Bhubanananda Odisha School of Engineering, Cuttack

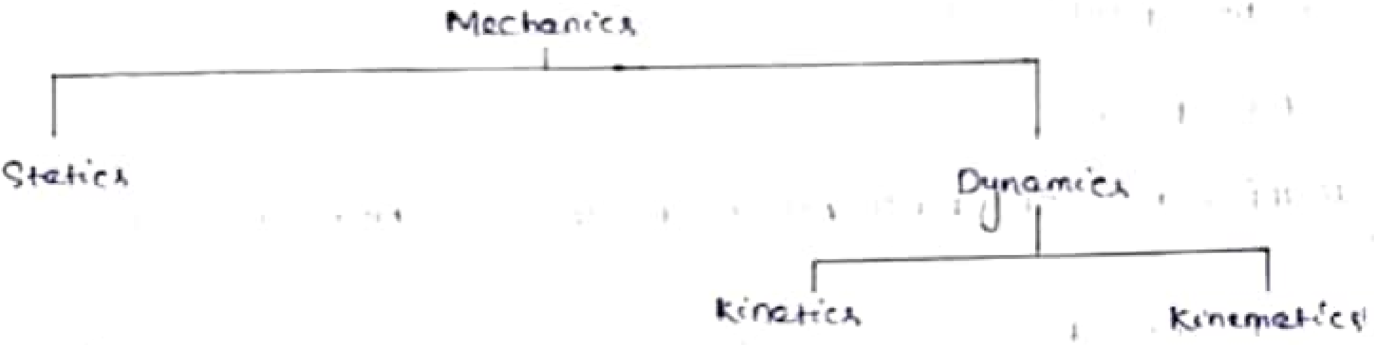
Department of Mechanical Engineering

SUBECT- ENGINEERING MECHANICS (TH-04)

PREPARED BY-PRASANTA KUMAR PADHI

1.1 FUNDAMENTALS

→ It is the branch of science which deals with the study of forces that act on the bodies and the resultant motion that those bodies experience.



Statics :- It deals with the forces and their effects, while acting on the bodies at rest.

Dynamics :- It deals with the forces and their effects, while acting upon the bodies in motion.

Kinetics :- It deals with the bodies in motion due to the application of forces.

Kinematics :- It deals with the bodies in motion without any reference to the forces which are responsible for motion (due to velocity, accel^n)

Rigid Body : Physical bodies are never absolutely but deform slightly under the action of loads. If the deformation is negligible as compared to size, the body is termed as rigid.

1.2. FORCE SYSTEM :-

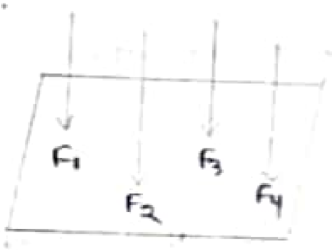
→ Force is defined as an agent which produces or destroys motion of the body to which it is applied.

→ It is also defined as the cause of change of state of motion of a pra particle or body.

→ Also $F = ma$.

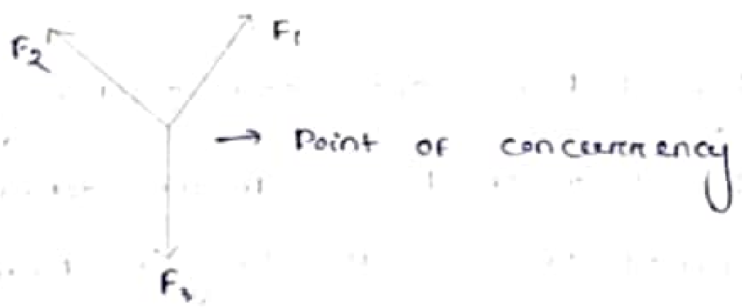
CLASIFICATION OF FORCE A/C TO PLANE OR LINE OF ACTION

(1) Coplanar Forces :- The forces whose line of action lie on the same plane.

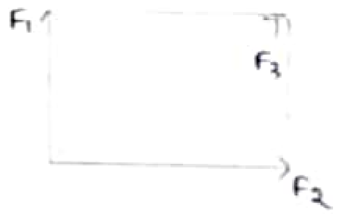


(2) Collinear Forces :- The forces whose line of action lie on the same line.

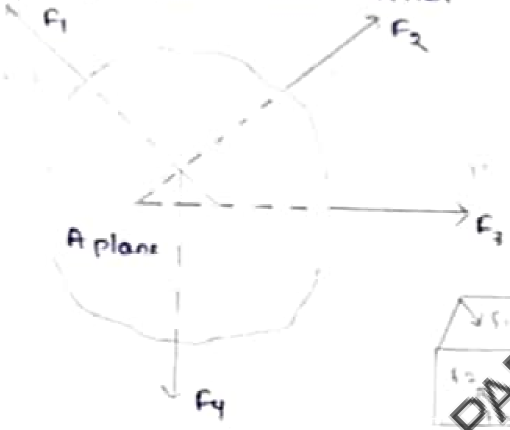
(3) Concurrent Forces :- The forces which meet at one point are known as concurrent forces.



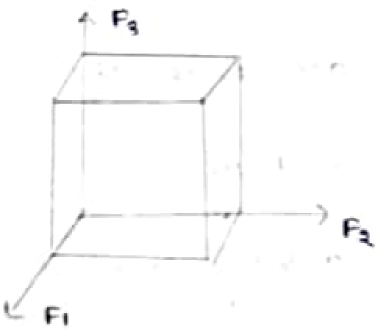
4) Coplannar and concurrent Forces :- The Forces which meet at one point and their lines of action also lie on the same plane



(5) Coplannar non concurrent Forces :- The Forces which do not meet at one point but their lines of action lie on the same plane



(6) Non Coplannar concurrent Forces :- The Forces which meet at one point but their lines of action do not lie on the same plane.



(7) Non coplannar non concurrent Forces :- The Forces which do not meet at one point and their lines of action do not lie on the same plane.

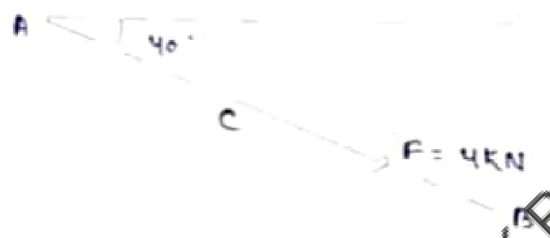
Characteristics of a Force :-

(1) Magnitude of the force

(2) Direction of the line along which the force acts. It is also known as the line of action of force.

