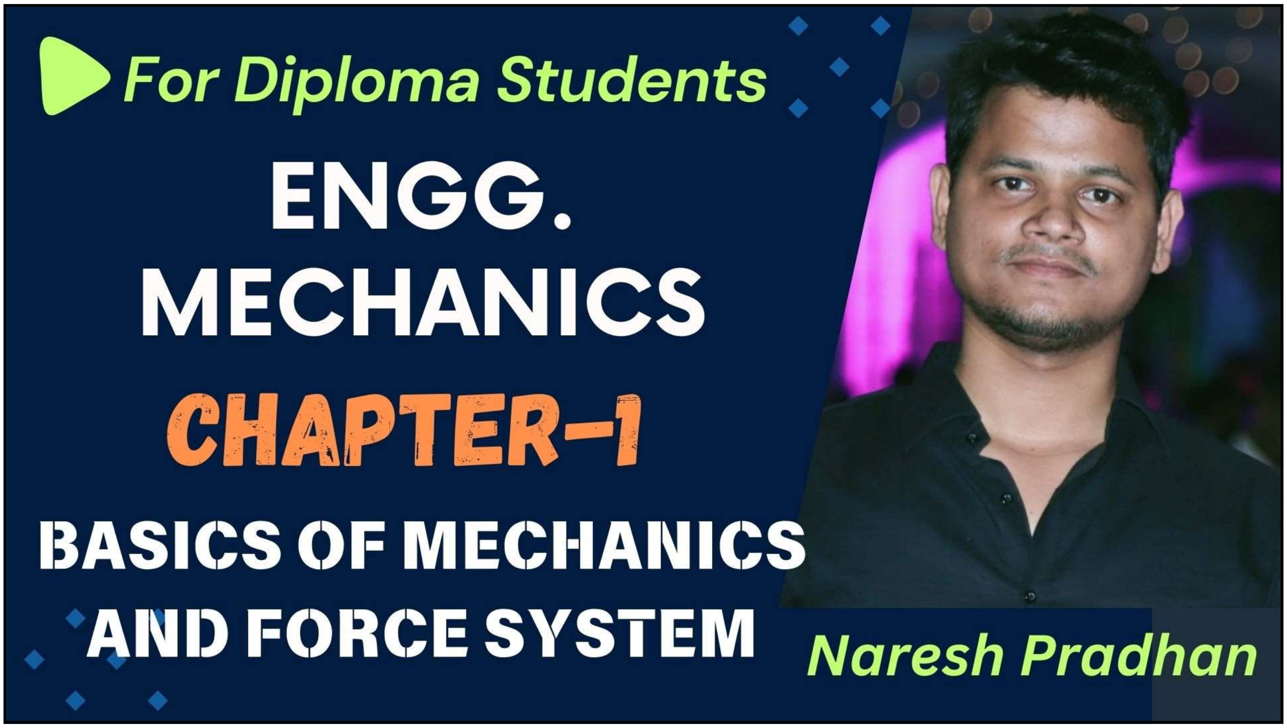
Bhubanananda Odisha School of Engineering, Cuttack

Department of Mechanical Engineering

LECTURER NOTES

SUBECT- ENGINEERING MECHANICS (TH-04)

PREPARED BY- NARESH PRADHAN



1. **FUNDAMENTALS OF ENGINEERING MECHANICS**

1.1 Fundamentals.

Definitions of Mechanics, Statics, Dynamics, Rigid Bodies,

1.2 Force

Force System.

Definition, Classification of force system according to plane & line of action.

Characteristics of Force & effect of Force. Principles of Transmissibility & Principles of Superposition. Action & Reaction Forces & concept of Free Body Diagram.

1.3 Resolution of a Force.

Definition, Method of Resolution, Types of Component forces, Perpendicular components & non-perpendicular components.

1.4 Composition of Forces.

Definition, Resultant Force, Method of composition of forces, such as

1.4.1 Analytical Method such as Law of Parallelogram of forces & method of resolution.

1.4.2. Graphical Method.

Introduction, Space diagram, Vector diagram, Polygon law of forces.

1.4.3 Resultant of concurrent, non-concurrent & parallel force system by Analytical & Graphical Method.

1.5 Moment of Force.

Definition, Geometrical meaning of moment of a force, measurement of moment of a force & its S.I units. Classification of moments according to direction of rotation, sign convention, Law of moments, Varignon's Theorem, Couple – Definition, S.I. units, measurement of couple, properties of couple.

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Mechanics

- Engineering mechanics is the branch of science which deals with the study of forces and its effect on the body.

- So it is basically study of *state of rigid body* under the action of external forces.

Rest Motion

Deformation = 0

Change in size & shape = 0

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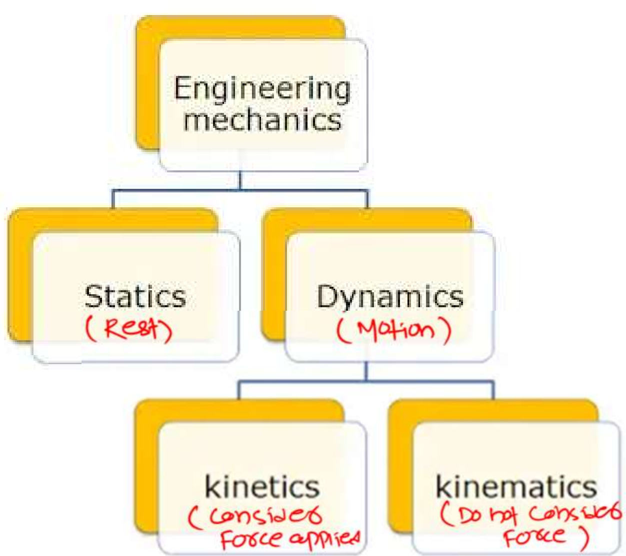
Classification of Engineering Mechanics



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Force

- Force is an agent, which produce or tends to produce, destroys or tends to destroy the state of body, which is either at rest or in motion.

