

LECTURE NOTES ON

THEORY OF MACHINE

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SYLLABUS


(P.K. SWAIN)

1. Simple mechanism

1.1 → Link, kinematic chain, mechanism, machine.

1.2 → Inversion, four bar link mechanism and its inversion.

1.3 → Lower pair & higher pair

1.4 → Cam and followers

3. POWER TRANSMISSION

3.1 concept of power transmission

3.2 Types of driver, belt, gear and chain drive.

3.3 computation of velocity ratio, length of belts (open & cross) with and without slip.

3.4 Ratio of belt tension, centrifugal tension and initial tension

3.5 power transmitted by the belt.

3.6 V-belt & V-belt pulleys.

3.8 gear driver & Terminology

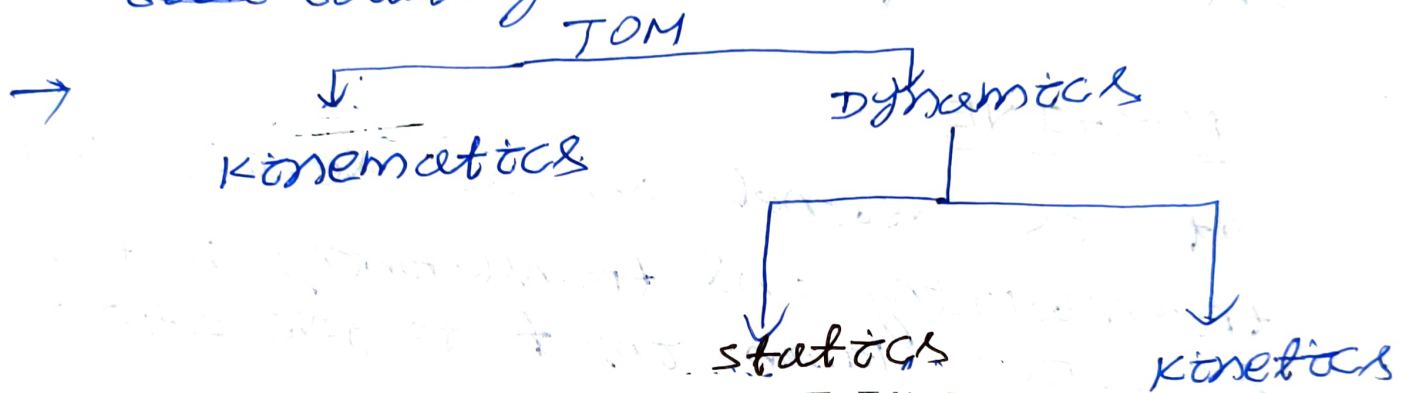
3.9 gear trains, working principle of simple, compound, reverted & epicyclic gear train.

THEORY OF MACHINE


(C.P.K. SWAIN)

→ Defⁿ:- It is defined as branch of engineering science, which deals with study of relative motion between various parts of a machine & forces which act on them.

→ Theory of machine divided into ~~following~~ following branches.



→ Statics:- It is the branch of TOM, which deals with the study of forces & their effects on machine in rest.

→ Dynamics:- It is the branch of TOM which deals with the study of forces & their effects, while acting on machine in motion or rest.

→ Kinematics:- It is the branch of TOM which deals with the relative motion between parts of machine.

→ study of position, displacement, speed, rotation, v , a .

→ Kinetics:- It is the branch of TOM which deals with the inertia

forces which arise from the combined effect of mass and motion of the machine parts.

STRUCTURE

→ It is defined as the combination of rigid or resistance bodies assembled having no relative motion between them and transmit only forces.

MACHINE

→ It is defined as a device for transforming and transmitting motion & force from input to give output.

→ It is defined as a device which receives energy and transforms it into some useful work.

MECHANISM

TYPES OF BODY

① Rigid body:- A body is said to be rigid if it does not suffer any distortion under the action of forces or distance b/w two points remains constant.

→ Study of rigid body called engineering mechanics.

RESISTING BODY :-

→ a body is said to be resisting body if it suffers small deformation under action of force EX → connecting rod, spring, belt.

→ study of resisting body called SOM.

(iii) FLEXIBLE BODY :- a body is said to be flexible, if it suffers large deformation under action of force.

EX → belt, spring.

1.1 LINK

→ It is a resistant body or assembly or resistant body of a machine connecting other parts of the machine with relative motion between them.

OR

It is defined as a resistant body or a group of resistant bodies with rigid connections preventing their relative movement.

EX → piston + piston rod + crankhead.

connecting rod with big and small end.

crank + crank shaft + flywheel.

cylinders + frame.

→ Link should have two characteristics.

① relative motion w.r.t. another link.

② must be a resistant body.

TYPES OF LINKS → according to medium by which motion is transmitted.

① RIGID LINK:- It is one which does not undergo any deformation while transmitting motion.

EX → connecting rod, crank.

② FLEXIBLE LINK:- It is one which is partly deformed in a manner not to affect the transmission of motion.

EX → belt, rope, chain

③ FLUID LINK:- It is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only.

EX → hydraulic press, jacks & brakes.

MACHINE

① parts of machine move relative to one another

② converts some ^{available} ~~work~~ energy into some useful work

③ Transmits both power & motion

STRUCTURE

① parts of ~~machine~~ structure do not move relative to one another.

② does not convert ~~available~~ energy into useful work.

③ Transmits forces only.

KINEMATIC CHAIN

→ Kinematic chain is a group of links either joined together or arranged in a manner that permits them to move relative to one another.

→ If the links are connected in such a way that no motion is possible, it results in a locked chain or structure.

TYPES OF JOINTS

① Binary joint:- If two links are joined at the same connection, it is called a binary joint.

② Ternary joint:- If three links are joined at a connection, it is known as ternary joint.

→ It can be considered as two binary joint.

③ Quaternary joint:- If four links are joined at a connection, known as quaternary joint.

→ It can be considered as three binary joint.

TYPES OF CONSTRAINED MOTION

→ There are three types of constrained motion.

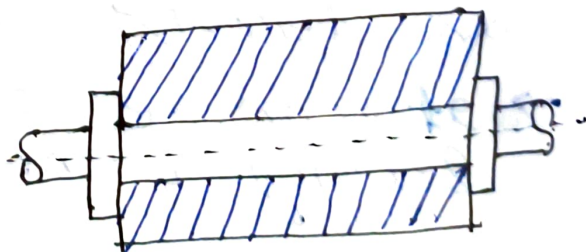
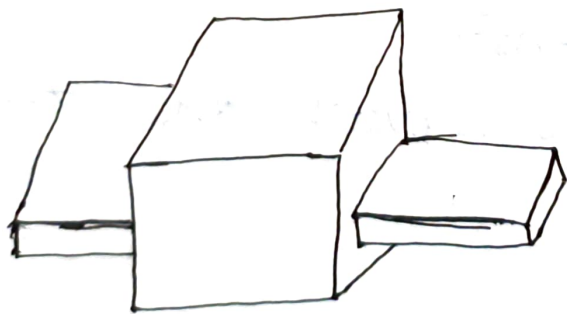
- ① completely constrained motion.
- ② incompletely constrained
- ③ successfully constrained.

completely constrained motion

→ When the motion b/w pair is limited to a definite direction irrespective of the direction force applied, then the motion is said to be a completely constrained motion.

→ EX →
① Motion of a square bar in a square hole.

② Motion of a shaft with collars at each end in a circular hole.



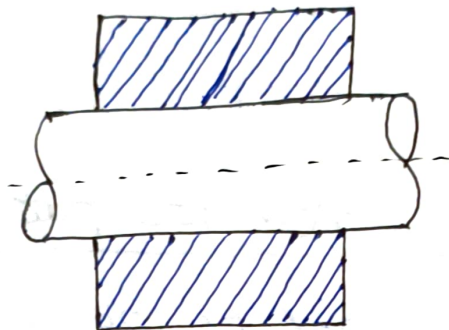
→ D.O.F is 1

incompletely constrained motion

→ When the motion between a pair can take place in more than

one direction, then the motion is called an incompletely constrained motion.

EX $\rightarrow$ shaft in a circular hole.