(3) Nature of force (Push or pull) This is denoted by placing an arrow head on the line action of force

(4) Point at which forces acts on the body



Effects of Force :-

(1) It may change the motion of a body i.e if a body is at rest, the force may set it in motion and if the body is already in motion, the force may accelerate it

(2) It may retard the motion of a body.

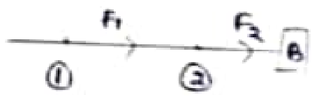
(3) It may change the direction of moving object

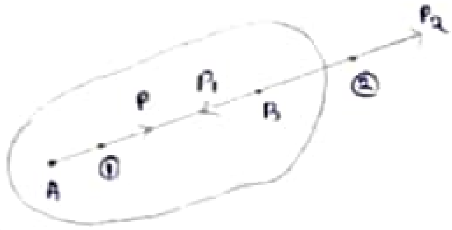
(4) It may stop moving of an object or brings it to rest.

(5) Change the shape of an object.

Principle of Transmissibility :

→ A/c (According) to this law the state of rest or motion of rigid body is unaltered (unchange) if a force acting on the body is replace by another force of same magnitude and direction but acting any where on the body along the line of action of replace force.



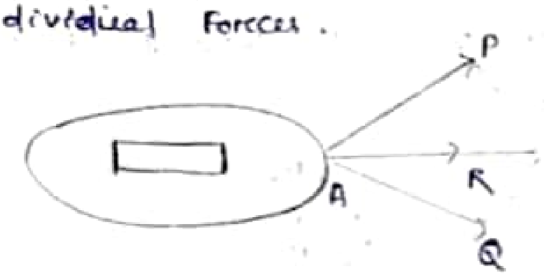


$P = P_1 = P_2$

P is replaced from point 1 to point 2

Principle of Superposition

→ The principle states that the combined effect of force system acting on a particle or rigid body is the sum of effects of individual forces.



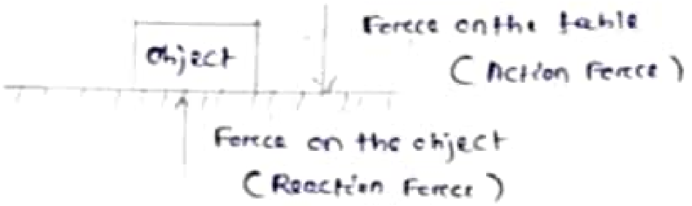
[Boat breaks down in the mid ocean & u have to pull it to shore]

Consider two forces P & Q acting at A on a boat. Let R be resultant of these 2 forces P & Q. Boat will move in the direction of 'R'.

ACTION AND REACTION FORCES

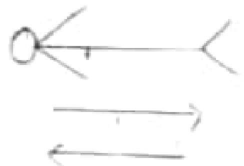
Newton's 3rd law of motion :

For every action Force there is an equal and opposite reaction force.



Both have same magnitude but directions are different.

While swimming Frog pushes water back and water pushes its body forward.



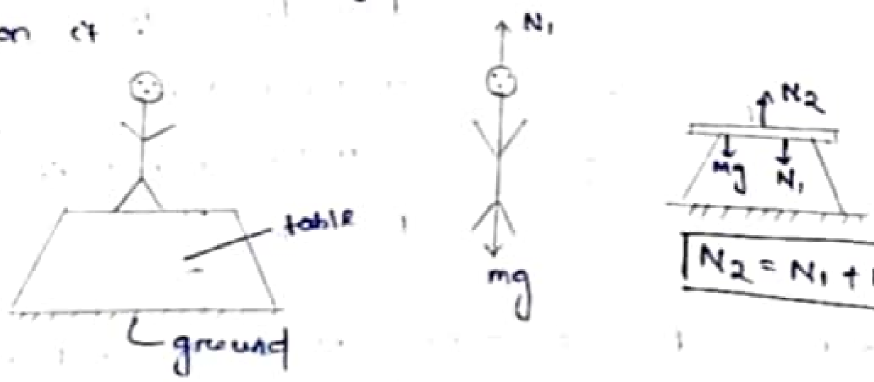
* The Forces which is acting in one direction is called action Force & equal Force acting in opposite dirⁿ is called reacⁿ Force.

Concept of Free Body Diagram

FBD [Free body Diagram] :- It is the sketch that shows only the action and reaction Forces acting on the body.

-> We will isolate the body and then we will show the Forces acting on it :

① $\boxed{\text{Case - 1}}$

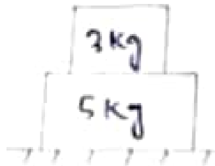


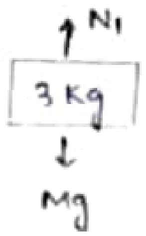
$$\boxed{N_2 = N_1 + Mg}$$

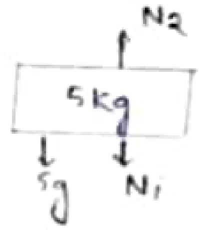
N_2 : Normal reacⁿ betⁿ ground and table

N_1 :- Normal reacⁿ betⁿ man and table

(2) $\boxed{\text{Case - 2}}$







$\boxed{N_1 = 3g}$

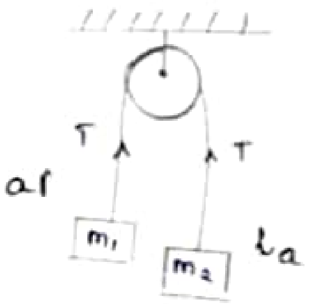
$\Rightarrow \boxed{N_1 - 3g = 0}$

$\boxed{N_2 = N_1 + 5g = 3g + 5g}$

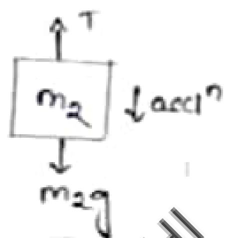
$\Rightarrow \boxed{N_2 = 8g}$

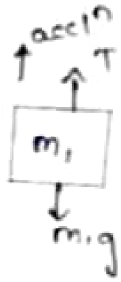
(3)