- Force is an external agent capable of changing a body's state of rest or motion.

$$\text{Force} = \text{Mass} \times \text{Acceleration} = m \times a = kg \times \frac{m}{s^2} = \text{Newton} = N$$

So $1N = \frac{1kg \cdot m}{s^2} = \frac{1000gm \times 100cm}{s^2} = \frac{10^3 gm \times 10^2 cm}{s^2} = \frac{10^5 gm \cdot cm}{s^2} = 10^5$ dyne

$\boxed{\therefore\ 1N = 10^5 \text{ dyne}}$

$\because$ $kg \cdot m/s^2 = N$ (S.I unit), $gm \cdot cm/s^2 =$ dyne (CGS unit)

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Characteristics of Force

a) The magnitude of force (eg → 1N, 5N, 10N, 500N etc)

b) The direction of the force (horizontal or vertical or Inclined)

c) The nature of force

Push / Pull

d) Line of action

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Effects of Force

1. Force can make a body that is at rest to move.
2. It can stop a moving body or slow it down.
3. It can accelerate the speed of a moving body.
4. It can also change the direction of a moving body.

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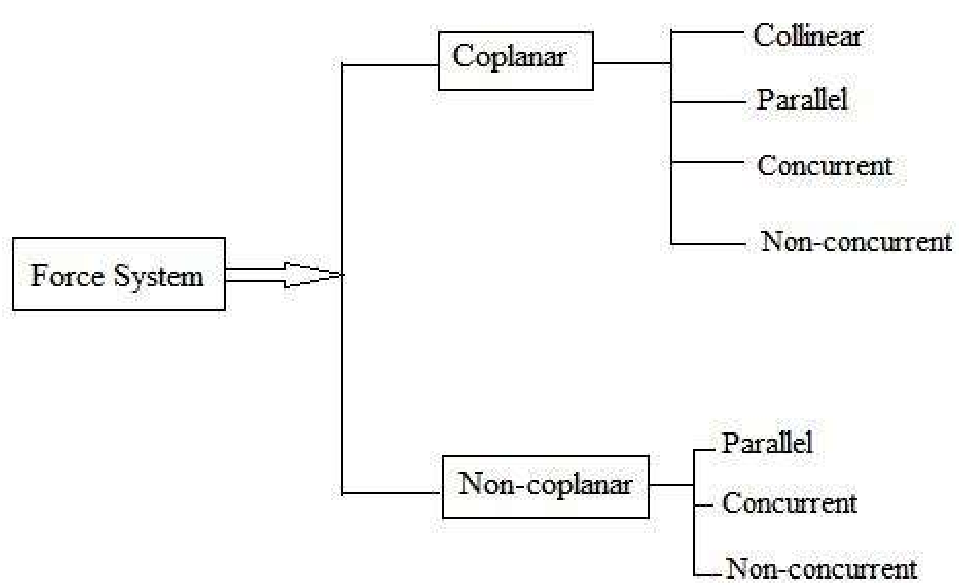

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Collinear Force System

When the lines of action of all the forces of a system **act along the same line**, this force system is called collinear force system.

Non-Collinear Force System

When the lines of action of all the forces of a system **do not act along the same line**, this force system is called collinear force system.

Coplanar Force System

When the lines of action of a set of forces **lie in a single plane** is called coplanar force system.

Non-Coplanar Force System

When the line of action of all the forces **do not lie in one plane**, is called Non-coplanar force system

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Concurrent Force System

- The forces when extended **pass through a single point** and the point is called point of concurrency.
- The lines of actions of all forces meet at the point of concurrency.
- Concurrent forces may or may not be coplanar.

Non-concurrent Force System

- When the forces of a system **do not meet at a common point** of concurrency, this type of force system is called non-concurrent force system.
- Parallel forces are the example of this type of force system.
- Non-concurrent forces may be coplanar or non-coplanar.

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Principle Of Transmissibility:

- It says the point of application of the force can be moved anywhere along the line of action of force without changing the external effects (magnitude) of the force on the body
- The force can be represented anywhere along the line of action without changing the effect of the forces on any rigid body

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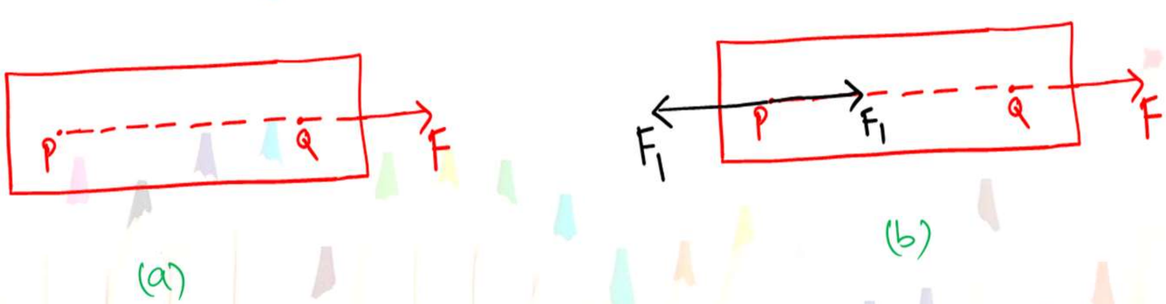
Principle Of Superposition:

- The action of a given system of forces on a rigid body will never changed if we add or subtract another system of forces in equilibrium



