Bhubanananda Odisha School of Engineering, Cuttack

Department of Mechanical Engineering

SUBECT-DESIGN OF MACHINE ELEMENTS (TH-2)

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LECTURE NOTES

Introduction

Definition of Machine design:

The subject Machine design is the creation of new and better machine and improving the existing one which is more ~~co~~ economical in the overall cost of production and operation.

Classification of Machine design

Machine design are classified various type

(I) Adaptive design
(II) Development design
(IV) New design.

* General procedure for Machine design

Need or Aim
↓
Synthesis (Mechanism)
↓
Analysis of forces
↓
Material selection
↓
Design of elements size & stress
↓
Modification
↓
Detail drawing
↓
production

Unit:

It is the comparision of one things or Materials ~~from~~ with an other material for exchange each other with same value.

> ~~Derive unit~~

> The units are verious type.

(i) Fundamental unit

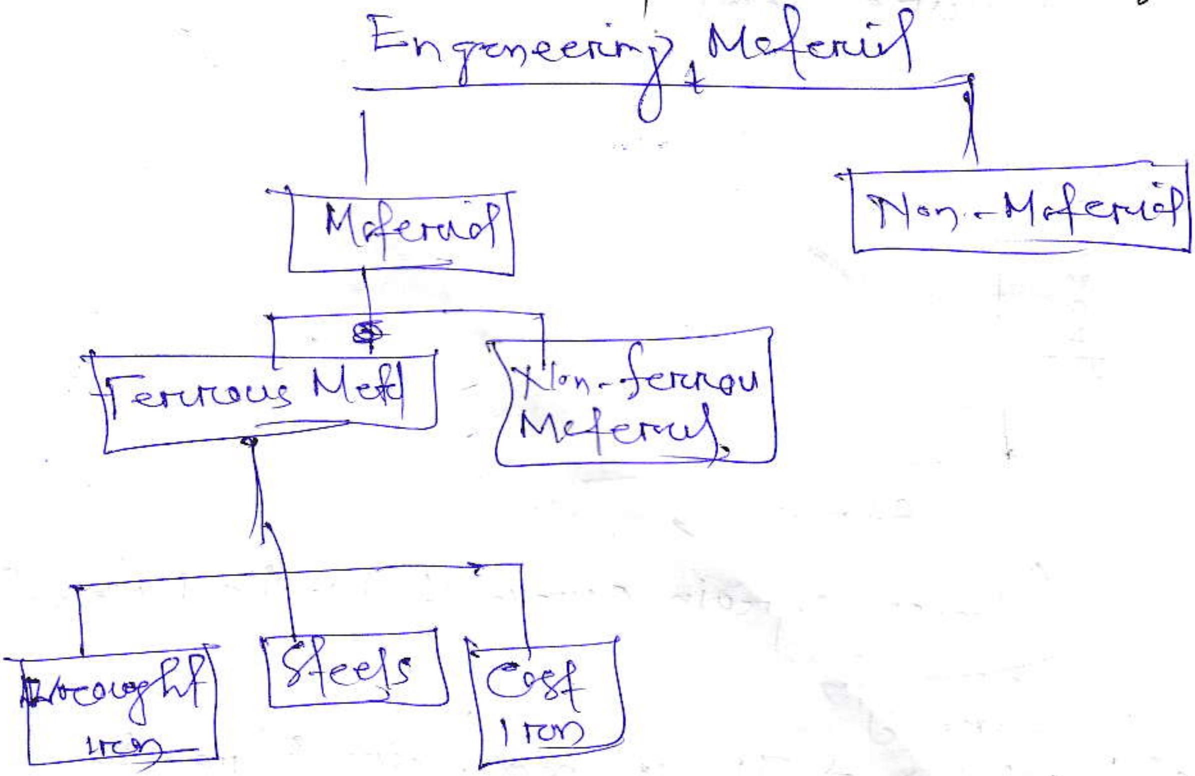
(ii) derived unit.

* Materials used in Machine design

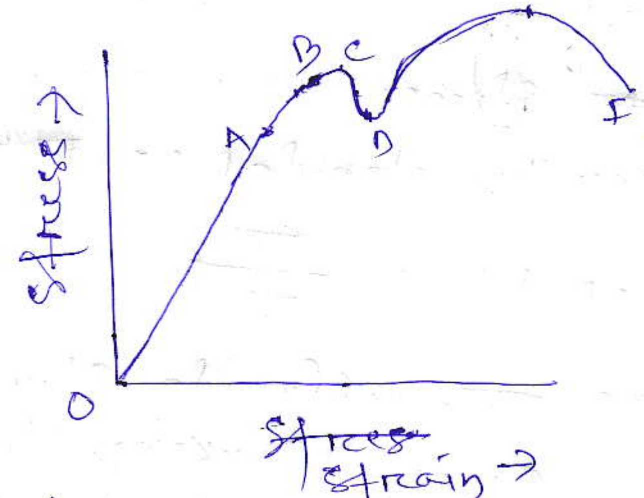
The properties of Material which is used in Machine design are.

1. Elastic
2. plastic
3. Ductility
4. Brittleness
5. Malleability
6. Stiffness
7. Hardness
8. Toughness
9. Creep.
10. Fatigue.

* The materials used in Machine design



* Stress - Strain curve



(Stress-strain curve of M. ductile Materia

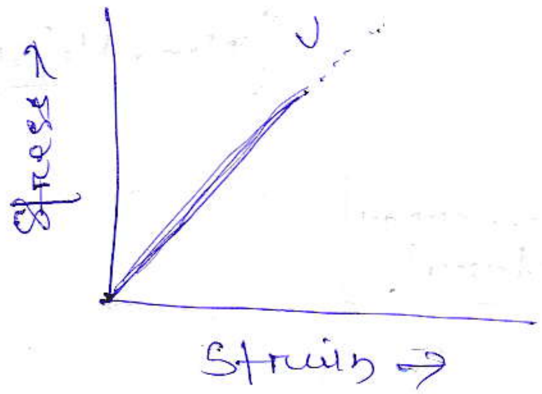
Where OA → proportional limit or Elastic limit

AB → plastic limit

C = upper yield point

D = Lower yield point

E = Necking point
F = Failure point



(Stress-strain curve of Brittle Material)

* Yield ~~point~~ ~~stress~~ strength → It is maximum strength of the material without deformation.

* Working stress → It is the required stress before deformation of material or it is also called design stress.