$\boxed{\text{Case - 3}}$



$(m_2 > m_1)$

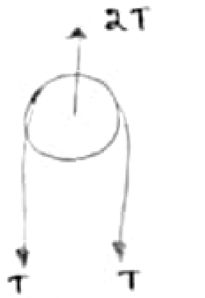




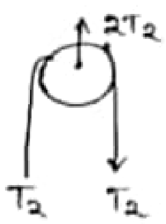
$\boxed{m_2g - T = m_2a}$

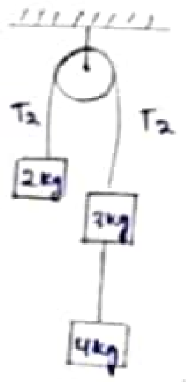
$(m_2g > T)$

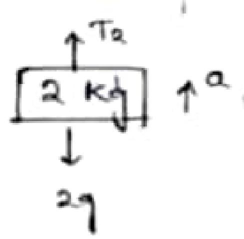
$\boxed{T - M_1g = M_1a}$



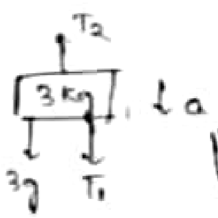
(Pulley)



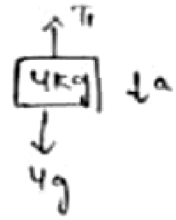




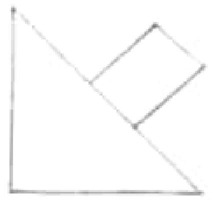
$\boxed{T_2 - 2g = 2a}$

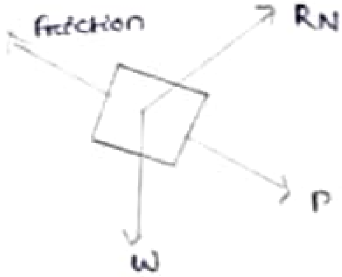


$\boxed{3g + T_2 = 3a}$



$4g - T_1 = 4a$





1.4 COMPOSITION OF FORCES :

Defination

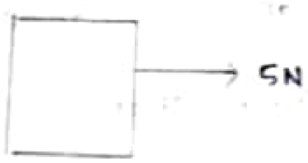
→ The Forces of Finding out the resultant Force, of a no. of given Forces is called composition of Forces or compounding of Forces.

Resultant Force :

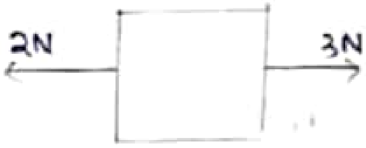
→ If a number of Forces P, Q, R ... etc are acting simultaneously on a particle, then it is possible to find out a single Force which could replace them i.e which would produce the same effect as produced by all the given Forces. The single Force is called resultant Force and the given Forces are called component Forces.

Ex -

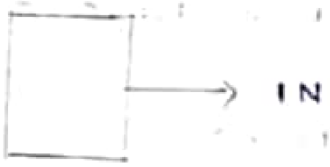
(Applied Forces)



(Resultant Force)



(Applied Force)



(Resultant Force)

Methods of composition of force

(1) Analytical Method

(2) Graphical Method

1.4.1 ANALYTICAL METHOD SUCH AS PARALLELOGRAM LAW OF FORCES AND METHOD OF RESOLUTION :

Analytical method for resultant force :

The resultant force of a given system of forces may be found out analytical by :-

(1) Parallelogram law of forces.

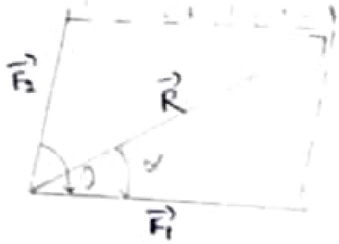
(2) Method of Resolution.

Parallelogram Law of Forces

It states that, If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjucent sides of a parallelogram, their resultant may be represented in magnitude and dirn by the diagonal of the parallelogram, which passes through their point of intersection.

Mathematically, Resultant Forces :-

$$\boxed{R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}}$$



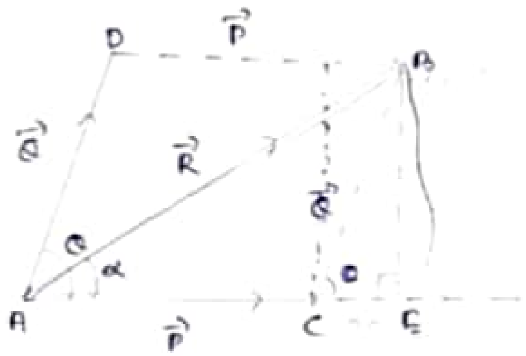
and $$\boxed{\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}}$$ → (Direction of Resultant Forces with horizontal)

F_1, F_2 = Forces whose resultant is reqd. to be found out

θ = Angle betn the forces F_1 & F_2

α = Angle which resultant force makes with one of the forces (say F_1)

Proof



In ΔABE

$$AB^2 = AE^2 + BE^2$$

$$= (AC + CE)^2 + BE^2$$

$$= (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$= P^2 + (Q\cos\theta)^2 + 2PQ\cos\theta + (Q\sin\theta)^2$$

$$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta$$

$$AB^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$\Rightarrow AB = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\Rightarrow \boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

In ΔABB, $\tan\alpha = \frac{BE}{AE} = \frac{BE}{AC+CE}$

$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}}$$

Case-1 $(\theta = 0^\circ)$

$$R = \sqrt{P^2+Q^2+2PQ\cos 0^\circ}$$

$$= \sqrt{P^2+Q^2+2PQ}$$

$$= \sqrt{(P+Q)^2}$$

$$\Rightarrow \boxed{R = P+Q}$$

Case-2 $(\theta = 90^\circ)$

$$R = \sqrt{P^2+Q^2+2PQ\cos 90^\circ}$$

$$\boxed{R = \sqrt{P^2+Q^2}}$$

Case-3 $(\theta = 180^\circ)$

$$R = \sqrt{P^2+Q^2+2PQ\cos(180^\circ)}$$

$$= \sqrt{P^2+Q^2-2PQ}$$

$$= \sqrt{(P-Q)^2}$$

$$\boxed{R = P-Q}$$

Case - 4 $(P = Q)$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{P^2 + P^2 + 2P \cdot P \cdot \cos\theta}$$

$$= \sqrt{2P^2 + 2P^2\cos\theta}$$

$$= \sqrt{2P^2(1 + \cos\theta)}$$

$$\boxed{R = P\sqrt{2(1+\cos\theta)}} \quad \text{——} (1)$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

Put $\theta = \frac{\theta}{2}$

$$\cos 2\left(\frac{\theta}{2}\right) = 2\cos^2\left(\frac{\theta}{2}\right) - 1$$

$$\Rightarrow \boxed{1 + \cos\theta = 2\cos^2\frac{\theta}{2}} \quad \text{——} (2)$$

Putting the value of $(1 + \cos\theta)$

$$R = P\sqrt{2 \times 2\cos^2\frac{\theta}{2}}$$

$$= P\sqrt{4 \times \cos^2\frac{\theta}{2}}$$

$$= 2P\sqrt{\cos^2\frac{\theta}{2}}$$

$$\boxed{R = 2P\cos\left(\frac{\theta}{2}\right)}$$

Q(1) Two forces of 100N and 150 N acting simultaneously at a point. What is the resultant of these two forces, if the angle betn them is 45° ?