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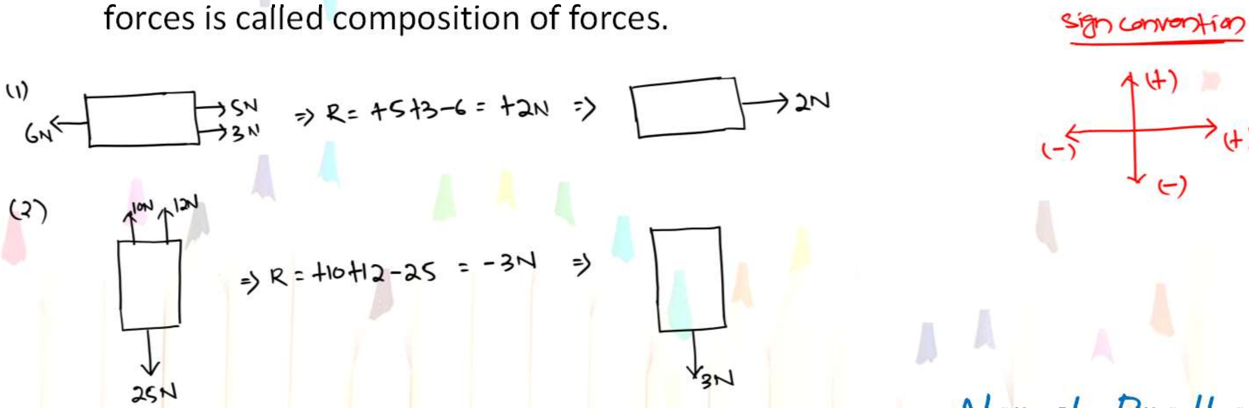
Resultant Of Forces

- A single force which produces the same effect as a number of forces acting together is called the resultant of forces.
- And the process of finding out the resultant force of a number of given forces is called composition of forces.



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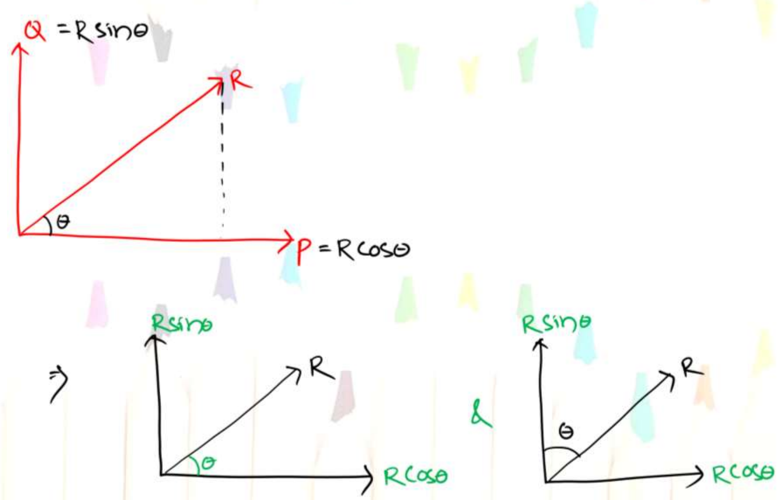
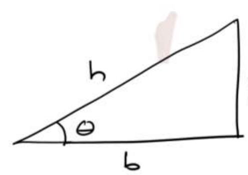
Resolution Of Forces

The process of splitting up the given forces into a number of components without changing its effect on a body is called resolution of force.





$\sin\theta = p/h$

$\Rightarrow p = h\sin\theta$ ✓

$\cos\theta = b/h$

$\Rightarrow b = h\cos\theta$ ✓

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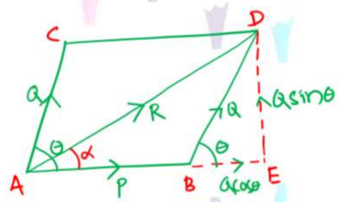
Parallelogram Law Of Forces

- If two forces are acting at a point and are represented in both magnitude and direction by the adjacent sides of the parallelogram, then the diagonal passing through their point of intersection, represents the resultant in both magnitude and direction.



In $\triangle ADE$, $(AD)^2 = (DE)^2 + (AE)^2$

$\Rightarrow (AD)^2 = (DE)^2 + (AB+BE)^2$

$\Rightarrow R^2 = (Q\sin\theta)^2 + (P + Q\cos\theta)^2$

$\Rightarrow R^2 = Q^2\sin^2\theta + P^2 + Q^2\cos^2\theta + 2PQ\cos\theta$

$\Rightarrow R^2 = P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta$

$\Rightarrow R^2 = P^2 + Q^2 \times 1 + 2PQ\cos\theta$

$\Rightarrow R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\Rightarrow \boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$

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Case-1 (If $\theta = 0°$)

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ\cos 0}$

$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ \times 1}$

$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ}$

$\Rightarrow R = \sqrt{(P+Q)^2}$

$\Rightarrow \boxed{R = P + Q}$

Case-2 (If $\theta = 180°$)

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{P^2 + Q^2 + 2PQ\cos 180}$

$= \sqrt{P^2 + Q^2 + 2PQ \times (-1)}$

$= \sqrt{P^2 + Q^2 - 2PQ}$

$= \sqrt{(P-Q)^2}$

$\Rightarrow \boxed{R = P - Q}$

Case-3 (If $\theta = 90°$)

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{P^2 + Q^2 + 2PQ\cos 90}$

$= \sqrt{P^2 + Q^2 + 2PQ \times 0}$

$= \sqrt{P^2 + Q^2 + 0}$

$\Rightarrow \boxed{R = \sqrt{P^2 + Q^2}}$

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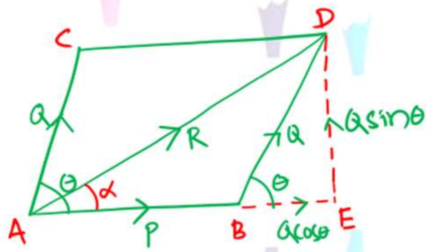

In ΔADE, $\tan\alpha = \frac{DE}{AE}$

$$\Rightarrow \tan\alpha = \frac{DE}{AB+BE}$$

$$\Rightarrow \tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)}$$

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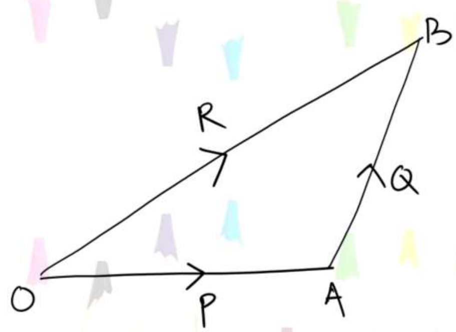
Triangle Law Of Forces

- If two forces are acting as the adjacent sides of a triangle taken in order, then the closing sides of the triangle represents the resultant in opposite direction.