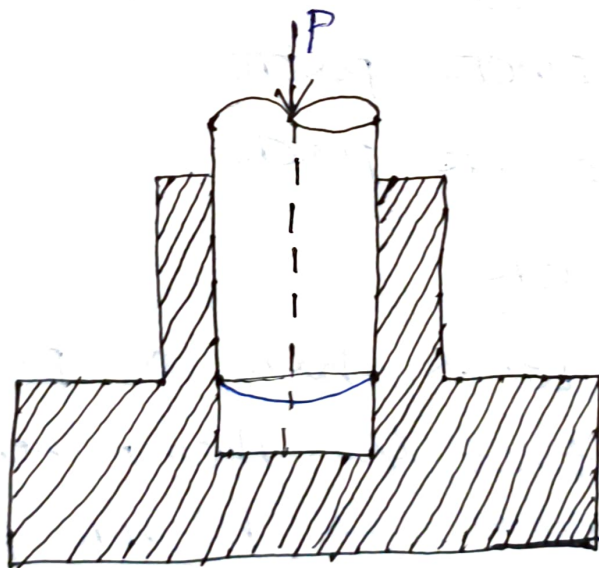


$\rightarrow$ D.O.F is more than 1

Successfully constrained motion / partially constrained

$\rightarrow$ ~~The~~ If the motion between the pair is not completely constrained by itself but by some other means, then the motion is called successfully constrained motion.

EX $\rightarrow$ shaft in a foot-step bearing.



EX $\rightarrow$ piston in cylinder of IC engine.

$\rightarrow$ conversion of more D.O.F to one D.O.F.

KINEMATIC PAIR

→ a joint of two links having relative motion between them is known as kinematic pair.

→ Classification of kinematic pairs

- ① According to nature of contact between links.
- ② According to type of relative motion b/w links.
- ③ According to nature of mechanical constraint b/w the links.

1.3 ① According to nature of contact

→ It is of two types.

② Higher pair.

③ Lower pair.

② Higher pair

→ If two links have a point or line contact b/w them, then kinematic pair is known as higher pair.

→ contact points of two links are not similar.

→ EX → Cam & follower.

① Lower pair

→ If two links has surface or area contact b/w them, then kinematic pair is known as Lower pair.

EX → Shaft rotating in a bearing.
Nut turning on a screw.

② According to type of relative motion b/w two links.

→ kinematic pairs are classified as

① Sliding pair

② Turning pair

③ Rolling pair

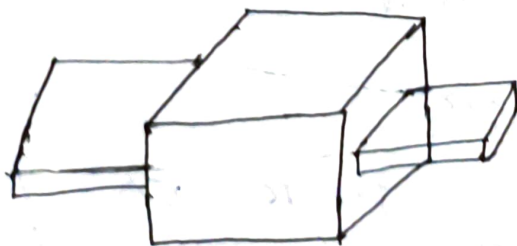
④ screw pair (helical pair)

⑤ spherical pair.

① Sliding pair

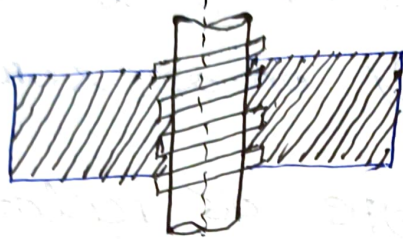
→ a kinematic pair is known as sliding pair if two links have a sliding motion relative to each other.

EX → rectangular rod in a rectangular hole.



⑧ Turning pair

→ a kinematic pair is known as turning pair if one link has a turning or revolving motion relative to other.



⑨ Rolling pair

→ a kinematic pair is known as rolling pair if one link has a rolling motion relative to the other.

EX → Ball & bearings

rolling wheel on flat surfaces

⑩ Screw pair

→ a kinematic pair is known as screw pair if two links have turning & sliding motion w.r. to them.

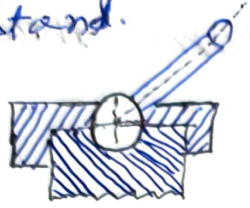
EX → Bolt & nut.

⑪ Spherical pair

→ a kinematic pair is known as spherical pair if one link is in the form of sphere turns inside a

fixed LINK

EX → ball & socket joint.
→ pen stand.



③ According to nature of mechanical constraint
b/w two links

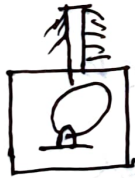
Ⓐ closed pair / ~~force closed pair~~ ^{self closed pair}

Ⓑ open pair / unclosed pair / force closed pair.

Ⓐ closed pair / self closed pair

→ 90 two elements of a pair held together mechanically called closed pair.

→ It can be separated by destroying the two joints.



Ⓑ open pair / force closed pair

→ 90 two elements of a pair are in contact due to gravity force or ~~by~~ due to spring force called open pair.

→ EX → cam & follower.



TYPES OF LINK.

- ① According to number of connections
- ② According to medium through which force/motion is transmitted.

① According to number of connections

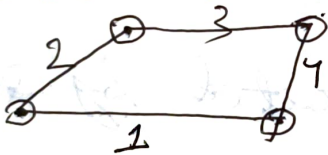
Ⓐ Binary Link.

Ⓑ Ternary Link.

Ⓒ Quaternary Link.

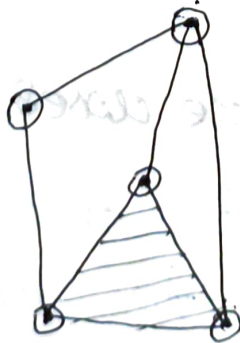
Ⓐ Binary Link

→ a link to which two links are connected known as binary link.



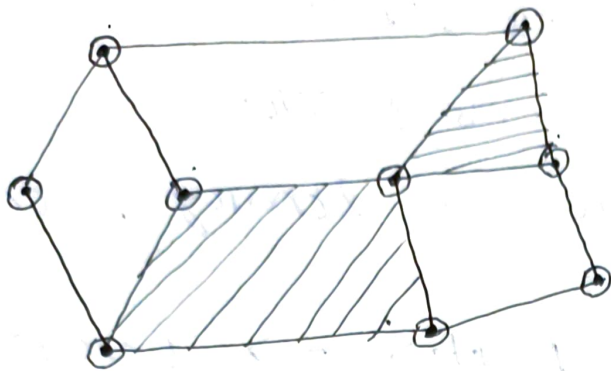
Ⓑ Ternary Link

→ a link to which three links are connected known as Ternary Link.



Ⓒ Quaternary Link

→ a link to which four links are connected, known as quaternary link.



DEGREES OF FREEDOM (D.O.F)

→ It is defined as the number of independent motion (both translation & rotational) a body can have.

$$\boxed{\text{D.O.F.} = 6 - \text{Number of constraints}}$$

→ D.O.F of plane mechanism:-

D.O.F of plane mechanism means the number of inputs needed to determine the configuration or position of all the links of the mechanism w.r.t. fixed link.

→ D.O.F also known as mobility of the mechanism.

→ no. of independent variable that must be specified to determine completely the condition of system.

RELATION B/W NO OF PAIRS & NUMBER OF LINK.

$$\begin{aligned} L &= 2P - 4 \\ j &= \frac{3}{2}L - 2 \end{aligned}$$

n = number of links

P = number of pairs

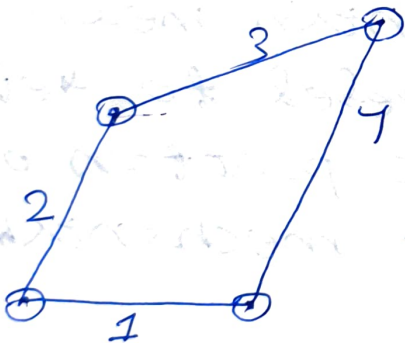
j = number of binary joints

① $L.H.S > R.H.S \rightarrow$ Locked chain

② $L.H.S = R.H.S \rightarrow$ chain is constrained.