Soln

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

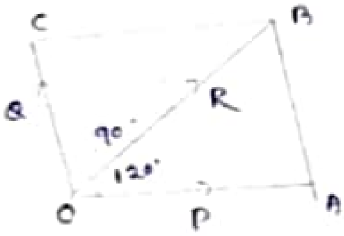
$$= \sqrt{(100)^2 + (150)^2 + 2 \times 100 \times 150 \times \cos 45^\circ}$$

$$= 232\ N$$

Q (2) Two forces act at an angle of 120. The bigger force is 40 N and the resultant is perpendicular to the smaller one. Find the smaller one ?

$$\alpha = 120^\circ - 90^\circ = 30^\circ$$

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$



$$\tan 30^\circ = \frac{F_2 \sin}{40 + F_2\cos 120^\circ}$$

$$\Rightarrow 0.577 = \frac{F_2 \times 0.866}{40 - 0.5F_2}$$

$$\Rightarrow 40 - 0.5F_2 = \frac{0.866F_2}{0.577} = 1.5F_2$$

$$\Rightarrow 2F_2 = 40N$$

$$\Rightarrow \boxed{F_2 = 20}$$

Q.(3) Find the magnitude of the 2 forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

$$\Rightarrow 10 = F_1^2 + F_2^2 \quad \text{———} (1)$$

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \times \frac{1}{2}}$$

$$\Rightarrow 13 = \underbrace{F_1^2 + F_2^2}_{10} + F_1F_2$$

$$\Rightarrow F_1F_2 = 3 \quad \text{———} (2)$$

$$(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2$$

$$= 10 + 6 = 16$$

$$F_1 + F_2 = 4 \quad \text{———} (3)$$

$$(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$$

$$F_1 - F_2 = 2 \quad \text{———} (4)$$

$$2F_1 = 6$$

$$\Rightarrow F_1 = 3N$$

$$2F_2 = 2$$

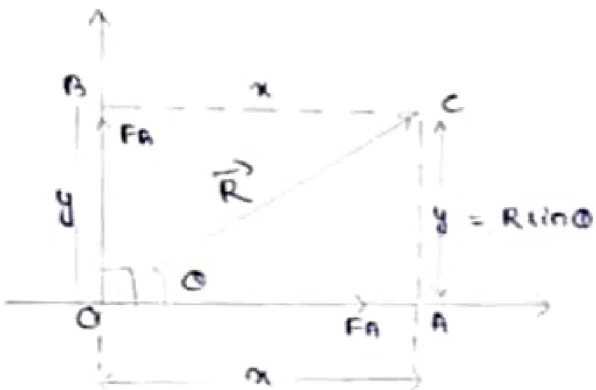
$$\Rightarrow F_2 = 1N$$

1.3 RESOLUTION OF FORCE

Resolution of a Force

The process of splitting up the given force into no. of components, without changing its effect on the body is called resolution of force.

A force is generally resolved along 2 mutually perpendicular direction.



Boat example

Segregation of result into components i.e resolution of force

(Retangular component mutually $\perp$ to each other)

In Δ OCA, $\sin\theta = \frac{AC}{OC} = \frac{y}{R}$

$\Rightarrow y = R\sin\theta$

$\cos\theta = \frac{OA}{OC} = \frac{x}{R} \Rightarrow x = R\cos\theta$

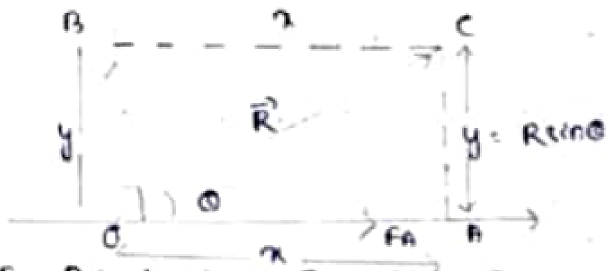
$$\text{Resultant}(R) = \sqrt{x^2 + y^2}$$

$$= \sqrt{(R\cos\theta)^2 + (R\sin\theta)^2}$$

$$= \pm R$$

Principle of Resolution

The algebraic sum of the resolved parts of a no. of forces, in a given direction is equal to the resolved place of their resultant in the same direction.



Method of Resolution for the Resultant Force :

(1) Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (ΣH).

(2) Resolve all the forces vertically and find the algebraic sum of all the vertical components (ΣV).

(3) Resultant 'R' of the given forces will be

$$\boxed{R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}}$$

(4) Resultant force will be inclined at angle 'θ' with the horizontal such that

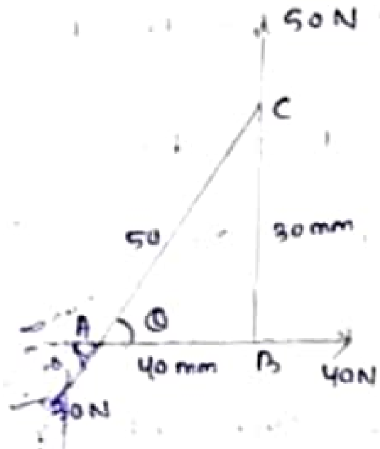
$$\boxed{\tan\theta = \frac{\Sigma V}{\Sigma H}}$$

Note

(1) When ΣV is +ve, the resultant makes an angle betn 0° & 180°. But when ΣV is -ve, the resultant makes an angle betn 180° & 360°.

(2) When ΣH is +ve, the resultant makes an angle betn 0° to 90° or 270° to 360°. But when ΣH is -ve the resultant makes an angle betn 90° to 270°.