* Factor of safety → It is the ratio of ultimate stress to working stress

→ It is generally denoted as FOS

So $FOS \text{ or } (FS) = \dfrac{\sigma_u}{\sigma_s}$

where σ_u = ultimate stress

σ_s = working stress or design stress

Mode of failure of design

(i) Failure due to elastic deflection
(ii) Failure by general yielding
(iii) Failure by fracture.

* Design of Fastening Elements

Fastening: Joining of Machine parts either permanently or temporarily.

→ Types of fastening

(i) Permanent fastening → Welded joint
  └ Riveted joint

(ii) Temporary fastening → Screw
  └ Nut & bolt.

* Classification of Joint

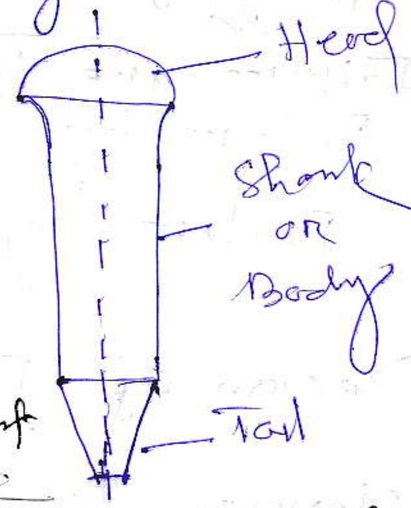
(i) Bolting
(ii) Riveting
(iii) Soldering
(iv) Brazing
(v) Seaming

* Rivet ⇒ A rivet is a short cylindrical bar with a head integral to it.
  → The cylindrical portion is called shank or body
  → Lower portion is called Tail.

Types of Rivet

→ Rivet is a permanent ~~ty~~ joint

→ Some time it also detachable types of joint



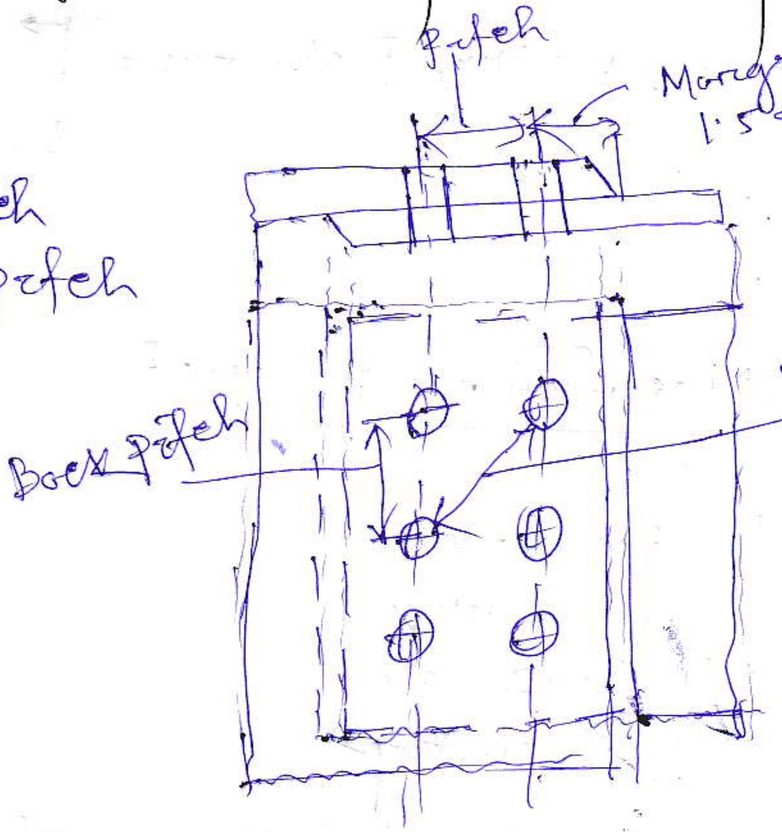
Rivet parts

* classification of Riveted joint

(I) Lab. joint
(II) Butt joint
(III) Zig-Zag joint

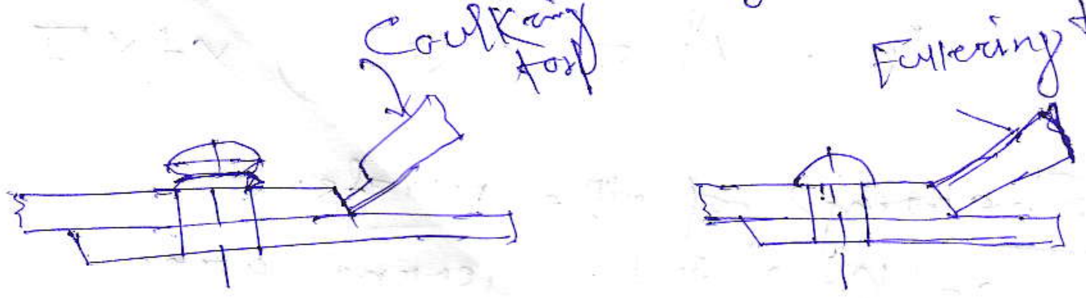
* Important ~~temp~~ terms used in Riveted joint

(I) Pitch
(II) Back pitch
(III) Diagonal pitch
(IV) Margin



* Caulking & Fullering

In order to make the joint leak proof or fluid tight in pressure vessel like steam boiler, air receiverse and tank etc a process known as caulking is employed.



(a) Caulking (b) Fullering

* Failures of a Riveted Joint

(1) Riveted joint are mainly failure due to following way.

(a) Tearing of plate at an edge

(b) Tearing of plate across a row of rivets

(2) Shearing of the rivets

(3) Crushing of the plate or rivets

* Tearing of plate

Due to the tensile stress in main plate the tearing occurred in the plate.

→ The resistance offered by the plate against tearing is known as tearing resistance or tearing strength or tearing value of the plate

Let p = pitch of the rivet

d = diameter of the riveted hole

t = thickness of the plate

σ_t = Permissible tensile stress on the we know that tearing area per pitch length

$$A_t = (p-d) \times t$$

Tearing resistance of the plate per pitch length

$$P_t = A_t \sigma_t = (p-d) \times t \times \sigma_t.$$

<u>Shearing of the rivets.</u>

we know that shearing area

$$A_s = \frac{\pi}{4} \times d^2 \text{ (In single shear)}$$

$$= 2 \times \frac{\pi}{4} \times d^2 \text{ (Double shear)}$$

Shearing resistance of the rivet per pitch length

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \leftarrow \text{Single sh}$$

$$= n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau \text{ (double she}$$

n = no. of rivet.