$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$\Rightarrow P + Q = R$$

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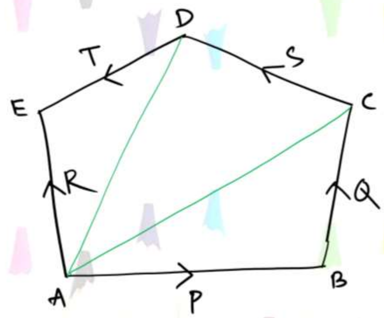
Polygon Law Of Forces

- If any number of coplanar concurrent forces can be represented in magnitude and direction by the sides of a polygon taken in order; then their resultant will be represented by the closing side of the polygon taken in opposite order.



$\vec{AC} = \vec{AB} + \vec{BC}$

$\vec{AD} = \vec{AC} + \vec{CD}$

$= \vec{AB} + \vec{BC} + \vec{CD}$

$\vec{AE} = \vec{AD} + \vec{DE}$

$= \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE}$

$\Rightarrow R = P + Q + S + T$

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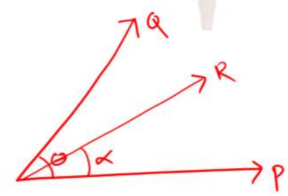
Q1: *Two forces 5N and 3N act an angle 60^0 with each other. Find the resultant of the forces and angle at which the resultant act.*

Given: Bigger Force = P = 5N , smaller Force = 3N ,
Angle between them = $\theta = 60^\circ$, Resultant = R = ? , α = ?



we know Resultant $= R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{5^2 + 3^2 + 2\times5\times3\times\cos 60}$

$= \sqrt{25 + 9 + 30\times\frac{1}{2}}$

$= \sqrt{25+9+15}$

$= \sqrt{49} = 7N \Rightarrow R = 7N$ Ans

Now $\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right) = \tan^{-1}\left(\frac{3\times\sin 60}{5+3\times\cos 60}\right) \Rightarrow \alpha = 21.78^\circ$ Ans

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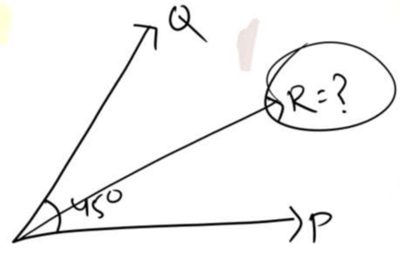
Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°

$P = 150N$, $Q = 100N$, $\theta = 45^\circ$, $R = ?$



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{(150)^2 + (100)^2 + 2 \times 150 \times 100 \times \cos 45}$$

$$= \sqrt{22500 + 10000 + 30000 \times \frac{1}{\sqrt{2}}}$$

$$= \sqrt{32500 + 21213.20}$$

$$= \sqrt{53713.20}$$

$$= \underline{231.76\ N} \approx 232N \ \text{Ans}$$

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The resultant of two forces i.e. P+Q and P-Q is $\sqrt{3P^2 + Q^2}$. Find the angle between the two forces.

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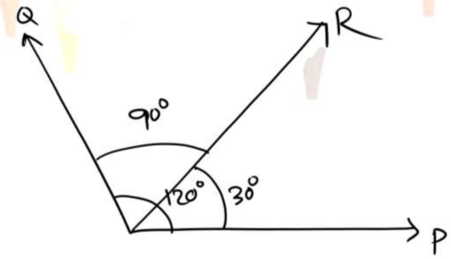
Two forces act an angle of 120^0. The bigger force is of 40N and the resultant is perpendicular to the smaller one. Find the smaller force.

$P = 40N$, $Q = ?$, $\alpha = 30^\circ$, $\theta = 120^\circ$



$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\Rightarrow \tan 30 = \frac{Q\sin 120}{40 + Q\cos 120}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{Q \times 0.866}{40 + Q \times (-0.5)}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{0.866Q}{40 - 0.5Q} \Rightarrow 40 - 0.5Q = 0.866 \times Q \times \sqrt{3}$$

$$\Rightarrow 40 - 0.5Q = 1.5Q \Rightarrow 40 = 1.5Q + 0.5Q \Rightarrow 40 = 2Q \Rightarrow Q = 20N$$

$Q = 20N$ Ans

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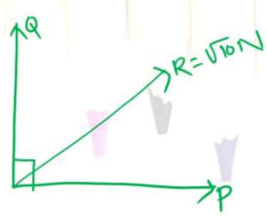
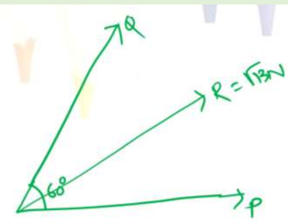
Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N . But if they Act at 60°, their resultant is $\sqrt{13}$ N



so $R = \sqrt{P^2 + Q^2}$

$\Rightarrow \sqrt{10} = \sqrt{P^2 + Q^2}$

$\Rightarrow P^2 + Q^2 = 10$ —(1)



$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow \sqrt{13} = \sqrt{P^2 + Q^2 + 2PQ\cos 60}$

$\Rightarrow \sqrt{13} = \sqrt{P^2 + Q^2 + 2PQ \times \frac{1}{2}}$

$\Rightarrow 13 = P^2 + Q^2 + PQ$

$\Rightarrow 13 = 10 + PQ$

$\Rightarrow PQ = 13 - 10 = 3$

$\Rightarrow PQ = 3$ —(2)

Now $(P+Q)^2 = P^2 + Q^2 + 2PQ = 10 + 2 \times 3 = 16$

$\Rightarrow (P+Q)^2 = 16 \Rightarrow P + Q = 4$ —(a)

