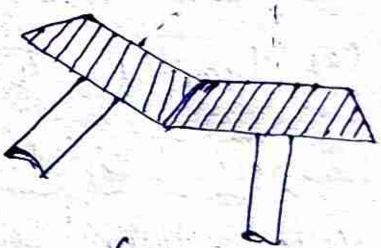
→ The double helical gears are known as Herring bone gears.

→ Two non parallel (or) intersecting but coplanar shafts connected by gears are called as Bevel gears.



Single Helical Double Helical

→ The two non intersecting and non parallel i.e non coplanar shaft connected by gears are called as skew bevel gears (or) Spiral gears.



2. According to the peripheral velocity of the gears :(Bevel Gears)

The gears, according to the peripheral velocity of the gears are classified as (a) Low velocity (b) Medium velocity (c) High velocity.

→ The Gears having velocity less than 3 m/s are termed as low velocity gears

→ The gears having velocity between 3 m/s and 15 m/s are known as medium velocity gears

→ The gears having velocity more than 15 m/s is known as High speed gears.

3. According to the type of gearing :-

The gears according to the type of gearing may be classified as (a) External gearing (b) Internal gearing (c) Rack and pinion.

→ In External gearing, the gears of two shafts mesh externally with each other. The larger of these two wheels is called spur wheel and the smaller wheel is called pinion. The motion of the two wheels is always unlike.

→ In internal gearing, the gears of the two shaft mesh internally with each other. The larger of these two wheels is called annular wheel and the smaller wheel is called pinion. The motion of the two wheels is always like.

→ The gear of a shaft meshes externally and internally with the gears in a straight line, such type of gear is called rack and pinion.

4. According to the position of teeth on the gear surface :-

The teeth on the gear surface may be classified as (a) straight (b) inclined (c) curved.

3.3 Computation of velocity ratio :-

* Velocity ratio of Belt drive :

→ It is the ratio between the velocities of the driver and the follower (or) driven. It may be expressed mathematically

d_1 = diameter of the driver

d_2 = diameter of the follower

N_1 = Speed of the driver in r.p.m.

N_2 = Speed of the follower in r.p.m.

→ length of the belt passes over the driver in one minute

$= \pi d_1 N_1$

→ length of the belt passes over the follower in one minute

$= \pi d_2 N_2$.

→ Since the length of the belt that passes over the driver is equal to the length of the ~~follower~~ belt passes over the follower

$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\Rightarrow \boxed{\frac{N_1}{N_2} = \frac{d_2}{d_1}} \rightarrow \text{velocity Ratio}$$

* when thickness(t) of the belt is considered then the velocity ratio

$$\boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}}$$

another method :-

Peripheral velocity of the belt on the driving pulley

$$v_1 = \frac{\pi d_1 N_1}{60} \text{ m/s}$$

Peripheral velocity of the belt on the follower pulley

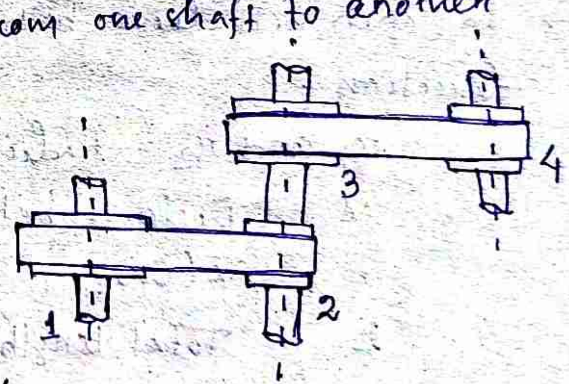
$$v_2 = \frac{\pi d_2 N_2}{60} \text{ m/s}.$$

when there is no slip $v_1 = v_2$

$$\Rightarrow \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \Rightarrow \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

Velocity ratio of a compound belt drive :

→ Sometimes the power is transmitted from one shaft to another through a number of pulleys



d_1 = Diameter of the pulley 1.
N_1 = Speed of the pulley 1 in rpm

d_2, d_3, d_4 and N_2, N_3, N_4 = corresponding values of pulleys 2, 3, 4.

We know that the velocity ratio of pulleys 1 and 2.

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{---(i)}$$

Similarly velocity ratio of pulleys 3 and 4.

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{---(ii)}$$

Multiplying eq^n (i) and (ii)

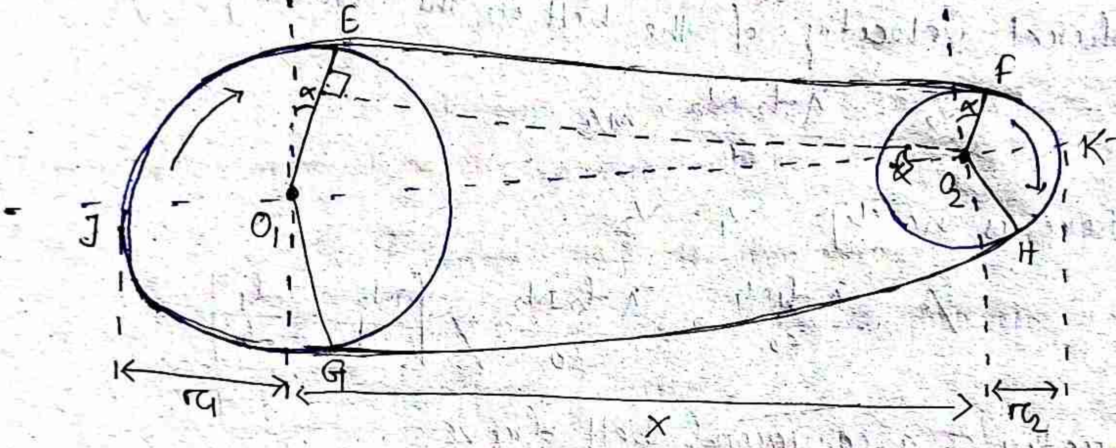
$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$$ ($\because$ $N_2 = N_3$ as the '2' and '3' are keyed to same shaft)

$\therefore$ A little consideration will show that if there are six pulleys then $\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$.

$\Rightarrow$ $\frac{\text{Speed of the last driven}}{\text{Speed of the first driven}} = \frac{\text{Product of diameters of drivers}}{\text{product of diameter of drivens}}$

Length of an open belt drive :-



→ In open belt drive both the pulleys rotate in the same direction.

r_1 and r_2 = Radii of larger and smaller pulley.

x = Distance between the centres of two pulley i.e (O_1O_2)

L = Total length of the belt

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H through O_2 draw O_2M parallel to EF.

→ from the geometry we find that O_2M will be perpendicular to O_1E.

→ Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2(\text{Arc } JE + EF + \text{Arc } FK) \quad \text{---} (1)$$

From the geometry, we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

Since 'α' is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x}$$

$$\text{Arc } JE = r_1 (\pi/2 + \alpha)$$

Similarly Arc $FK = r_2 (\pi/2 - \alpha)$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \cdots\right] = x - \frac{(r_1 - r_2)^2}{2x}$$

Substituting the values of arc JE, arc FK and EF in eq^n (1)

$$L = 2\left[r_1 (\pi/2 + \alpha) + x - \frac{(r_1 - r_2)^2}{2x} + r_2 (\pi/2 - \alpha)\right]$$

$$= 2\left[r_1 \times \pi/2 + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \pi/2 - r_2\alpha\right]$$

$$= 2\left[\pi/2 (r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 - r_2}{x}$

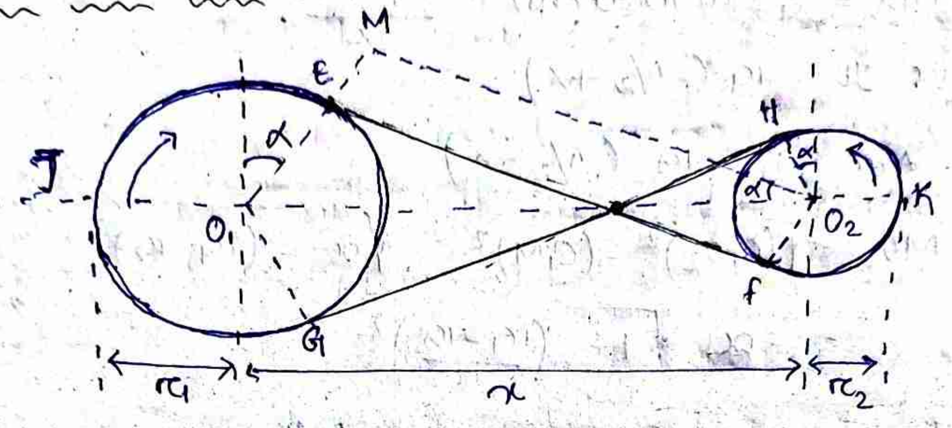
$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}$$

$$= \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1-d_2)^2}{4x}$$

Length of a cross belt drive :-



→ Cross belt drive, both the pulleys rotate in opposite direction

r_1, r_2 = Radii of larger and smaller pulley respectively

x = Distance between the centre of the two pulleys (i.e O_1O_2)

L = total length of the belt.

→ Let the belt leaves the larger pulley at E and G and the smaller pulley at f and H. Through O_2, draw O_2M parallel to Ef.

→ from the geometry, we found that O_2M will be perpendicular to O_1E.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt

L = Arc GJE + Ef + Arc fKH + HG

= 2(Arc JE + Ef + Arc fK)

from the geometry we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1+r_2}{x}.$$

$\sin\alpha$ is very small, therefore putting

$$\sin\alpha = \alpha \text{ (radians)} = \frac{r_1+r_2}{x}$$

Arc JE $= r_1 (\pi/2 + \alpha)$

Similarly Arc FK $= r_2 (\pi/2 + \alpha)$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1+r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1+r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1+r_2}{x}\right)^2 + \cdots\right] = x - \frac{(r_1+r_2)^2}{2x}$$

Substituting the value of arc JE, arc FK and EF we get

$$L = 2\left[r_1(\pi/2 + \alpha) + x - \frac{(r_1+r_2)^2}{2x} + r_2(\pi/2 + \alpha)\right]$$

$$= 2\left[r_1 \times \pi/2 + r_1\alpha + x - \frac{(r_1+r_2)^2}{2x} + r_2\,\pi/2 + r_2\alpha\right]$$

$$= 2\left[\pi/2\,(r_1+r_2) + \alpha\,(r_1+r_2) + x - \frac{(r_1+r_2)^2}{2x}\right]$$

$$= \pi(r_1+r_2) + 2\alpha(r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}.$$

Substituting the value of $\alpha = \frac{r_1+r_2}{x}$ we will get

$$L = \pi(r_1+r_2) + \frac{2(r_1+r_2)}{x} \times (r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + \frac{2(r_1+r_2)^2}{x} + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$$

$$= \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1+d_2)^2}{4x}$$

Velocity ratio with slip :-

→ Sometimes the frictional grip between pulley and belt becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driving pulley with it.

→ This is called slip of the belt and generally expressed as a percentage. The result of belt slipping is to reduce the velocity ratio of the system.

Let $S_1\%$ = slip between the driver and the belt.

$S_2\%$ = slip between the belt and the follower.