③ $L.H.S < R.H.S \rightarrow$ chain is unconstrained

①



$$n = 4$$

$$P = 4$$

$$j = 4$$

$$n = 2P - j$$

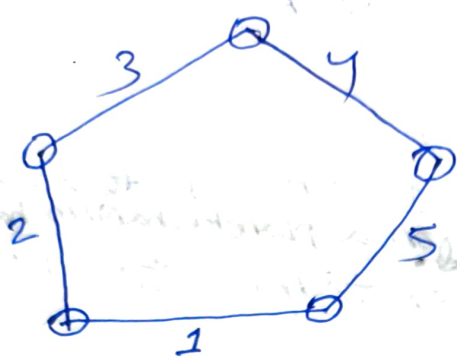
$$\Rightarrow 4 = 2 \times 4 - 4$$

$$L.H.S = 4$$

$$R.H.S = 2P - j = 2 \times 4 - 4 = 8 - 4 = 4$$

$$\boxed{L.H.S = R.H.S} \quad (\text{constrained})$$

②



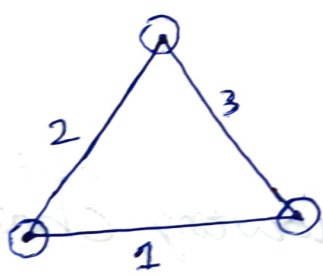
$$\begin{aligned} n &= 5 \\ J &= 5 \\ P &= 5 \end{aligned}$$

$$L.H.S = 5$$

$$R.H.S = 2P - 4 = 2 \times 5 - 4 = 6$$

$L.H.S < R.H.S$ → unconstrained.

③



$$\begin{aligned} n &= 3 \\ P &= 3 \\ J &= 3 \end{aligned}$$

$$L.H.S = 3$$

$$R.H.S = 2P - 4 = 2 \times 3 - 4 = 2$$

$L.H.S > R.H.S$
(Locked chain)

KUTZBACH EQUATION OF D.O.F

$$D.O.F = 3(n-1) - 2J - h$$

n = no of links

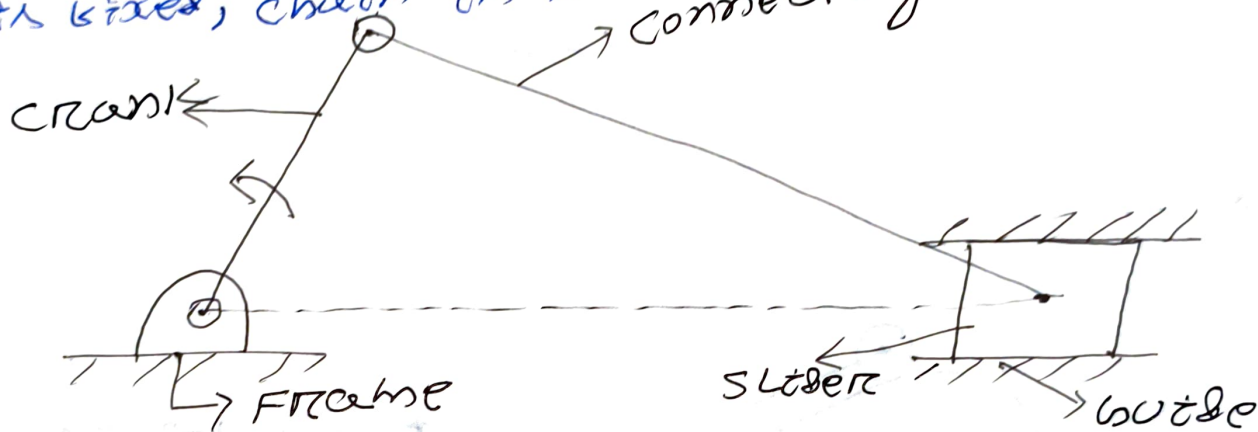
J = no of binary joint

$n-1$ = no of movable links

h = no of higher pair.

MECHANISM

- a combination of rigid or restraining bodies which are so shaped and connected that they move upon each other with definite relative motion is known as mechanism.
- When one of the links of a kinematic chain is fixed, chain is known as mechanism.



- it converts reciprocating motion of the slider into a rotary motion of crank or vice versa.
- $n \geq 4$ → simple mechanism
 $n > 4$ → compound mechanism.

MACHINE

- a machine is a mechanism or a combination of mechanisms which not only imparts definite motions to the parts but also transmits & modifies the available mechanical energy into some kind of useful energy.

INVERSION

- Method of obtaining different mechanism by fixing different links in a kinematic chain, is known as inversion of the mechanism.
- Relative motion ^{of various links} does not change but absolute motion ~~may~~ is changed.

DIFFERENT TYPES OF KINEMATIC CHAIN AND THEIR INVERSIONS

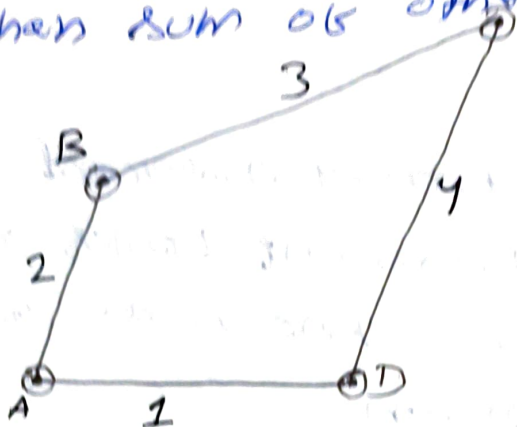
→ Different types of kinematic chains with four kinematic pairs are

- ① Four bar chain
- ② single slider crank chain
- ③ double slider crank chain.

FOUR BAR CHAIN / QUADRIC CYCLE CHAIN

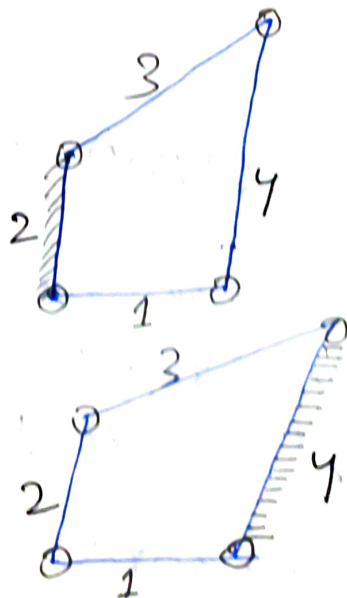
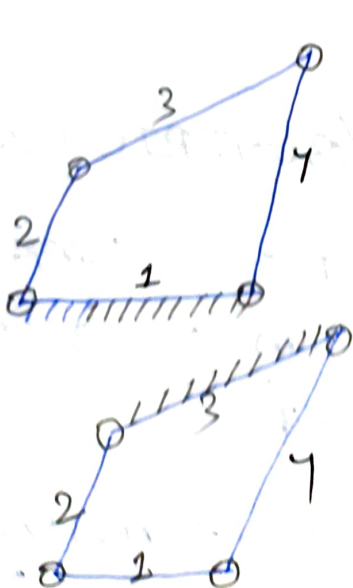
- It consists of four rigid links which are connected in the form of quadrilateral by four pin joints.
- It consists of four turning pairs.

condⁿ:- Length of one of the links should always be less than sum of other three links.



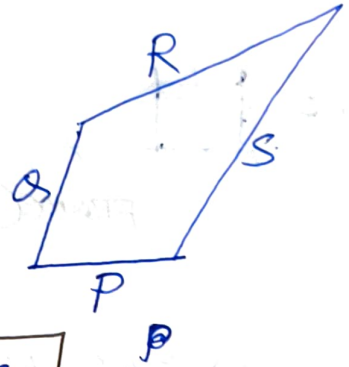
- The Link that makes complete revolution is known as crank.
- The fixed Link is known as frame.
- Link opposite to the fixed Link is known as coupler or connecting rod.
- Fourth Link is known as Lever or rocker.
- If different links of four-bar mechanism are fixed, four different mechanisms (inversion) will be obtained.

→



GRASHOF'S LAW

→ In a four-bar mechanism, the sum of lengths of shortest and longest link should be less than the sum of length of other two links for continuous relative motion between two links.



$$P + R \leq Q + S$$

P = shortest link.

R = longest link.

