BHUBANANANDAODISHASCHOOLOFENGINEERING, CUTTACK

DEPARTMENT OF CIVIL ENGINEERING

LECTURENOTEON: HYDRAULICS AND IRRIGATION

(TH-1) 4TH SEMESTER

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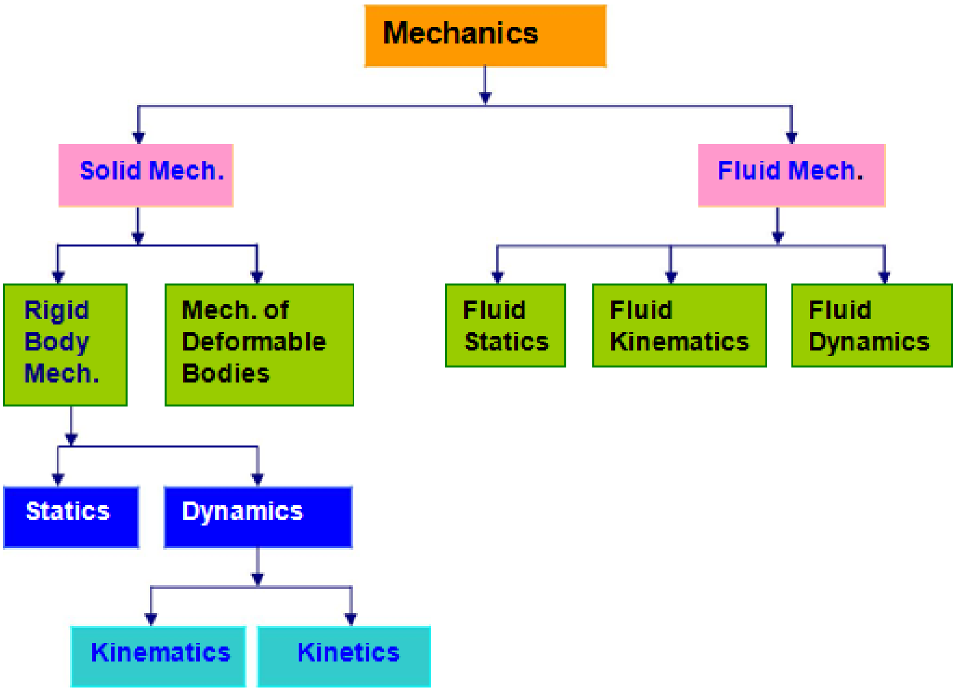
Introduction to Fluids Mechanics

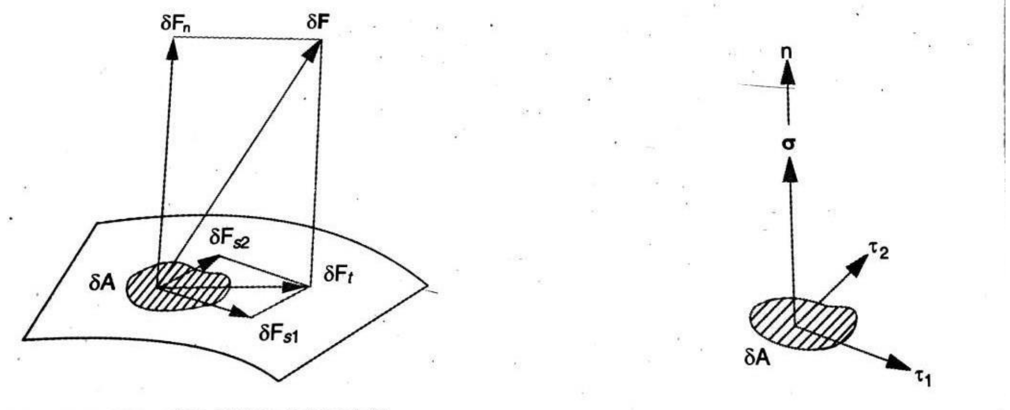


Fluid mechanics is the study of fluids either in motion (fluid *dynamics*) or at rest (fluid *statics*) and the interaction of fluids with solids or other fluids at the boundaries. Both gases and liquids are classified as fluids, and the number of fluid engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few. The study of fluid mechanics is categorized as:

- *Hydrodynamics*: It deals with the study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds).
- *Hydraulics*: It deals with the liquid lows in pipes and open channels.
- *Gasdynamics*: Itdealswiththeflowoffluidsthatundergosignificantdensity changes, such as the flow of gases through nozzles at high speeds.
- *Aerodynamics*:Itdealswiththeflowofgases(especiallyair)overbodiessuchas aircraft, rockets, and automobiles at high or low speeds.

Definitionofstress LetusconsiderasmallelementδAonthesurfaceofabodyuponwhich a force is acting asδF

Theforcecanberesolvedintotwocomponents,δF_nandδF_t.



(a) NORMAL AND SHEAR FORCES (b) NORMAL AND SHEAR STRESSES

δF_t=Normalforceactingonthearea

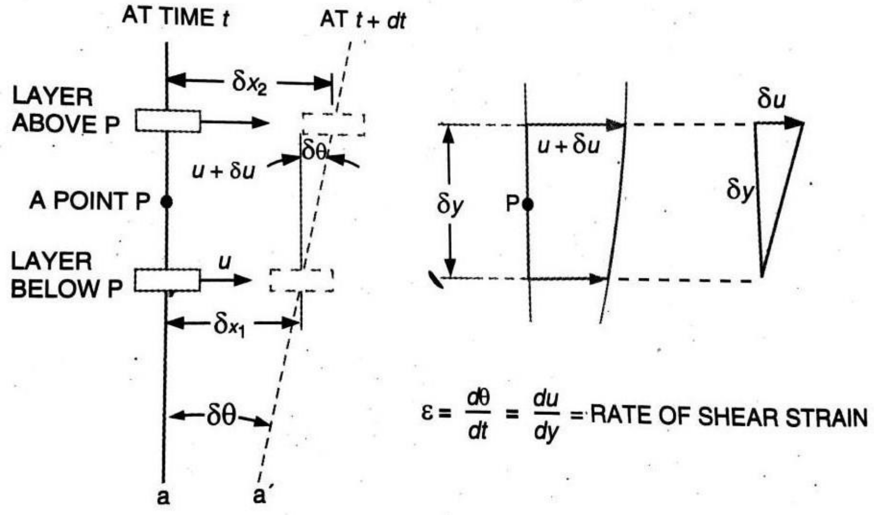
δF_t=horizontalforcetothearea,i.e.,(tangentialforceorshearforce)

Normalforce$\sigma=\lim_{\delta A\to 0}\left(\frac{\delta F_n}{\delta A}\right)$

Tangentialorsheerstress$\tau=\lim_{\delta A\to 0}\left(\frac{\delta F_t}{\delta A}\right)$

Rateof strain

Therateofstrain atappointinafluidreferstothe rateofshearingof the adjacentlayer atthat point This may be interpreted in terms of the rate of change of angle$\theta$of a line a-a joining the identified fluid elements in layer just above the just the point P. Consider the changes that takes place over an interval of timeδt.



Thiselementinthelayerbelow P,movingwithavelocityu,translatesadistanceδx_1tothe

Dottedpositionsuchthat$\delta x_1=u.\delta t$

The element above P moving with a velocity $(u+\delta u)$ translates a distance δx_2 to the dotted position such that $\delta x_2=(u+\delta u).\delta t$

The line element a-a therefore, shifts to a new position a'-a' such that the inclination $\delta\theta$

$$\delta\theta \cong \tan\delta\theta = \frac{\delta x_2 - \delta x_1}{\delta y} = \frac{(u+\delta u).\delta t - u.\delta t}{\delta y}$$

The rate of shear strain is given by $\varepsilon = \dfrac{d\theta}{dt} = \lim_{\delta t \to 0} \dfrac{\delta\theta}{\delta t} = \dfrac{du}{dt}$

Solids and Fluids:

- **Solid**: A solid can resist a shear stress by a static deflection; The amount of deflection is proportional to the magnitude of applied stress upto some limiting condition. When the applied shear force is constant, a solid stops deforming.

  The molecules of a solid are more closely packed. Hence the force of attraction between the molecules of solid is more larger than that of a fluid.

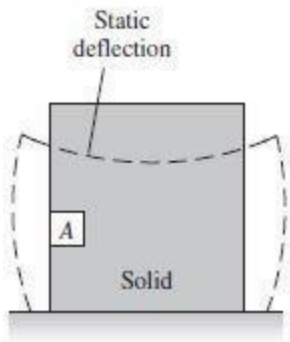


Fig.1 Deformation in a solid

- **Fluid**: Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied. When constant shear force is applied, the fluid continues to deform and stops at certain strain rate. In fluids the applied stress is proportional to rate of deformation. A fluid may be either liquid or gas.
  - ✓ **Liquid**: A liquid is composed of relatively close-packed molecules with strong cohesive forces which tends to retain its volume and will form a free surface in a gravitational field if unconfined from above.

    Examples: water, oil, mercury, gasoline, alcohol etc.
  - ✓ **Gas**: A gas, expands until it encounters the walls of the container and fills the entire available space. This is because the gas molecules are widely spaced, and the cohesive forces between them are very small. A gas has no definite volume, and when left to itself without confinement, it forms an atmosphere that is essentially hydrostatic.

    Examples: air, helium, hydrogen, steam etc.,

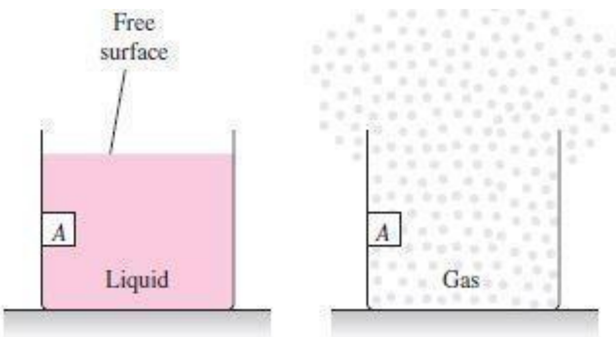


Fig.2Liquidand Gas

Solidscanresisttangentialstressunderstaticconditionswhereasfluidscandoitonly under dynamic conditions.

BasicFluidProperties:

Characteristicsofafluidwhichareindependentof motionarecalledbasicproperties.

1. **Density(ρ):**Densityof a fluidis its mass perunit volume.

   Units: kg/m^{3}in SIsystem

   The density of most gases is proportional to pressure and inversely proportional to temperature. Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible. Thus liquid flows are analytically treated as incompressible.