∴ Velocity of the belt passing over the driver per second

$$V = V_1 - S_1\% \, V_1$$

$$V = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right) \quad -(1)$$

and velocity of the belt passing over the follower per second

$$\frac{\pi d_2 N_2}{60} = V - V \times \frac{S_2}{100} = V\left(1 - \frac{S_2}{100}\right)$$

Substituting the value of eq^n (1) in the above eq^n we will get

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

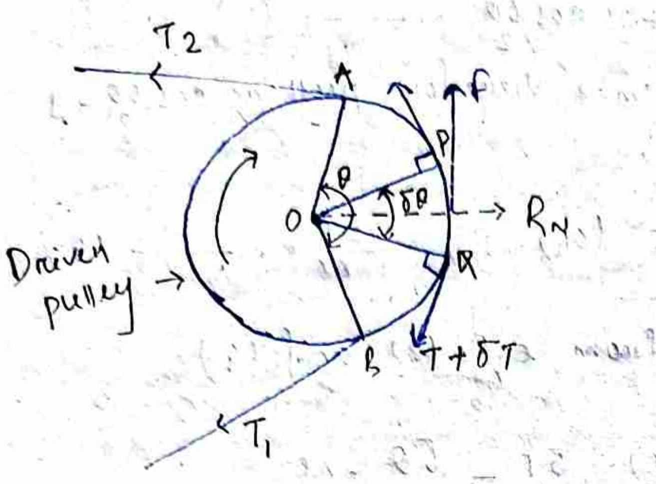
$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1}{100} - \frac{S_2}{100}\right) \qquad \left(\because \text{Neglecting } \frac{S_1 \times S_2}{100 \times 100}\right)$$

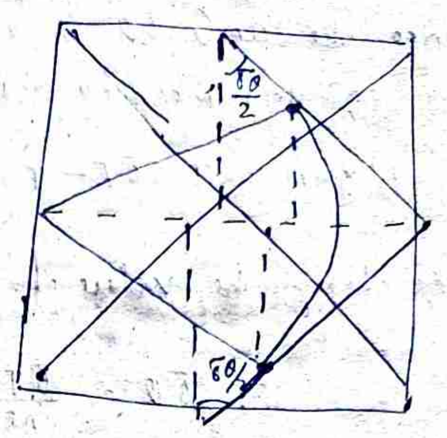
$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1 + S_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)$$

If the thickness of the belt is considered then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{S}{100}\right)$$

3.4 Ratio of the belt tension





T_1 = Tension in the belt on the right side

T_2 = Tension in the belt on the slack side

θ = Angle of contact in radians (i.e angle substended by the arc AB, along which the belt touches the pulley at the centre).

→ Now consider a small portion of belt PR, sustending an angle $\delta\theta$ at the centre of the pulley. The belt 'PR' is in equilibrium under the following forces.

1. Tension 'T' in the belt at 'P'.
2. Tension $(T+\delta T)$ in the belt at R
3. Normal reaction R_N
4. frictional force $f = \mu \times R_N$, where 'μ' is the coefficien of friction between the belt and pulley.

→ Resolving all the forces horizontally and equating the same

$$R_N = (T+\delta T)\sin\left(\frac{\delta\theta}{2}\right) + T\sin\left(\frac{\delta\theta}{2}\right) \quad \text{--- (1)}$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$ in eq^n (1)

$$R_N = (T+\delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = \frac{T\cdot\delta\theta}{2} + \frac{\delta T\cdot\delta\theta}{2} + \frac{T\cdot\delta\theta}{2} = T\cdot\delta\theta \quad \text{--- (2)}$$

Neglecting $\left(\frac{\delta T\cdot\delta\theta}{2}\right)$ we will get the above eq^n.

→ Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \quad \text{———} \quad (3)$$

Since the angle $\delta\theta$ is very small therefore putting $\cos\frac{\delta\theta}{2} = 1$ in eq^n (3) we will get

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{(OR)} \quad R_N = \frac{\delta T}{\mu} \quad \text{———} \quad (4)$$

→ Equating the value of R_N from eq^{n}s (2) and (4)

$$T\,\delta\theta = \frac{\delta T}{\mu} \quad \text{(OR)} \quad \frac{\delta T}{T} = \delta\theta \times \mu$$

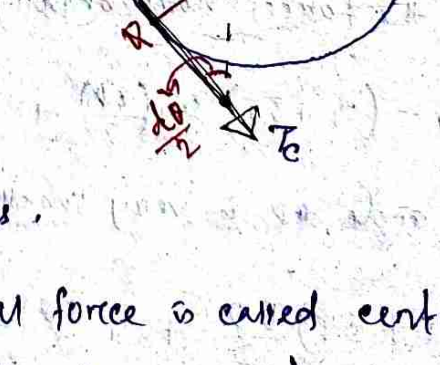
→ Integrating both sides between the limits T_2 and T_1 and from '0' to 'θ' respectively

$$\int_{T_1}^{T_2} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \quad \Rightarrow \log_e\left(T_1/T_2\right) = \mu\theta$$

$$\Rightarrow \log_e [T]_{T_1}^{T_2} = \mu[\theta]_0^{\theta} \quad \Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\theta}} \quad \text{———} \quad (5)$$

→ Eq^n (5) can be expressed in terms of corresponding logarithm to the base '10' i.e $\boxed{2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta}$.

CENTRIFUGAL TENSION :-



→ Since the belt continuously runs over the pulleys, therefore some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as slack sides.

→ The tension caused by centrifugal force is called centrifugal tension. At lower belt speed (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s) its effect is considerable and these should be taken into account.

→ Consider a small portion PQ of the belt subtending an angle $d\theta$ at the centre of the pulley.

let m = mass of the belt per unit length in kg.

v = Linear velocity of the belt in m/s

r = Radius of the pulley over which the belt runs in meter.

T_c = Centrifugal tension acting tangentially at 'P' and 'Q' in newtons.

→ We know that the length of the belt $PQ = r\,d\theta$

∴ mass of the belt $PQ = m.r.d\theta$.

centrifugal force acting on the belt PQ,

$$f_c = (m r\,d\theta)\frac{v^2}{r} = m\,d\theta\,v^2$$

→ The centrifugal tension T_c acting tangentially at P and Q keeps the belt in equilibrium.

→ Now resolving the forces (i.e centrifugal force and centrifugal tension) horizontally and equating the same we have

$$T_c \sin\left(\frac{d\theta}{2}\right) + T_c \sin\left(\frac{d\theta}{2}\right) = f_c = m.d\theta.v^2$$

→ Since the angle $d\theta$ is very small, therefore, putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$ in the above expression.

$$2T_c\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \qquad \text{(or)} \quad T_c = mv^2$$

NOTE

1. When the centrifugal tension is taken in to account, then total tension in the tight side $T_{t1} = T_1 + T_c$.

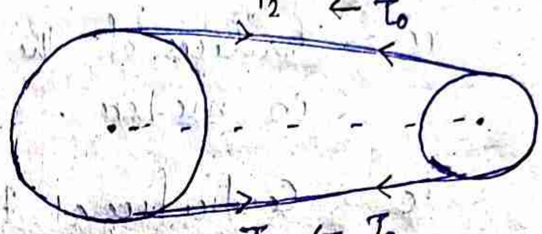
and tension in the slack side $T_{t2} = T_2 + T_c$.

2. Power transmitted $P = (T_{t1} - T_{t2})v$.

$\Rightarrow [(T_1 + T_c) - (T_2 + T_c)] \times v \Rightarrow (T_1 - T_2)v$.

$\Rightarrow$ There is no effect of centrifugal tension on power transmitted.

INITIAL TENSION :-



→ The belt always has an initial tension when installed over the pulleys.

→ The initial tension is same throughout the belt length when there is no motion. During rotation of the drive, tight side tension is higher than the initial tension and slack side tension is lower than the initial tension.

→ when the belt enters the driving pulley, it is elongated and while it leaves the pulley it contracts. Hence the driving pulley receives a larger length of the belt than it delivers.

→ The average belt velocity of the driving pulley is slightly lower than the speed of the pulley surface. On the other hand driven pulley receives a shorter belt than it deliver.

→ The avg. belt velocity on the driven pulley is slightly higher than the speed of the pulley surface.

Let the initial tension on the belt is T_0.

∴ Tight side Elongation $\propto (T_1 - T_0)$

slack side contraction $\propto (T_0 - T_2)$

Since the belt length remain same i.e elongation is equal to the contraction.

$$(T_1 - T_0) = (T_0 - T_2)$$

$$\Rightarrow T_1 + T_2 = 2T_0 \Rightarrow \boxed{T_0 = \frac{T_1 + T_2}{2}}$$

NOTE
* It is noted that with increase in initial tension power transmission can be increased.
* Max^m power transmission will occur when $T_2 = 0$ and $T_1 = 2T_0$.

3.5 POWER TRANSMITTED BY THE BELT :-

→ We know that the driving pulley pulls the belt from one side and delivers the same to the other side. It is also obvious that tension in the tight side is greater than the slack side.

T_1, T_2 = Tension in the tight and slack side of the belt respectively in newtons.

r_1, r_2 = Radii of the driver and follower respectively.

v = velocity of the belt in m/s.

→ The effective turning (driving) force at the circumference of the follower is the difference between the two tension

i.e $(T_1 - T_2)$

∴ work done per second = $(T_1 - T_2)v$ N m/s

∴ Power transmitted ⇒ $P = (T_1 - T_2)v$ W.

→ Torque transmitted in the driving pulley = $(T_1 - T_2) r_1$

Torque transmitted in the driven pulley = $(T_1 - T_2) r_2$.

3.6 Determine belt thickness and width for given permissible stress considering centrifugal tension :-

→ A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1})

Let σ = Maximum safe stress.

b = Width of the belt.

t = thickness of the belt.

→ We know that maximum tension in the belt

T = maximum stress × cross-sectional area = $\sigma \times b \times t$

→ when centrifugal tension is neglected then

T (or) $T_{t_1} = T_1$ i.e tension in the tight side of the belt

→ when the centrifugal tension is considered then

T (or) $T_{t_1} = T_1 + T_c$.

3.7 V belts and V belts pulleys :

→ It is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other.

→ The 'V' belts are made of fabric and cords moulded in rubber and covered with fabric and rubber.

→ These belts are moulded to a trapezoidal shape and are made endless. These are particularly suitable for short drives. i.e when the shafts are at short distance apart

→ The included angle for the 'V' belt is 30°- 40°.

→ In case of 'V' belt drive the rim is grooved in which the 'V' belt runs. The effect of groove is to increase the frictional grip of the V-belt on the pulley and thus to reduce the tendency of slipping.

→ A clearance must be provided at the bottom of the groove in order to prevent touching to the bottom as it becomes narrower from wear.

→ According to Indian standards, the V-belts are made in five types - A, B, C, D and E.

→ The pulleys for 'V' belts may be made of cast iron (or) pressed steel in order to reduce the weight.

3.8 Concept of crowning of pulleys :-

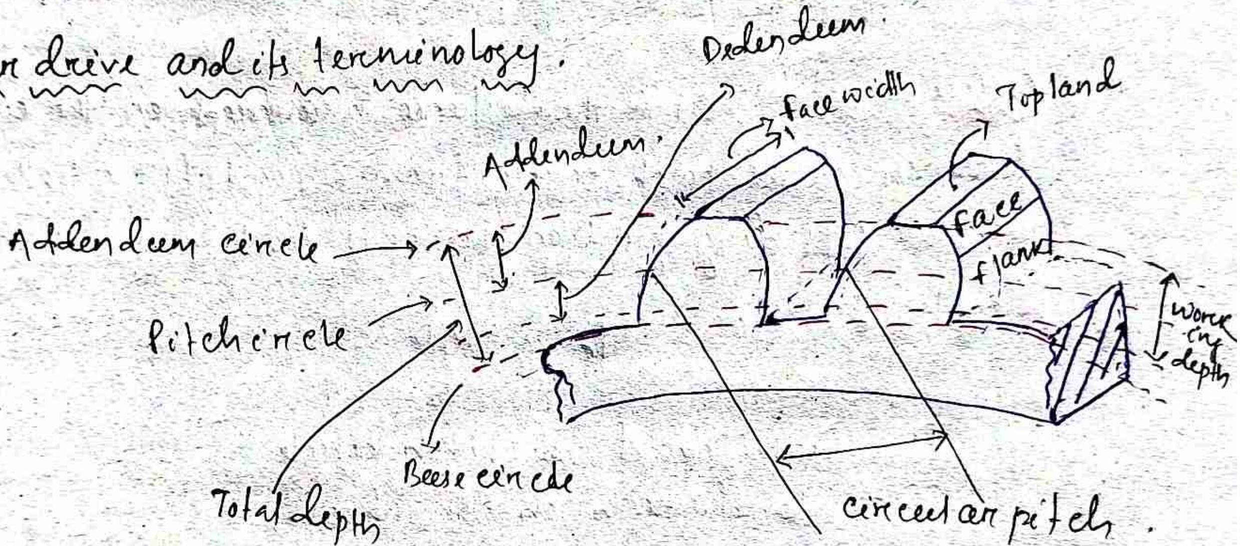
→ A crowned pulley is a pulley which has two tappered edges which helps the belt drive to run without wobbling and with much higher efficiency.