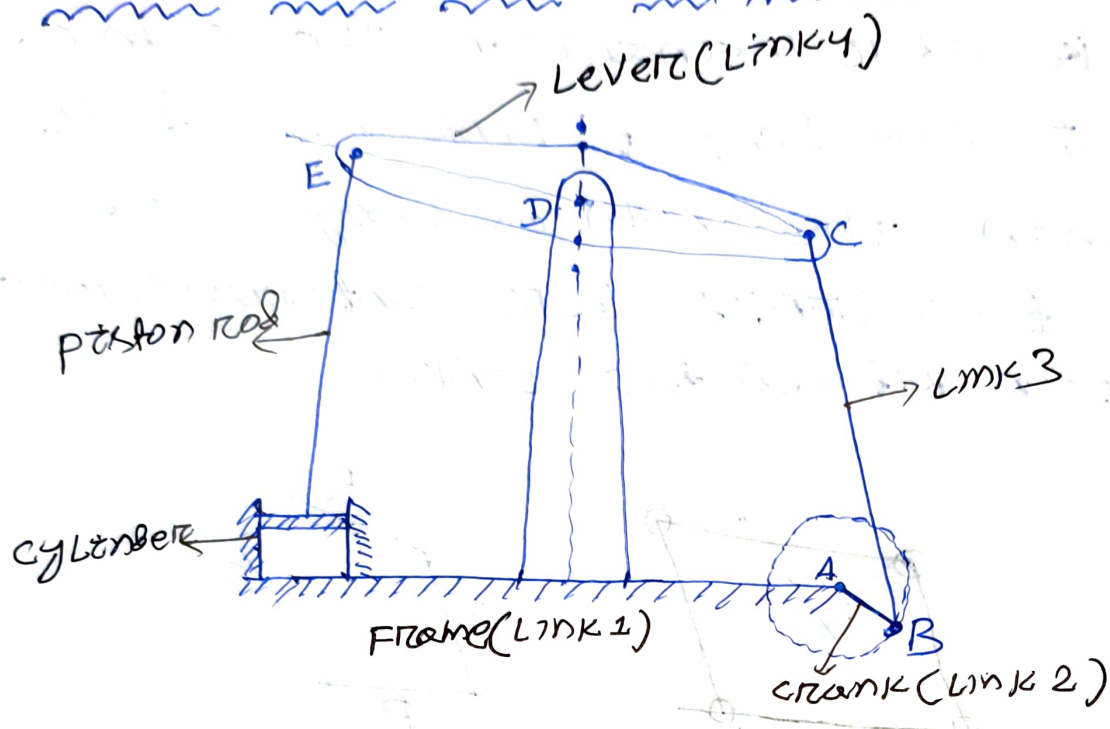
→ one of the links (shortest link) should make a complete revolution relative to the other three links.

INVERSION OF FOUR BAR CHAIN

→ There are ~~three~~ ^{different} types of inversion of four-bar chain as follows.

- ① crank and lever mechanism (Beam engine)
- ② Double crank mechanism (coupling rod of a locomotive)
- ③ Double lever mechanism (Watts indicator mechanism)

① Crank and Lever mechanism (Beam engine)

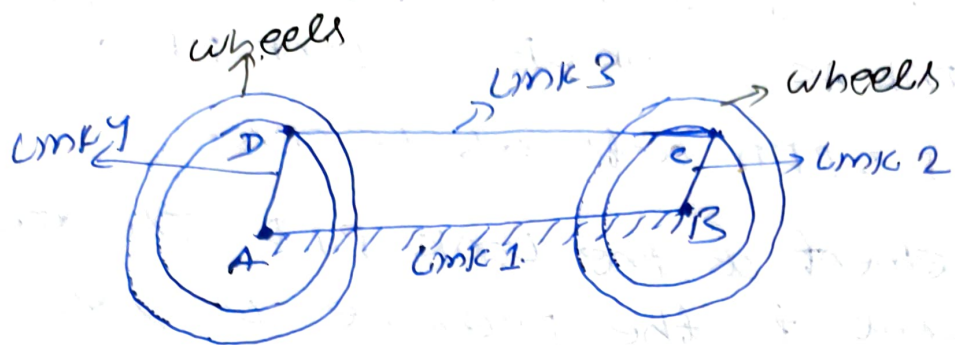


→ It consists of four links & purpose of this mechanism is to convert rotary motion into reciprocating motion.

→ When crank rotates about fixed centre A, Lever oscillates about a fixed centre D, the end E of the Lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

② Double Crank mechanism (coupling rod of a locomotive)

→ It consists of four links as shown below.

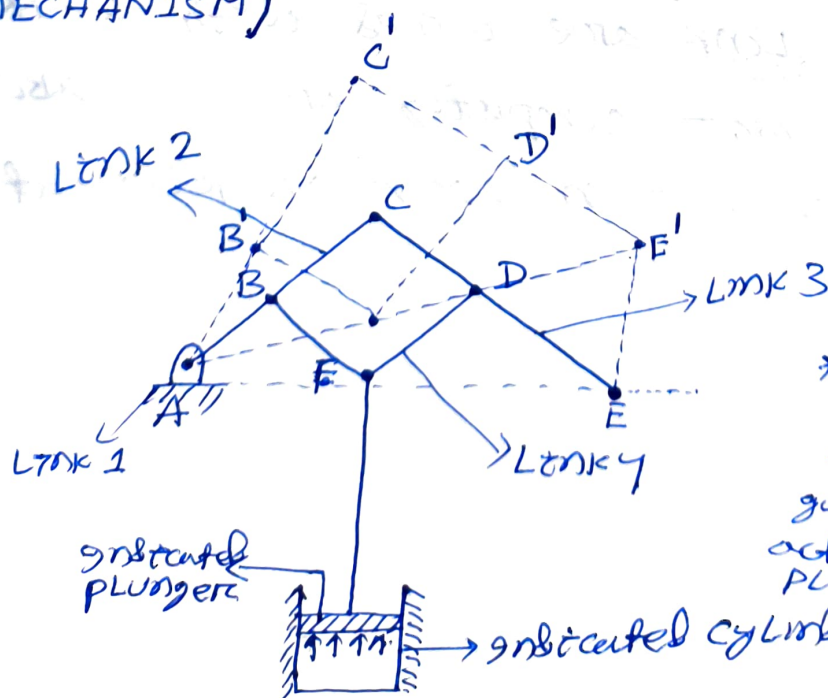


→ In this mechanism, the Link AD & BC (having equal length) acts as cranks and connected to the respective wheels.

→ The Link CD acts as a coupling rod & Link AB is fixed in order to maintain a constant centre to centre distance b/w them.

→ This mechanism is used for transmitting rotary motion from one wheel to the other wheel.

③ Double Lever mechanism (WATT'S INDICATOR MECHANISM)



* Dotted line shows the position of mechanism when gas or steam pressure acts on the grindicated plunger.

- It consists of four links i.e. fixed link at A, Link AC, Link CE & Link BFD.
- Link CE & BFD acts as levers.
- Displacement of the Link BFD is directly proportional to the pressure of gas on steam which acts on the indicated plunger.
- on any small displacement of the mechanism, the tracing point E at the end of the Link CE traces out approx. straight line.

-
- ① Simple mechanism ($n=4$, $DOF=1$)
 - ② Compound mechanism ($n \geq 6$, $DOF=1$)
 - ③ Complex mechanism ($n \geq 6$, $DOF > 1$)

KINEMATIC CHAIN

- When the kinematic pairs are so connected that last link are joined with first link to transmit completely or successfully constrained motion known as kinematic chain.

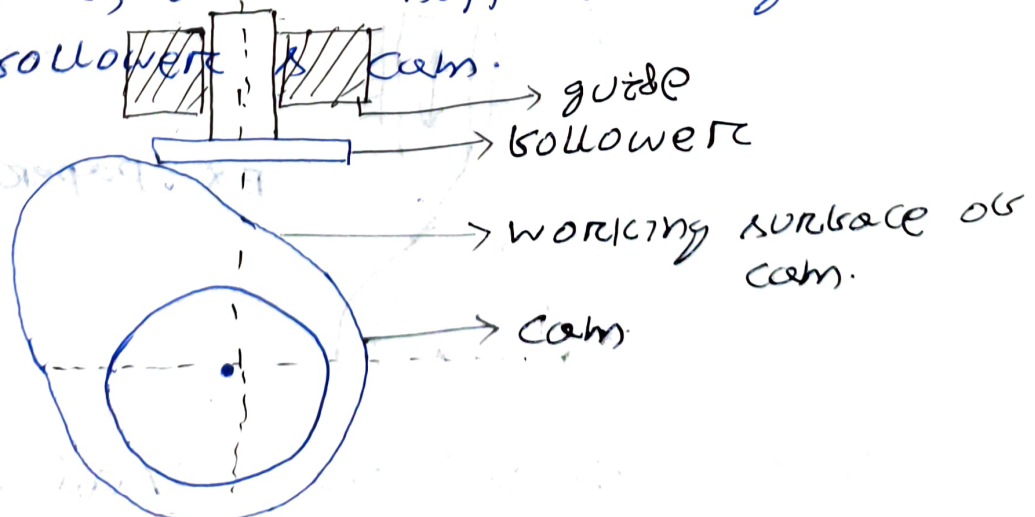
CAM AND FOLLOWER

- Defⁿ: a Rotating machine element, which gives reciprocating or oscillating motion to a second element is known as cam.
 - second element is called follower
 - cam rotates usually at constant speed ~~set~~ and drives the follower whose motion depends upon the shape of cam.
 - Always cam is driver & follower is the driven.
- EX → cams are commonly used in IC engine for operating the inlet and exhaust valve
- printing machinery, machine tool, automa
 - tic machine etc.
- In a cam mechanism, there are three essential parts. i.e.