<u>Crushing of the plate of rivet</u>

Let

d = diameter of rivet ho

t = thickness of plate

σ_c = Safe permissible crushing stress

n = no. of rivet per pitch length.

We know that crushing Area
$A_c = dt$

Total crushing area $= n \cdot dt$

Crushing resistance of the rivet per pitch length.

$$P_c = n \cdot d \cdot t \cdot \sigma_c$$

* Efficiency of a Riveted Joint

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

We have already discussed that strength of the riveted joint
$= \text{Least of } P_t, P_s \text{ \& } P_c$

Strength of the un-riveted or solid plate per pitch length
$$P = p \times t \times \sigma_t$$

Efficiency of the riveted joint

$$\eta = \frac{\text{Least } P_t, P_s \text{ \& } P_c}{p \times t \times \sigma_c}$$

where p = Pitch of the rivets
t = thickness of the plate
σ_t = permissible tensile stress of the plate material

Problem: Find the efficiency of the following riveted joint:

(a) Single riveted lap joint of 6 mm plate with 20 mm diameter rivets having a pitch of 50 mm.

(2) Double riveted lap joint of 6 mm plate with 20 mm diameter rivets having a pitch of 50 mm.

Assume permissible tensile stress = 120 MPa

" shearing " = 90 MPa

" crushing " = 180 MPa

Sol^n

Data given: $t = 6$ mm $d = 20$ mm $\sigma_t = 120$ MPa

$\tau = 90$ MPa $\sigma_c = 180$ MPa

(a) **Efficiency of the joint**

pitch $p = 50$ mm.

(1) **Tearing resistance of the plate**

we know that tearing resistance

$P_t = (p-d) \times t \times \sigma_t$

$= (50-20) \times 6 \times 120 = 21600$ N

(2) **Shearing resistance of the rivet** ~~plate~~

we know that shearing resistance

$P_s = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times (20)^2 \times 90$

$= 28278$ N

(3) **Crushing resistance of the rivet**

we know that crushing resistance

$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180$

$= \cancel{3600}$ 2160

Strength of the joint:

strength Least of P_t, P_s & P_c = 21600 N

strength solid plate = $p \times t \times \sigma_t = 50 \times 6 \times 120 = 36000$

Efficiency of the joint

$$\eta = \frac{\text{Least of } P_t\ P_s\ \&\ P_c}{P}$$

$$= \frac{21600}{36000} = 0.60 \text{ or } 60\%$$

* Efficiency of the second joint

pitch $p = 65$ mm

(I) Tearing resistance

$$P_t = (p-d) \times t \times \sigma_t$$

$$= (65-20) \times 6 \times 120 = 32400 \text{ N}$$

(II) Shearing resistance

$$P_s = \frac{\pi}{4} \times d^2 \times \tau \times n =$$

$$= \frac{\pi}{4} \times (20)^2 \times 90 \times 2 = 56556 \text{ N}$$

(III) Crushing resistance

$$P_c = d \times t \times n \times \sigma_c$$

$$= 20 \times 6 \times 2 \times 180 = 43200 \text{ N}$$

Strength of the joint = Least of P_t P_s & P_c
= 32400 N

strength of un-riveted plate

$$P = p \times t \times \sigma_t = 65 \times 6 \times 120$$
$$= 46800 \text{ N}$$

efficiency η = Least of P_t P_s & P_c

$$\eta = \frac{32400}{46800} = 0.692 \text{ or } 69.2\%$$

* Welded Joint

Welding: welding is a process of joining of two homogeneous similar or dis similar plates of same family.

→ It is done by with application of heat or and pressure.

Classification of welding

welding are various type

- Welding process
  - Fussion welding
    - Arc weld
      - → pulsed arc
      - → electrod gas
      - → Spray transfer
      - stud
    - oxy fuel gas weld
      - (i) Oxyacetylene
      - (ii) Pressure gas
    - Resistance welding
      - → Spot
      - → Seam
      - → Projection
  - Non fussion welding / Solid state
    - → Diffusion
    - → Friction
    - → ultrasonic
    - → Forge
    - → cold
    - → Explosion
    - → Roll weld

*. ~~Wet~~ Types of welding joint

(1) Lap joint
(2) Butt joint
- (1) Square butt joint
- Single V- butt joint
- Single U-butt joint
- Double V - butt joint.
- Double U - butt joint.

(3) Corner joint
(4) Edge joint
(5). T - joint.

* Advantages ~~are~~ of welding joint compared to riveted joint

(1) The welded structure is lighter than riveted structure
(2) The welded joints provide maximum efficiency
(3) As the welded structure is smooth in appearance
(4) A welded joint has a great strength
(5) The process of welding takes less time than the riveting

* Disadvantages

(1) Since there is an uneven heating & cooling during fabrication
(2) It requires highly skilled labour and ~~sup~~ supervision
(3) Since no provision is kept for expansion and contraction in the frame therefore there is possibility of crank developed in it.
(4) The inspection of welding work is more

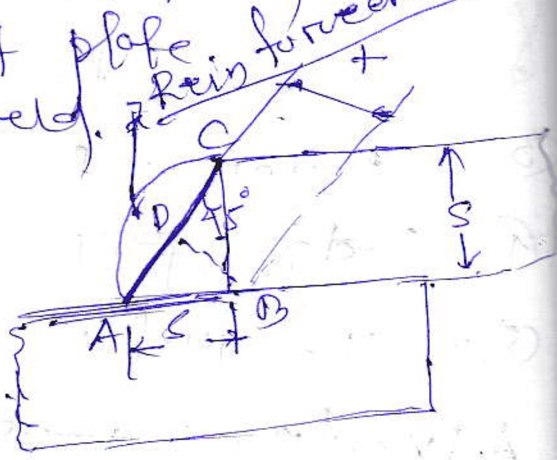
Strength of transverse fillet welded joint

Let t = throat thickness

s = leg size or size of weld

= Thickness of plate

L = Length of weld.