→ It also helps in increasing stability when more drives are run.

→ A crowned pulley is used when the drive is flat belt drive.

→ A crowned pulley is used to increase the stability of the belt on pulley and to run at the centre of pulley without slipping off from edges.

3.9 Gear drive and its terminology.



1. Pitch circle :- It is the imaginary circle which by pure rolling action would give the same motion as the actual gear.

2. Pitch circle diameter :- It is the diameter of the pitch circle. The size of the gear usually specified by the pitch circle diameter. It is also known as pitch diameter.

3. Pitch point :- It is a common point of contact between two pitch circle.

4. Pitch surface :- It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle (or) angle of obliquity : It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is noted by "ϕ".

6. Addendum :- It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. Dedendum :- It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Addendum circle :- It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle :- It is the circle drawn through the bottom of the teeth. It is also called as root circle.

Root circle diameter = Pitch circle diameter × $\cos\phi$.

10. Circular pitch :- It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by 'P_c'.

$$P_c = \frac{\pi D}{T}$$

D = Diameter of the pitch circle
T = No. of teeth on the wheel.

* Two gears will mesh together correctly if the two gears have the same circular pitch.

11. Diametrical pitch :- It is the ratio of no. of teeth to the pitch circle diameter in "mm". It is denoted by P_d.

$$P_d = \frac{T}{D} = \frac{\pi}{P_c}$$

12. Module :- It is the ratio of the pitch circle diameter in "mm" to the no. of teeth. It is usually denoted by 'm'.

$$m = \frac{D}{T}$$

13. clearance :- It is the radial distance from the top of the tooth to the bottom of the tooth in a meshing gear. A circle passing through the top of the meshing gear is known as clearance circle.

14. Total depth :- It is the radial distance between the addendum and dedendum circle of a gear. It is equal to sum of addendum and dedendum.

15. Working depth :- It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. Tooth thickness :- It is the width of the tooth measured along the pitch circle.

17. Tooth space :- It is the width of space between the two adjacent teeth measured along the pitch circle.

18. Back lash :- It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle.

19. face of tooth :- It is the surface of the gear tooth above the pitch surface.

20. flank of the tooth :- It is the surface of gear tooth below the pitch surface.

21. Top land :- It is the surface of the top of the tooth.

22. face width :- It is the width of the gear tooth measured parallel to its axis.

23. Profile :- It is the curve formed by face and flank of the tooth.

24. Path of the contact :- It is the path traced by the point of contact of two teeth from the begining to the end of the engagement.

25. Length of path of contact :- It is the length of the common normal cut-off by the addendum circle of the wheel and pinion.

26. Arc of contact :- It is the path traced by a point on the pitch circle from the begining to the end of engagement of a given pair of teeth. The arc of contact consists of two parts. (a) Arc of approach (b) Arc recess.

(a) Arc of approach :- It is the portion of the path of the contact from the begining of the engagement to the pitch point.

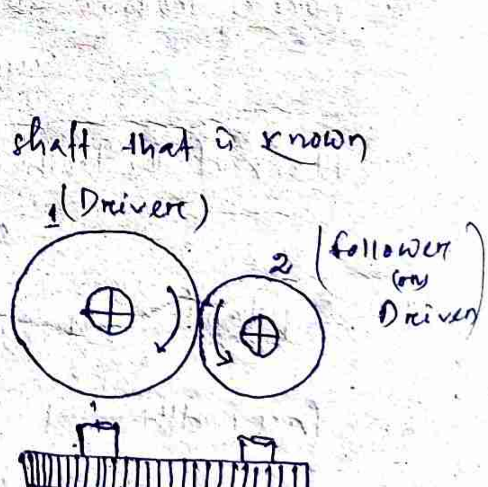
(b) Arc of recess :- It is the portion of the path of the contact from the pitch point to the end of the engagement of the pair of tooth.

3.10 GEAR TRAINS :-

* As per the arrangement of wheels following are the types of gear train.
  (a) Simple gear train (b) compound gear train
  (c) Reverted gear train (d) Epicyclic gear train.

Simple gear train :-

→ when there is only one gear on each shaft that is known as simple gear train.



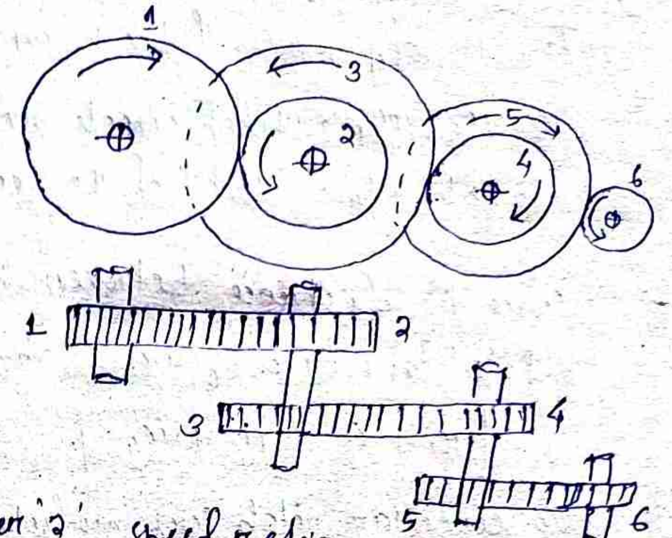
Let N_1 = Speed of gear '1' in r.p.m
N_2 = Speed of gear '2' in r.p.m.
T_1 = No. of teeth in gear 1.
T_2 = No. of teeth in gear '2'.

$$\therefore \text{Speed ratio} = \frac{\text{Speed of the driver}}{\text{Speed of the follower}} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\therefore \text{Train value} = \frac{\text{Speed of the follower}}{\text{Speed of the driver}} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

Compound gear train :-

→ when there are more than one gear on a shaft, it is called compound gear train.



Let N_1 = Speed of driven gear

T_1 = No. of teeth on driving gear 1.

$N_2, N_3 \cdots N_6$ = Speed of the respective gears in r.p.m.

$T_2, T_3 \cdots T_6$ = No. of teeth on respective gears.

Since gear '1' is meshed with gear '2', speed ratio

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly gear '3', and '4' are meshed

$$\frac{N_3}{N_4} = \frac{T_4}{T_3}.$$

for gear '5' and '6' $\quad \frac{N_5}{N_6} = \frac{T_6}{T_5}.$

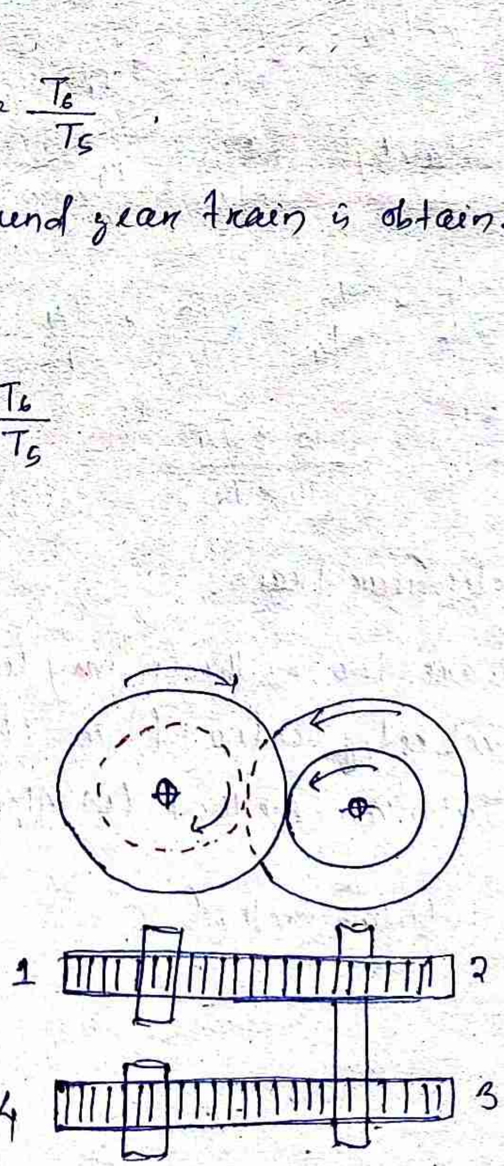
∴ The speed ratio of the compound gear train is obtained by multiplying the above eqn.

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$\Rightarrow \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

Reverted gear train :-

→ when the axes of the 1st gear (i.e driver) and the last gear (i.e last driven (or) follower) are co-axial then the gear train is known as reverted gear train.



T_1 = No. of teeth gear '1'
r_1 = Pitch circle radius of gear 1.
N_1 = Speed of the gear '1' in r.p.m.

T_2, T_3, T_4 = No. of teeth on respective gears.
r_2, r_3, r_4 = Pitch circle radius of respective gears.
N_2, N_3, N_4 = Speed of the respective gears in r.p.m.

→ Since the distance between the centre of the shafts gear '1' and '2' as well as gear '3' and '4' is same therefore

$$\boxed{r_1 + r_2 = r_3 + r_4}$$

→ Also circular pitch (or) module of all gears is assumed to be same, therefore the no. of teeth on each gear is directly proportional to its circumference (or) radius.

$$\boxed{T_1 + T_2 = T_3 + T_4}$$

Speed ratio = $\dfrac{N_1}{N_2} = \dfrac{T_2}{T_1}$

Similarly $\dfrac{N_3}{N_4} = \dfrac{T_4}{T_3}$

$\Rightarrow \dfrac{N_1}{N_2} \times \dfrac{N_3}{N_4} = \dfrac{T_2}{T_1} \times \dfrac{T_4}{T_3}$ ($\because N_2 = N_3$ as Gear '2' and '3' are mounted on a single shaft)

$\Rightarrow \dfrac{N_1}{N_4} = \dfrac{T_2 \times T_4}{T_1 \times T_3}$

Epicyclic Gear train :-

There are two methods may be used used for finding out the velocity ratio of an epicyclic gear train.

(a) Tabular method (b) Algebraic method.