① cam

② follower

③ frame, which supports a guide, the follower & cam.



21. TYPES OF FOLLOWER

→ The followers which are having either reciprocating or oscillating motion, are generally classified as follows.

① According to their shape or that part which is in contact with the cam

Ⓐ knife-edge follower

Ⓑ roller follower

Ⓒ flat faced follower or mushroom follower

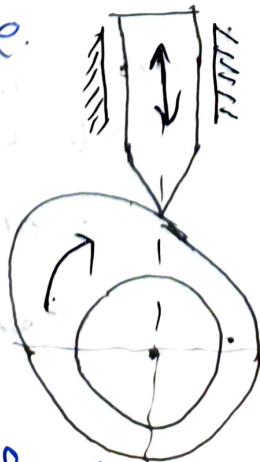
Ⓓ spherical faced follower.

Ⓐ knife-edge follower

→ When the contacting end of the follower has the sharp knife edge, it is called knife edge follower.

→ It is not used in practice bcz the small area of contacting surface results in excessive wear.

→ In knife edge follower a considerable side thrust exists b/w the follower & guide.

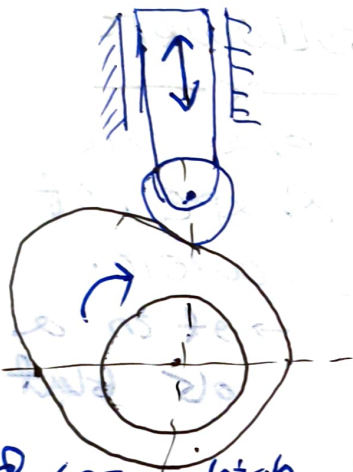


EX - Paper cutting machine.

→ It is used in slow speed application.

② Roller Follower

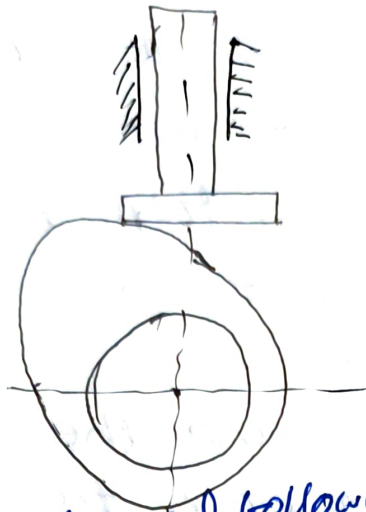
- When the contacting end of the follower is a roller, called a roller follower.
- Since the rolling motion takes place b/w the contacting, rate of wear is considerably reduced.
- In roller follower also the side thrust exist b/w the follower & the guide.



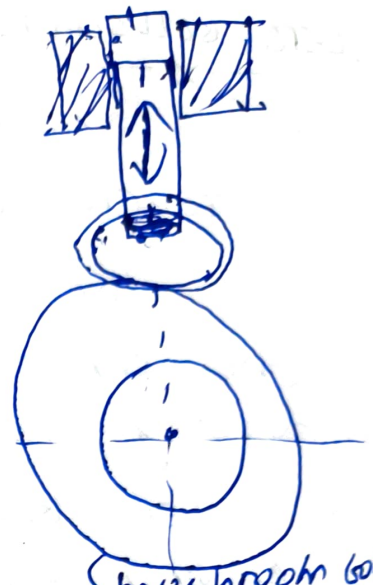
→ It is used for high speed application.

③ Flat faced / Mushroom Follower

- When contacting end of the follower is perfectly flat faced it is called flat faced follower.
- In this follower side thrust is reduced. b/w follower & guide.
- When flat faced follower is circular called mushroom follower.



(flat faced follower)

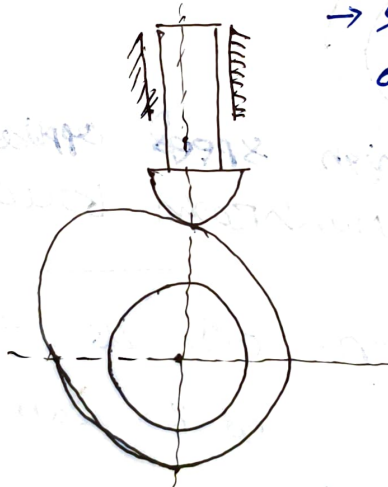


(mushroom follower)

③ spherical faced follower

→ When contacting end of follower is of spherical shape, it is called spherical faced follower.

→ It is a modification of flat faced follower.



② According to motion of follower

① Reciprocating or translatory follower

② Oscillating follower

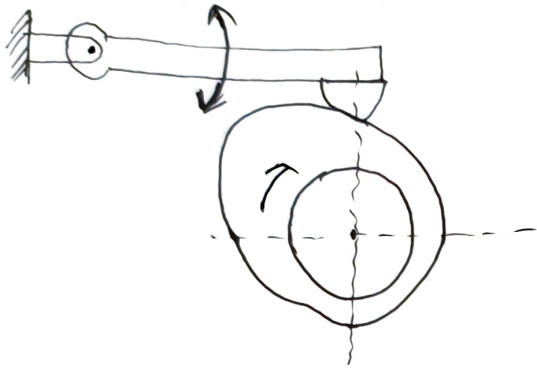
① Reciprocating follower

→ In reciprocating follower, follower reciprocates as the cam rotates uniformly.

EX → knife edge follower, roller follower, flat faced follower, spherical follower.

② Oscillating follower

→ In oscillating follower, follower oscillates as cam rotates uniformly.



③ According to path of motion of the follower.

(a) Radial follower / in line follower

(b) offset follower.

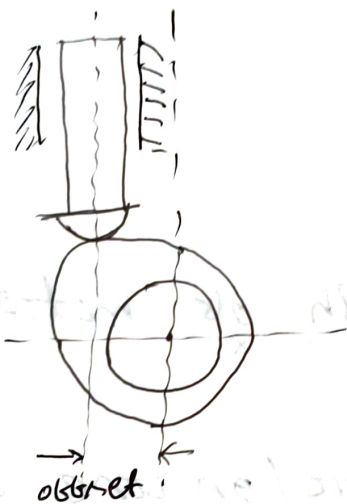
④ Radial follower

→ When the motion of the follower is along an axis passing through the centre of cam, known as radial/in line follower.



⑧ offset follower

- When motion of the follower is along an axis away from the axis of the cam centre, called offset follower.
- In this case wear reduced substantially.