& $(P-Q)^2 = P^2 + Q^2 - 2PQ = 10 - 2 \times 3 = 4 \Rightarrow P - Q = \sqrt{4} \Rightarrow P - Q = 2$ —(b)

From eqn (a) & (b)

$$\begin{array}{r} P + \cancel{Q} = 4 \\ P - \cancel{Q} = 2 \\ \hline 2P = 6 \end{array} \Rightarrow P = 3N$$

& $P + Q = 4$

$\Rightarrow 3 + Q = 4$

$\Rightarrow Q = 4 - 3 \Rightarrow Q = 1N$

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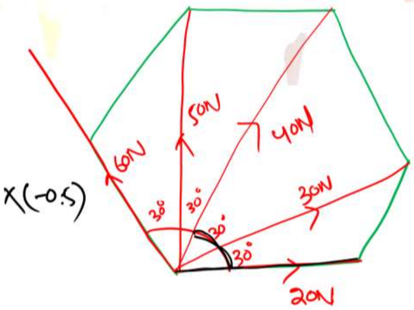
The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$



$$\Sigma H = 20\cos 0 + 30\cos 30 + 40\cos 60 + 50\cos 90 + 60\cos 120$$

$$= 20\times 1 + 30\times 0.866 + 40\times 0.5 + 50\times 0 + 60\times(-0.5)$$

$$= 20 + 25.98 + 20 + 0 - 30$$

$$= 35.98 = 36\text{ N}$$

$$\Sigma V = 20\sin 0 + 30\sin 30 + 40\sin 60 + 50\sin 90 + 60\sin 120$$

$$= 151.60\text{ N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36)^2 + (151.6)^2} = 155.8\text{ N}$$

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{151.6}{36}\right) = 76.64°$$

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Forces 10N, 20 N, 30 N, 40 N, 50 N and 60 N are acting along the side of a regular hexagon. Find out the resultant force.

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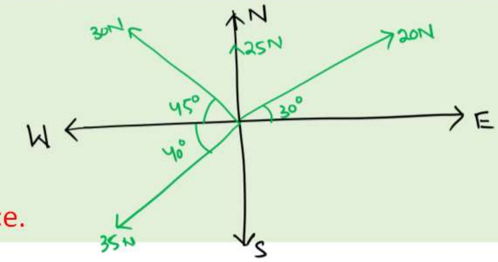
Imp

The following forces act at a point :
(i) 20 N inclined at 30° towards North of East,
(ii) 25 N towards North,
(iii) 30 N towards North West, and
(iv) 35 N inclined at 40° towards South of West.
Find the magnitude and direction of the resultant force.

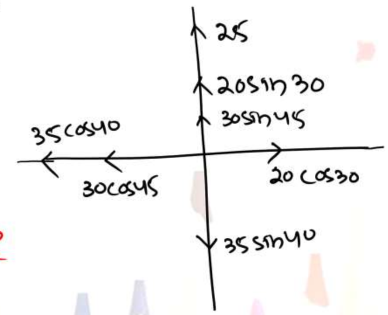


Sol

Resolving all the forces in horizontally (ΣH) & vertically (ΣV)

$$\Sigma H = 20\cos 30 - 35\cos 40 - 30\cos 45$$
$$= -30.7N$$

$$\Sigma V = 25 + 20\sin 30 + 30\sin 45 - 35\sin 40$$
$$= 33.7N$$



$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} \Rightarrow R = 45.6N$$ Ans

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{33.7}{-30.7}\right) \Rightarrow \alpha = -47.67^\circ$$

$$\Rightarrow \alpha = 180 - 47.67 = 132.33^\circ$$ Ans

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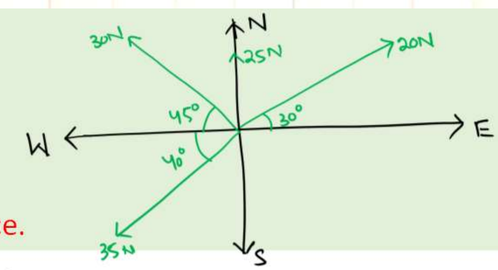
The following forces act at a point :
(i) 20 N inclined at 30° towards North of East,
(ii) 25 N towards North,
(iii) 30 N towards North West, and
(iv) 35 N inclined at 40° towards South of West.
Find the magnitude and direction of the resultant force.



Resolving all the forces horizontally (ΣH) & vertically (ΣV)

$$\Sigma H = 20\cos 30 + 25\cos 90 + 30\cos 135 + 35\cos 220$$
$$= -30.7N$$

$$\Sigma V = 20\sin 30 + 25\sin 90 + 30\sin 135 + 35\sin 220$$
$$= 33.7N$$

Now Resultant = $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} \Rightarrow R = 45.6N$ Ans

& Direction $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{33.7}{-30.7}\right) = -47.67^\circ$

$$\Rightarrow \alpha = 180 - 47.67 = 132.33^\circ$$ Ans

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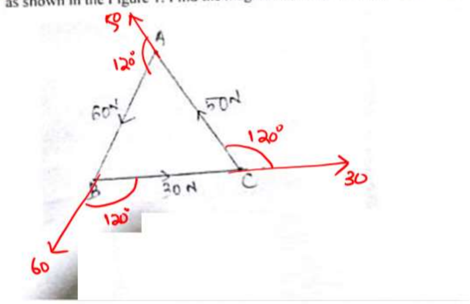
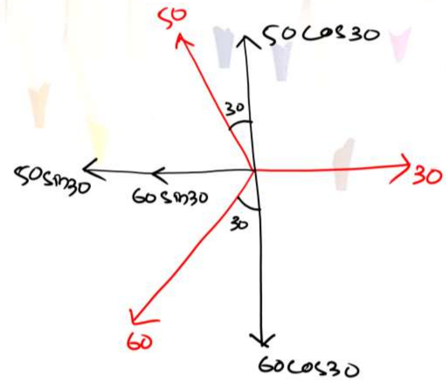
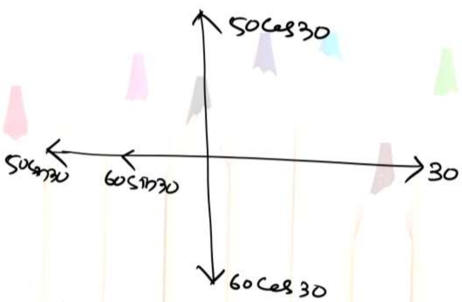
b. State and explain the Lami's Theorem.

c. Force 30N, 50N and 60N are acting along the three sides of an equilateral triangle ABC of sides 50cm as shown in the Figure 1. Find the magnitude, direction and position of resultant force.