(a) Tabular method :-

→ T_A = No. of teeth on gear A
T_B = No. of teeth on gear B

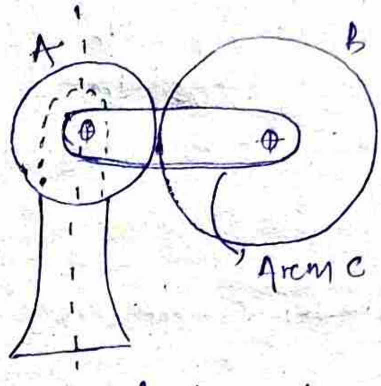


Table of motion

Step No	Conditions of motion	Revolutions of element		
		Arm C	Gear A	Gear B.
1.	Arm fixed gear 'A' rotates through +1 revolution i.e 1 rev. anticlockwise	0	+1	$-\frac{T_A}{T_B}$
2.	Arm fixed gear 'A' rotates through +x revolution.	0	+x	$-x\frac{T_A}{T_B}$
3.	Add +y revolutions to all elements.	+y	+y	+y
4.	Total motion	+y	x+y	$y-x\frac{T_A}{T_B}$

(2) Algebraic method:-
let the arm 'c' be fixed in an epicyclic gear train, Therefore speed of the gear 'A' relative to the arm 'c' = $N_A - N_c$.
and the speed of the gear 'B' relative to the arm 'c' = $N_B - N_c$.
→ Since the gear 'A' and 'B' are meshing directly, therefore they will revolve in opposite directions.

$$\frac{N_B - N_c}{N_A - N_c} = \frac{T_A}{T_B}$$

Since the arm 'c' is fixed, therefore its speed $N_c = 0$

$$\frac{N_B}{N_A} = \frac{-T_A}{T_B}$$

(58) If the gear 'A' is fixed, then $N_A = 0$

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad (OR) \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

FRICTION

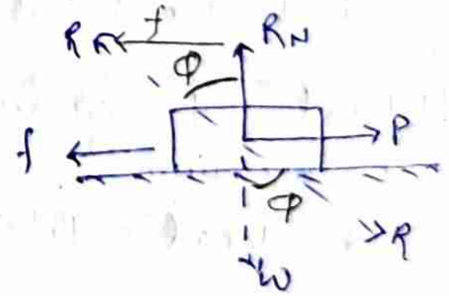
- 2.1. friction between nut and screw for square thread, screw jack
- 2.2. Bearing and its classification, Description of roller, needle roller and ball bearing.
- 2.3. Torque transmission in flat pivot and conical pivot bearings.
- 2.4. Flat collar bearing of single and multiple types.
- 2.5. Torque transmission for single and multiple clutches
- 2.6. Working of simple frictional brakes.
- 2.7. Working of absorption type of dynamometer.

2.1 Friction :-

→ The resisting force which acts in the opposite direction of the moving body due to its motion is called frictional force.

→ $f = \mu N$

f = frictional force
μ = coefficient of friction
N = Normal reaction.



Limiting angle of friction :-

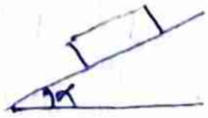
W = load of the body.
R_N = Normal reaction.
P = force applied
R = Resultant of W and P.

→ The angle between the 'R' and R_N is called limiting angle of friction (ϕ).

$$\tan\phi = \frac{f}{R_N} \Rightarrow \boxed{\mu = \frac{f}{R_N}} \Rightarrow \boxed{\tan\phi = \mu}$$

Angle of Repose :-

If the angle of inclination 'α' of the plane to the horizontal is such that the body begins to move down the plane, then the angle 'α' is called angle of Repose.



Screw friction :

→ If the threads are cut at the outer surface of the solid rod then those are called External threads.

→ If the threads are cut on the internal surface of a hollow rod, then those are called Internal threads.

→ The screw threads are mainly two types i.e v-threads and square threads. In general v-threads are used for the purpose of tightening pieces together i.e bolts and nuts. But the square threads are used in screw jack.

(a) Helix :- It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at uniform rate.

(b) Pitch :- It is the distance from a point of screw to a corresponding point on the next thread measured parallel to the axis of the screw.

(c) Depth of thread :- It is the distance between the top and bottom surfaces of the thread (also known as crest and root of the thread)

(d) Lead :- It is the distance, a screw thread advances axially in one turn.

* Lead = Pitch × Number of threads.

(e) Helix angle :- It is the slope or inclination of the thread with the horizontal.

$$\tan\alpha = \frac{\text{Lead of the screw}}{\text{circumference of the screw}}$$

$$= \frac{p}{\pi d} \quad (\text{or}) \quad \frac{np}{\pi d} \rightarrow \text{for multi thread screw.}$$

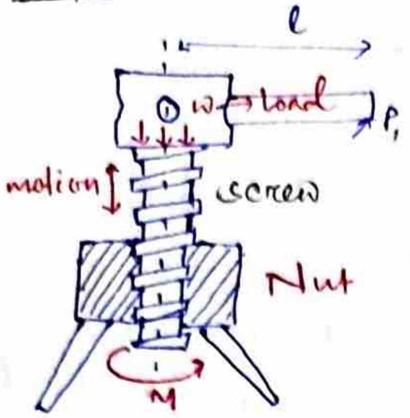
α = Helix angle

p = Pitch of the screw

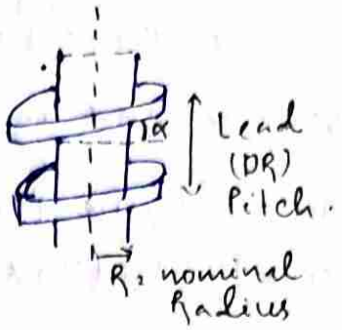
d = diameter of the screw

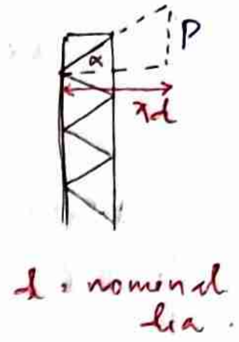
n = no. of threads in one coil.

SCREWJACK :

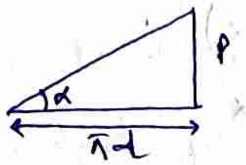


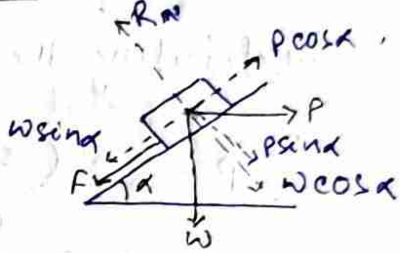
α, Helix angle





TORQUE REQUIRED TO LIFT THE LOAD BY A SCREWJACK :





p : Pitch of the screw

d : Mean dia of the screw.

α : Helix angle.

P : Effort applied at the circumference of the screw to lift the load.

W : Load to be lifted.

μ : coefficient of friction between the screw and nut = $\tan\phi$.

φ : friction angle.

* from geometry we know that $\tan\alpha = \frac{p}{\pi d}$.

→ Since the load is being lifted, therefore the force of friction ($f = \mu R_N$) will act downwards.

All the forces acting on the screw are shown in the fig. above.

Resolving the forces along the plane

$$P\cos\alpha = W\sin\alpha + f = W\sin\alpha + \mu R_N \quad \text{——— (1)}$$

Resolving the forces perpendicular to the plane

$$R_N = P\sin\alpha + W\cos\alpha \quad \text{——— (2)}$$

Substituting this value of R_N in equation (1)

$$P\cos\alpha = W\sin\alpha + \mu(P\sin\alpha + W\cos\alpha)$$
$$= W\sin\alpha + \mu P\sin\alpha + \mu W\cos\alpha.$$

$$P\cos\alpha - \mu P\sin\alpha = W\sin\alpha + \mu W\cos\alpha$$
$$\Rightarrow P(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$$

$$\Rightarrow P = \frac{W(\sin\alpha + \mu\cos\alpha)}{\cos\alpha - \mu\sin\alpha}$$

Substituting the value of $\mu = \tan\phi$ in the above eq^n, we get

$$P = \frac{W(\sin\alpha + \tan\phi\cos\alpha)}{\cos\alpha - \tan\phi\sin\alpha}$$

Multiplying the numarator and denominator by $\cos\phi$.

$$P = W\frac{\sin\alpha\cos\phi + \cos\alpha\sin\phi}{\cos\alpha\cdot\cos\phi - \sin\phi\sin\alpha} = \frac{W\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$\boxed{P = W\tan(\alpha+\phi)}$$

$\therefore$ Torque required to overcome friction between the screw and nut

$$T_1 = P\times\frac{d}{2} = W\tan(\alpha+\phi)\times\frac{d}{2}$$

* If an effort P_1 is applied at the end of the lever arm length 'l', then the total torque required to overcome friction must be equal to the torque applied at the end of the lever.

$$\boxed{T = P\times\frac{d}{2} = P_1\times l}$$

2.2 BEARING AND ITS CLASSIFICATION :-

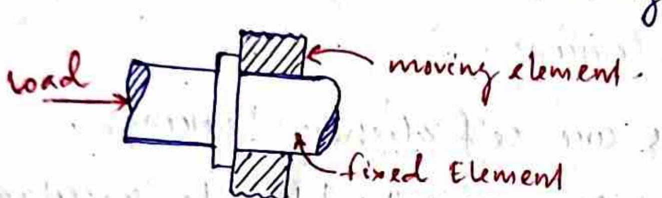
→ Bearing is a machine element which support another moving machine element (known as journal). It permits the relative motion between the contact surfaces of the members, while carrying the load.

Classification of Bearing:-

*(a) Depending up on the direction of load to be supported :-

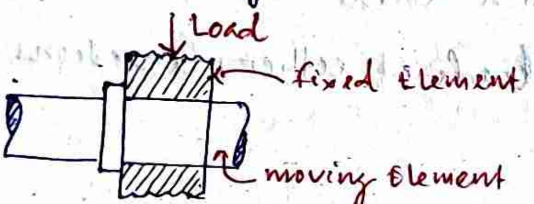
(i) Radial Bearing (ii) Thrust bearing

(i) Radial Bearing :- In this type of bearing the load acts perpendicular to the direction of motion of the moving element.



(ii) Thrust Bearing :-

In this type of bearing the load acts along the axis of rotation.

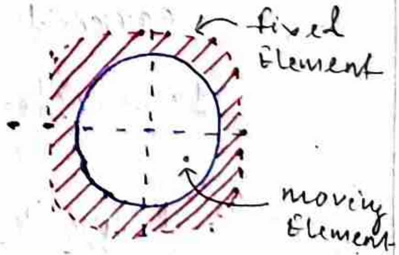


(b) Depending up on the nature of contact :-

(i) Sliding contact Bearing :-

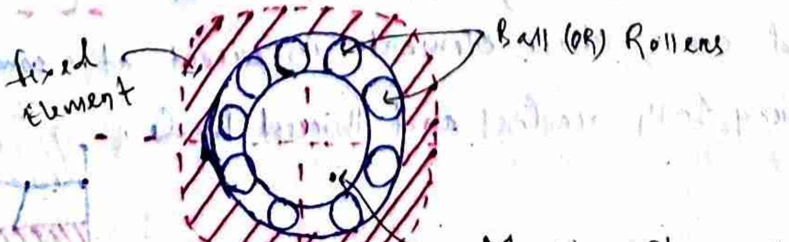
In - this sliding takes place along the surfaces of contact between the moving element and fixed element.

→ These are also known as plain bearing.



(ii) Rolling contact Bearing :-

In this the steel balls (or) Rollers are interposed between the moving and the fixed element.



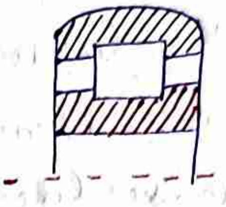
(14)

Types of Roller Bearing..

1. cylindrical roller bearing:..

These bearings have short rollers guided in cage. These bearings are relatively rigid against radial motion and have the lowest coefficient of friction of any form of heavy duty rolling contact bearings.

→ These are used in high speed service.

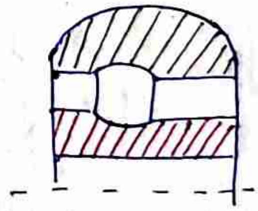


2. Spherical roller bearing:..

→ These bearings are self aligning bearings.