Throat thickness

$t = s \times \sin 45° = 0.707s$

Minimum area of weld or throat area

A = Throat thickness × Length of weld

$= 0.707 \times s \times L$

Tensile strength of single fillet weld

P = Throat Area × Allowable tensile stress

$= 0.707 \times s \times L \times \sigma_t$

Tensile strength for double fillet weld

$P = 2 \times 0.707 \times s \times L \times \sigma_t$

Strength of parallel fillet welded joint

Minimum throat Area

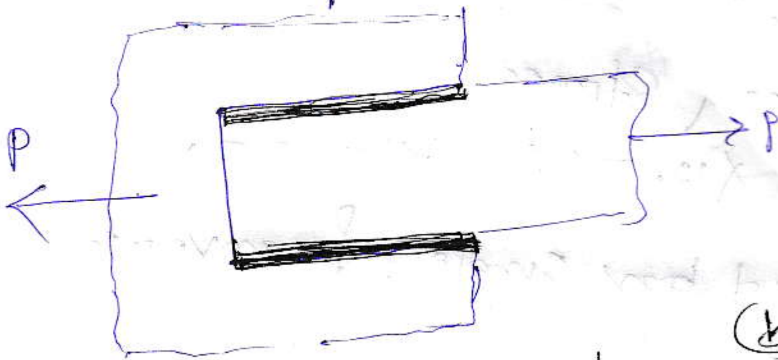
$$A = 0.707 \times S \times L$$

Shear strength of single parallel fillet weld

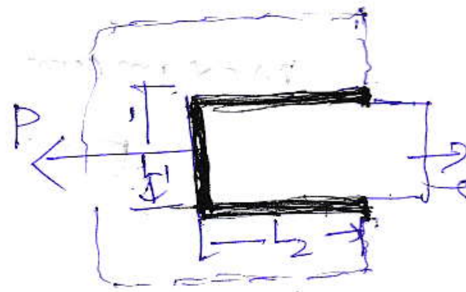
$$P = 0.707 \times S \times L \times \tau$$

Shear strength of double parallel fillet weld

$$P = 2 \times 0.707 \times S \times L \times \tau$$



(Double parallel fillet weld)



(b) Combination of transverse and parallel fillet weld

problem

* A plate 75 mm wide and 12.5 mm thickness is ~~joint~~ joined with another plate by a single transverse weld and double parallel fillet weld. The maximum tensile and shear stress are 70 MPa and 56 MPa respectively.
Find the length of each parallel fillet weld. If the joint is subjected to to both static and fatigue Loading?

Soln

$w = 75\ mm$

$t = 12.5\ mm$

$\sigma_t = 70\ MPa$

$\tau = 56\ MPa$

The effective length of weld (L_1) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate

$L_1 = 75 - 12.5 = 62.5$ mm.

Length of each parallel fillet for static Loading

L_2 = length of each parallel fillet

Maximum load

P = Area X Stress

$= 75 \times 12.5 \times 70 = 65625$ N.

Load carried by single transverse weld

$P_t = 0.707 \times s \times L_1 \times \sigma_t$

$= 0.707 \times 12.5 \times 62.5 \times 70$

$= 38664$ N.

The load carried by double parallel fillet weld

$P_2 = 1.414\, s \times L_2 \times 2$

$= 1.414 \times 12.5 \times L_2 \times 56$

$= 990\, L_2$ N

Load carried by the joint (P)

$65625 = P_1 + P_2$

$= 38664 + 990\, L_2$

$L_2 = 27.2$ mm

Adding 12.5 mm for starting and stopping of weld run we have

$$L_2 = 27.2 + 12.5 = 39.7 \text{ say } 40 \text{ mm}$$

* Length of each parallel fillet for fatigue Loading:

Permissible tensile stress

$$\sigma_t = \frac{70}{1.5} = 46.7 \text{ N/mm}^2$$

Permissible shear stress

$$\tau = \frac{56}{2.7} = 20.74 \text{ N/mm}^2$$

Single transverse weld

$$P_1 = 0.707 \times s \times l_1 \times \sigma_t$$
$$= 0.707 \times 12.5 \times 62.5 \times 46.7$$
$$= 25795 \text{ N}$$

Load carried by double parallel fillet weld

$$P_2 = 1.414 \times s \times L_2 \times \tau$$
$$= 1.414 \times 12.5 \times L_2 \times 20.74$$
$$= 366 L_2 \text{ N}$$

Load carried by the joint (P)

$$65625 = P_1 + P_2$$
$$= 25795 + 366 L_2$$
$$L_2 = 108.8 \text{ mm.}$$

Adding 12.5 mm for starting and stopping weld run, we have

$$L_2 = 108.8 + 12.5 = 121.3 \text{ mm}$$

Design of Shafts and Keys

Shaft → A shaft is a rotating machine element which is used to transmit power from one place to another.

* Material used for shafts

The material used for shaft should have the following properties.

(I) It should have high strength
(II) It should have good Machinability
(III) It should have low notch sensitivity factor
(IV) It should have good heat treatment properties
(V) It should have high wear resistant properties.

* Types of shaft:

The following two of shaft are important.

(I). Transmission shaft
(II) Machine shaft.

* Design of Shaft

The shaft may be designed on the basis of

(a) Strength
(b) Rigidity and Stiffness

In designing shafts on the basis of strength the following case may be considered.

(a) Shaft subjected to twisting moment

(b) Shaft subjected to bending moment only

(c) Shaft subjected to combined twisting & bending moments.

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

* Shaft Subjected to twisting moment only

Torque eqn.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

where T = Torque
J = polar moment of Inertia
τ = Shear stress
R = Radius = $\frac{d}{2}$
G = Modulus of rigidity
θ = Angle of twist.
L = distance.

considering $\frac{T}{J} = \frac{\tau}{R}$ - (1)

polar moment of Inertia $J = \frac{\pi}{32} \times d^4$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$= \frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{d/2} \text{ or } T = \frac{\pi}{16} \times \tau \times d^3$$

for hollow Shaft

$$J = \frac{\pi}{32}\left[\left(d_o^4 - d_i^4\right)\right]$$

where d_o = outside dia
d_i = Inner dia

$$T = \frac{\pi}{16} \times \tau \left[\frac{d_o^4 - d_i^4}{d_o}\right] \qquad \boxed{K = \frac{d_i}{d_o}}$$

$$T = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

Power transmitted

$$P = \frac{2\pi N T}{60} \quad T = \frac{P \times 60}{2\pi N}$$

3. In case of belt drives, the twisting moment (T) is given ~~you~~ you.

$$T = (T_1 - T_2) R$$

T_1 & T_2 = Tension in tight side & slack side

R = Radius of the pulley

Problem

* A line shaft rotating at 200 r.p.m to transmit 20kW. The shaft may be assumed to be made of mild steel at an allowable shear stress of 42 MPa. Determine the diameter of the shaft. Neglecting the bending moment on the