   TheheaviestcommonliquidisMercurywithdensity, $\rho=13{,}580\,{}^{kg}\!/_{m^3}$ andthe lightestgasinHydrogen withdensity, $\rho=0.0838\,{}^{kg}\!/_{m^3}$.

2. **Specificweight** (ω): Itistheweightofthefluid tothevolumeoccupiedwithit.

   Specificweight,$\omega=\rho\times g=\frac{m\times g}{v}\,{}^{N}\!/_{m^3}$

3. **Specificvolume(V):**Itisthe volume occupiedby the unitmassof fluidandisthe reciprocal of density.

   *Units***:** m^3/kginSIsystem

4. **Specificgravity(G):** Itistheratioofdensityoffluidtothedensityofstandardfluid. It is also referred as relative density.

   Ingeneral,wateristhe standardfluidforliquids whileairisforgases.

   Forliquids,$G=\frac{\rho_{liquid}}{\rho_{water}}$ Forgases,$G=\frac{\rho_{gas}}{\rho_{air}}$

5. **Staterelationsforliquids:**

Liquidsarenearlyincompressibleandhaveasingle,reasonablyconstantspecific heat. Thus an idealized state relation for a liquid is

$$\rho \approx \text{constant} \qquad c_p \approx c_v \approx \text{constant} \qquad dh \approx c_p dT$$

The density of a liquid usually decreases slightly with temperature and increases moderatelywithpressure.Ifweneglectthetemperatureeffect,anempiricalpressure– density relation for a liquid is given as

$$\frac{p}{p_a} \approx (B+1)\left(\frac{\rho}{\rho_a}\right)^n - B$$

where B and n are dimensionless parameters that vary slightly with temperature andp_a and ρ_a are standard atmospheric values. Water can be fitted approximately to the values B=3000 and n=7.

Viscosity:It is a quantitative measure of a fluid's resistance to flow. It determines the strain rate in a fluid that is generated by a given applied shear stress. This property is due to the cohesive forces of attraction between the fluid molecules.

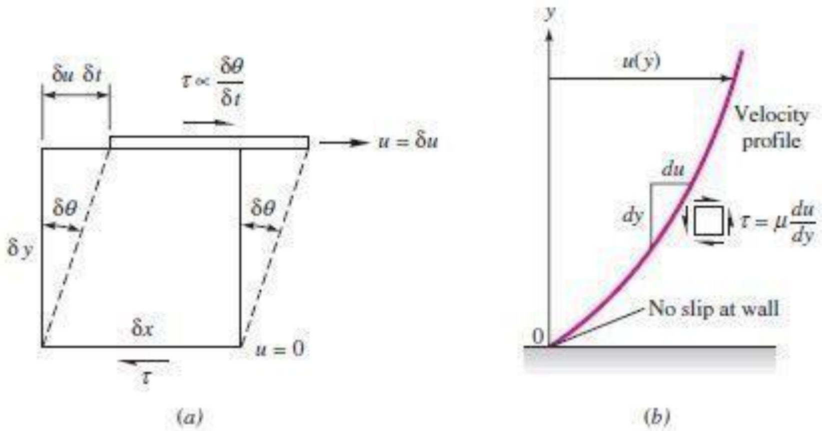


Fig.3(a)Fluid elementstraining at rate $\frac{d\theta}{dt}$

Fig.3(b)Newtonianshear distributionina shear layernear a wall.

Consider a fluid element sheared in one plane by a single shear stress τ, as in fig. 3(a). The shearstrainangleδθwillcontinuouslygrowwithtimeaslongasthestress τismaintained, theuppersurfacemovingatspeedδ$u$largerthanthelower.Suchcommon fluidsaswater, oil,andairshowalinear relationbetweenappliedshearandresultingstrainrate:

$$\tau \alpha \frac{\delta\theta}{\delta t}$$

Fromfig.3(a),wecanwrite,$\tan\delta\theta = \frac{\delta u \delta t}{\delta y}$

Inthelimitofinfinitesimalchanges,thisbecomesarelationbetweenshearstrainrateand velocity gradient:

$$\frac{\delta\theta}{\delta t}=\frac{\delta u}{\delta y}$$

Then,theappliedshearisalsoproportionaltothevelocitygradientforthecommonlinear fluids. The constant of proportionality is the viscosity coefficient μ:

$$\tau=\mu\frac{d\theta}{dt}=\mu\frac{du}{dy}$$

μ-absoluteviscosityor dynamicviscosity.

UnitforAbsoluteviscosity:N-sec/m2orPa-sec

Common unit is poise. 1 poise = 0.1 pa-sec.

From the fig. 3(b), it has to be observed that at the solid boundary, i.e. y=0, the fluid velocity is zero indicating the no-slip condition where the velocity gradient is zero. With increase in the distance from the solid boundary, the velocity increases gradually i.e. velocity gradient increases.

*Kinematicviscosity:*It istheratio of dynamicviscositytothedensityof the fluid.

$$\gamma=\frac{\mu}{\rho}$$

Units:Itisexpressedinm^2/secorcm^2/sec. 1 cm^2/sec = 1 stoke

Parameter	**Liquids**	**Gases**
Pressure	For liquids, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usuallydisregarded,exceptat extremelyhigh pressures.	For gases, dynamic viscosity is independent of pressure (at low to moderate pressures), but not for kinematicviscositysincethedensityof a gas is proportional to its pressure.
Temperature	The viscosity of liquids decreases with temperature. This is because in a liquid the molecules possess more energy at higher temperatures, and theycanopposethelargecohesive intermolecularforcesmorestrongly.	the viscosity of gases increases with temperature. This is because, in a gas the intermolecular forces are negligible, making gas molecules movingrandomlyathighervelocities athightemperatures.Thisresultsin

	Asaresult,theenergizedliquid molecules can move more freely.	moremolecularcollisionsperunit volume per unittime andtherefore in greater resistance to flow.
Governing equations	Viscosityisgiven as: $\ln\left(\frac{\mu}{\mu_0}\right)\approx a+b\left(\frac{T_0}{T}\right)+c\left(\frac{T_0}{T}\right)^2$	Viscosityisgiven as: $\frac{\mu}{\mu_0}\approx\begin{cases}\left(\frac{T}{T_0}\right)^2 \text{...... } \mathit{Powerlaw} \\ \left(\frac{T}{T_0}\right)^{3/2}\frac{(T_0+s)}{(T+s)} \text{..} \mathit{Sutherlandlaw}\end{cases}$

- ***Newtonianfluids***:Thesearethefluidsforwhichtherateofdeformationis proportional to the shear stress.
  Examples:Air,water,kerosene
- ***Non-Newtonianfluids***:Thesearethefluidsforwhichtherateofdeformationisnot directlyproportionaltotheshearstressi.e.$\tau=k\left(\frac{du}{dy}\right)^n$,wherek-consistencyindex, n-flowbehavior index. This equationcan be rewritten as $\tau=k\left(\frac{du}{dy}\right)^n=k\left(\frac{du}{dy}\right)^{n-1}\left(\frac{du}{dy}\right)=\eta\left(\frac{du}{dy}\right)$,whereapparentviscosity,$\eta=k\left(\frac{du}{dy}\right)^{n-1}$.

  Non-Newtonianfluidsareclassified astimeindependent andtimedependent fluids.

  Timeindependentfluids

  1. *Pseudoplastic fluids:* Fluids in which the apparent viscosity decreases with increasing deformation rate (n<1).
     Examples: Polymer solutions, colloidal suspensions, paper pulp in water, gelatine, blood, milk
  2. *Dilatant fluids*: Fluids in which the apparent viscosity increases with increasing deformation rate (n>1).
     Examples:Suspensionsofstarch,sugarinstarch

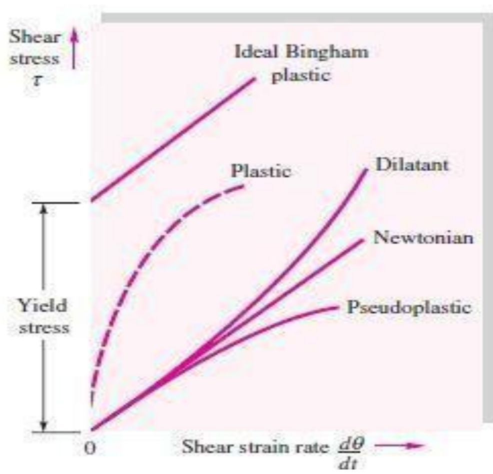


Fig.4StressvsStrainrate

  3. *Binghamplastic*:Itisafluidwhichbehaves asasoliduntilminimumyieldstress

(τ_y)isexceededandsubsequentlyexhibitsalinearrelationshipbetweenstress

andrateofdeformation.Therelationshipisgivenas$\tau=\tau_y+\mu_p\left(\frac{du}{dy}\right)$

Examples:Claysuspensions,drillingmuds

Timedependentfluids

1. *Thixotropic fluids*: Fluids whose apparent viscosity decreases with time under a constant applied stress.
   Examples:Most of the paints
2. *Rheopectic fluids*: Fluids whose apparent viscosity increases with time under a constant applied stress.
3. *Viscoelastic fluids*: These fluids partially returntotheiroriginalshapeafterdeformation, whentheappliedstressisreleased.

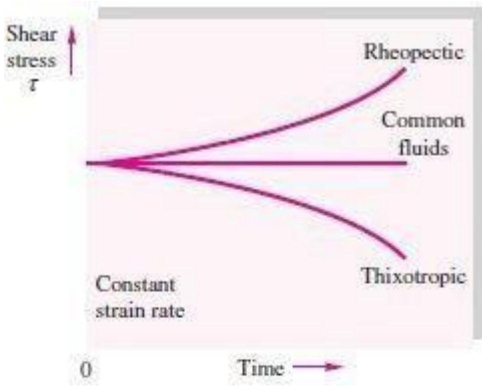


Fig.5Shearstressvstime

SurfaceTension

Molecules deep within the liquid repel each other because of their close packing. Molecules at the surface are less dense and attract each other. Since half of the neighboring molecules are missing, the mechanical effect is that the surface is in tension. This tensileforce actsparallel tothesurface andtowardsthe interioroftheliquid.Themagnitudeofthisforceper unit length is called surface *tension*, σ_s.

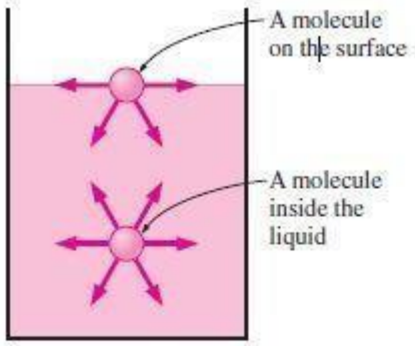


Fig.6Attractiveforcesinliquid

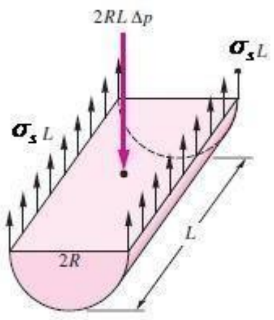
Units: In SI system unit is N/m.