Q. A ΔABC has its sides AB = 40 mm along +ve X-axis and side BC = 30 mm along +ve Y-axis, 3 forces of 40N, 50N, 30 N act along AB, BC, CA respectively. Determine the magnitude of the resultant of such a system of forces.



$$AC = \sqrt{(40)^2 + (30)^2}$$

$$= 50 \text{ mm}$$

$$\sin\theta = \frac{30}{50} = 0.6$$

$$\cos\theta = \frac{40}{50} = 0.8$$

Resolving all forces horizontally

$$\Sigma H = 40 - 30\cos\theta$$

$$= 40 - (30 \times 0.8) = 16 \text{ N}$$

Resolving all forces vertically

$$\Sigma V = 50 - 30\sin\theta$$

$$= 50 - (30 \times 0.6)$$

$$= 32 \text{ N}$$

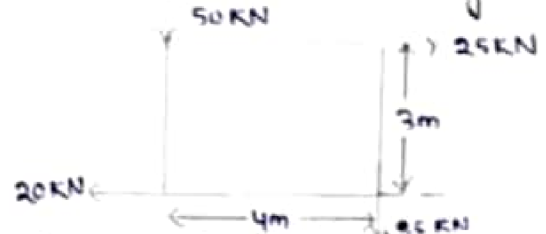
Magnitude of the resultant force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(16)^2 + (32)^2}$$

$$= 35.8 \text{ N}$$

Q. A system of forces are acting at the corners of a block.



Determine the magnitude and dirn of the resultant force?

$\Sigma H = 25 - 20 = 5KN$

$\Sigma V = (-50) + (35) = -85 N$

Magnitude of resultant force

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{5^2 + (85)^2}$

$= 85.15 KN$

Direction of the resultant force

$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-85}{5} =$

$\theta = 86.6°$

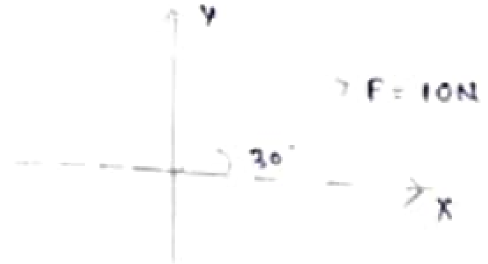
Since ΣH is +ve and ΣV is -ve, the resultant lines betn 270° to 360°

Actual angle of resultant force

$= 360° - 86.6°$

$= 273.4°$

Q.(1)

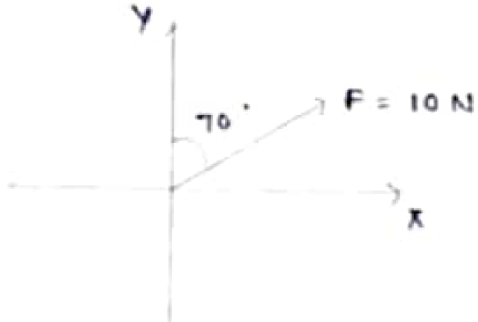


Resolve the force into 2 components?

$F_x = 10 \cos 30°$

$F_y = 10 \sin 30°$

Q.(2)

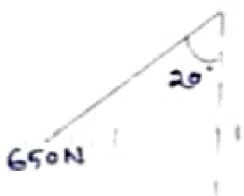


Resolve the Force?

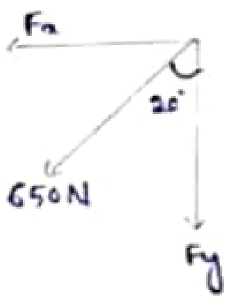
$F_x = 10 \sin 70°$

$F_y = 10 \cos 70°$

Q.(3)



Resolve the Force into 2 component?

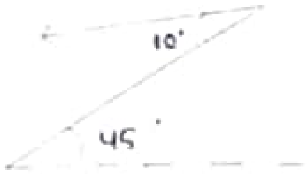


$F_y = 650 \cos 20 = 610.8 \text{ N}$

$F_x = 650 \sin 20°$

$= 22.3 \text{ N}$

Q.(4) Resolve the Force in to components ?

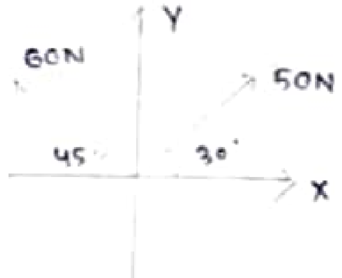


Ans

$F_x = 25 \cos 35°$

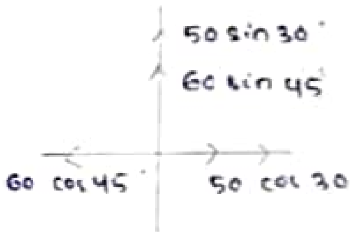
$F_y = 25 \sin 35°$

Q.(5)



Find Fx, Fy, Resultant Force ?

Ans



$\Sigma F_x = 50 \cos 30° - 60 \cos 45$

$= 50 \times \frac{\sqrt{3}}{2} - 60 \times \frac{1}{\sqrt{2}}$

$= 25\sqrt{3} - 30\sqrt{2} = 0.9 \text{ N}$

$\Sigma F_y = 50 \sin 30° + 60 \sin 45°$

$= 50 \times \frac{1}{2} + 60 \times \frac{1}{\sqrt{2}}$

$= 25 + 30\sqrt{2} = 67.4 \text{ N}$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

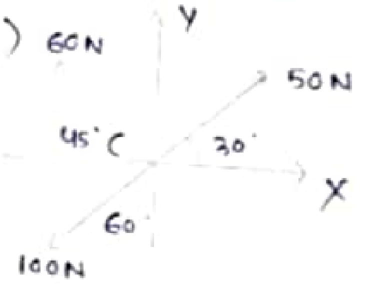
$= \sqrt{(0.9)^2 + (67.4)^2}$

$= \sqrt{0.81 + 4542.80}$

$= \sqrt{4543.6}$

$= 67.4 \text{ N}$

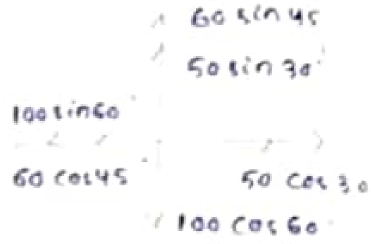
Q.(6)



Find F_x, F_y & Resultant ?