Data given $N = 200$ rpm

$P = 20$ kW $= 20 \times 10^3$ watt

$\tau = 42$ MPa $= 42$ N/mm²

d = diameter of shaft

Torque transmitted (T) $= \frac{P \times 60}{2\pi N}$

$$= \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955$$

$= 955 \times 10^3$

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3$$

$d = 48.7$ say 50 mm (Ans)

* ## Shaft Subjected to Bending Moment only

We known that bending eqn

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

where M = Bending Moment

I = Moment of Inertia

σ_b = Bending stress

y = distance

E = Young Modulus

R = Radius

Considering $\frac{M}{I} = \frac{\sigma_b}{y}$ — (1)

we know that $I = \frac{\pi}{64} \times d^4$

$$y = \frac{d}{2}$$

Now Substituting the value of I & y in eqn (1)

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}}$$

$$M = \frac{\pi}{32} \times \sigma_b \times d^3$$

* In case hollow shaft

$$M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1-K^4)$$

* Shaft Subjected to combined twisting moment and Bending moment

(1) Maximum shear stress theory or Guest theory

(2) Maximum normal stress theory or Rankine theory.

Let τ = shear stress

σ_b = Bending stress

According to maximum shear stress theory

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{M\,32}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\left(\sqrt{M^2 + T^2}\right)$$

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

when $\sqrt{M^2 + T^2}$ = equivalent twisting moment

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3$$

According to maximum Normal Stress.

$$\sigma_{b\,max} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4 \times \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left(\frac{1}{2}M + \sqrt{M^2 + T^2}\right)$$

$$\frac{\pi}{32} \times \sigma_{b(max)} \times d^2 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

$\frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$ = equivalent Bending moment.

$$M_e = \frac{1}{2}\left(M^2 + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b \times d^3$$

* In case hollow shaft.

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\, (d_o^3)\,(1 - k^4)$$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

* Keys & Coupling

Keys:- A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect those together in order to prevent relative motion.

Types of Keys

1. Sunk Key
2. Saddle Key
3. Tangent key
4. Round key
5. Splines Key.

Scenk key

Scenk key also devoted in many typ

(i) Rectangular sunk key

(ii) Square sunk key

(iii) Parallel sunk key

(iv) Gib-head key

(v) Feather key

width of key $w = \frac{d}{4}$

$t = \frac{2w}{3} = \frac{d}{6}$

Strength of a Sunk key

Let T = Torque transmitted by the shaft

F = Tangential force

d = diameter of shaft

L = Length of key

w = width of key

t = thickness of key

τ & σ_c = shear & crushing stress

Considering shearing stress

Tangential shearing force

F = Area of resisting shear × shear stress

= $L \times w \times \tau$

Torque transmitted

$$T = F \times \frac{d}{2}$$

$$\boxed{T = L \times \omega \times \tau \times \frac{d}{2}} \quad \text{--- (I)}$$

Considering crushing of the stress

Tangential crushing force

F = Area resisting crushing X Crushing stress

$$= L \times \frac{t}{2} \times \sigma_c$$

Torque transmitted

$$T = F \times \frac{d}{2} = L \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$\boxed{T = L \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}} \quad \text{--- (II)}$$

equating eqn (I) & (II) we found

$$L \times \omega \times \tau \times \frac{d}{2} = L \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$\boxed{\frac{\omega}{t} = \frac{\sigma_c}{2\tau}}$$

we know that Shearing strength of key

$$T = L \times \omega \times \tau \times \frac{d}{2} \quad \text{--- (II)}$$

Torsional Shear strength of the shaft

$$T = \frac{\pi}{16} \times \tau \times d^3 \quad \text{--- (II)}$$

Now substituting the value of T in eqn (III)

$$L \times \omega \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau \times d^3$$

$$L = \frac{\pi d}{2} \times \frac{d}{2}$$

When the key material is same as that of the shaft then $\tau = \tau_1$

$l = 1.571d$.

* Effect of Keyways

The following relation is used for finding out the ~~ke~~ effect ~~of~~ of keyway

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

~~where e = effect of~~

where e = Effect key ways

w = width of keys

d = diameter of shaft

h = Depth of keyway

= thickness of key (t)

$$\boxed{h = \frac{t}{2}}$$

* In case the keyway is too long and the key of sliding type

$$K_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where K_θ = Reduction factor for angular twist.

Problem

A 15 KW 960 r.p.m motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear & crushing stresses for the mild steel key are 56 Mpa and 112 Mpa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.

Soln Data given

$p = 15\ KW = 15 \times 10^3\ W$

$N = 960\ rpm$

$d = 40\ mm$

$L = 75\ mm$

$\tau = 56\ MPa = 56 \times 10^{\cancel{6}}\ N/mm^2$

$\sigma_c = 112\ MPa = 112\ N/mm^2$

We know that Torque transmitting

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2 \times 960 \times \pi} = 149 \times 10^3\ N\text{-}mm$$

Let w = width of the key.

considering shear of the key

$$T = L \times w \times \tau \times \frac{d}{2}$$

$$149 \times 10^3 = 75 \times w \times 56 \times \frac{40}{2}$$

$$w = 1.8\ mm$$

This width of keyway is too small. The width of the keyway should be ~~atleas~~ atleast $\frac{d}{4}$

$$w = \frac{d}{4} = \frac{40}{4} = 10\ mm$$

Since $\sigma_c = 2\tau$, therefore ~~a squ~~ the key is in square

So $w = t = 10$ mm

According to H. F moore, the shaft factor :

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) \quad \left[h =\right.$$

$$= 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right)1.1$$

$$= 0.812.$$

Strength of shaft with keyway

$$= \frac{\pi}{16} \times \tau \times d^3 \times e$$

$$= \frac{\pi}{16} \times 56 \times (40)^3 \times 0.812 = 5718$$

Shearing strength of the key

$$= l \times w \times \tau \times \frac{d}{2}$$

$$= 75 \times 10 \times 56 \times \frac{40}{2} = 840000 \text{ N}$$

$$\therefore \frac{\text{Shear Strength of the key}}{\text{Normal Strength of the Shaft}} = \frac{8400}{5718}$$

$$= 1.47$$

Shaft coupling

Shafts are usually available upto 7 metres Length due to inconvenience in transport. In order to have a great length, it becomes necessary to joint two or more piece of the shaft by means of a coupling.