This force (surface tension) is balanced by the repulsive forces from the molecules below the surface that are being compressed. The resulting compression effect causes the liquidtominimizeitssurfacearea.Thisisthereasonforthetendencyoftheliquiddropletsto attain a spherical shape, which has the minimum surface area for a given volume.

This effect is also called ***surface energy*** expressed in J/m^2, which represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

Applications:Surfacetensiondeterminesthesizeoftheliquiddropletsthat form.

1. *Pressureinsidea cylindrical jet:*



Consider a jet of water exiting as a cylinder with radius 'R' and length 'L'. When one half of the cylinder is considered, as shown in the fig. 7, two forces can be observed. One force is surface tensionactingalongthecircumferenceand anotherduetotheinternalpressure(Δpin excess of atmosphere pressure).

Fig.7Sectionofliquidcylinder

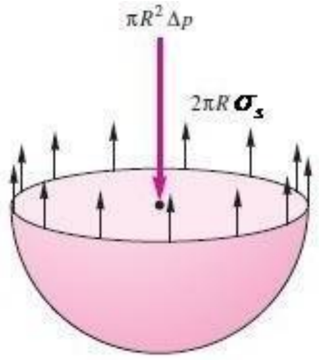
Surfacetension forcealongthe lengthof thecylinder, $F_t = \sigma_S(L+L) = 2L\sigma_S$

Forceduetointernal pressure, $F_p = \Delta p(L\times 2R) = 2LR(\Delta p)$

Forequilibrium,both forcesshould besamei.e. $F_p = F_t \Rightarrow 2LR(\Delta p) = 2L\sigma_S$

Excesspressureinsideacylindricaljet, $\Delta p = \dfrac{\sigma_S}{R}$

2. *Pressureinsidea spherical droplet:*



Consider a spherical droplet of diameter 'R' subject to internal pressure (Δpin excess of atmosphere pressure) balanced by surface tension acting along the circumference.

Surfacetensionforcealongthecircumference, $F_t = \sigma_S(2\pi R)$

Forceduetointernal pressure, $F_p = \Delta p(\pi R^2)$

Fig.8Sectionofsphericaldroplet

For equilibrium, both forces should be same i.e. $F_p = F_t \Rightarrow \Delta p(\pi R^2) = (2\pi R)\sigma_S$

Excess pressureinside asphericaldroplet,$\Delta p = \dfrac{2\sigma_S}{R}$

3. *Pressureinsidea soapbubble:*

In the case of a soap bubble, it has twointerfaces with air; one at inner surface and another at outer surface. Hence the excess pressure is given as

$$(\Delta p)_{soap\,bble} \approx 2(\Delta p)_{spherical\,bub\,droplet} = \frac{4\sigma_S}{R}$$

BulkModulusandCompressibility

Bulk modulus, K is defined as the ratio of the change in pressure to the rate of change of volume due to the change in pressure or the ratio of change in pressure to the volumetric strain.

$$K=\frac{-dp}{\left(\frac{dv}{v}\right)}$$

wheredpisthechangeinpressurecausingachangeinvolumedvwhentheoriginalvolume was v. The unit is the same as that of pressure.

Also,wecanexpressvolumetricstrainintermsofrelativechangeindensity,dv/v=– dρ/ρ.

Bulkmodulus can also beexpressed in termsofchangeof densityas$K=\frac{-dp}{\left(\frac{\Delta v}{v}\right)}=\frac{dp}{\left(\frac{d\rho}{\rho}\right)}$.

Thenegative signindicatesthatifthepressureisincreases,thenthevolumeofthesubstance decreases.

This definition can be applied to liquids as such, without any modifications. In the case of gases, the value of compressibility will depend on the process law for the change of volume and will be different for different processes.

The bulk modulus for liquids depends on both pressure and temperature. The value increases with pressure as dv will be lower at higher pressures for the same value of dp. With temperature the bulk modulus of liquids generally increases, reaches a maximum and then decreases. For water the maximum is at about 50°C. The value is in the range of 2000MN/m^2.

Pressure

Pressure is a measure of force distribution over any surface associated with the force. Pressure is a surface phenomenon and it can be physicallyvisualized or calculated onlyif the surfaceoverwhichitacts isspecified.***Pressuremaybedefinedas theforceacting along the normal direction on unit area of the surface.***

$$P=\frac{dF}{dA}$$

TheforcedFin thenormal directionon the areadA dueto thepressureP isdF=P dA.

TheunitofpressureintheSIsystemisN/m^2,calledasPascal (Pa).Asthe magnitudeissmall kN/m^2(kPa)andMN/m^2(Mpa)aremorepopularlyused.Theatmosphericpressureis approximately 10^5 N/m^2 and is designated as ‘‘bar’’. This is also a popular unit of pressure.In the metric system the popular unit of pressure is kg-f/cm^2. This is approximately equal to the atmospheric pressure or 1 bar.

Pascal'sLaw

A fluid element when isolated from the surroundings experiences two types of external forces: (a) Body force and (b) Surface force

Body force: These forces act throughout the body of the fluid element and are distributedover the entire mass or the volume of the element.

Ex:Gravitationalforce,Magneticforceetc.,

Surface force: It includes all the forces exerted on the fluid element by its surroundings through direct contact at the surface.

Ex:Normalforceandshear force

Forafluid element atrest,shearforceiszero(shearstress =0)and henceonlynormal stresses exist.

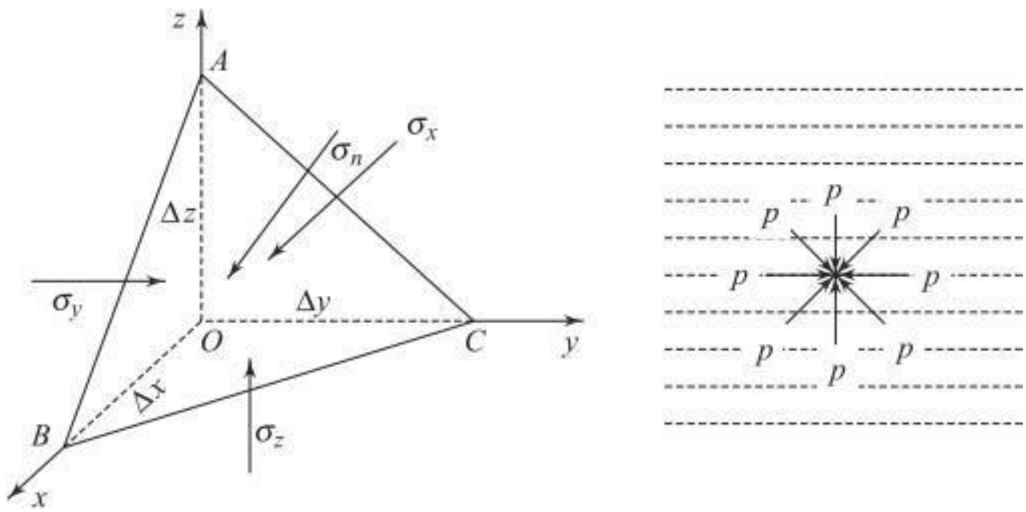


Fig.9Stateofstressona fluidelementatrest,stateofnormalstresson afluidelementatrest

Considerastationaryfluidelementoftetrahedronalshapewiththreeofitsfaces coincidingwiththecoordinateplanesasshowninthefigure.Hencethenatureofnormal stress acting on the fluid element will be of compressive in nature. Considering gravity as the onlyexternalbodyforce,theequationsofstaticequilibriumforthetetrahedronalfluid elementarewrittenas:

$$\Sigma F_x = \sigma_x \left(\frac{\Delta y\, \Delta z}{2}\right) - \sigma_n\, \Delta A \cos\alpha = 0$$

$$\Sigma F_y = \sigma_y \left(\frac{\Delta x\, \Delta z}{2}\right) - \sigma_n\, \Delta A \cos\beta = 0$$

$$\Sigma F_z = \sigma_z \left(\frac{\Delta x\, \Delta y}{2}\right) - \sigma_n\, \Delta A \cos\gamma - \frac{\rho g}{6}(\Delta x\, \Delta y\, \Delta z) = 0$$

Where $\sum F_X, \sum F_Y$ and $\sum F_Z$ arethenetforcesactingonthefluidelementinpositiveX,Y and Z directions respectively, cosα, cosβ and cosγ are the direction cosines of the normal to the plane of area ΔA.

Therefore,

$\Delta A \cos\alpha = (\Delta y.\Delta z)/2$

$\Delta A \cos\beta = (\Delta x.\Delta z)/2$

$\Delta A \cos\gamma = (\Delta x.\Delta y)/2$.

Substitutingtheseequations inforceequilibriumequations gives,

$\sigma_x = \sigma_y = \sigma_z = \sigma$

This equation concludes that the normal stress at any point in a fluid at rest are directed towards the point from all directions and are of equal magnitude. These stresses are definedas hydrostatic pressure or thermodynamics pressure. This is known as Pascal's law of hydrostatics. With convention of the positive sign for the tensile stress, the above statement can be analytically expressed as

$\sigma_x = \sigma_y = \sigma_z = -P$

PressureMeasurement

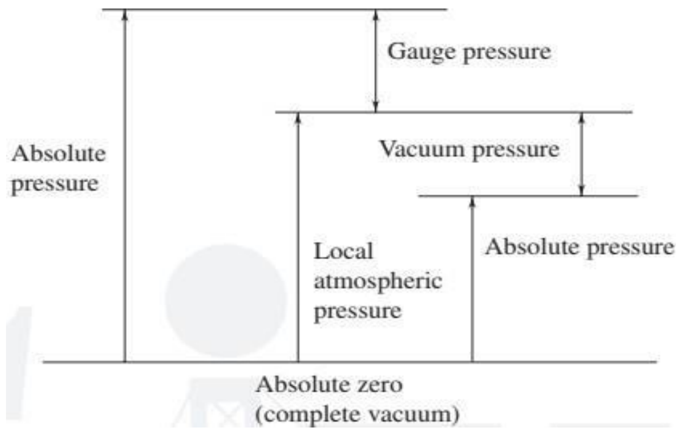


Fig.ScaleofPressure

- Pressure is usually expressed with reference to either absolute zero pressure (complete vacuum) or local atmospheric pressure.