Soln

$\Sigma F_x = 50\cos 30 - 60\cos 45° - 100\sin 60$



$= 50 \times \frac{\sqrt{3}}{2} - 60 \times \frac{1}{\sqrt{2}} - 100 \times \frac{\sqrt{3}}{2}$

$= 25\sqrt{3} - 30\sqrt{2} - 50\sqrt{3}$

$= -30\sqrt{2} - 25\sqrt{3} = -(30\sqrt{2} + 25\sqrt{3}) = -85.7 \text{ N}$

$\Sigma F_y = 60\sin 45° + 50\sin 30 - 100\cos 60$

$= 60 \times \frac{1}{\sqrt{2}} + 50 \times \frac{1}{2} - 100 \times \frac{1}{2}$

$= 30\sqrt{2} + 25 - 50 = 30\sqrt{2} - 25 = 17.4 \text{ N}$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(-85.7)^2 + (17.4)^2}$

$= \sqrt{7344.5 + 302.8}$

$= \sqrt{7647.3} = 87.4 \text{ N}$

Q.(7) The following Forces act at a point ?

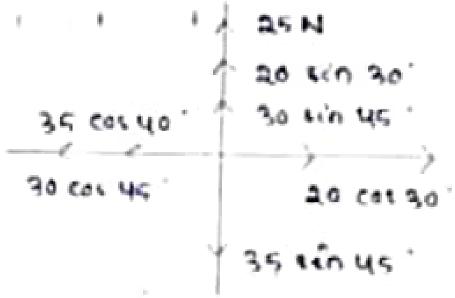
(i) 20 N inclined at 30° towards north of east

(ii) 25 N towards north.

(iii) 30 N towards north west at 45°

(iv) 35 N inclined at 40° towards south of west

Find the resultant ?



$\Sigma F_x = 20\cos 30 - 30\cos 45^\circ - 35\cos 40$

$= 20 \times \frac{\sqrt{3}}{2} - 30 \times \frac{1}{\sqrt{2}} - 35\cos 40$

$= 10\sqrt{3} - 15\sqrt{2} - 26.8 = -30.7\ N$

$\Sigma F_y = 25 + 20\sin 30^\circ + 30\sin 45^\circ - 35\sin 45$

$= 25 + 20 \times \frac{1}{2} + 30 \times \frac{1}{\sqrt{2}} - 35 \times \frac{1}{\sqrt{2}}$

$= 25 + 10 + 15\sqrt{2} - 17.5\sqrt{2}$

$= 35 + 15\sqrt{2} - 17.5\sqrt{2} = 35 - 2.5\sqrt{2} = 31.5\ N$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

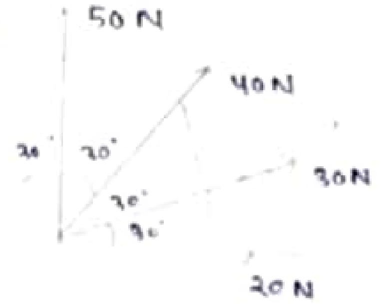
$= \sqrt{(-30.7)^2 + (31.5)^2}$

$= \sqrt{942.5 + 992.2}$

$= \sqrt{1934.7}$

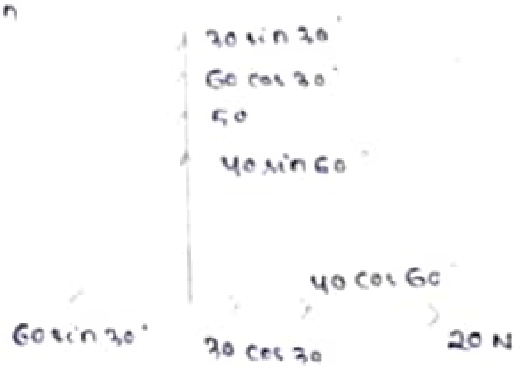
$= 43.1\ N$

Q.(8)



Find F_x, F_y & Resultant ?

Soln



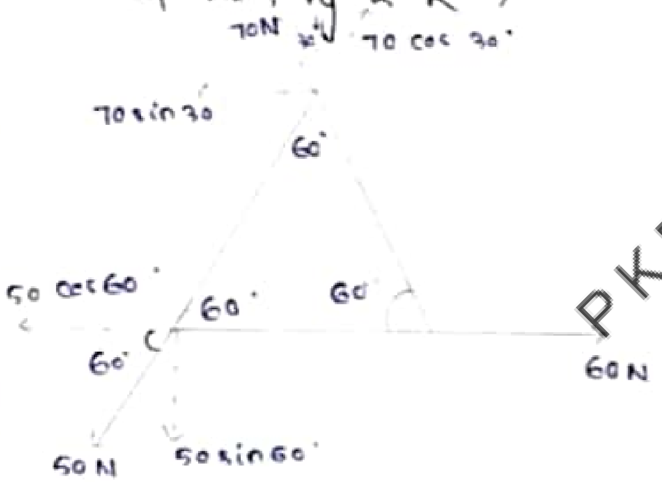
$\Sigma F_x = 36\ N$

$\Sigma F_y = 151.6\ N$

$R = 155.8\ N$

Q.(9) In a equilateral Δ, Forces are acting as shown below.

Find F_x, F_y & R ?



$\Sigma F_x = 60 - 50 \cos 60° - 70 \sin 30°$

$= 60 - 50 \times \frac{1}{2} - 70 \times \frac{1}{2}$

$= 60 - 25 - 35$

$= 0$

$\Sigma F_y = 70 \cos 30° - 50 \sin 60°$

$= 70 \times \frac{\sqrt{3}}{2} - 50 \times \frac{\sqrt{3}}{2}$

$= 35\sqrt{3} - 25\sqrt{3} = 10\sqrt{3}$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(0)^2 + (10\sqrt{3})^2}$

$= \sqrt{300}$

$= 10\sqrt{3}\ N$

TRIANGLE LAW OF FORCES

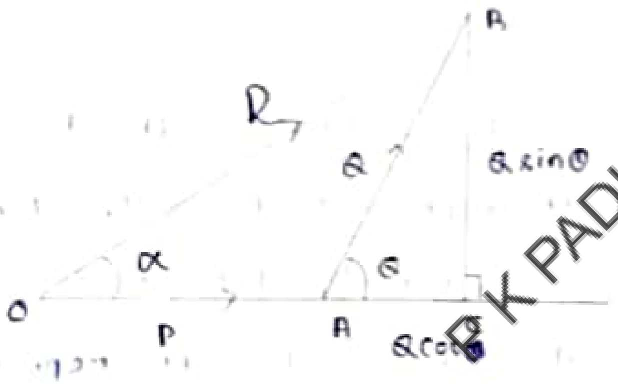
It states that if two forces are acting as the two adjacent sides of a triangle taken in a order, then the closing sides of the triangle represents the resultant in opposite direction.

$$\boxed{\vec{R} = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\Rightarrow \boxed{\vec{P} + \vec{Q} = R}$$

Proof



Draw a ⊥r line from point B whose line of action meet at point C.