$\Sigma H = 0$:- $30 - 50\sin 30 - 60\sin 30 = -25\,N$

$\Sigma V = 0 \Rightarrow 50\cos 30 - 60\cos 30 = -8.66\,N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 26.45\,N$

$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 19.1°$

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The resultant of two forces P and Q is R. if Q is doubled then the new resultant is perpendicular to P. then proof that Q =R.

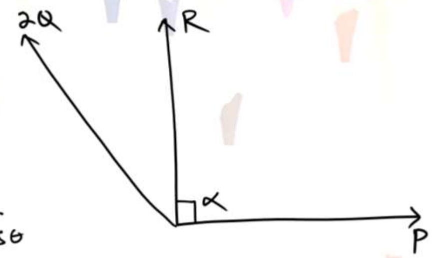
We know

$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$

$\Rightarrow \tan 90 = \frac{2Q\sin\theta}{P+2Q\cos\theta}$

$\Rightarrow \frac{\sin 90}{\cos 90} = \frac{2Q\sin\theta}{P+2Q\cos\theta} \Rightarrow \frac{1}{0} = \frac{2Q\sin\theta}{P+2Q\cos\theta}$

$\Rightarrow P+2Q\cos\theta = 0$



Now Resultant = $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\Rightarrow R = \sqrt{Q^2+P^2+2PQ\cos\theta}$

$\Rightarrow R = \sqrt{Q^2+P(P+2Q\cos\theta)}$

$\Rightarrow R = \sqrt{Q^2+P\times 0}$

$\Rightarrow R = \sqrt{Q^2+0} \Rightarrow R = \sqrt{Q^2} \Rightarrow \boxed{R = Q}$ Proved

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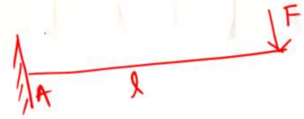
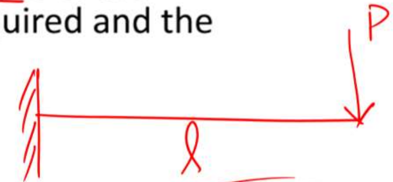
Moment



M

The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.



$$M = P \times l$$

- *P* = Force acting on the body, and
- *l* = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

$M = P \times l$

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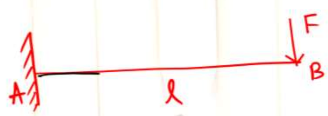
Moment



M = Force × perpendicular distance

$M_A = F \times l = Fl$, $M_B = F \times 0 = 0$

Clockwise

+ve

Anti-clockwise
or
Counter-clockwise

-ve

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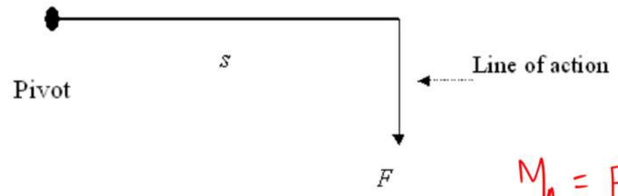

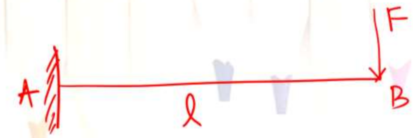

$M_A = F \times l$
$= N \times m$
$= N \times mm$
$= N \times cm$
$= kN \times m$
$= kN \times mm$
$= kN \times cm$

UNITS OF MOMENT

- kN-m
- N-mm
- N-m

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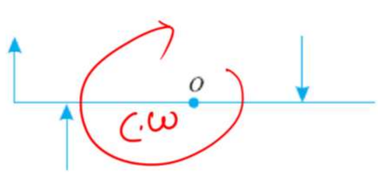
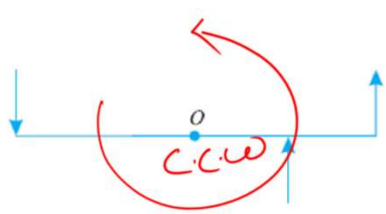
TYPES OF MOMENTS

- 1. Clockwise moments. (C·W)
- 2. Anticlockwise moments. (counter clockwise moment) (C.C.W)



(a) Clockwise moments



(b) Anticlockwise moments

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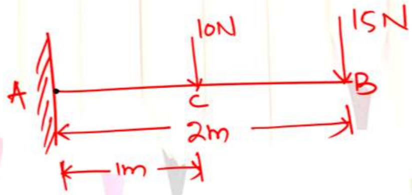
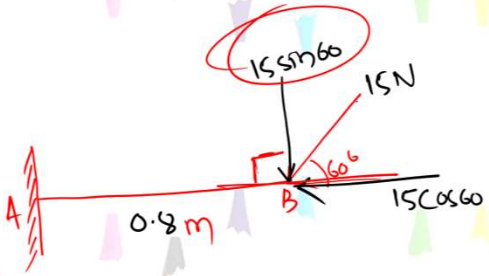


$$M_A = 15 \times 2 + 10 \times 1$$
$$= 40\,N{\cdot}m$$



$$M_A = 15\sin 60 \times 0.8 + 15\cancel{\cos 60 \times 0} = 10.39\,Nm$$
$$M_B = 15\cos 60 \times 0 + 15\sin 60 \times 0 = 0$$