- The absolute pressure isthe pressure expressedasa difference betweenitsvalue and the absolute zero pressure.
- Whenthepressureisexpressedasadifferencebetweenitsvalueandthelocal atmospheric pressure, it is known as gauge pressure.

$$P_{gauge}=P-P_{atm}$$

- IfthepressurePislessthanthelocalatmosphericpressure,thegaugepressurebecomes negative and is called vacuum pressure.
- At sea level, the international standard atmospheric pressure is given as 101.325kN/m^2.

Pressure is generally measured using a sensing element which is exposed on one side to thepressuretobemeasuredandontheothersidetothesurroundingatmosphericpressureor other reference pressure.

Piezometer

It is used to measure the gauge pressure i.e. system pressure greater than the local atmospheric pressure. It contains a vertical tube with an opening at the top to atmosphere. Thus when connected to a pipe in which some fluid is flowing, there occurs a rise in the liquid level in the vertical tubecalled as pressure head.

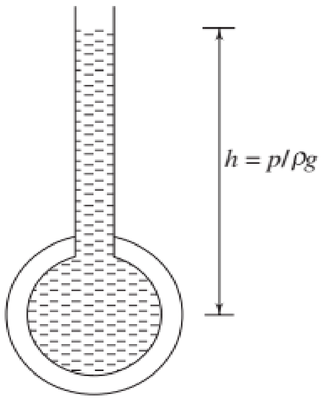


Fig.Piezometer

FromtheHydrostaticlaw, thePressureat apoint'A'is

P_A=External pressure +Pressuredueto rise of liquid level in thetube

$P_A = Patm + \rho gh$

Gaugepressure $= P_A - Patm = \rho gh$

Barometer

It is used to measure the atmospheric pressure. Mercury is used as the working fluid because of its very small vapour pressure at normal temperature and its high density for a relatively short column to be obtained.

When an inverted tube is dipped into a vessel containing fluid like mercury, there occurs a rise in level inthetubeabovethesurfacelevelinthevessel,asshown inthefigure.

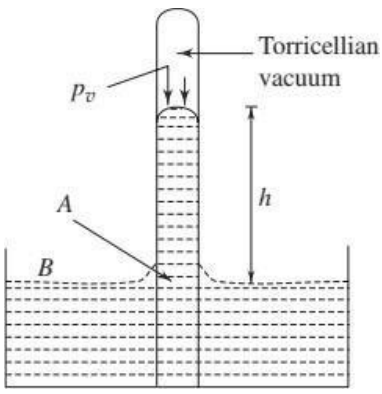


Fig.Barometer

At the level above the mercury in the tube, vacuum is created thus forming mercury vapourhaving pressure equal to the vapour pressure of mercury at its existing temperature. Fromthefigure,wecanobservethatpointsAandBlieonthesamehorizontalplane. Therefore we can write, $P_B = P_{atm} = Pv + \rho gh$.

Thevapourpressurewillbesmallincomparisontoatmosphericpressureandhenceitis neglected. Hence $P_B = P_{atm} = \rho gh$.

BourdonTube PressureGauge:

In the Borden gauge a tube of elliptical section bent into circular shape is exposed on the inside to the pressure to be measured and on the outside to atmospheric pressure. The tube will tend to straighten under pressure. The end of the tube will move due to this action and will actuate through linkages the indicating pointer in proportion to the pressure. Vacuum also can be measured by such a gauge. Under vacuum the tube will tend to bend further inwards and as in the case of pressure, will actuate the pointertoindicatethevacuumpressure.Thescaleisobtainedbycalibrationwithknownpressure source.

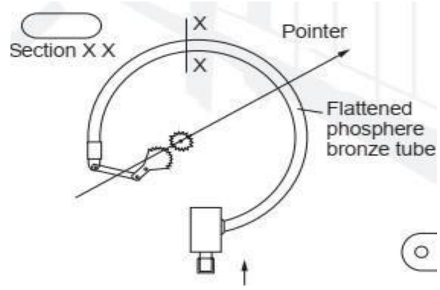


Fig. 9 Bourdon Tube Pressure Gauge

Manometers: Manometer is a device to measure pressure or mostly difference in pressure using a column of liquid to balance the pressure. It is a basic instrument and is used extensively in flow measurement. It needs no calibration. Very low pressures can be measured using micromanometers.

Thebasicprincipleofoperationofmanometersisthatatthesamelevelin continuousfluidatrest,thepressureis thesame. Thepressuredueto aconstantdensityliquid (ρ) column if height h is equal to $\rho\, gh$.

SimpleUtube Manometer

It is a transparent 'U' tube as shown in figure. One of its ends is connected to a pipe or a container having fluid (A) whose pressure is to be measured while the other end is open to atmosphere. The lower part of the tube contains a liquid immiscible with the fluid A and is of greater than that of A. This fluid is called manometric fluid. The pressureattwopointsPandQinahorizontalplanewithin thecontinuousexpanseofthesamefluidmustbeequal.

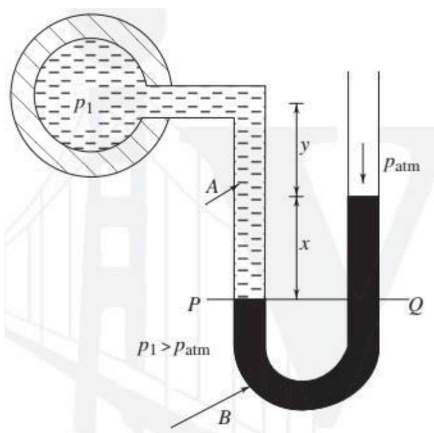


Fig.10Manometertomeasuregauge pressure

PressureatthepointP=PressureatthepointQ

$$P_1+\rho_A g(y+x)=P_{atm}+\rho_B gx$$

$$P_1-P_{atm}=gx(\rho_B-\rho_A)-\rho_A gy$$

When the pressure of the fluid in the container is lower than the atmospheric pressure, the liquid levels in the manometer would be adjusted as shown in the figure. Hence it becomes

PressureatthepointP = Pressureat thepoint Q

$$P_1+\rho_A gy+\rho_B gx=P_{atm}$$
$$P_{atm}-P_1=-\rho_A gy-\rho_B gx$$

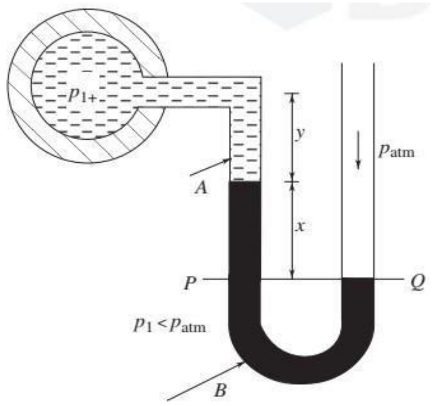


Fig.11Manometermeasuring vacuum

DifferentialUtubemanometer

Amanometerisfrequentlyusedtomeasurethepressuredifference,incourseofflow,across a restriction in a horizontal pipe as shown in figure.

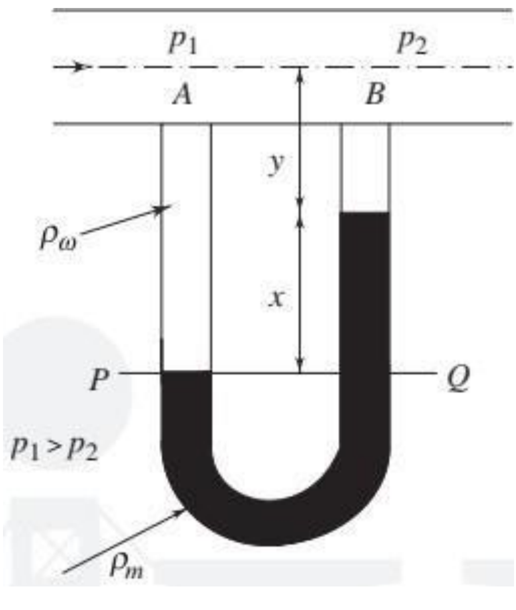


Fig.12Manometermeasuringpressure differential

It is important that the axis of each connecting tube at A and B to be perpendicular to the direction of the flow and also for the edges of the connection to be smooth. From the principle of hydrostatics, Pressure at the point P = Pressure at the point Q

$$P_1+\rho_w g(y+x)=P_2+\rho_w gy+\rho_m gx$$

$$P_1-P_2=gx(\rho_m-\rho_w)$$

Whereρm-densityofmanometricfluidandρwis thedensityofworkingfluid. Other configurations of manometers are:

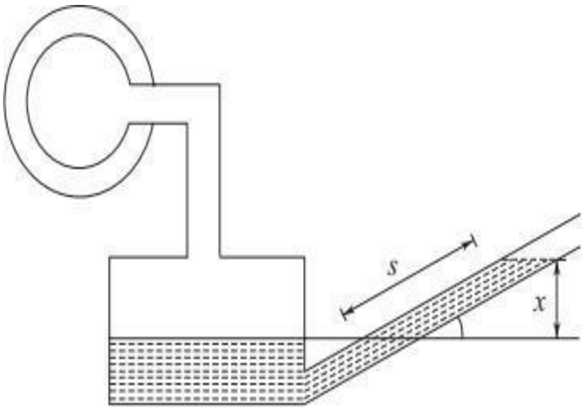


Fig.13Inclinedtube manometer

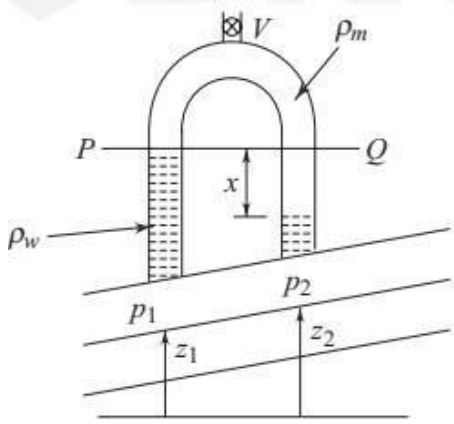


Fig.14Invertedtube manometer

FLUIDMECHANICS

UNIT-II

FLUIDKINEMATICS

Introduction

Fluid Kinematics gives the geometry of fluid motion. It is a branch of fluid mechanics, which describes the fluid motion, and it's consequences without consideration of the nature of forces causing the motion. Fluid kinematics is the studyof velocity as a function of space and time in the flow field. From velocity, pressure variations and hence, forces acting on the fluid can be determined.