→ These bearings can normally tolerate angular misalignment in order of ±1½°.

→ when used with a double row of rollers, these can carry thrust loads in either directions.



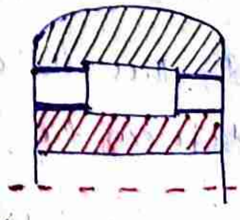
3. Needle Roller Bearing:

→ These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed.

→ These bearings are used when heavy loads are to be carried with a oscillatory motion.

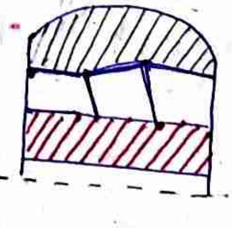
Ex: Piston pin bearing in heavy duty diesel engine.

4. Tapered roller bearing..



→ The rollers and race ways of these bearings are truncated cones whose elements intersect at a common point.

→ These can carry both radial and thrust loads.



* Rolling contact bearing are of two types 1. Ball Bearing 2. Roller Bearing.

→ The ball and rolling bearings consists of an inner race which is mounted on the shaft (or) journal and outer race which is carried by housing or casing.

→ In between the inner and outer race there are balls (or) rollers.

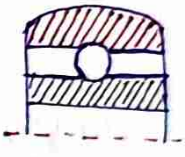
Types of Ball Bearing :-

1. Single row deep groove bearing :-

→ During assembly of this bearing, the races are offset and the maximum number of balls are placed between the races.

→ The races are then centered and the balls are symmetrically located by the use of a retainer cage.

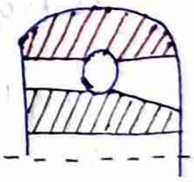
→ The deep groove ball bearings are used due to their high load carrying capacity and suitability of high running speed.



2. Filling notch bearing :-

→ These bearings have notches in the inner and outer races which permit more balls so to be inserted than in a deep groove ball bearing.

→ Since this type of bearing contains larger number of balls than the corresponding unnotched one, therefore it has a larger bearing load capacity.

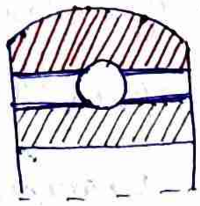


3. Angular contact bearing :-

→ These bearings have one side of the outer race cut away to permit the insertion of more balls than in deep groove bearing but without having a notch cut in to both races.

→ This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load.

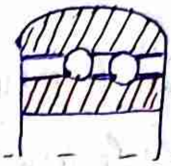
→ It may also carry thrust load in either directions.



(4) Double row bearing :-

These bearings may be made with radial (or) angular contact between the balls and races.

→ The load capacity of these bearings are slightly less than that of twice that of single row bearing.



(5) Self aligning bearing :-

→ These bearings permit shafts deflection within 2-3 degrees.

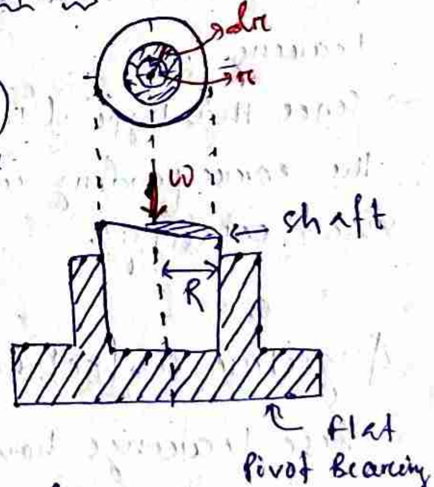
→ If the unit is assembled with shaft misallignment present then the bearing will subjected to a load that may be in excess of the design value and premature failure may occure.

→ These are of two types
* External self aligning bearing
* Internally self aligning bearing

2.3 TORQUE TRANSMISSION IN FLAT PIVOT AND CONICAL PIVOT BEARINGS

FLAT PIVOT BEARING :-(OR) FOOTSTEP BEARING :-

→ when a vertical shaft rotates in a pivot bearing (known as footstep bearing) the sliding friction will be along the surface contact between the shaft and bearing.



W = load transmitted over the bearing surface.

R = radius of the bearing surface

p = intensity of pressure per unit area of bearing surface between rubbing surfaces.

μ = coefficient of friction.

→ we will consider the following two cases.

1. Considering uniform pressure :-

when the pressure is uniformly distributed over the bearing area than $p = \frac{W}{\pi R^2}$

Considering a ring of radius 'r' and thickness 'dr' of the bearing area.

→ Area of the bearing surface $A = 2\pi r\, dr$

Load transmitted to the ring $\delta W = P \times A = P \times 2\pi r . dr$.

Frictional resistance to sliding on the ring acting tangentially at radius 'r' $\quad f_r = \mu\, \delta W = \mu \times P \times 2\pi r . dr$.

Frictional torque on the ring

$$T_r = f_r \times r = r \times 2\pi \mu P . r\, dr = 2\pi \mu P r^2\, dr .$$

Integrating this with in the limit '0' to 'R' the total frictional torque on the pivot bearing.

$\therefore$ Total frictional torque $T = \int_0^R T_r = \int_0^R 2\pi\mu P r^2\, dr$.

$$= 2\pi\mu P \int_0^R r^2\, dr = 2\pi\mu P \left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu P \frac{R^3}{3} = \frac{2}{3}\pi\mu P R^3 = \frac{2}{3} \times \pi \times \mu \frac{W}{\pi R^2} R^3$$

$$\boxed{T = \frac{2}{3}\mu W R} \qquad \left(\because P = \frac{W}{\pi R^2}\right)$$

$\therefore$ when shaft rotates at 'ω' rad/sec the power lost in the friction

$$\text{Power} = T\omega = T \times \frac{2\pi N}{60} \qquad \because \omega = \frac{2\pi N}{60}$$

N = revolutions per minute

(2) Considering uniform wear theory :

→ The rate of wear depends up on the intensity of pressure (P) and velocity of rubbing surfaces (v) (i.e P.v)

→ Since the velocity of rubbing surfaces increases with the distance (i.e radius 'r') from the axis of the bearing. Therefore uniform wear

$p \cdot r = C \Rightarrow p = \frac{C}{r}$ (C = constant)

and the load transmitted to the ring

$\delta W = p \times 2\pi r \, dr$

$= \frac{C}{r} \times 2\pi r \, dr = 2\pi C \, dr$

Total load transmitted to the bearing

$W = \int_0^R \delta W = \int_0^R 2\pi C \, dr = 2\pi C [r]_0^R = 2\pi C R$

$\Rightarrow \boxed{C = \frac{W}{2\pi R}}$ * the value of the constant

we know that frictional torque acting on the ring

$T_r = 2\pi \mu p r^2 dr = 2\pi \mu \frac{C}{r} r^2 dr$

$= 2\pi \mu C r \, dr$

Total frictional torque on the bearing

$T = \int_0^R T_r = \int_0^R 2\pi \mu C r \, dr = 2\pi \mu C \int_0^R r \, dr$

$= 2\pi \mu C \left[\frac{r^2}{2}\right]_0^R = \pi \mu C R^2 = \pi \mu \frac{W}{2\pi R} R^2$

$\boxed{T = \frac{\mu W R}{2}}$ $\left(\because C = \frac{W}{2\pi R}\right)$

Q. A vertical shaft 150 mm in diameter rotating at 100 rpm rest on a flat end footstep bearing. The shaft carries a vertical load of 20 KN. Assuming uniform pressure distribution and coefficient of friction equal to 0.05. Estimate power loss in friction.

Sol. Given data

$D = 150 \text{ mm} \Rightarrow R = 75 \text{ mm} = 0.075 \text{ m}$

$N = 100 \text{ rpm} \Rightarrow \omega = \frac{2\pi N}{60} = 10.47 \text{ rad/sec}$

$W = 20 \text{ KN} = 20 \times 10^3 \text{ N}$, $\mu = 0.05$

we know that uniform pressure distribution

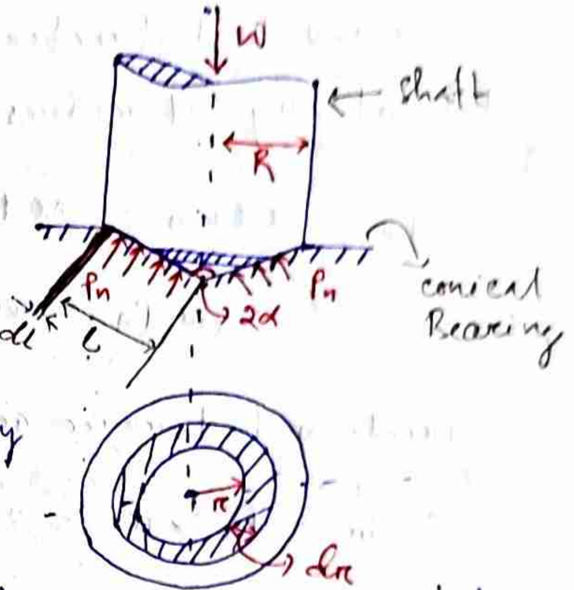
Torque $T = \frac{2}{3} \times \mu W R = \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 0.075 = 50\,Nm$

Power lost in friction

$P = T\omega = 50 \times 10.47 = 523.5$

CONICAL PIVOT BEARING :-

The conical pivot bearing supporting a shaft carrying a load 'W'



P_n = Intensity of pressure normal to the cone.

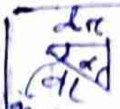
α = Semi angle of cone.

μ = coefficient of friction between the shaft and the bearing

R = Radius of the shaft.

consider a small ring of radius 'r' and thickness 'dr'. Let 'dl' is the length of the ring along the cone such that

$$dl = dr \operatorname{cosec}\alpha.$$



$\rightarrow dl \cos(90-\alpha) = dr$

$\rightarrow dl \sin\alpha = dr$

$\rightarrow dl = dr \operatorname{cosec}\alpha$

Area of the ring $A = 2\pi r\, dl = 2\pi r\, dr \operatorname{cosec}\alpha$.

1. Considering uniform pressure theory :-

we know that normal load acting on the ring

δW_n = Normal pressure $\times$ Area

$= P_n \times 2\pi r\, dr \operatorname{cosec}\alpha$.

and vertical load acting on the ring

δW = vertical component of $\delta W_n = \delta W_n \sin\alpha$

$= P_n \times 2\pi r\, dr \operatorname{cosec}\alpha \cdot \sin\alpha$.

$$\boxed{\delta W = P_n \times 2\pi r\, dr}$$

(20)

Total vertical load transmitted to the bearing

$$W = \int_0^R P_n \times 2\pi r\, dr = 2\pi P_n \int_0^R r\, dr$$

$$= 2\pi P_n \left[\frac{r^2}{2}\right]_0^R = \pi P_n R^2.$$

$$\Rightarrow \boxed{P_n = \frac{W}{\pi R^2}}$$

we know that frictional force on the ring acting tangentially at radius 'r'

$$f_r = \mu\, \delta W_n = \mu P_n\, 2\pi r\, dr \operatorname{cosec}\alpha$$

$$= 2\pi \mu P_n \operatorname{cosec}\alpha\, r\, dr$$

frictional torque acting on the ring

$$T_r = f_r \times r = 2\pi\mu P_n \operatorname{cosec}\alpha\, r\, dr \times r$$

$$= 2\pi\mu P_n \operatorname{cosec}\alpha\, r^2\, dr$$

Integrating this with in the limit '0 to R' the total frictional torque on the bearing

$$T = \int_0^R T_r = \int_0^R 2\pi\mu P_n \operatorname{cosec}\alpha\, r^2\, dr$$

$$= 2\pi\mu P_n \operatorname{cosec}\alpha \left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{2\pi}{3}\mu P_n \operatorname{cosec}\alpha\, R^3$$

Substituting the value of 'P_n' in the eq^n

$$T = \frac{2}{3}\pi\mu \frac{W}{\pi R^2} \times \operatorname{cosec}\alpha\, R^3$$

$$T = \frac{2}{3}\mu W R \operatorname{cosec}\alpha = \frac{2}{3}\mu W R \operatorname{cosec}\alpha$$

$$\boxed{T = 2/3\, \mu W l} \qquad (\because l = R \operatorname{cosec}\alpha)$$

(2) Considering uniform wear :

→ Let 'P_r' be the normal intensity of pressure at a distance 'r' from the central axis.