* **Coupling ~~are~~ done for following purposes**

1. To provide for the connection of shafts of units are manufactured separately.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock load from one shaft to an other.
4. To introduce protection against overloads.

* **Requirement of a Good shaft coupling**

A good shaft coupling should have following requirement.

(i) It should be easy to connect or disconnect
(ii) It should transmit the full power one shaft to an other shaft without loses
(iii) It should hold the shaft in perfect alignment
(iv) It should reduce the transmission of shock loads from one shaft to ~~an oth~~ an other shaft
(v) It should have no project of any parts.

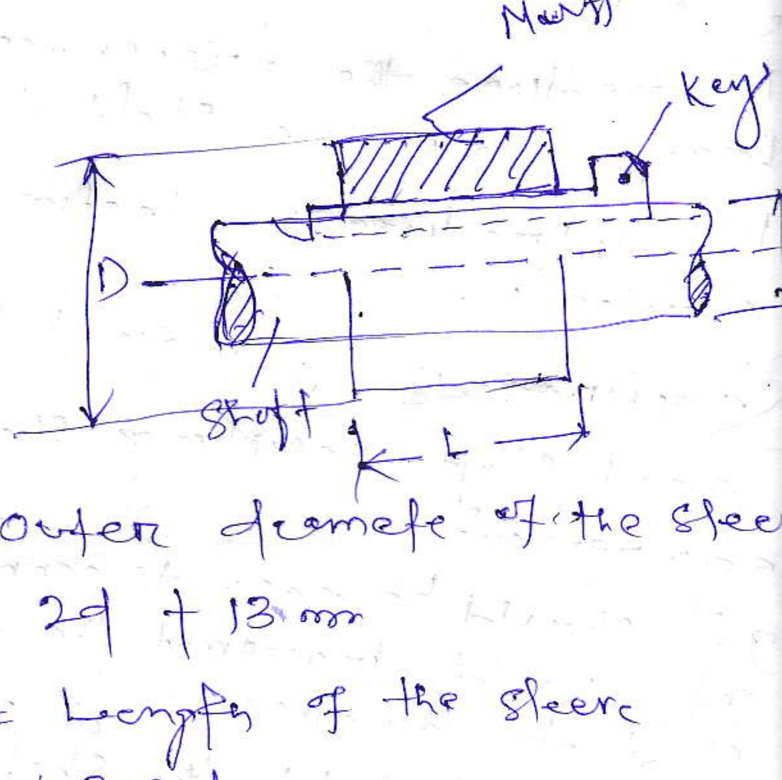
* Types of shaft couplings

It is mainly two types

(i) Rigid coupling → Sleeve or Muff coupling, clamp or compression coupling, Flange coupling

(ii) Flexible coupling → Bushed pin..., → Universal c..., → Oldham co...

* Sleeve or Muff - coupling



Let D = Outer diameter of the slee[ve]

= 2d + 13 mm

L = Length of the sleeve

= 3.5 d.

1. Design for sleeve

Let T = Torque transmitted

τ_c = permissible shear stress

We know that torque transmitted

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right)$$

$$T = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \quad \left[\because k = \frac{d}{D} \right]$$

Design for Key

The length of the coupling ~~i~~ key is atleast equal to the length of the sleeve (i.e 3.5d)
The ~~length~~ coupling key is usually made into two parts so that the length of key in each shaft $l = \frac{L}{2} = \frac{3.5d}{2}$

Torque transmitted

$$T = L \times w \times \tau \times \frac{d}{2} \quad \text{(shearing of the key)}$$

$$= L \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{(Crushing of the key)}$$

Problem: Design and make neat dimensioned sketch a muff coupling which is used to connect two steel shafts transmitting 40 KW at 350 r.p.m. The materials for the shaft and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa, respectively. The material for muff is cast iron for which the allowable shear stress may be assume as 15 MPa

Sol^n Data given

$P = 40\ KW = 40 \times 10^3\ W$

$N = 350$ r.p.m

$\tau_s = 40\ MPa = 40\ N/mm^2$

$\sigma_{cs} = 80\ MPa = 80\ N/mm^2$

$\tau_c = 15\ MPa = 15\ N/mm^2$

1. Design for shaft

Let d = diameter of shaft

we know that

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350}$$

$$= 1100 \text{ N-m} = 1100 \times 10^3$$

Again ~~H~~ =

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3$$

$$d = 52 \text{ mm say } 55$$

2. Design ~~f~~for sleeve

Outer diameter of the Muff

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say}$$

or 125 mm

L = Length of the Muff

$$= 3.5d = 3.5 \times 55 = 192.5 \text{ mm} \simeq 195 \text{ m}$$

we know that Torque (T)

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

$$= \frac{\pi}{16} \times \tau_c \left(\frac{125^4 - 55^4}{125} \right)$$

$$\tau = 2.97 \text{ N/mm}^2$$

3. Design for Key

~~From~~ According to diameter 55 mm we found from data book $w = 18$ m and the Key is Square type. $[\because \sigma_c = 2\tau]$

So, $t = w = 18$ mm

The length of key in each shaft

$$l = \frac{L}{2} = \frac{195}{2} = 97.5 \text{ m}$$

we know that ~~(T)~~ torque (T)

$$1100 \times 10^3 = L \times w \times \tau_s \times \frac{d}{2}$$ (for shearing of the key)