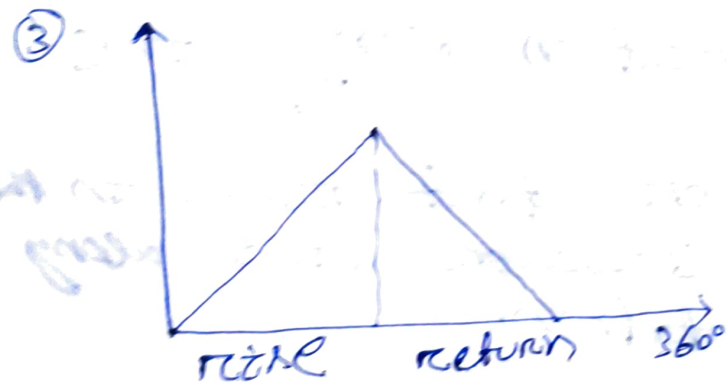
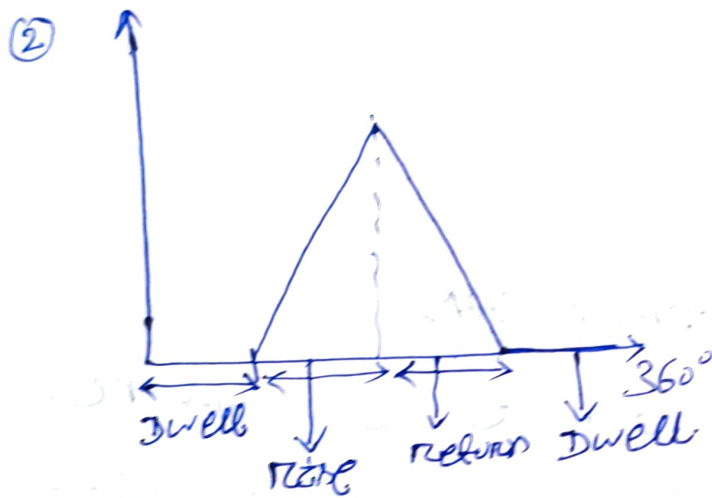
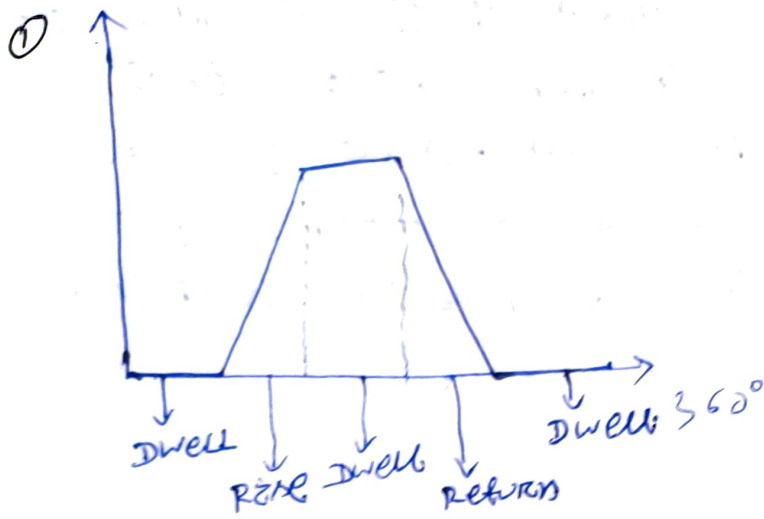
TYPES OF CAM

→ It is divided into two categories.

- ① According to follower motion
- ② According to shape.

According to follower motion

- ① Dwell-Rise-Dwell cam.
- ② Dwell-Rise-Return-Dwell cam
- ③ Rise-Return-Rise cam.



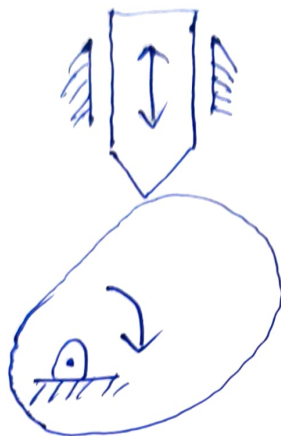
② According to shape

① Radial or Disc cam / plate cam

→ It is the simplest and most common type of cam.

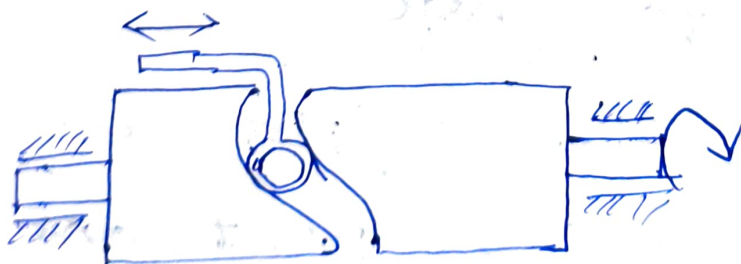
→ This type of cam is mounted on a shaft or plate.

→ The radial distance from the centre of the disk is varied throughout the circumference of the cam. Allowing a follower to ride on this outer edge gives the follower a radial motion.



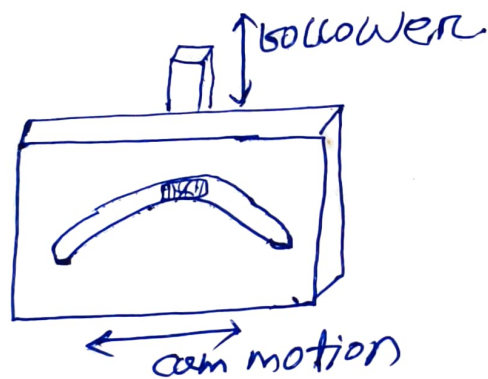
⑧ CYLINDRICAL OR DRUM CAM

- It is formed on a cylinder.
- a groove is cut into the cylinder with a varying location along the axis of rotation.
- Attaching a follower that rides in the groove gives the follower motion along the axis of rotation.



③ ~~Linear~~ Flat cam / Linear cam

- This type of cam is formed on a translated block.
- a groove is cut into the block with a distance that varies from the plane of translation.
- Attaching a follower that rides in the groove gives the follower motion perpendicular to the plane of translation.



POWER TRANSMISSION (P.K. SWAIN)

- power transmission is a process to transmit motion from one shaft to another by using some connection between them like belt, rope, chain & gear.
- To connect the shaft, mainly two types of connectors are used, one is flexible & other is rigid.
- In flexible types of connection there is relative velocity between shaft and connectors due to slip and strain produced in the connectors.
- But in rigid connection, there is no relative velocity between connector & shaft.
- Belt, rope and chain are flexible connectors where as gears are rigid connectors.

TYPES OF DRIVE

- generally there are two types of drives present.
 - ① belt drive.
 - ② rope drive.

③ chain drive.

④ gear drive.

Belt drive

→ belt drive is a mechanical transmission that utilizes a flexible belt that is tensioned on a pulley for power transmission.

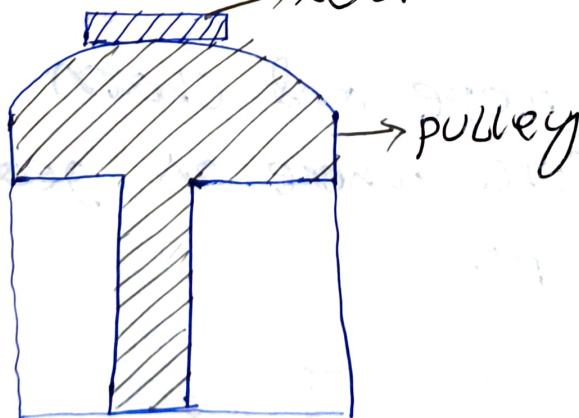
→ It consists of a drive wheel, a driven wheel and a flexible endless belt that is tensioned on the two pulleys.

→ There are ^{generally} three types of belt ~~drive~~.

① Flat belt ~~drive~~.

→ rectangular in section.

→ 8 m distance

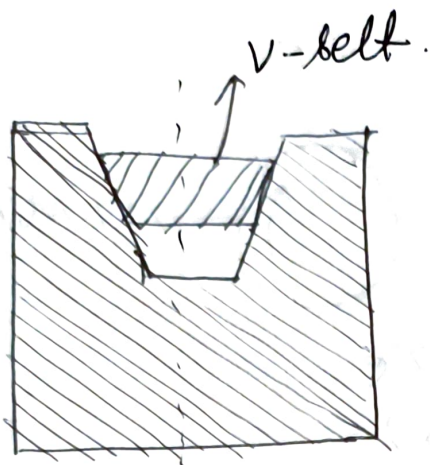


EX → Factory, workshop

② V-belt drive

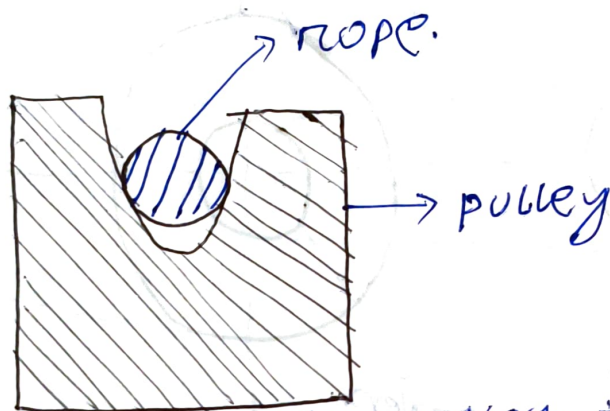
→ trapezoidal in section

→ very near to each other



- Materials of belt
- ① Leather belt:-
 - ② cotton belt
 - ③ Rubber belt
 - ④ Balata belt

③ circular belt or rope drive.