Velocityfield

Velocity at a given point is defined as the instantaneous velocity of the fluid particle, which at a given instant is passing through the point. It is represented by V=V(x,y,z,t). Vectorially, V=ui+vj+wk where u,v,w are three scalar components of velocity in x,y and z directions and (t) is the time. Velocity is a vector quantity and velocity field is a vector field.

Velocityandacceleration

Let V= Resultant velocity at any point in a fluid flow. (u,v,w) are the velocity componentsinx,yandz directionswhicharefunctionsofspacecoordinatesand time.

u=u(x,y,z,t);v=v(x,y,z,t);w=w(x,y,z,t).

Resultant velocity=V=ui+vj+wk

$|V|=(u^2+v^2+w^2)^{1/2}$

Leta$_x$,a$_y$,a$_z$arethetotalaccelerationsinthex,y,zdirectionsrespectivelya$_{x=}$ [du/ dt]= $[\partial u/ \partial x]\ [\partial x/ \partial t]+ [\partial u/ \partial y][\partial y/ \partial t]+ [\partial u/ \partial z][\partial z/ \partial t]+ [\partial u/ \partial t]$

$a_{x=}[du/dt]=u[\partial u/\partial x]+v[\partial u/\partial y]+w[\partial u/\partial z]+[\partial u/\partial t]$

Similarly,

$a_{y=}\ [dv/ dt] = u[\partial v/ \partial x]+ v[\partial v/ \partial y]+w[\partial v/ \partial z]+[\partial v/\partial t]$

$a_{z=}[dw/dt]=u[\partial w/\partial x]+v[\partial w/\partial y]+w[\partial w/\partial z]+[\partial w/\partial t]$

1. ConvectiveAccelerationTerms–Thefirstthreetermsintheexpressionsfora$_x$,a$_y$, a$_{z.}$

Convective acceleration is defined as the rate of change of velocity due to change of position of the fluid particles in a flow field

2. Local Acceleration Terms- The 4th term, $[\partial\,(\ )/\,\partial t]$ in the expressions for a_x, a_y, a_z. Local or temporal acceleration is the rate of change of velocitywith respect to time at a given point in a flow field.

MaterialorSubstantialAcceleration=ConvectiveAcceleration+LocalorTemporal Acceleration

Inasteadyflow,temporalorlocalaccelerationiszero.In uniform flow, convective acceleration is zero.

Forsteadyflow,$[\partial u/\partial t]=[\partial v/\partial t]=[\partial w/\partial t]=0$ $a_x=$ $[du/dt]= u[\partial u/\partial x]+v[\partial u/\partial y]+w[\partial u/\partial z]$

$a_y=$ $[dv/dt]= u[\partial v/\partial x]+v[\partial v/\partial y]+w[\partial v/\partial z]$

$a_z=[dw/dt]=[\partial w/\partial x]+v[\partial w/\partial y]+w[\partial w/\partial z]$

AccelerationVector=$a_x i+a_y j+a_z k$;$|A|=[a_x^2+a_y^2+a_z^2]^{1/2}$

Methodofdescribingfluidmotion

Twomethodsofdescribingthefluidmotionare:(a)Lagrangianmethodand (b)Eularianmethod.

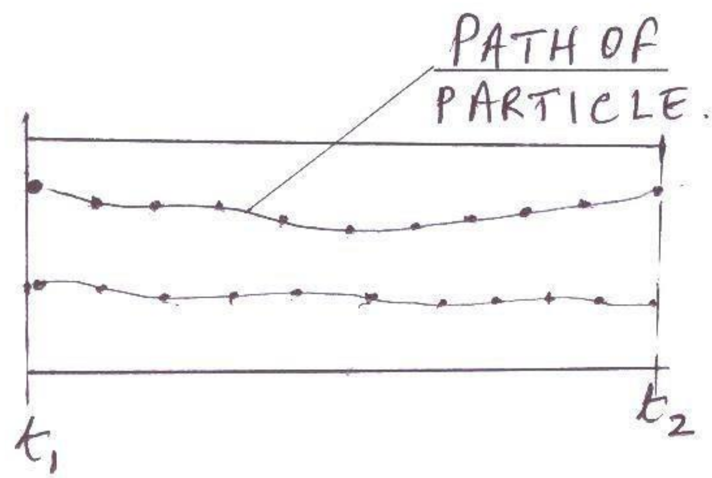


Fig.Lagrangianmethod

A single fluid particle is followed during its motion and its velocity, acceleration etc. are described with respect to time. Fluid motion is described by tracing the kinematics behavior of each and every individual particle constituting the flow. We follow individual fluid particle as it moves through the flow. The particle is identifiedbyitspositionatsomeinstantandthetimeelapsedsincethatinstant.Weidentifyand follow small, fixed masses of fluid. To describe the fluid flow where there is a relative motion, we need to follow many particles and to resolve details of the flow; we need a large number of particles. Therefore, Langrangian method is very difficult and not widely used in Fluid Mechanics.

Eularianmethod

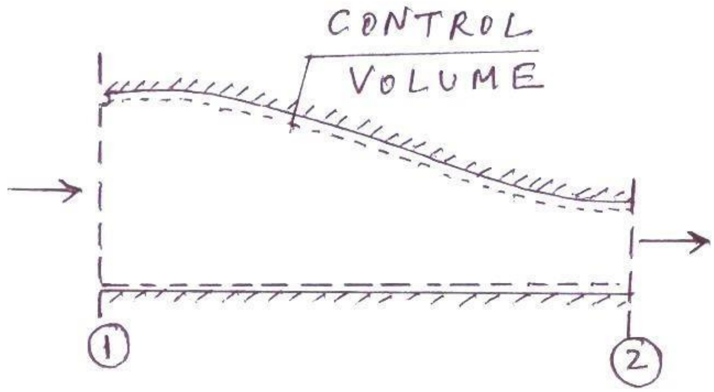


Fig.EulerianMethod

The velocity, acceleration, pressure etc. are described at a point or at a section as a function of time. This method commonly used in Fluid Mechanics. We look for field description, for Ex.; seek the velocity and its variation with time at each and every location in a flow field. Ex., V=V(x,y,z,t). This is also called control volume approach. We draw an imaginary box around a fluid system. The box can be large or small, and it can be stationary or in motion.

Typesoffluid flow

1. SteadyandUn-steadyflows
2. UniformandNon-uniformflows
3. LaminarandTurbulent flows
4. CompressibleandIncompressibleflows
5. RotationalandIrrotationalflows
6. One,Twoand Threedimensional flows

1. Steadyandunsteadyflow

Steady flow is the type of flow in which the various flow parameters and fluid properties atanypointdo notchange withtime. In asteadyflow, anypropertymayvary from point to point in the field, but all properties remain constant with time at every point.$[\partial V/\partial t]_{x,y,z} = 0$; $[\partial p/\partial t]_{x,y,z} = 0$. Ex.: V=V(x,y,z); p=p(x,y,z) . Time is a criterion.

Unsteadyflowisthetypeofflowinwhichthevariousflowparametersandfluid properties at any point change with time. $[\partial V/\partial t]_{x,y,z} \neq 0$; $[\partial p/\partial t]_{x,y,z} \neq 0$, Eg.:V=V(x,y,z,t), p=p(x,y,z,t) or V=V(t), p=p(t) . Time is a criterion

2. <u>Uniformandnon-uniformflows</u>

UniformFlow is thetypeofflowin which velocityand otherflowparameters at any instant of time do not change with respect to space. Eg., V=V(x) indicates that the flow is uniform in ‘y’ and ‘z’ axis. V=V (t) indicates that the flow is uniform in ‘x’, ‘y’ and ‘z’ directions. Space is a criterion.

Uniform flow field is used to describe a flow in which the magnitude and direction of the velocity vector are constant, i.e., independent of all space coordinates throughout the entire flow field (as opposed to uniform flow at a cross section). That is, $[\partial V/\partial s]_{t=constant} = 0$, that is ‘V’ has unique value in entire flow field

Non-uniformflow is the type of flow in which velocity and other flow parameters at any instant change with respect to space.
$[\partial V/\partial s]_{t=constant}$isnotequaltozero.Distanceorspaceisa criterion

3. Laminarandturbulent flows

Laminar Flow is a type of flow in which the fluid particles move along well-defined paths or stream-lines. The fluid particles move in laminas or layers gliding smoothly over one another. The behavior of fluid particles in motion is a criterion.

Turbulent Flow is a type of flow in which the fluid particles move in zigzagway in the flow field. Fluid particles move randomly from one layer to another. Reynoldsnumberis acriterion.Wecanassumethatforaflowinpipe,forReynoldsNo. less than 2000, the flow is laminar; between 2000-4000, the flow is transitional; and greater than 4000, the flow is turbulent.

4. Compressibleandincompressibleflows

IncompressibleFlowis atypeofflowinwhichthedensity(ρ)isconstantinthe flowfield.ThisassumptionisvalidforflowMachnumberswithin0.25.Machnumber is used as a criterion. Mach Number is the ratio of flow velocity to velocity of sound waves in the fluid medium

Compressible Flow is the type of flow in which the densityof the fluid changes in the flow field. Density is not constant in the flow field. Classification of flow based on Mach number is given below:

M<0.25–Lowspeed M

< unity – Subsonic

Maroundunity–Transonic M

> unity – Supersonic

M>>unity,(say7) –Hypersonic

5. Rotationalandirrotationalflows

Rotational flow is the type of flow in which the fluid particles while flowing along stream-lines also rotate about their own axis.

Ir-rotational flow is the type of flow in which the fluid particles while flowing along stream-lines do not rotate about their own axis.

6. One,twoandthreedimensionalflows

The number of space dimensions needed to define the flow field completely governs dimensionality of flow field. Flow is classified as one, two and three-dimensional depending upon the number of space co-ordinates required to specify the velocity fields.

One-dimensional flow is the type of flow in which flow parameters such as velocity is a function of time and one space coordinate only.

ForEx.,V=V(x,t)–1-D,unsteady;V=V(x)–1-D, steady

Two-dimensional flow is the type of flow in which flow parameters describing the flow vary in two space coordinates and time.

ForEx.,V=V(x,y,t)–2-D,unsteady;V=V(x,y)–2-D,steady

Three-dimensional flow is the type of flow in which the flow parameters describing the flow vary in three space coordinates and time.