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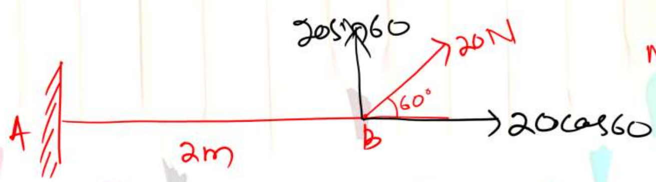
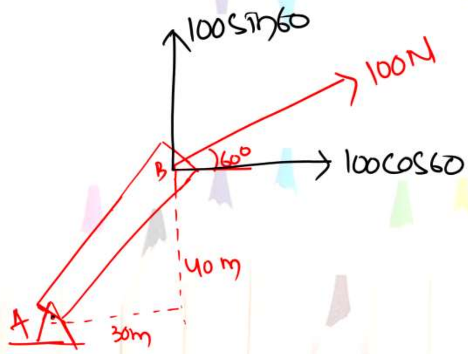


$$M_A = 20\sin 60 \times 2 + 20\cos 60 \times 0$$
$$= 34.64\,N{\cdot}m$$



$$M_A = -100\sin 60 \times 30 + 100\cos 60 \times 40$$
$$= -598.08\,Nm$$

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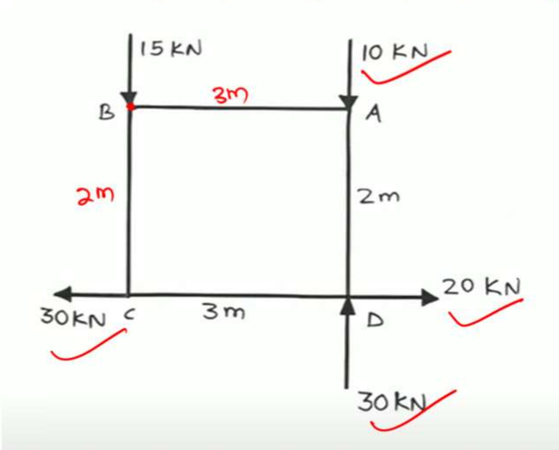
Calculate the moment about B for this force system.



$M_B = 10\times3 - 20\times2 - 30\times3 + 30\times2$

$= 30 - 40 - 90 + 60$

$= -40 \text{ kN·m}$

$= 40 \text{ kN·m}$ (Anti-clockwise)

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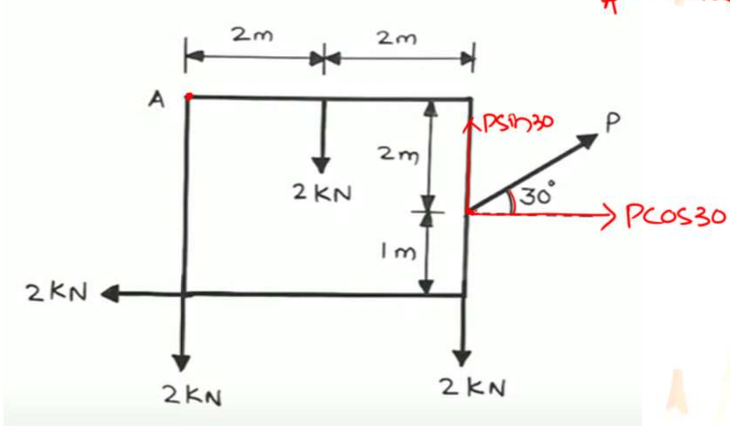
Find the value of force P so that moment about point A is zero.



$M_A = 2\times2 + 2\times4 + 2\times3$

$- P\sin30 \times 4 - P\cos30 \times 2 = 0$

$18 - P(4\sin30 + 2\cos30) = 0$

$\Rightarrow 18 - P \times 3.73 = 0$

$\Rightarrow 18 = 3.73P$

$\Rightarrow P = \frac{18}{3.73}$

$\Rightarrow P = 4.82 \text{ KN}$ Ans

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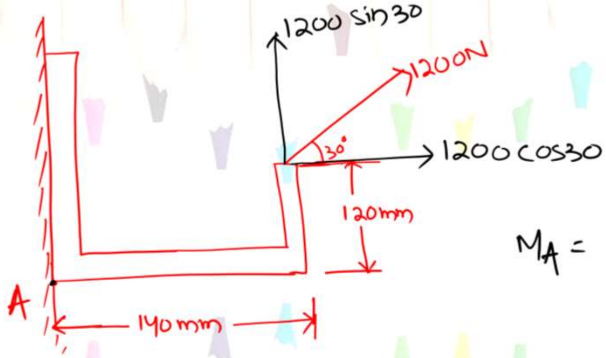

$$M_A = -1200\sin 30 \times 140 + 1200\cos 30 \times 120$$

$$= \underline{40707.65 \text{ N}\cdot\text{mm}} \quad (\text{Clockwise})$$

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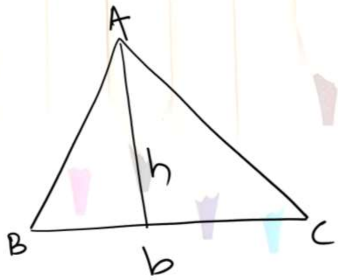

$$\text{Area of } \Delta \text{ ABC} = \frac{1}{2} \times b \times h = \frac{bh}{2}$$

Moment of b about A

$$M_A = b \times h$$

$$\text{Area} \times 2 = bh = M_A$$

$$\therefore \boxed{M = 2 \times \text{Area of } \Delta}$$

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VARIGNON'S PRINCIPLE OF MOMENTS (OR LAW OF MOMENTS)

- It states, "If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."