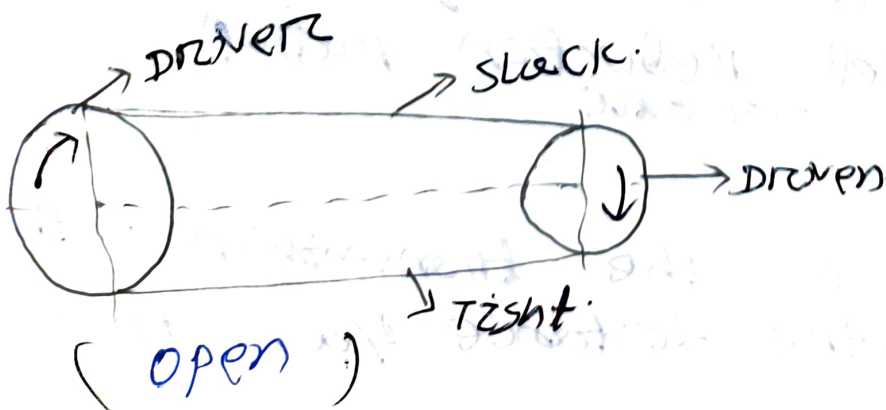
→ when distance b/w pulley is more than 8 m.

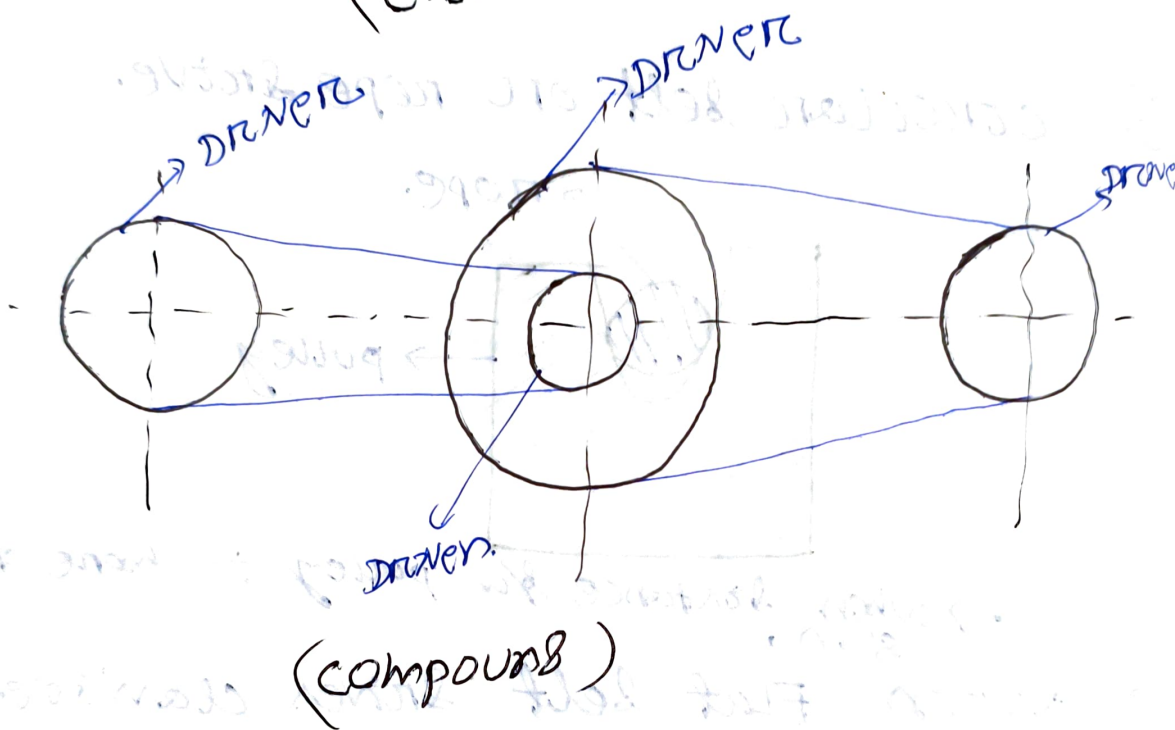
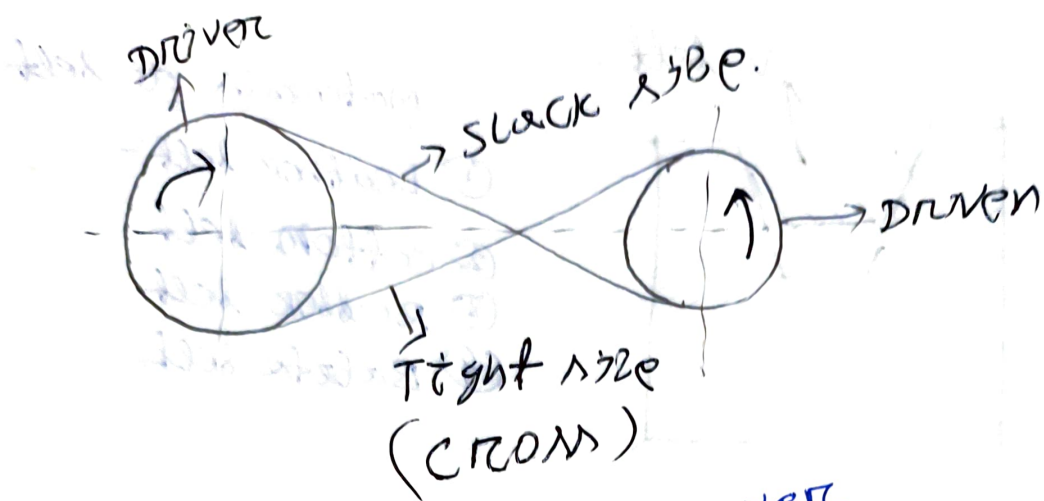
→ again Flat belt drives classified into three categories.

① open belt drive.

② cross belt drive.

③ compound belt drive.





→ selection of belt drive depends upon

- ① centre distance b/w the shaft
- ② power to be transmitted.
- ③ speed of driving & driven shaft
- ④ speed reduction ratio.
- ⑤ space available.

PROS

→ applicable to the transmission with large centre distance b/w two shaft.

- slip during overload to prevent damage to other parts.
- simple structure & low cost.

CONS

- belt has a short life span
- Transmission η is low.
- Due to slip, constant velocity ratio cannot be guaranteed.
- outer dimension of the transmission are large.

CHAIN DRIVE

- Chain drive is a transmission method in which the movement and power of a drive sprocket having a special tooth shape is transmitted to a driven sprocket having a special tooth shape through chain.

→



(etc.)

PROS

- higher η than belt drive.
- Transmission power is large, the ~~overload capability~~ & transmission size under same working condⁿ is small.
- required tension is small and the pressure acting on the shaft is small. $\therefore$
- it can work in harsh environment such as high temp^r, humidity, dust and pollution.

→

CONS

- can be only used for transmission b/w two parallel shaft.
- high cost.
- easy to wear, easy to stretch, & poor transmission stability.

- additional dynamic loads, vibration, shock and noise are generated during operation
- it should not be used in a cost reverse drive.

GEAR DRIVE

- gear is a kind of machine element in which teeth are cut around cylindrical or cone shaped surfaces with equal spacing.
- by meshing a pair of these elements; they are used to transmit rotations and forces from the driving shaft to the driven shaft.
- ~~it can be classified as~~



PROS

- compact structure, suitable for short distance transmission.
- Transmission ratio is accurate, stable & efficient.
- high reliability & long life.
- it can transmit the power b/w the parallel axis; intersecting axis or any angle & staggered axis or any angle.