In ΔOBC $\quad OB^2 = OC^2 + BC^2$

$$= (OA + AC)^2 + BC^2$$

$$= OA^2 + AC^2 + 2 \cdot OA \cdot AC + BC^2$$

$$= P^2 + Q^2\cos^2\theta + 2 \cdot P \cdot Q\cos\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta$$

$$= P^2 + Q^2 + 2PQ\cos\theta$$

$$\Rightarrow R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \text{ (proved)}$$

$$\tan\alpha = \frac{BC}{OC} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)}$$

POLYGON LAW OF FORCES

It is an extension of triangle law of forces for more than two forces which states that if a number of forces are acting simunltaneously on a particle be represented in both magnitude and direction by the sides of a polygon taken in order, then the resultant of all these forces may be represented in magnitude & direction by the closing side of the polygon taken in opposite order.

Proof

Consider the forces P_1, P_2, P_3 and P_4 acting on the body at point 'A' as shown in figure. Line AB represent force P_1, Line BC represents force P_2, CD for P_3 and DE for P_4. Polygon is completed by drawing the closing line AE. This closing line AE represent the resultant of given system.

In Δ ABC $\quad \vec{AC} = \vec{AB} + \vec{BC}$ (According to triangle law of forces)

$\quad = P_1 + P_2$

In Δ ACD $\quad \vec{AD} = \vec{AC} + \vec{CD}$

$\quad = \vec{AB} + \vec{BC} + \vec{CD}$ $\quad (\vec{AC} = \vec{AB} + \vec{BC})$

$\quad = P_1 + P_2 + P_3$

In ΔADE $\quad \vec{AE} = \vec{AD} + \vec{DE}$

$$= \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} \qquad (\because \vec{AD} = \vec{AB} + \vec{BC} + \vec{CD})$$

$$= P_1 + P_2 + P_3 + P_4$$

$$\Rightarrow \boxed{\vec{R} = P_1 + P_2 + P_3 + P_4} \qquad \text{(proved)}$$

MOMENT

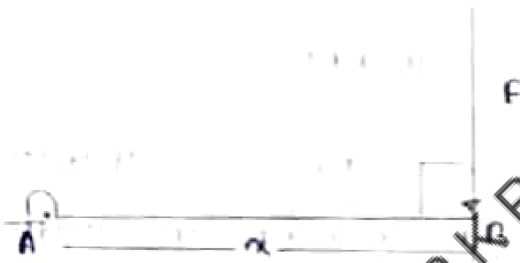
→ Moment is the turning effect produced by the force on the body on which act.

→ Moment of force is defined as the product of the force and the perpendicular distance betn force and the point about which moment is required.

$$\boxed{\text{Moment } (M) = F \times L}$$

where F = Force applied

L = perpendicular length



Taking moment about A

$$\therefore (M)_{\text{at point } A} = \text{Force} \times \text{perpendicular distance}$$

$$= F \times x \quad (\text{Clockwise})$$

Units

$M = F \times L$

$= N \times m$

$= Nm$

CGS - dyne x cm

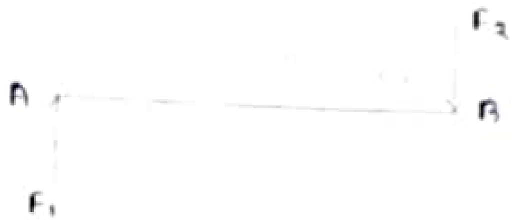
TYPES OF MOMENT

(i) Clock wise moment

(ii) Anticlock wise / counter clock wise moment

(i) CLOCKWISE MOMENT

→ It is the moment of force whose effect rotate the body about the point in clockwise direction.

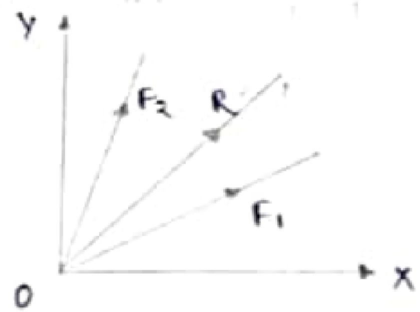


(ii) ANTICLOCKWISE MOMENT

→ It is the moment of force whose effect rotate the body about the point in Anticlockwise direction.

LAWS OF MOMENT / VARIGNON'S THEOREM

If a no. of coplanner forces are acting simunltaneously on a particle then the algebric sum of the moments of all the forces about any point is equals to the moment of their resultant force about the same point.



moment of force F_1 about at point 'O' = $(M)_1 = F_1 \times x_1$

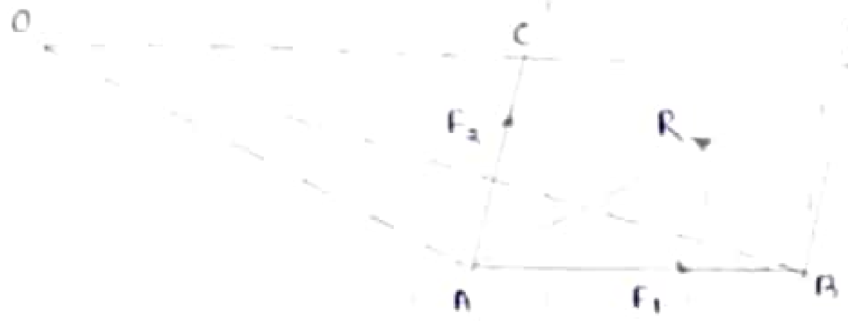
moment of force F_2 about point 'O' = $(M)_2 = F_2 \times x_2$

Moment of force F_1 & F_2 about point O

= moment of Resultant force about point O

⇒ $\Sigma(M_1 + M_1) = R \times a$

Proof



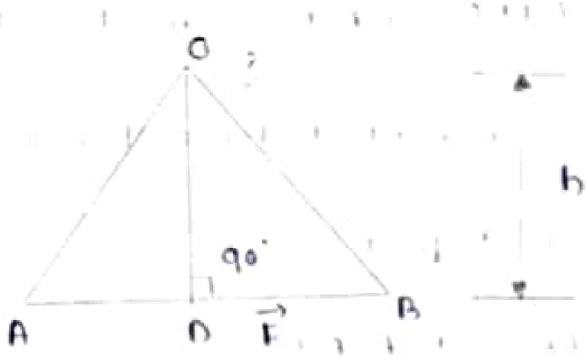
* Considering point O about which the moment is to be taken After that from point O, draw a line which is parallel to the line of action Force F_1 or base AB and intersecting the line at point C

* Here AB and AC adjacent sides of parallelogram. Now complete the ACDB parallelogram. The diagonal of parallelogram representing magnitude and direction by the resultant of Force F_1 & F_2.