$$M_{P_O} + M_{Q_O} = M_{R_O}$$

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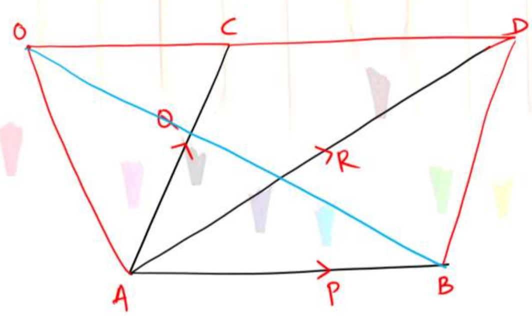

$$\boxed{M_{P_O} + M_{Q_O} = M_{R_O}}$$

$M_{P_O} = 2 \times$ Area of Δ AOB

$M_{Q_O} = 2 \times$ Area of Δ AOC

$M_{R_O} = 2 \times$ Area of Δ AOD

AOB = ABD = ACD

$$M_{P_O} + M_{Q_O} = 2 \times \Delta AOB + 2 \times \Delta AOC$$
$$= 2 \times \Delta ACD + 2 \times \Delta AOC$$
$$= 2(\Delta ACD + \Delta AOC)$$
$$= 2 \Delta AOD$$
$$= M_{R_O}$$ Proof

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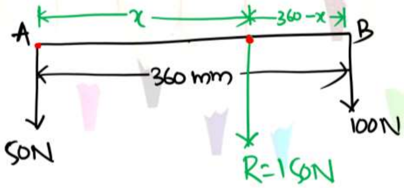
Two like parallel forces of 50 N and 100 N act at the ends of a rod 360 mm long. Find the magnitude of the resultant force and the point where it acts.



$R = -50 - 100 = -150N$

$x = 240mm$

Applying Varignon's Theorem,

$M_A = 100 \times 360 = 150 \times x$

$\Rightarrow x = \frac{100 \times 360}{150} = 240mm$

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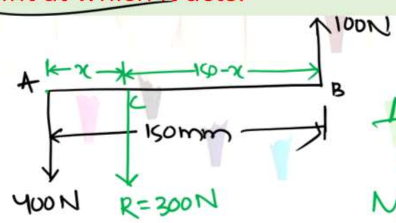
Two unlike parallel forces of magnitude 400 N and 100 N are acting in such a way that their lines of action are 150 mm apart. Determine the magnitude of the resultant force and the point at which it acts.



$R = -400 + 100 = -300N$

Applying Varignon's theorem

$M_A = 100 \times 150 = 300 \times x$

$\Rightarrow x = 50mm$

or $AC = 50mm$

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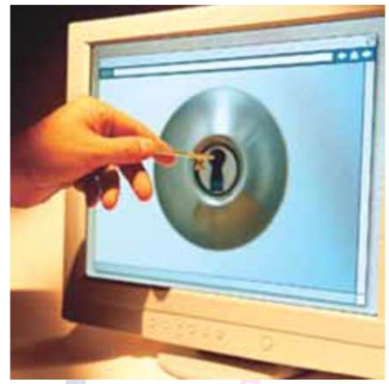
COUPLE

- A pair of two equal and unlike parallel forces (i.e. forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.



- As a matter of fact, a couple is unable to produce any translatory motion (i.e., motion in a straight line). But it produces a motion of rotation in the body, on which it acts.
- The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

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ARM OF A COUPLE

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as arm of the couple.

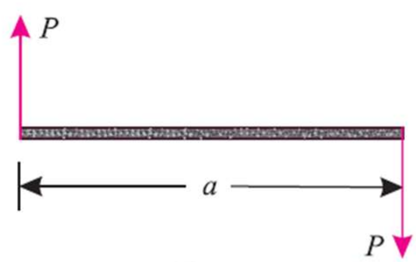
MOMENT OF A COUPLE

The moment of a couple is the product of the force (i.e., one of the forces of the two equal and opposite parallel forces) and the arm of the couple.

Mathematically:

Moment of a couple = $P \times a$

where P = Magnitude of the force, and
a = Arm of the couple.



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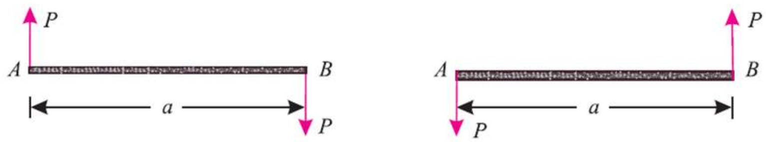
4.11. CLASSIFICATION OF COUPLES

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts :

1. Clockwise couple, and 2. Anticlockwise couple.

4.12. CLOCKWISE COUPLE

A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. 4.12 (*a*). Such a couple is also called positive couple.



(*a*) Clockwise couple (*b*) Anticlockwise couple

Fig. 4.12.

4.13. ANTICLOCKWISE COUPLE

A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. 4.12 (*b*). Such a couple is also called a negative couple.

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CHARACTERISTICS OF A COUPLE

A couple (whether clockwise or anticlockwise) has the following characteristics:

1. The algebraic sum of the forces, constituting the couple, is zero.

2. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.

3. Any no. of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

4. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.

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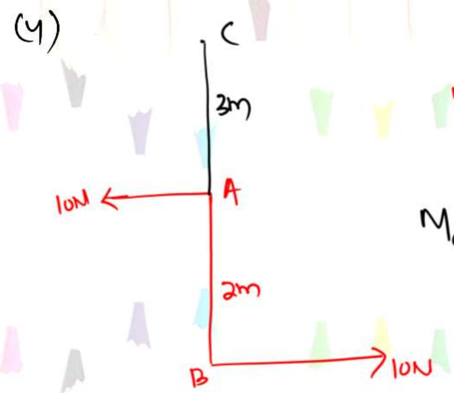

Magnitude of couple $= -10 \times 2$
$= \underline{-20 N\cdot m}$

$$M_C = -10 \times 5 + 10 \times 3$$
$$= -50 + 30$$
$$= \underline{\underline{-20 N\cdot m}}$$

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