$$= 97.5 \times 18 \times \tau_s \times \frac{55}{2}$$

$$\tau_s = 22.8 \text{ N/mm}^2$$

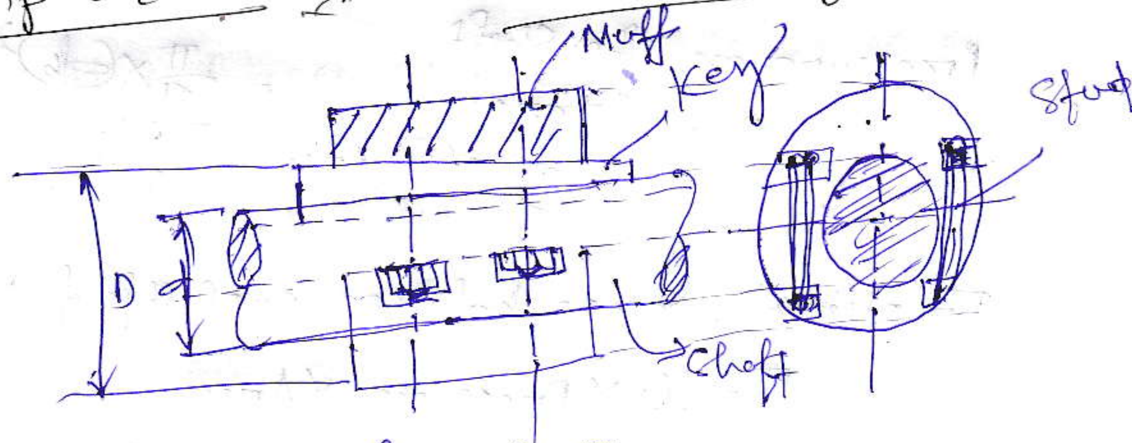
According crushing

$$T = L \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2}$$

$$\Rightarrow 1100 \times 10^3 = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2}$$

$$\sigma_{cs} = 45.6 \text{ N/mm}^2$$ (Ans

* Clamp or Compression coupling



Let D = Outer diameter of Sleeve

$= 2d + 13$ mm

L = length of muff

$= 3.5d$

d = diameter of the shaft.

1. <u>Design of Muff and Key</u>

It is done in previous discussion

2. <u>Design of clamping bolts</u>

Let T = Torque transmitted

d = diameter of shaft

d_b = Root or effective diameter of

n = No. of bolt

σ_t = permissible tensile stress

μ = coefficient of friction

L = Length of Muff.

We know the force exerted by the b

$$= \frac{\pi}{4} \times (d_b)^2 \times \sigma_t$$

Force exerted by the bolt on each side of

$$= \frac{\pi}{4} \times (d_b)^2 \times \sigma_t \times \frac{n}{2}$$

Pressure on the shaft

$$p = \frac{\text{Force}}{\text{projected Area}} = \frac{\frac{\pi}{4} \times (d_b)^2 \times \sigma_t}{\frac{1}{2} L \times d}$$

Frictional force between each shaft & Muf

$$F = \mu \times \text{Pressure} \times \text{Area}$$

$$= \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4} (d_b)^2 \times \sigma_t \times \frac{n}{2}}{\frac{1}{2} \times L \times d} \times \frac{1}{2} \pi d \times$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \times \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8} \times (d_b)^2 \times \sigma_t \times n$$

Torque transmitted by the coupling

$$T = F \times \frac{d}{2}$$

$$= \mu \times \frac{\pi^2}{8} \times (d_b)^2 \times \sigma_t \times n \times \frac{d}{2}$$

$$\boxed{T = \frac{\pi^2}{16} \times \mu \times (d_b)^2 \times \sigma_t \times n \times d}$$

* Spring:

A Spring is defined as an elastic body whose function is to distort when loaded and to recover its original shape when the load is removed.

Various application of Spring

(I) To cushion, absorb or control energy due to either shock

(II) To apply forces, as brake, clutches and spring loaded valves.

(III) To control motion by maintaining contact between two element as cam & follower

(IV) To measure forces as in spring balances and engines indicators

(V) To store energy as in watches, toys etc

Types of Springs

There are various type of spring

1. Helical spring
2. Conical and volute springs
3. Torsion spring
4. Leaf spring
5. Disc or belleville spring
6. Special ~~purp~~ purposes spring.

* Material for helical spring

1. The material of the ~~sph~~ spring should high fatigue strength, high ductility, high resilience and it should be creep resistant.

* Terms used in Compression Spring

1. Solid length → under loaded condition
2. ~~Solo~~ Solid length of spring (L_s)

$$L_s = n' d$$

where n' = no. of coil

d = diameter of wire

2. Free length (L_f) =

= Solid length + Maximum compression + clearance b/w two adjacent coils

$$= n'd + \delta_{max} + 0.15\,\delta_{max}$$

The following relation are used find out the Free length of spring

$L_f = n'd + \delta_{max} + (n'-1) \times 1\,mm$

(3) Spring index (C) $= \frac{D}{d}$

where C = Spring index

D = Mean dia of coil

d = diameter of wire

(4) Spring rate (K) $= \frac{W}{\delta}$

where K = Spring rate

W = Load

δ = deflection of the spring.

* The spring having loop on both end.

The total no of turn $\boxed{n' = n + 1}$

Stresses in Helical spring of circular wire

Let D = Mean diameter of coil

d = dia of wire

n = no of turn of coil

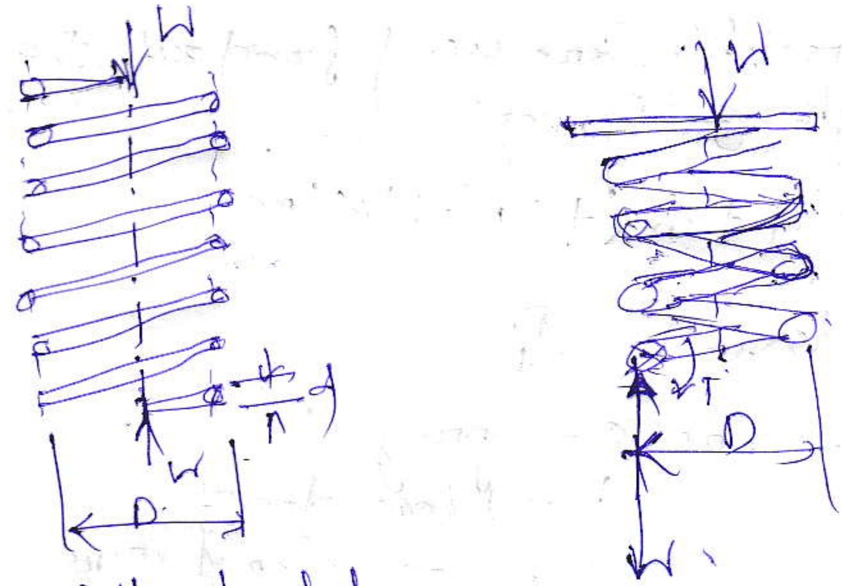
G = Modulus of rigidity

W = Axial load

C = Spring index

p = pitch of the coil

δ = deflection of spring



(a) Axially loaded helical spring (1) Torsional shear of the helical spring

We know that twisting moment

$$T = W \times \frac{D}{2}$$

$$\frac{\pi}{16} \times \tau_1 \times d^3 = W \times \frac{D}{2}$$

$$\tau_1 = \frac{8WD}{\pi d^3}$$

* The following stresses are also acting on the wire

1. Direct shear stress due to the load
(2) ~~Shear~~ stress due to curvature of wire

direct shear stress due to load W

$$\tau_2 = \frac{\text{Load}}{\text{cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2}$$

So total or resultant stress ~~produce~~ induced on the wire

$$\tau = \tau_1 + \tau_2$$

$$= \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2}$$

~~Maximum~~ maximum shear stress induced in the wire.