ForEx.,V=V(x,y,z,t)–3-D,unsteady;V=V(x,y,z)–3D,steady

Flowpatterns

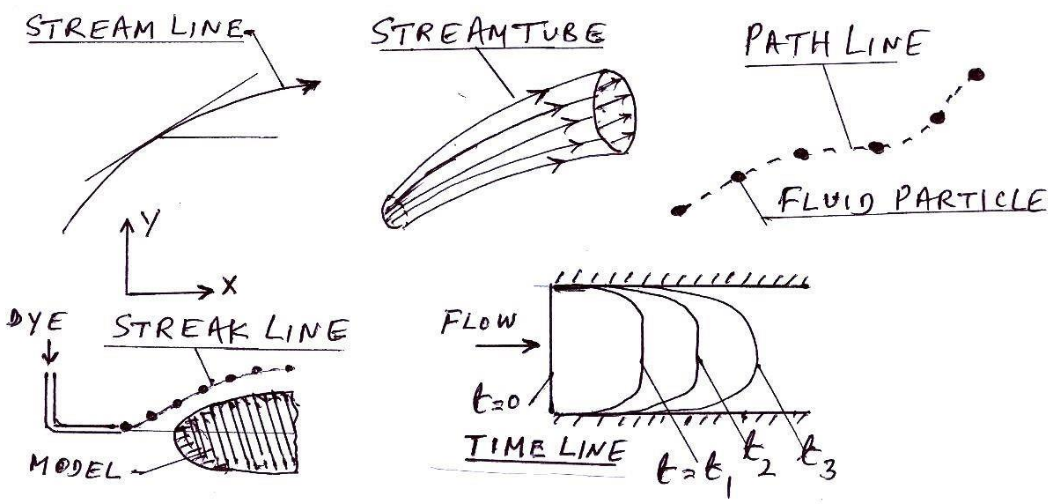


Fig.FlowPatterns

Fluid Mechanics is a visual subject. Patterns of flow can be visualized in several ways. Basic types of line patterns used to visualize flow are streamline, path line, streak line and time line.

(a) **Stream line** is a line, which is everywhere tangent to the velocity vector at a given instant.

(b) **Pathline**istheactualpathtraversedbyagiven particle.

(c) **Streakline**isthelocusofparticlesthathaveearlierpassedthroughaprescribed point.

(d) Timelineisaset offluidparticlesthatformalineata given instant.

Streamline is convenient to calculate mathematically. Other three lines are easier to obtain experimentally. Streamlines are difficult to generate experimentally. Streamlines and Time lines are instantaneous lines. Path lines and streak lines are generated by passage of time. In a steady flow situation, streamlines, path lines and streak lines areidentical. In Fluid Mechanics, the most common mathematical result for flow visualization is the streamline pattern – It is a common method of flow pattern presentation.

Streamlines are everywhere tangent to the local velocity vector. For a streamline, (dx/u) = (dy/v) = (dz/w). Stream tube is formed by a closed collection of streamlines. Fluid within the stream tube is confined there because flow cannot cross streamlines. Stream tube walls need not be solid, but may be fluid surfaces

ContinuityEquation

Rateofflowordischarge(Q)isthevolumeoffluidflowingpersecond.For incompressible fluids flowing across a section,

Volumeflowrate,Q=A×Vm^3/swhereA=crosssectionalareaandV=average velocity.

Forcompressiblefluids,rateofflowisexpressedasmassoffluidflowingacrossa section per second.

Massflowrate (m)=(ρAV)kg/swhereρ= density.

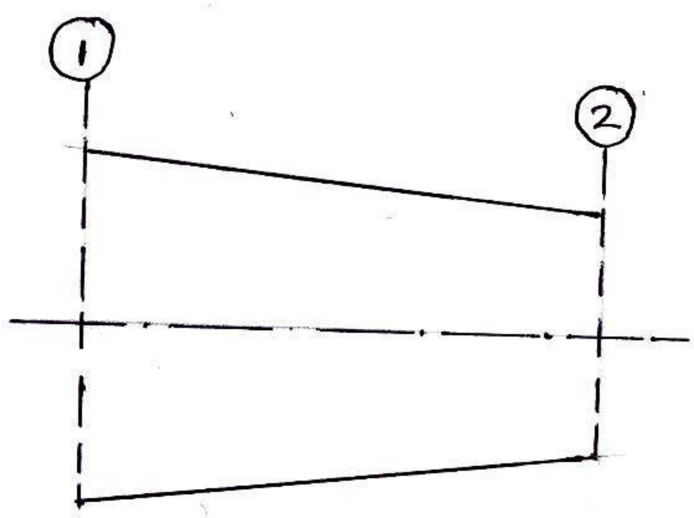


Fig.ContinuityEquation

Continuity equation is based on Law of Conservation of Mass. For a fluid flowing through a pipe, in a steady flow, the quantity of fluid flowing per second at all cross-sections is a constant.

Letv_1=averagevelocityat section[1],

ρ_1=densityoffluidat[1],

A_1=area of flow at [1];

Letv_{2},ρ_{2},A_2 becorrespondingvalues atsection[2].

Rateofflowatsection [1]=$\rho_1 A_1 v_1$

Rateofflowatsection [2]=$\rho_2 A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Thisequationisapplicabletosteadycompressibleorincompressiblefluidflowsandis called Continuity Equation. If the fluid is incompressible, $\rho_1 = \rho_2$ and the continuity equation reduces to $A_1 v_1 = A_2 v_2$

Forsteady,onedimensionalflowwithone inletandone outlet,

$\rho_1 A_1 v_1 - \rho_2 A_2 v_2 = 0$

Forcontrolvolumewith Ninletsand outlets

$\sum_{i=1}^{N}(\rho_i A_i v_i) = 0$where inflows are positive and outflows are negative .

Velocitiesarenormaltotheareas.Thisisthecontinuityequationforsteadyone dimensional flow through a fixed control volume

When densityisconstant, $\sum_{i=1}^{N}(A_i v_i) = 0$

Continuityequationin3-dimensions

(Differentialform,Cartesianco-ordinates)

Considerinfinitesimalcontrolvolumeasshownofdimensionsdx,dyanddzin x,y,and z directions

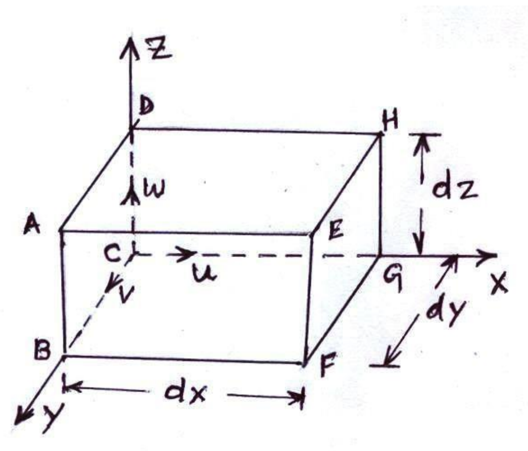


Fig.ContinuityEquationinThree Dimensions

u,v,warethevelocitiesinx,y,zdirections.

Mass of fluid entering the face ABCD =

Density×velocityin x-direction×AreaABCD =ρudy.dz

Massoffluidleavingthe face EFGH=ρudy.dz+[∂(ρudy.dz)/ ∂x](dx)

Therefore,netrateofmassefflux inx-direction= −[∂(ρudy.dz)/∂x](dx)=

-[∂(ρu)/∂x](dxdydz)

Similarly,thenetrateofmasseffluxin y-

direction= - [∂ (ρv)/ ∂y](dxdydz) z-

direction= - [∂ (ρw)/ ∂z](dxdydz)

Therateofaccumulationofmasswithinthecontrolvolume=∂(ρdV)/∂t= ρ∂/∂t(dV)

wheredV=Volumeoftheelement=dxdydzanddVisinvarientwithtime.

Fromconservationofmass,thenetrateofefflux=Rateofaccumulationofmass within the control volume.

-[$\partial(\rho u)/\partial x+\partial(\rho v)/\partial y+\partial(\rho w)/\partial z$](dxdydz)=$\rho\partial/\partial t$(dxdydz)OR $\rho\partial/\partial t + \partial (\rho u)/\partial x +\partial (\rho v)/\partial y + \partial (\rho w)/\partial z =0$

Thisis thecontinuityequationapplicable for

(a) Steadyand unsteadyflows

(b) Uniformandnon-uniform flows

(c) Compressibleandincompressibleflows.

Forsteadyflows,($\partial/\partial t$)=0and[$\partial(\rho u)/\partial x +\partial(\rho v)/\partial y+\partial(\rho w)/\partial z$]=0 If the fluid is incompressible, ρ= constant [$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z$] =0 This is the continuity equation for 3-D flows.

For2-Dflows,w=0and[$\partial u/\partial x+\partial v/\partial y$]=0

<u>Velocitypotentialandstreamfunction</u>

Velocity Potential Function is a Scalar Function of space and time co-ordinates suchthat its negative derivatives with respect to any direction give the fluid velocity in that direction.

$\Phi = \Phi$ (x,y,z) forsteadyflow.

u= -($\partial\Phi/\partial x$); v= -($\partial\Phi/\partial y$); w= -($\partial\Phi/\partial z$) where u,v,w are the components of velocity inx,y and z directions.

In cylindrical co-ordinates, the velocity potential function is given by u_r= ($\partial\Phi/\partial r$), u_θ = (1/r)($\partial\Phi/\partial\theta$)

Thecontinuityequationforanincompressibleflowinsteadystateis ($\partial u/\partial x + \partial v/\partial y + \partial w/\partial z$) = 0

Substitutingforu,vandwandsimplifying, ($\partial^2\Phi /\partial x^2 + \partial^2\Phi /\partial y^2 + \partial^2\Phi/\partial z^{2)}$ = 0

WhichisaLaplaceEquation.For2-D Flow,($\partial^2\Phi /\partial x^2 +\partial^2\Phi/\partial y^{2)}$=0

IfanyfunctionsatisfiesLaplaceequation,itcorrespondstosomecaseofsteady incompressible fluid flow.

Irrotationalflowandvelocitypotential

Assumption of Ir-rotational flow leads to the existence of velocity potential. Consider the rotation of the fluid particle about an axis parallel to z-axis. The rotation component is defined as the average angular velocity of two infinitesimal linear segments that are mutually perpendicular to each other and to the axis of rotation.

Considertwo-linesegmentsδx,δy.TheparticleatP(x,y)hasvelocitycomponentsu,v inthe x-yplane.

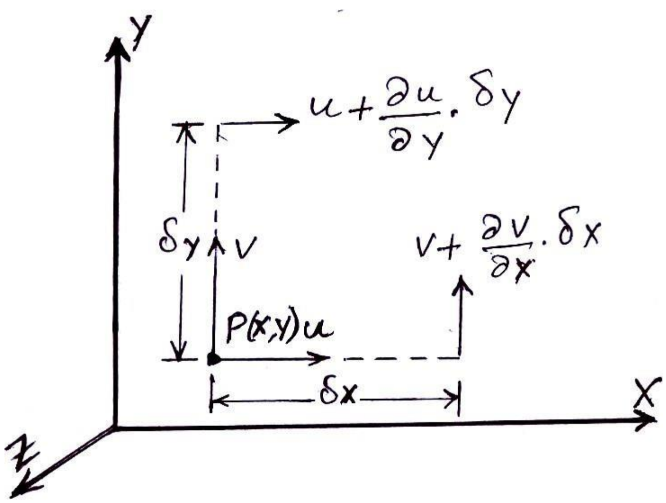


Fig.Rotationofafluid partical.