→ We know that in-case of uniform wear the intensity of pressure varies inversely with the distance.

$$P_r \cdot r = C \Rightarrow \frac{C}{r} = P_r$$

→ The load transmitted to the ring

$$\delta W = P_r \times 2\pi r\, dr = \frac{C}{r}\, 2\pi r\, dr = 2\pi C\, dr.$$

Total load transmitted to the bearing $W = \int_0^R \delta W = 2\pi C R$

$$\Rightarrow \boxed{C = \frac{W}{2\pi R}}$$

Torque acting on the ring

$$T_r = 2\pi \mu P_r \operatorname{cosec}\alpha\, r^2\, dr$$

$$= 2\pi \mu \frac{C}{r} \operatorname{cosec}\alpha\, r^2\, dr$$

$$= 2\pi \mu C \operatorname{cosec}\alpha\, r\, dr.$$

∴ Total frictional torque acting on the bearing

$$T = \int_0^R 2\pi \mu C \operatorname{cosec}\alpha\, r\, dr$$

$$= 2\pi \mu C \operatorname{cosec}\alpha \left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi \mu C \operatorname{cosec}\alpha \frac{R^2}{2}$$

$$\boxed{T = \pi \mu C R^2 \operatorname{cosec}\alpha}$$

Substituting the value of 'C'

$$T = \pi \mu \times \frac{W}{2\pi R} R^2 \operatorname{cosec}\alpha$$

$$= \frac{1}{2} W \mu R \operatorname{cosec}\alpha$$

$$\boxed{T = \frac{1}{2} W \mu l}.$$

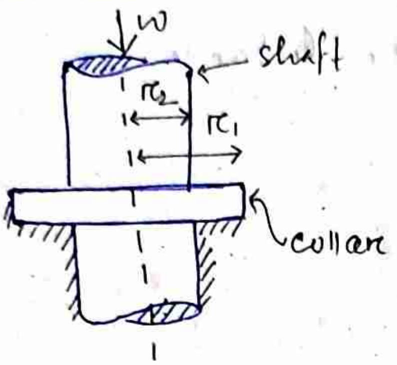
(22)

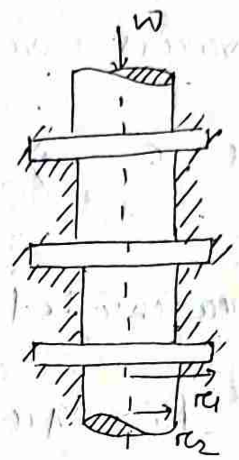
2.4 FLAT COLLAR BEARING :-

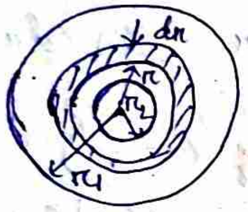
→ collar bearings are used to take the axial thrust of the rotating shafts.

→ There may be single (OR) multiple collar bearing.

→ These are also called as THRUST BEARING.







consider a flat collar bearing supporting a shaft

Let r_1 = External radius of the collar

r_2 = Internal radius of the collar

Area of the bearing surface

$$A = \pi(r_1^2 - r_2^2)$$

1. Considering uniform pressure :

when the pressure is uniformly distributed over the bearing surface then the intensity of pressure

$$p = \frac{W}{A} = \frac{W}{\pi(r_1^2 - r_2^2)}$$

we have seen that the frictional torque on the ring of radius 'r' and thickness 'dr'

$$T_r = 2\pi\mu p r^2 dr$$

Integrating this equation with in the limit 'r_2' to 'r_1' for the total frictional torque on the collar.

Total frictional torque $= \int_{r_2}^{r_1} 2\pi \mu P r^2 dr$

$$= 2\pi \mu P \left(\frac{r^3}{3}\right)_{r_2}^{r_1} = 2\pi \mu P \frac{(r_1^3 - r_2^3)}{3}$$

Substituting the value of 'P' we will get

$$T = 2\pi \mu \frac{W}{\pi(r_1^2 - r_2^2)} \times \frac{(r_1^3 - r_2^3)}{3}$$

$$= \frac{2}{3} \mu W \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$$

MULTI-COLLARED :-

NOTE :- In order to increase the amount of rubbing surfaces so as to reduce the intensity of pressure, it is better to use two (or) more collars, rather than one large collar.

* In case of multi collared bearings with say 'n' collars, the intensity of the uniform pressure,

$$P = \frac{\text{Load}}{\text{No. of collar} \times \text{Bearing area of one collar}} = \frac{W}{n\pi(r_1^2 - r_2^2)}$$

* The total torque transmitted in a multi collared shaft remains constant i.e $T = \frac{2}{3} \mu W \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$

(2) Considering uniform wear :-

Load transmitted on the ring in case of uniform wear we know that $\delta W = P_r \, 2\pi r \, dr = \frac{C}{r} 2\pi r \, dr = 2\pi C \, dr$

The total load transmitted to the collar

$$W = \int_{r_2}^{r_1} 2\pi C \, dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

$$\boxed{C = \frac{W}{2\pi(r_1 - r_2)}}$$

We also know that frictional torque on the ring

$$T_r = \mu \, \delta W \, r = \mu \, 2\pi c \, dr \, r = 2\pi\mu c \, r \, dr$$

Total frictional torque on the bearing

$$T = \int_{r_2}^{r_1} 2\pi\mu c \, r \, dr = 2\pi\mu c \left(\frac{r^2}{2}\right)_{r_2}^{r_1}$$

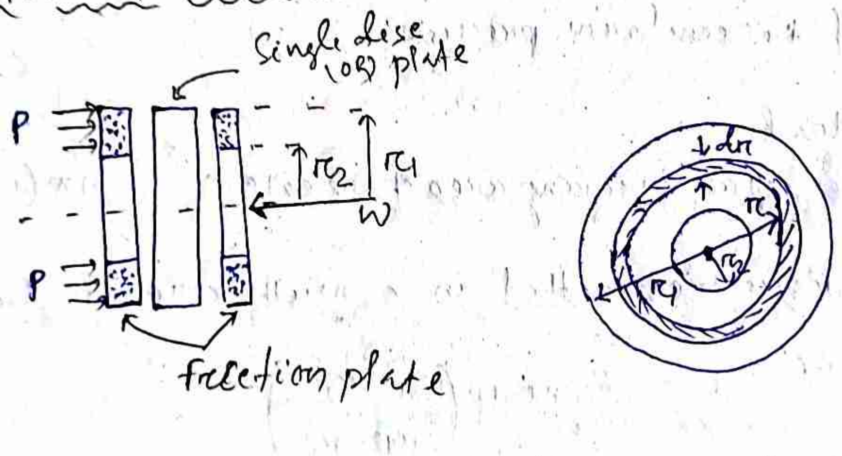
$$= 2\pi\mu c \frac{(r_1^2 - r_2^2)}{2} = \pi\mu c (r_1^2 - r_2^2)$$

Substituting the value of 'c'

$$T = \pi\mu \frac{W}{2\pi(r_1 - r_2)} \times (r_1^2 - r_2^2)$$

$$\boxed{T = \tfrac{1}{2} \mu W (r_1 + r_2)}$$

2.5. TORQUE TRANSMISSION FOR SINGLE PLATE CLUTCH:



Let us consider two friction surfaces maintained in contact by an axial thrust 'W'

T = torque transmitted by the clutch

p = intensity of axial pressure with which the contact surfaces are held together.

r_1 and r_2 = External and internal radii of friction faces.

μ = coefficient of friction.

Consider an elementary ring of radius 'r' and thickness 'dr'

we know that area of contact surface (or) friction surface

$= 2\pi r \cdot dr$.

Normal (or) axial force on the ring

$$\delta W = \text{Pressure} \times \text{Area} = P \times 2\pi r\, dr.$$

frictional force on the ring acting tangentially at radius 'r'

$$F_r = \mu \delta W = \mu P\, 2\pi r\, dr.$$

frictional torque acting on the ring

$$T_r = F_r \times r = \mu P \times 2\pi r\, dr = 2\pi \mu P r^2\, dr.$$

Now we will consider the two cases of (a) uniform pressure theory
(b) uniform wear theory

1. <u>Uniform pressure theory</u> :-

when the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure

$$P = \frac{W}{\pi(r_1^2 - r_2^2)}$$

, W = Axial thrust with which the contact (or) friction surfaces are held together.

The frictional torque on the elementary ring of radius 'r' and thickness 'dr'.

$$T_r = 2\pi \mu P r^2\, dr$$

Integrating this equation with in the limits from r_2 to r_1 for the total frictional torque

Total frictional torque acting on the friction surface (or) on the clutch :-

$$T = \int_{r_1}^{r_2} 2\pi \mu P r^2\, dr$$

$$T = 2\pi\mu p \left(\frac{r^3}{3}\right)_{r_2}^{r_1} = 2\pi\mu p \frac{(r_1^3 - r_2^3)}{3}.$$

Substituting the value of 'p' we will get

$$T = 2\pi\mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \times \frac{(r_1^3 - r_2^3)}{3}$$

$$\boxed{T = \frac{2}{3} \times \mu W \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)}$$

$$\boxed{T = \mu W R}\,,$$

R = mean radius of the friction surfaces.

$$\boxed{R = \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)}$$

2. Considering uniform wear :

→ let 'p' be the normal intensity of pressure at a distance 'r' from the axis of the clutch.

→ Since the intensity of pressure varies inversely with the distance, therefore

$$p \cdot r = C \Rightarrow \boxed{p = \frac{C}{r}}$$

→ the normal force acting on the ring

$$\delta W = p \cdot 2\pi r\, dr = \frac{C}{r} 2\pi r\, dr = 2\pi C\, dr$$

Total force acting on the friction surfaces

$$W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C\, [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

$$\therefore \boxed{C = \frac{W}{2\pi(r_1 - r_2)}}$$

we know that frictional torque acting on the ring

$$T_r = 2\pi\mu p r^2 dr = 2\pi\mu \times \frac{C}{r} \times r^2 dr = 2\pi\mu C r\, dr.$$

Total frictional torque on the friction surface

$$T = \int_{r_2}^{r_1} 2\pi\mu C r \, dr = 2\pi\mu C \left(\frac{r^2}{2}\right)_{r_2}^{r_1}$$

$$= 2\pi\mu C \frac{(r_1^2 - r_2^2)}{2} = \pi\mu C (r_1^2 - r_2^2)$$

$$= \pi\mu \frac{W}{2\pi(r_1 - r_2)} \cdot (r_1^2 - r_2^2)$$

** $\boxed{T = \frac{1}{2} \mu W (r_1 + r_2)}$

$T = \mu W R$ where the value of $R = \frac{r_1 + r_2}{2}$.