= ~~Total~~ Torsional ~~shear~~ shear stress + Direct shear stress

$$= \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2}$$

$$= \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2}$$

$$= \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2}$$

$$= \frac{8WD}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8WD}{\pi d^3}\left(1 + \frac{1}{2c}\right)$$

$$= K_s \times \frac{8WD}{\pi d^3} \quad \left(\frac{D}{d} = c\right)$$

where K_s = shear stress factor

$= 1 + \frac{1}{2c}$

Maximum shear stress induced in the wire

$$\tau = K_s \times \frac{8WD}{\pi d^3} = K \frac{8WC}{\pi d^2}$$

where $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$ → , Wahl's

The values of K for a given spring may be obtained ~~fro~~

The wahl's stress factor (K) may be as composed of two sub-factor K_s such that

$$K = K_s \times K_c$$

where K_s = stress factor due to sh

K_c = stress concentration fa due to curvature

* Deflection of Helical Springs of Circ

Total active length of the wire

L = Length of one coil × no of active

$= \pi D \times n$.

θ = Angular deflection

Let

Axial deflection of the spring

$$\delta = \theta \times \frac{D}{2} \quad \text{--- (i)}$$

We know that Torque eqn

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

Considering $\cancel{\frac{T}{J}} = \cancel{\frac{\tau}{d/2}}$ $\boxed{\frac{T}{J} = \frac{G\theta}{l}}$

$$\theta = \frac{Tl}{GJ} \quad \text{(ii)}$$

where J = polar moment of inertia of the spring

$$= \frac{\pi}{32} \times d^4$$

G = Modulus of rigidity.

Now substituting the value of J & G in eqn (ii)

$$\theta = \frac{Tl}{JG} = \frac{\left(W \times \frac{D}{2}\right)\pi D n}{\frac{\pi}{32} \times d^4 G} = \frac{16 W D^2 n}{G d^4}$$

Substituting the value of θ in eqn (i)

$$\delta = \frac{16 W D^2 n}{G d^4} \times \frac{D}{2} = \frac{8 W D^3 n}{G d^4}$$

$$= \frac{8 W C^3 n}{G d} \quad \left[\because C = \frac{D}{d}\right]$$

The stiffness of the spring or spring rate

$$\frac{W}{\delta} = \frac{G d^4}{8 D^3 n} = \frac{G d}{8 C^3 n} = \text{constant}.$$

* Eccentric loading of springs (e)

When the load is offset by a distance e from spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor $\frac{D}{}$ when D = Mean dia of

Buckling of Compression Springs

The critical axial load (W_{cr}) may be cal[culated] by following relation

$$W_{cr} = K \times K_B \times L_F$$

where K = spring rate or spring stiff[ness] $= \frac{W}{\delta}$

L_F = Free length of spring

K_B = Buckling factor depen[ding] upon the ratio $\frac{L_F}{D}$

* Surge in Springs

when one end of a helical spring is [resting] on a rigid support and the other end [is] loaded suddenly, then all the coils [of the] spring will not suddenly deflected equally, because some time is requ[ired] for the propagation of stress along the [spring] wire. A little consideration will be show [that at] the beginning, the end coils of the spri[ng] in contact with the applied load take [up] whole deflection and then it transmits large parts of its deflection to the adjacent coils. This phenomenon is [known as] surge in spring.

It has been found that the natur[al] frequency of spring should be atle[ast] twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency of spring clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 n}\sqrt{\frac{6Gg}{\rho}} \text{ cycle/s}$$

where f_n = Natural frequency
d = diameter of wire
D = diameter of coil
n = no. of active coil
G = Modulus of rigidity
g = Accⁿ ~~due~~ due to gravity
ρ = density

* A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is ~~up~~ 350 MPa and Modulus of rigidity 84 KN/mm², Find the axial load which the spring ~~carry~~ can carry and deflection per active turn.

Solⁿ $d = 6\,mm$
$D_o = 75\,mm$
$\tau = 350\,MPa = 350\,N/mm^2$
$G = 84\,KN/mm^2 = 84 \times 10^3\,N/mm^2$
we know that Mean diameter of coil
$D = D_o - d = 75 - 6 = 69\,mm$

Spring Index $c = \frac{D}{d} = \frac{69}{6} = 11.5$

Let W = Axial load

δ/n = Deflection per active turn

1. Neglecting the effect of curvature

We know that shear stress factor

$$K_s = 1 + \frac{1}{2c} = 1 + \frac{1}{2 \times 11.5} = 1$$

Maximum shear stress induced in the

$$\tau \quad 350 = K_s \times \frac{8WD}{\pi d^3}$$

$$= 1.043 \times \frac{8W \times 69}{\pi \times 6^3} = 0$$

$$W = 412.7\ N \quad \text{Ans}$$

We know that deflection of the spr

$$\delta = \frac{8WD^3 n}{Gd^4}$$

Deflection per active turn

$$\frac{\delta}{n} = \frac{8WD^3}{Gd^4} = \frac{8 \times 41}{84 \times}$$

$$= 9.96\ mm \quad \text{Ans}$$

2. Considering the effect of curvature.

We know that Wahl's factor.

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5}$$

$$= 1.123$$

The maximum shear stress induced in wire (τ)

$$350 = K \times \frac{8 W . C}{\pi d^2}$$

$$= 1.123 \times \frac{8 \times W \times 11.5}{\pi \times (6)^2}$$

$$W = 383.4 N \text{ (Ans}$$

Deflection of the spring

$$\delta = \frac{8 W D^3 n}{G d^4}$$

Deflection per active turn

$$\frac{\delta}{n} = \frac{8 W D^3}{G d^4}$$

$$= \frac{8 \times 383.4 \times (69)^3}{84 \times 10^3 \times 6^4}$$

$$= 9.26 \text{ mm (Ans}$$