Theangularvelocitiesofδx and δyaresought.

The angular velocity of (δx) is $\{[v+ (\partial v/\partial x)\ \delta x - v] / \delta x\} = (\partial v/\partial x)$ rad/sec

The angular velocity of (δy) is $-\{[u+ (\partial u/\partial y)\ \delta y - u] / \delta y\} = -(\partial u/\partial y)$ rad/sec

Counter clockwise direction is taken positive. Hence, by definition, rotation component(ω_z)is$\omega_z=1/2\{(\partial v/\partial x)-(\partial u/\partial y)\}$.Theothertwocomponentsare $\omega_x=$ $1/2\ \{(\partial w/\partial y)-$
$(\partial v/\partial z)\}$ $\omega_y=1/2$
$\{(\partial u/\partial z)-(\partial w/\partial x)\}$

Therotation vector=$\omega= i\omega_x+j\omega_y+k\omega_z$.

Thevorticityvector(Ω)isdefinedastwicethe rotationvector =2ω

Propertiesofpotentialfunction

$\omega_z=1/2\{(\partial v/\partial x)-(\partial u/\partial y)\}$

$\omega_x=1/2\{(\partial w/\partial y)-(\partial v/\partial z)\}$

$\omega_y=1/2\{(\partial u/\partial z)-\partial w/\partial x)\}$;

Substituting $u=-(\partial\Phi/\partial x)$;

$v=-(\partial\Phi/\partial y)$;

$w=-(\partial\Phi/\partial z)$;

weget$\omega_z=1/2\{(\partial/\partial x)(-\partial\Phi/\partial y)-(\partial/\partial y)(-\partial\Phi/\partial x)\}$

$=½\{-(\partial^2\Phi/\partial x\partial y)+(\partial^2\Phi/\partial y\partial x)\}=0$sinceΦisa continuousfunction.

Similarly,$\omega_x=0$and$\omega_y=0$

All rotational components are zero and the flow is irrotational.– Therefore, irrotational flow is also called as Potential Flow.

If the velocity potential (Φ) exists, the flow should be irrotational. If velocity potential function satisfies Laplace Equation, It represents the possible case of steady, incompressible, irrotational flow. Assumption ofavelocitypotential is equivalent to the assumption of irrotational flow.

Laplaceequationhasseveralsolutionsdependinguponboundaryconditions. If

Φ_1 and $\Phi_2$areboth solutions, $\Phi_1+\Phi_2$ is also a solution

$\nabla^2(\Phi_1)=0,\nabla^2(\Phi_2)=0,\nabla^2(\Phi_1+\Phi_2)=0$

Alsoif$\Phi_1$isasolution,$C\Phi_1$ isalso asolution(where C=Constant)

Streamfunction(ψ)

Stream Function is defined as the scalar function of space and time such that its partialderivativewithrespecttoanydirectiongivesthevelocitycomponentatright angles to that direction. Stream function is defined only for two dimensional flows and 3-D flows with axial symmetry.

$(\partial\psi/\partial x)=v$;

$(\partial\psi/\partial y)=-u$

InCylindricalcoordinates

,$u_r=(1/r)(\partial\psi/\partial\theta)$and

$u_\theta=(\partial\psi/\partial r)$

Continuityequation for2-D flowis$(\partial u/\partial x)+(\partial v/\partial y)$

$=0(\partial/\partial x)(-\partial\psi/\partial y)+(\partial/\partial y)(\partial\psi/\partial x)=0$

$-(\partial^2\psi/\partial x\partial y)+(\partial^2\psi/\partial y\partial x)=0;$

Therefore, continuity equation is satisfied. Hence, the existence of (ψ) means a possible case of fluid flow. The flow may be rotational or irrotational. The rotational component are:

$\omega_z=1/2\ \{(\partial v/\partial x)-(\partial u/\partial y)\}$

$\omega_z=1/2\ \{(\partial/\partial x)(\partial\psi/\partial x)-(\partial/\partial y)(-$

$\partial\psi/\partial y)\}$ $\omega_z=½\{(\partial^2\psi/\partial x^2)+(\partial^2\psi/\partial y^2)\}$

Forirrotational flow,$\omega_z=0$.Hencefor2-Dflow, $(\partial^2\psi/\partial x^2)+(\partial^2\psi/\partial y^2)=0$whichis a Laplace equation.

Propertiesofstreamfunction

1. IftheStreamFunction(ψ) exists,itisa possiblecaseof fluid flow,which may be rotationalorirrotational.
2. IfStreamFunctionsatisfiesLaplaceEquation,itisapossiblecaseofanirrotational flow.

Equi-potential&constantstreamfunctionlines

Onanequi-potentialline,thevelocitypotentialisconstant,Φ=constantord(Φ)=0. $\Phi = \Phi(x,y)$ for steady flow.

$d(\Phi)=(\partial\Phi/\partial x)\,dx+(\partial\Phi/\partial y)dy.$

$d(\Phi)=-udx-vdy=-(udx+vdy)=0$. For equi-

potential line, $u\,dx + v\,dy = 0$

Therefore,$(dy/dx)=-(u/v)$whichisaslopeofequi-potentiallines For

lines of constant stream Function,

ψ = Constant or $d(\psi)=0$, $\psi = \psi(x,y)$

$d(\psi)=(\partial\psi/\partial x)dx+(\partial\psi/\partial y)dy=vdx-udy$

Since $(\partial\psi/\partial x) = v$; $(\partial\psi/\partial y) = -u$

Therefore,$(dy/dx)=(v/u)$=slopeoftheconstantstreamfunctionline.Thisisthe slope of the stream line.

Theproductoftheslopeoftheequi-potentiallineandtheslopeoftheconstantstream function line (or stream Line) at the point of intersection = -1.

Thus,equi-potentiallinesandstreamlinesareorthogonalatallpointsofintersection.

Flownet

A grid obtained bydrawingaseries of equi-potential lines andstreamlines is called a FlowNet.AFlowNetisanimportanttoolinanalyzingtwo-dimensionalir-rotational flow problems.

Examplesofflownets

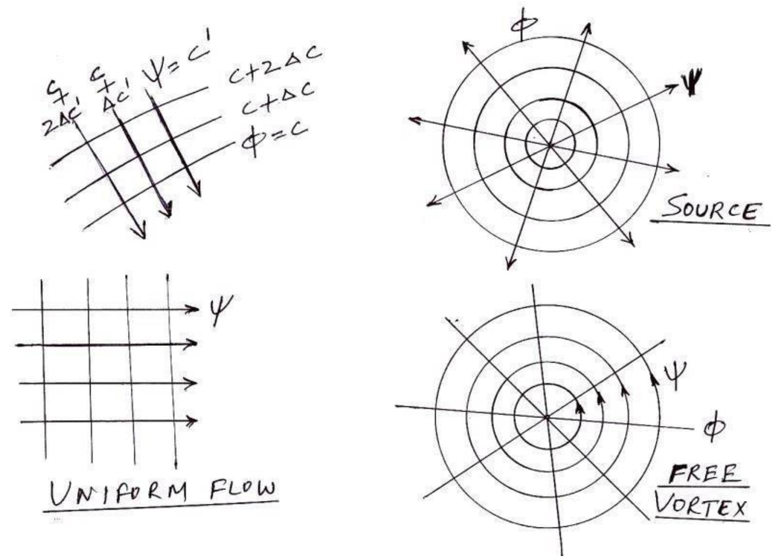


Fig.Flow Nets

Examples:Uniformflow,Linesourceandsink,Linevortex
Two-dimensionaldoublet– alimitingcaseof alinesource approachingalinesink

Relationshipbetweenstreamfunctionandvelocity Potential

$u=-(\partial\Phi/\partial x), v=-(\partial\Phi/\partial y)$

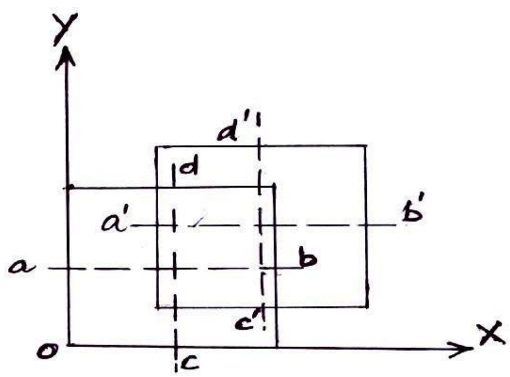
$u= -(\partial\psi/\partial y)$, $v= (\partial\psi/\partial x)$; Therefore, -
$(\partial\Phi/\partial x)=-(\partial\psi/\partial y)$ and $-(\partial\Phi/\partial y)=(\partial\psi/\partial x)$
Hence, $(\partial\Phi/\partial x)=(\partial\psi/\partial y)$ and $(\partial\Phi/\partial y)= -(\partial\psi/\partial x)$

Typesofmotion

AFluidparticlewhilemovinginafluidmayundergoanyoneoracombinationofthe following four types of displacements:

1. Linearorpuretranslation
2. Lineardeformation
3. Angulardeformation
4. Rotation.

(1) Linear Translation is defined as the movement of fluid element in which fluid element moves from one position to another bodily – Two axes ab & cd and a'b'& c'd' are parallel (Fig. 8a)



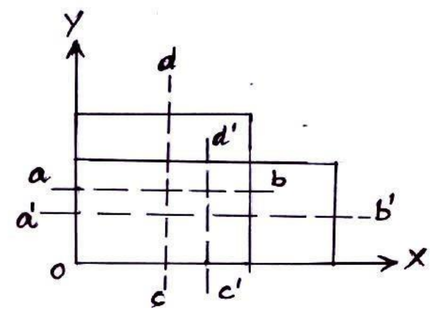
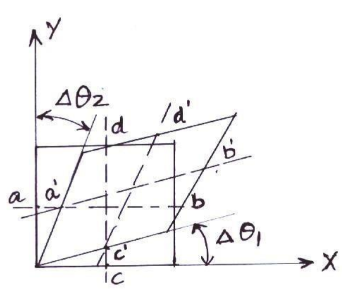
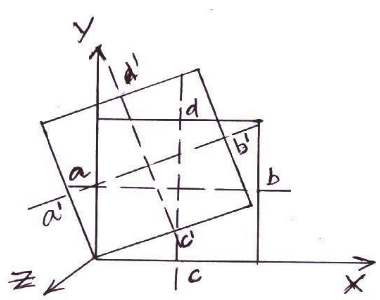


Fig.Lineartranslation. Fig.Lineardeformation.