MULTIPLE PLATE CLAUTCH :-

→ This is used when large torque is to be transmitted.
→ These are mostly used in motor cars, machine tools.

n_1 = No. of discs on the driving shaft
n_2 = No. of discs on the driven shaft

∴ No. of pairs of contact surfaces.

$$\boxed{n = n_1 + n_2 - 1}$$

Total frictional torque acting on the friction surface of the clutch $T = n\mu W R$

$\boxed{R = \frac{2}{3} \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}}$ ⟶ for uniform pressure

$\boxed{R = \frac{r_1 + r_2}{2}}$ ⟶ for uniform wear

Ex: Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4KN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.

sol: Given data :-

$W = 4\text{ KN} = 4 \times 10^3 \text{ N}$; $r_2 = 50\text{ mm}$ $r_1 = 100\text{ mm}$.

let Maximum pressure is P_{max}.

since the intensity of pressure is maximum at the inner radius (r_2)

therefore $P_{max} \times r_2 = C$ $\Rightarrow C = 50 \times P_{max}$

we know that total force on the contact surface (W)

$$4 \times 10^3 = 2\pi C (r_1 - r_2) = 2\pi \times 50 \, P_{max} (100-50)$$

$$\Rightarrow 4 \times 10^3 = 15710 \, P_{max}$$

$$\Rightarrow P_{max} = \frac{4 \times 10^3}{15710} = 0.2546 \ N/mm^2$$

let minimum pressure P_{min}

since the intensity of pressure is minimum at the outer radius r_1

$P_{min} \times r_1 = C$ $\Rightarrow C = P_{min} \times 100$

we know that the total force on the contact surface 'w'

$$4 \times 10^3 = 2\pi C (r_1 - r_2) = 2\pi \times 100 P_{min} (100-50)$$

$$\Rightarrow 4 \times 10^3 = 31420 \, P_{min}$$

$$\Rightarrow P_{min} = \frac{4 \times 10^3}{31420} = 0.1273 \ N/mm^2$$

Average pressure = $\dfrac{\text{Total normal force on contact surface}}{\text{cross-sectional area of contact surfaces}}$

$$= \frac{W}{\pi (r_1^2 - r_2^2)} = \frac{4 \times 10^3}{\pi (100^2 - 50^2)} = 0.17 \ N/mm^2$$

2.6 WORKING OF SIMPLE FRICTIONAL BRAKES:-

→ A brake is a device by means of which artificial frictional resistance is applied to a moving machine member in order to absorb (or) stop the motion of a machine.

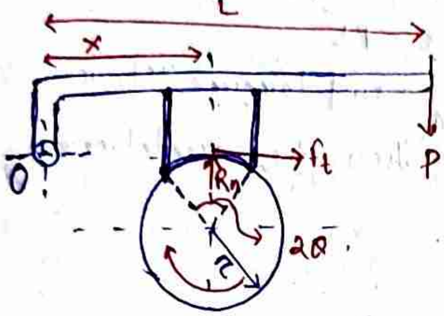
→ Types of brakes
(a) Hydraulic brake
(b) Electric brake
(c) Mechanical brake.

Single block (or) shoe brake : (Working principle)

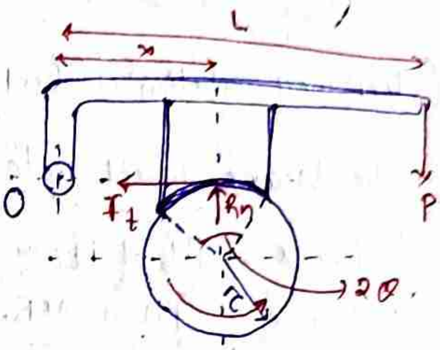
→ It consists of a block (or) shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel.

→ The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retarded the rotation of the wheel.

→ The block is pressed against the wheel by a force applied to one end of the lever to which the block is rigidly fixed. The other end of the lever is pivoted on a fixed fulcrum.



clock wise rotation of brake wheel.
(a)



Anticlock wise rotation of the brake wheel.
(b)

let P = force applied at the one end of the lever.

R_N = Normal force pressing the brake block on the wheel.

r = Radius of the wheel.

2θ = Angle of contact surface of the block.

μ = coefficient of friction

f_t = Tangential breaking force (or) the frictional force acting at the contact surface of the block and the wheel.

$\therefore f_t = \mu \times R_N$.

Torque (Resistive / frictional torque) = $f_t \times r = \mu R_N r$

Case-1 (fig(a))

when the line of action of tangential braking force (f_t) passes through the fulcrum 'O' of the lever, and the brake wheel rotates clockwise then for equilibrium, taking moments about the fulcrum 'O', we have

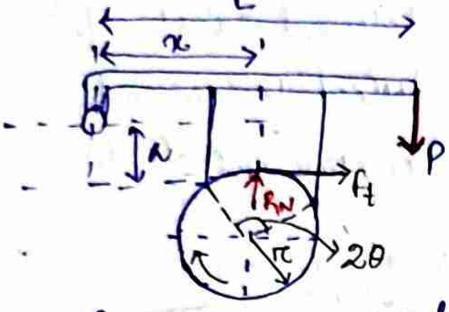
$$R_N \times x = P \times L$$

$$\Rightarrow R_N = \frac{P \times L}{x}$$

$\therefore$ Braking torque $T_B = \mu \times R_N \times r = \mu \times \frac{PL}{x} \times r = \frac{\mu P L r}{x}$.

** when the brake wheel rotates anticlockwise then the braking torque is same.

Case-2:-



(clockwise rotation of wheel)

→ when the line of action of the tangential braking force (f_t) passes through a distance 'a' below the fulcrum 'O', and the brake wheel rotates clockwise then for eqm taking moment about 'O'

$R_N \times x + f_t \times a = PL$

$\Rightarrow R_N = \frac{PL}{x + \mu a}$ and torque $= \mu R_N r$

when the brake wheel rotates anticlockwise, then the equilibrium

$R_N \cdot x = P.L + f_t \cdot a$

$\Rightarrow R_N x = P.L + \mu R_N a$

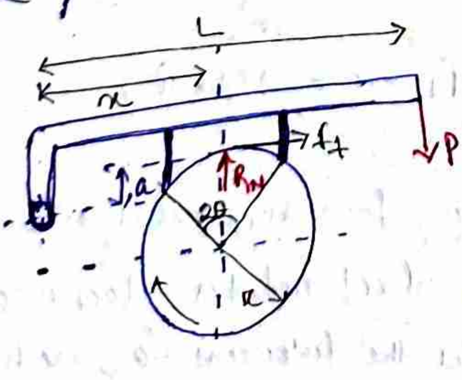
$\Rightarrow R_N (x - \mu a) = P.L$

$\Rightarrow R_N = \frac{P.L}{x - \mu a}$

$\therefore$ Braking Torque $\Rightarrow T_B = F_t \times r = \mu R_N r = \mu \times \frac{PL}{(x - \mu a)} r$

Case-3:-

when the line of action of the tangential braking force (f_t) passes through a distance 'a' above the fulcrum 'O', and the brake wheel rotates clockwise, then for equilibrium taking moment about 'O'



$R_N x = PL + f_t a$

$\Rightarrow R_N (x - \mu a) = PL$

$\Rightarrow R_N = \frac{PL}{x - \mu a}$

* Braking torque = $T_B = \mu R_N r \Rightarrow T_B = \frac{\mu PL}{x - \mu a} r$

Self energizing brake and self locking brake :

from the above equation $PL = R_{N1}(x - \mu a)$

→ if $x < \mu a$ then the value of 'P' will be negative i.e there will be a negative effort required to apply the brake.

→ This means we donot need ~~any~~ to apply any effort to brake the vehicle i.e with zero effort brake will be applied. This is called self locking brake.

→ If $x > \mu a$ then the value of 'P' will be positive i.e there will be need of effort to apply the brake. This is called self energizing brake.

[2.7] Working of absorption type of Dynamometer :

→ Dynamometers are used to measure the brake power of the engine. There are two types of dynamometer

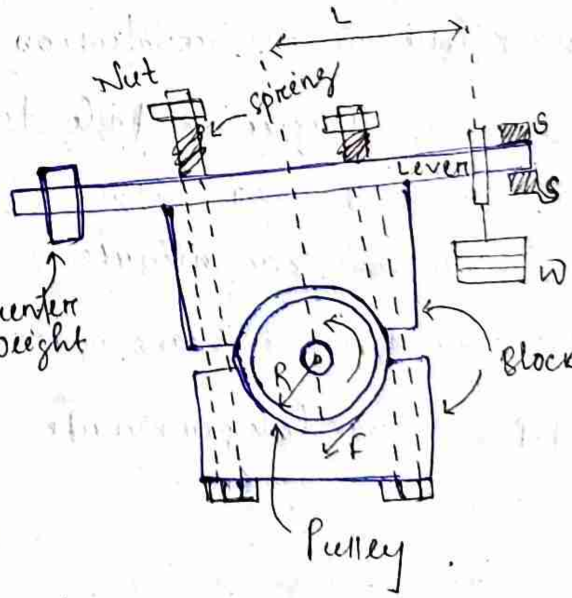
(i) Absorption type dynamometer (ii) Transmission type dynamometer

classification of absorption type dynamometer :

There are mainly two types of absorption type dynamometer

1. Prony brake dynamometer 2. Rope brake dynamometer.

1. Prony brake dynamometer :



→ It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured.

→ The blocks are clamped by means of two bolts and nuts. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed.

→ The upper block has a long lever attached to it and carries a weight 'W' at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded.

→ Two stops 's, s' are provided to limit the motion of the lever.

→ when the brake is to be put in operation, the long end of the lever is loaded with suitable weight 'w' and nuts are tightened until the engine shafts run at a constant speed and the lever is in horizontal position, the moment due to the weight 'w' must balance the moment of the frictional resistance between the blocks and the pulley.

Let W = weight at the outer end of the lever in newton.

L = Horizontal distance of the weight 'w' from the centre of the pulley in meters.

F = frictional resistance between the blocks and pulley in newtons.

R = Radius of the pulley in meter.

N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance (or) torque on the shaft

$T = W.L = F.R$ N.m.

work done in one revolution

= Torque × Angle turned in radians.

= $T \times 2\pi$ N.m.

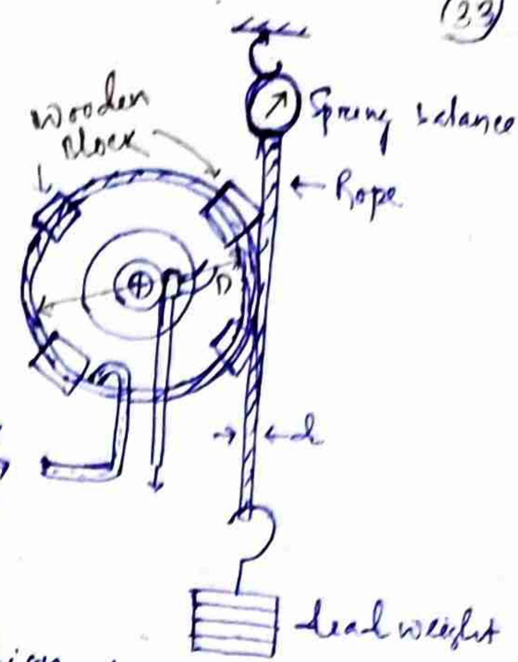
∴ Work done per minute = $T \times 2\pi \times N$

We know that the brake power of the engine

$$B.P = \frac{\text{Work done per minute}}{60} = \frac{T \times 2\pi N}{60} = \frac{WL \times 2\pi N}{60} \text{ watts.}$$

2. Rope brake dynamometer :-

→ It is commonly used for measuring the brake power of the engine.



→ It consists of one, two (or) more ropes wound around the flywheel (or) rim of a pulley fixed rigidly to shaft of an engine.