Here R is the resultant Force.

Now joint the point O from point A & B (Join OA and OB)

* According to graphical representation of the moment, i.e The moment of Force about a point is equal to twice of the area of triangle.



moment of Force $\vec{F}$ about point O

$$(M)_{\text{At point O}} = \vec{F} \times h \quad \text{---- (1)}$$

$$\text{Area of the Triangle} = \frac{1}{2} \times AB \times OD$$

$$= \frac{1}{2} \times \vec{F} \times h$$

$$\Rightarrow 2 \times \text{Area of triangle} = \vec{F} \times h \quad \text{---- (2)}$$

$\Rightarrow$ Equating the eq^n (1) and eq^n (2), we get

$$\boxed{\text{Moment of Force} = 2 \times \text{Area of triangle}}$$

From Geometrical configuration :

moment of Force F_1 about 'O' point

$= 2 \times$ Area of ΔAOB

moment of Force F_2 about point 'O'

$= 2 \times$ Area of ΔAOC

moment of force R about point 'O'

$= 2 \times$ Area of ΔAOD

Now $\Delta AOD = \Delta AOC + \Delta ACD$

$\Rightarrow \Delta AOD = \Delta AOC + \Delta ABD$

$(\because \Delta ACD = \Delta ABD)$

parallelogram, 2 equal Δ same

$\Rightarrow \Delta AOD = \Delta AOC + \Delta AOB$

$(\because \Delta ABD = \Delta AOB$ same base line$)$

putting this ΔAOD value in moment of Force R, we get

$\therefore$ Moment of Force R about point O

$= 2 \times$ Area of ΔAOD

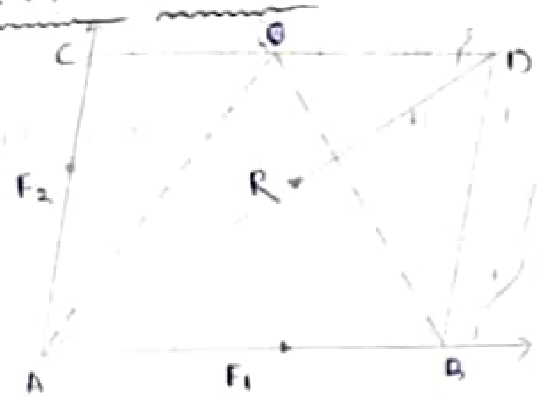
$= 2 \times$ Area of $(\Delta AOC + \Delta AOB)$

$\Rightarrow$ moment of force R about point O

= moment of Force F_1 & F_2 about point O

$\Rightarrow \{\Sigma (m_1 + m_2)\}_{\text{about o point}} = R \times x$

POINT OF ROTATION INSIDE



In Δ OAB moment of $\vec{F_1}$ about 'O' point = $-2 \times$ Area of Δ OAB

In Δ OAC moment of $\vec{F_2}$ about 'O' point

$= 2 \times$ Area of Δ OAC

In Δ OAD moment of $\vec{R}$ about 'O' point

$= -2 \times$ Area of Δ OAD

Algebric sum of moment of F_1 and F_2 about 'O' point

$= -2 \times$ Area of Δ OAB $+ 2 \times$ Area of Δ OAC

$= 2 \times$ Area of Δ OAC $- 2 \times$ Area of Δ OAB

$= 2 \times$ Area of OAC $- 2 \times$ Area of Δ BAD

($\because$ Δ OAB = Δ BAD

same base and height)

$= 2 \times$ Area of OAC $- 2 \times$ Area of Δ ACD

($\because$ parallelogram

2 equal Δ same

so Δ BAD = Δ ACD

$= -(2 \times$ Area of Δ ACD $- 2 \times$ Area of Δ OAC$)$

$= -2 \times$ Area of OAD (prooved)

COUPLE

→ A pair of 2 equal and unlike parallel forces (equal in magnitude (a... and line of action parallel to each other and acting opposite direction) called couple.

→ Couple is a rotational effect of force.

Example of Couple

(1) Winding of a clock.

(2) Unscrewing the cap of a bottle

(3) Locking or unlocking of a lock without key.

(4) Opening or closing of a water tap.

Arm of Couple

→ Perpendicular distance betn the lines of action of two equal and opposite forces is called Arm of couple.

Arm of couple = $r + r = 2r$

Magnitude of Couple

= Force × Arm of the couple

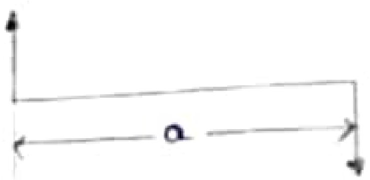
= $F' \times r + F \times r$

= $2Fr$

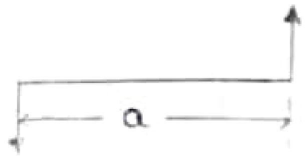
= $F \cap$ N.m

Clarification of couple

(1) Clock wise Couple



(2) Anticlockwise couple



CHARACTERISTICS OF A COUPLE

(1) Algebraic sum of forces producing couple is zero

$\Rightarrow \Sigma F = F_A + F_B$

$= 5N + (-5N)$

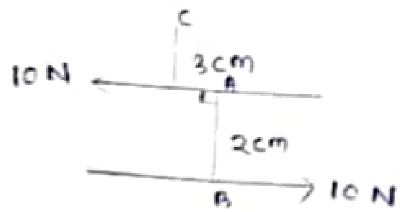
$= 0$

(2) Translatory effect of couple is zero.

(3) A couple can not be balanced by a single force

(4) Any number of co-planner couples can be reduced to a single couple whose magnitude will be equal to algebraic sum of the moments of all the couples.

(5) The algebraic sum of the moment of force producing couple about any point equal to the moment of couple.



$10\,N \rightarrow M_A = -20\,N$

$10\,N \rightarrow M_B = -20\,N$

$10\,N \rightarrow M_C = 30\,N - 50\,N = -20\,N$

moment of Couple $= 10N \times 2 = -20\,N$

(6) Two coplanner couples whose moments are equal and opposite, balance each other.

(7) Any two couples whose moments are equal and of same sign are equivalent both in magnitude and direction.