(2) Lineardeformationisdefinedasdeformationoffluidelementinlineardirection– axesareparallel,butlengthchanges.

(2)Angulardeformation,alsocalledsheardeformationisdefinedasthe average change in the angle contained by two adjacent sides. The angular deformation or shear strain rate = $\frac{1}{2}(\Delta\theta_1 + \Delta\theta_2) = \frac{1}{2}(\partial v/\partial x + \partial u/\partial y)$





FigAngulardeformation Fig.Rotation

(4) Rotation is defined as the movement of the fluid element in such a way that both its axes (horizontal as well as vertical) rotate in the same direction. Rotational components are:

FluidMechanics

Unit-III

FluidDynamics

- Fluid Dynamics is the branch of fluid science that deals with the study of fluid motion andthe associated external forces and internal resistances.
- Based on Newton's second law, it is possible to derive certain differential equations, called equations of motion or momentum conservation, which completely define the Dynamic behavior of the body.

TYPESOFFORCESINFLUID MOTION:

Thedifferent forcesactingon thefluid mass inmotion maybe classified asfollows;

i. BodyforcesorVolumeforces(Weights,GravityForce $\mathbf{F_g}$,compressibilityorelastic force $\mathbf{F_e}$)
ii. Surfaceforces(PressureforcesF $\mathbf{F_p}$,ShearForces, viscousforce $\mathbf{F_v}$,TurbulentForce $\mathbf{F_t}$)
iii. LinearForces(Capillaryorsurfacetensionforces $\mathbf{F_s}$)

Ifacertainmassoffluidinmotionisinfluencedbyalltheabovementionedforces,then according to newtons second law, the following equations of motion can be written.

$M\times a= F_g+F_e+ F_p+F_v+F_t+F_s$ -- eq.(1)

WhereM=Massofthefluidinflow a=

Acceleration of the flow

Furtherbyresolvingthevariousforcesinx,y&z directions,weget $M \times a_x$

$= F_{gx}+ F_{ex}+ F_{px}+ F_{vx}+ F_{tx}+ F_{sx}$

$M\times a_y=F_{gy}+F_{ey}+F_{py}+F_{vy}+F_{ty}+F_{sy}$ $M\times a_z$

$= F_{gz}+ F_{ez}+ F_{pz}+ F_{vz}+ F_{tz}+$

F_{sz}***Reynold's equation of motion:***

In most of the problemsof fluids inmotion,the surface tension forces and compressibilityforces are not significant. Hence these forces can be neglected and eq. (1) becomes

$M\times a= F_g+F_p+F_v+F_t$ -- eq.(2)

Theabove eqisknownasReynold'sequationof motion.

Navier–stokes equationof motion:

Further for laminar or viscous flows, turbulent forces also become less significant. Hence eq. (2) becomes

$M\times a = F_g + F_p + F_v$ -- eq.(3)

Theaboveeqis known asNavier – stokesequation ofmotion.

Euler'sequationofmotion:

Furtheriftheviscousforcesarealsooflittlesignificance,thentheseforcesmayalsobe neglected. Hence eq. (3) becomes

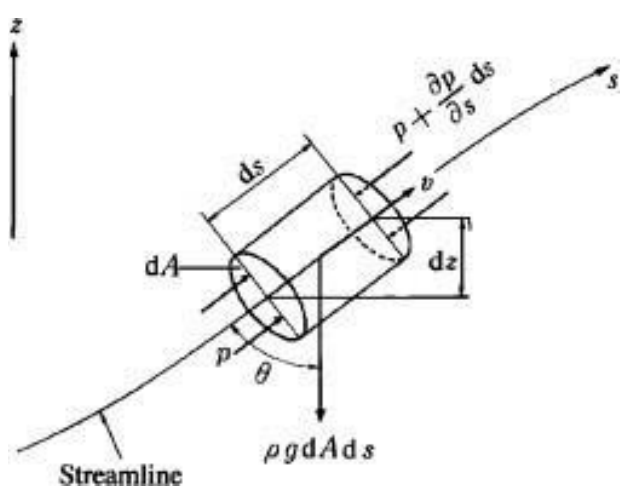
$M\times a = F_g + F_p$ -- eq.(4)

Theabove equationisknownasEuler's equationof motion.

EULER'SEQUATIONFOR 1-D FLOW:

General flows are three dimensional, but many of them may be studied as if they are one dimensional. For example, whenever a flow in a tube is considered, if it is studied in terms of mean velocity, it is a one-dimensional flow which is studied very simply. Let us apply the principle of conservation of momentum to the infinitesimal, cylindrical element of fluid having the cross-sectional area dA and length ds which lay along the streamline

- Wealsoassumethattheflowissteady,itisthatalllocaltimederivativesareequalzero, $\partial v/\partial t = 0$.
- Wealso assumethatflow is invicid (ideal).



Around a cylindrical element of fluid having the cross-sectional area 'dA' and length 'ds' is considered.

- Let p be the pressure acting on the lower face, and pressure {p+ dp} acts on the upper face a distance ‘ds’ away.
- The gravitational force acting on this element is its weight, [p.g.dA.ds]. Applying Newton’s second law of motion to this element, the resultant force acting on it, and producing acceleration along the streamline is the force due to the pressure difference across the streamline and the component of any other external force (in this case onlythe gravitational force) along the streamline. Therefore the following equation is obtained:
- Weknowthattheexternalforcestendingtoacceleratethefluidelementinthedirection of the streamline
$$= P.dA - (P + dP)dA$$
$$= -dP.dA$$
- **(1)**
- Wealsoknow thattheweightof the fluid element,
$$dW = \rho g.dA.ds$$
- Fromthegeometryofthefigure,wefindthatthecomponentoftheweightofthefluid element in the direction of flow,
$$= \rho g.dA.ds\cos\theta$$
$$= \rho g.dA.ds\frac{dz}{ds}$$
$$= \rho g.dA.dz$$
- **(2)**
- $\therefore$ Mass of the fluid element $= \rho.dA.ds$
- Weseethat the accelerationofthe fluid element
$$\frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v\frac{dv}{ds}$$
- **(3)**
- Now,as per Newton'ssecond lawof motion,weknow thatForce=Mass *Acceleration
$$\Rightarrow (-dp.dA) - (\rho g.dA.dz) = \rho.dA.ds \times v.\frac{dv}{ds}$$
- Dividingboth sides by $(-\rho dA)$
$$\Rightarrow \frac{dP}{\rho} + g.dz = -v.dv$$
- or,
$$\Rightarrow \frac{dP}{\rho} + g.dz + v.dv = 0$$
- **(4)**
- ThisistherequiredEuler's equationformotionasintheformofadifferential equation.

BERNOLLIESEQUATIONFROMEULER’SEQUATION

- Integratingtheabove equation,

$$\frac{1}{\rho}\int dP + \int g.dz + v.dv = Constant$$
$$\Rightarrow \frac{P}{\rho} + gz + \frac{v^2}{2} = Constant$$
$$\Rightarrow P + wz + \frac{wv^2}{2g} = Constant$$
$$\Rightarrow \frac{P}{w} + Z + \frac{v^2}{2g} = Constant$$

- orin other words,

$$\frac{P_1}{w_1} + Z_1 + \frac{v_1^2}{2g} = \frac{P_2}{w_2} + Z_2 + \frac{v_2^2}{2g}$$

MOMENTUM EQUATION:

It is based on law of conservation of momentum, which states that the net force actingon thefluidmassisequaltothechangeinmomentumofflowperunittimeinthatdirection. The force acting on fluid mass ‘m’ is given by Newton’s second law

F=m×a

Buta= dv/dt

ThereforeF=m [dv/dt]

⇨ F=[d(m.v)/dt]--------------------(1)momentum principle

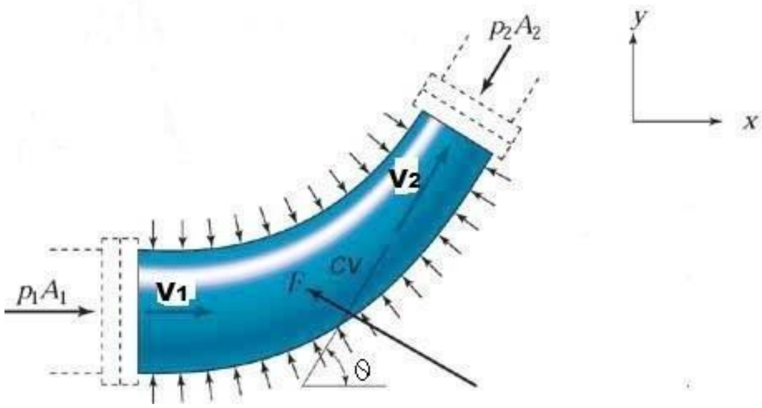
⇨ F×dt=d(m.v)-------------------- (2)Impulse–momentumequation

Impulse momentum equation states that the impulse of a force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum d (m.v) in the direction of force.

Application:

TheImpulsemomentumequationisusedtodeterminetheresultanytforceexertedby flowing fluid on a pipe bend.

Considerapipebend with twosections 1and 2 asshown in figure.



Let V_1 and V_2 be the velocities at inlet and outlet respectively. P_1 and P_2 be the pressures at inlet and outlet respectively.

Thecontinuityequationyields $$\rho V_1 A_1 = \rho V_2 A_2 = \dot{m}$$

Carryingout aforcebalancein x-direction,wehave

$$p_1 A_1 - p_2 A_2 \cos\theta + F_x = \rho V_2 A_2 V_2 \cos\theta - \rho V_1 A_1 V_1$$

$$= \dot{m} V_2 \cos\theta - \dot{m} V_1 = \dot{m} (V_2 \cos\theta - V_1)$$

Inthey-directionwe have,

$$0 - p_2 A_2 \sin\theta + F_y = \rho V_2 A_2 V_2 \sin\theta - 0$$

giving $$p_2 A_2 \sin\theta + F_y = \dot{m} V_2 \sin\theta$$

Thusthe forcecomponents actingon thebend are

$$F_x = p_2 A_2 \cos\theta - p_1 A_1 - \dot{m} (V_2 \cos\theta - V_1)$$

$$F_y = -p_2 A_2 \sin\theta + \dot{m} V_2 \sin\theta$$