→ The upper end of the rope is attached to a spring balance while the lower end of the rope is kept in position by applying a dead weight

→ In order to slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

→ In the operation of the brake, the engine is made to run at constant speed. The frictional torque due to rope must be equal to the torque being transmitted by the engine.

Let W = dead weight in Newton

S = Spring balance reading in newton.

D = Diameter of the wheel in meter.

d = Diameter of the rope in meter.

N = Speed of the engine in r.p.m.

Net load on the brake = $(W-S)$ N

We know that distance moved in one revolution

$= \pi(D+d)$ meter

∴ Work done per revolution $= (W-S)\pi(D+d)$ N-m.

∴ work done per minute $= (W-S)\pi(D+d)N$ N-m.

∴ Brake power of the engine $= \dfrac{\text{Work done per min}}{60} = \dfrac{(W-S)\pi(D+d)N}{60}$

If the dia of rope is neglected than $B.P = \dfrac{(W-S)\pi DN}{60}$ watt chapter-1 SIMPLE MECHANISM

1.1 Link, kinematic chain, mechanism, machine
1.2 Inversion, four bar link mechanism, and its inversion.
1.3 Lower pair and higher pair.
1.4. Cam and followers.

1.1. Link :- Each part of a machine, which moves relative to some other part is known as kinematic link (or) link (or) element.

* A link (or) element need not be a rigid body, but it should be a resistant body.

* Links are of three types (a) Rigid link → Does not undergo deformation
(b) Flexible link → Partly deformed.
(c) Fluid link → motion through fluid by pressure/compression

Kinematic pair :-

→ The two links (or) elements of a machine, when in contact with each other are said to form a pair.

→ If the relative motion between them is completely (or) successfully constrained (i.e in a definite direction) the pair is known as kinematic pair.

Kinematic chain :-

When the kinematic pairs are coupled in such a way that the last link is joined to the 1st link to transmit definite motion (i.e completely (or) successfully constrained motion), it is called as kinematic chain.

* Ex :- The crankshaft of an engine forms a kinematic pair with the bearings which are fixed in a pair, the connecting rod with the crank forms a second kinematic pair, the piston with the connecting rods forms a third pair and the piston with the cylinder forms a forth pair. The total combination of these links is a kinematic chain.

Mechanism :-

→ when one of the links of the kinematic chain is fixed, the chain is known as mechanism.

→ It may be used for transforming (or) transmitting motion. i.e engine indicator, type writers etc.

Machine :-

→ It is a device which receives energy and transforms it into some useful work.

→ It is the assembly of various mechanism of the various parts.

1.3 Inversion :-

→ The method of obtaining different mechanisms by fixing different links in a kinematic chain is known as Inversion of the mechanism.

→ Every inversion may be known as mechanism.

→ No. of inversion of a chain is equal to no. of links.

Four bar link mechanism and its inversion :-

→ According to their movements links are classified as

(a) Frame (or) fixed link

(b) Crank → The link which can execute full circular motion.

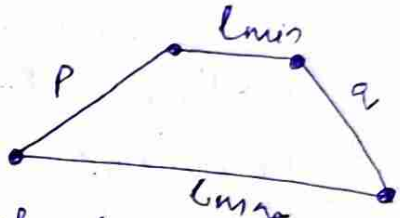
(c) Rocker → The link which oscillates (i.e which can not execute full circular motion)

(d) Coupler → The link which connects input and output and are opposite to the fixed link is called so.
* Its job is to transfer the motion.

I/P	O/P	Mechanism
(i) crank	crank	Double crank
(ii) Crank	Rocker	crank - Rocker
(iii) Rocker	Rocker	Double Rocker
(iv) Rocker	crank	Rocker - crank

* Condition for becoming a kinematic chain :-

$l_{max} < l_{min} + p + q$.

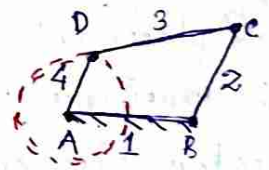


* Grashof's law :-

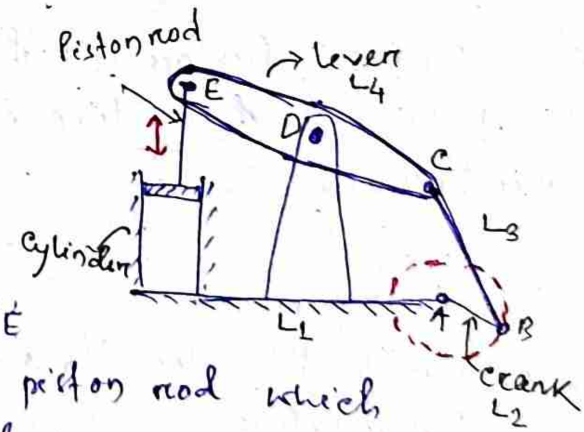
→ For a four bar mechanism "the sum of shortest and the longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be a continuous relative motion between the two links".

INVERSION OF FOUR BAR CHAIN :

(1) BEAM ENGINE (CRANK AND LEVER MECHANISM) :-



→ A part of the mechanism of a beam engine (also known as crank and lever mechanism) which ~~also~~ consists of four links.

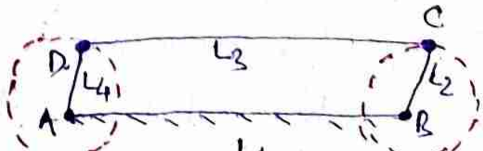


→ In this mechanism, when the crank rotates about the fixed centre 'A', the lever oscillates about the fixed centre 'D'. The end 'E' of the lever EDE is connected to a piston rod which reciprocates due to the rotation of the crank.

* Purpose is to convert rotary motion in to reciprocating motion.

(2) Coupling rod of locomotive (Double crank mechanism) :-

→ The mechanism of a coupling rod of a coupling rod is also known as double crank mechanism.

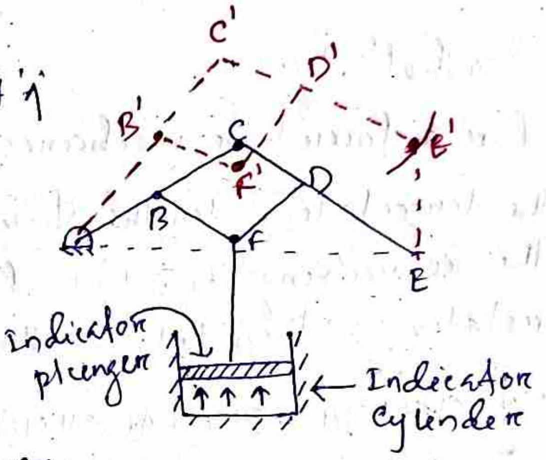


→ In this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link AB is fixed inorder to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

(3) Watts indicator mechanism (Double lever mechanism):-

→ A watt's indicator mechanism is also known as watt's straight line mechanism (or) double lever mechanism. which consists of four link.

→ The four links are: fixed link at A, link AC, link CE and link "BFD".



* It may be noted that BF and FD form one link as there is no relative movement between them.

* The link CE and BFD act as lever.

→ The displacement of the link BFD is directly proportional to the pressure of the gas (or) steam which acts on the indicator plunger.

→ On any small displacement of the mechanism, the tracing point 'E' at the end of the link CE traces out approximately a straight line

1.3: Lower pair:-

→ when the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair is called as lower pair.

→ Ex: sliding pair, turning pair, screw pair.

Higher pair:-

→ when two elements of a pair have a line (or) point contact when relative motion takes place then that is called higher pair.

→ Ex: Toothed gearing, belt and rope drive, ball and roller bearing cam and follower.

~~CAM AND FOLLOWERS~~

TYPES OF PAIR:

According to the relative motion between the elements pairs may be classified as

(a) Sliding pair:

→ when two elements of a pair are connected in such a way that one slide relative to other, the pair is known as sliding pair.

Ex. Piston - cylinder, ram and guides in shaper, tail stock on the lathe bed.

→ sliding pair has a completely constrained motion.

(b) Turning pair:

→ when two elements of a pair are connected in such a way that one can turn (or) revolve about a fixed axis of another link, the pair is known as turning pair.

→ A turning pair gives a completely constrained motion.

Ex: A shaft both ends fitted into a holes, the crankshaft in an journal bearing in an engine.

(c) Rolling pair:

→ when the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair.

Ex. Ball and roller bearing.

(d) Screw pair:

when the two elements of a pair are connected in such a way that one can turn about the other by screw threads is known as screw pair.

Ex. Leadscrew on lathe, Nut and bolt.

(ii) Spherical pair :-

when two elements of a pair are connected in such a way that one element (with spherical shape) turns (or) swivels about the other fixed element, the pair formed is called spherical pair.

Ex :- Ball and socket joint, attachment of a car mirror

1.4. Cam and follower :-

CAM :- It is a rotating machine element which gives reciprocating (or) oscillating motion to another element known as follower.

→ The cam and follower have a line contact and constitute a higher pair.

→ The cams are usually rotated at uniform speed by the shaft, but the follower motion is predetermined and will be according to the shape of the cam.

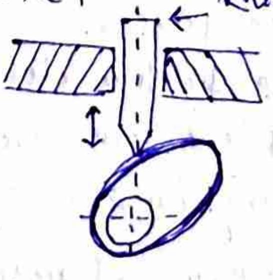
Classification of followers :-

1. According to surface in contact :-

(a) Knife edge follower :-

when the contacting end of the follower has a sharp knife edge, it is called as knife edge follower.

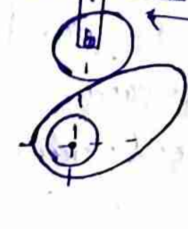
→ Sliding motion takes place between cam and follower.



(b) Roller follower :-

when the contacting end of the follower is a roller, it is called as roller follower.

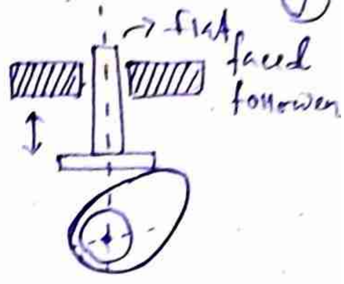
→ Rolling motion takes place between the contact surfaces.



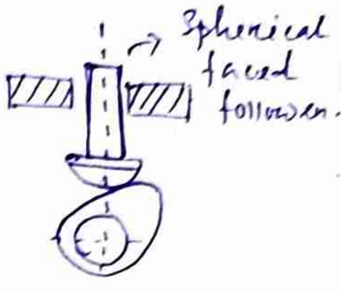
(c) Flat faced (or) mushroom follower :-

when the contact surface is completely flat face, it is called as flat face follower

*Note: when the flat faced follower is circular then it is called as mushroom follower.



(d) Spherical faced follower :-
when the contacting end of the follower is spherical shape it is called as spherical faced follower.



2. According to the motion of the follower

(a) Reciprocating (or) translating follower :-
when the follower reciprocates in guides as the cam rotates uniformly it is known as reciprocating (or) translating follower.

(b) Oscilating (or) Rotating follower :-
when the uniform rotary motion of the cam is converted in to predetermined ossillatory motion of the follower. It is called as oscillating (or) rotating follower.

3. According to the path of motion of the follower :-

(a) Radial follower :-
when the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower.

(b) off-set follower :-
when the motion of the follower is along an axis away from the axis of the cam centre, it is called off set follower.

Classification of cams :-

(a) Radial (OR) Disc cam :-
In radial cams, the follower reciprocates (or) oscilates ~~the follower~~ in a direction perpendicular to the cam axis.

(b) Cylindrical cam :-
In cylindrical cams, the follower reciprocates (or) oscillates in a direction parallel to the cam axis.