CONS

- high manufacturing & installation cost
- not suitable for transmission b/w two axes at a long distance.
- no overload protection.

BELT DRIVE

TYPES OF FLAT BELT DRIVE

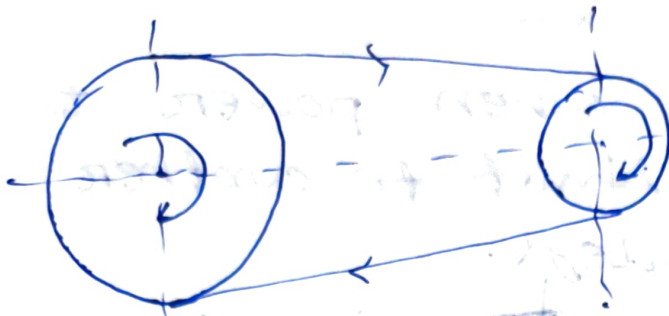
① OPEN BELT DRIVE

- It is used with shafts arranged parallel and rotating in same direction.

- In this case, driver pulls the belt from one side (i.e. lower side) and delivers it to the other side (i.e. upper side).

- So tension in lower side will be more & upper side will be less.

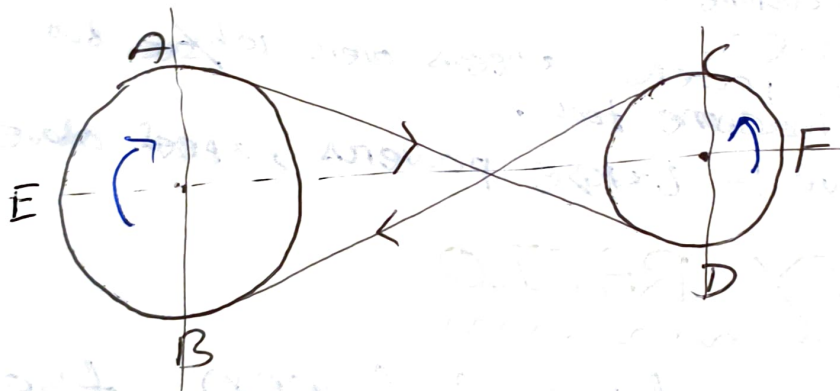
- So lower side is known as tight side & upper side known as slack side.



② CROSS BELT DRIVE

→ It is used with shafts arranged parallel and rotating in opposite direction.

→



BC → Tight side.

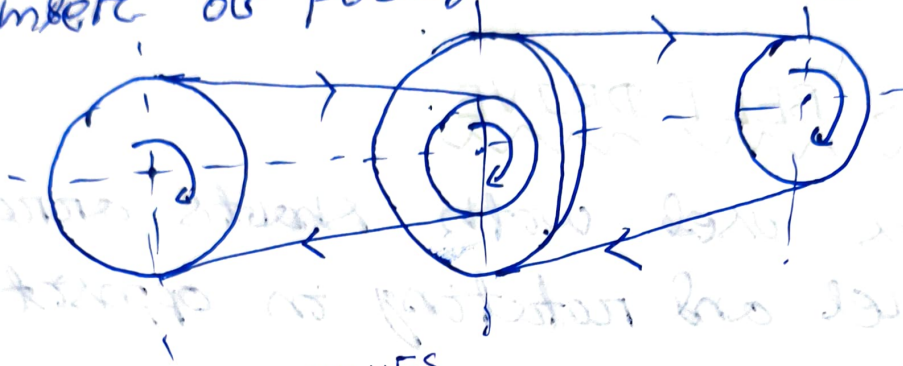
AD → Slack side.

→ at the point where ~~the~~ belt crosses each other, there is excessive wear and tear.

→ in order to avoid this, shaft is placed at a distance of $20b$,
 b = width of belt.

② COMPOUND BELT DRIVE

→ it is used when power is transmitted from one shaft to another through number of pulleys.



TYPES OF BELT DRIVES

- ② Light drive :- This is used to transmit small powers at belt speeds upto about 10 m/sec
EX → agriculture machine, small machine tool.
- ③ Medium drive :-
mean power, speeds over 10 m/sec but upto 22 m/sec
EX → machine tools.
- ④ Heavy drive :- Large powers, speed above 22 m/sec

VELOCITY RATIO

→ it is the ratio between the velocities of driver & driven or follower.

→ velocity ratio of belt drive

$$= \frac{\text{Velocity of follower (driven)}}{\text{Velocity of driver}}$$

→ Let N_1 = speed of driver in r.p.m.

d_1 = diameter of driver pulley

N_2 = speed of driven pulley in r.p.m.

d_2 = diameter of driven pulley.

→ Length of belt passing over the driver in 1 min.
 $= \pi d_1 N_1$

→ Length of belt passing over the follower in 1 min.
 $= \pi d_2 N_2$

→ but $\pi d_1 N_1 = \pi d_2 N_2$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}} \rightarrow \text{velocity ratio.}$$

→ Effect of belt thickness (t) on velocity ratio.

$$\text{Let } \omega_1 = \text{angular velocity of driver} \\ = \frac{2\pi N_1}{60}$$

$$\omega_2 = \text{angular velocity of driven} \\ = \frac{2\pi N_2}{60}$$

r_{m1} = mean radius of rotation of driver pulley $= r_1 + \frac{t}{2}$

r_{m2} = mean radius of rotation of driven pulley $= r_2 + \frac{t}{2}$

* peripheral velocity of two pulley should be same i.e. equal to velocity of belt (v)

$$v = \omega_1 r_{m1} = \omega_2 r_{m2}$$

$$\Rightarrow \omega_1 \left(r_1 + \frac{t}{2} \right) = \omega_2 \left(r_2 + \frac{t}{2} \right)$$

$$\Rightarrow \frac{\omega_2}{\omega_1} = \frac{r_1 + \frac{t}{2}}{r_2 + \frac{t}{2}}$$

$$\Rightarrow \frac{\omega_2}{\omega_1} = \frac{2r_1 + t}{2r_2 + t}$$

$$\Rightarrow \boxed{\frac{\omega_2}{\omega_1} = \frac{d_1 + t}{d_2 + t}}$$

→ VELOCITY RATIO CONSIDERING SLIP
Let v = velocity of belt passing over the driver pulley/min

N_1 = speed of driver pulley

N_2 = speed of driven

S_1 = slip between driver and belt

S_2 = slip between driven and belt.

→ peripheral velocity of driver pulley

$$= \omega_1 r_1$$

$$= \frac{2\pi N_1}{60} \times \frac{d_1}{2} = \frac{\pi d_1 N_1}{60}$$

→ now due to slip b/w driver pulley and belt, the velocity of belt passing over the driver pulley will decrease.

$$\therefore \text{velocity of belt} = \frac{\pi d_1 N_1}{60} - \left(\frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} \right)$$

$$= \frac{\pi d_1 N_1}{60} \left(1 - \frac{S_1}{100} \right) \quad \text{--- (1)}$$

→ peripheral velocity of follower/driven

$$= \text{velocity of belt} - \left(\text{velocity of belt} \times \frac{S_2}{100} \right)$$

$$= \text{velocity of belt} \left(1 - \frac{S_2}{100} \right)$$

putting value of eq (1) in above.

$$= \frac{\pi d_1 N_1}{60} \left(1 - \frac{S_1}{100} \right) \left(1 - \frac{S_2}{100} \right) \quad \text{--- (2)}$$

→ peripheral velocity of follower pulley

$$= \omega_2 r_2$$

$$= \frac{2\pi N_2}{60} \times \frac{d_2}{2} = \frac{\pi d_2 N_2}{60} \quad \text{--- (1b)}$$

→ equating eqn (ii) & (iii)

$$\frac{\cancel{K} d_2 N_2}{60} = \frac{\cancel{K} d_1 N_1}{60} \left(1 - \frac{s_1}{100}\right) \left(1 - \frac{s_2}{100}\right)$$

$$\Rightarrow d_2 N_2 = d_1 N_1 \left(1 - \frac{s_1}{100}\right) \left(1 - \frac{s_2}{100}\right)$$

$$\Rightarrow d_2 N_2 = d_1 N_1 \left(1 - \frac{s_2}{100} - \frac{s_1}{100} + \frac{s_1 s_2}{10000}\right)$$

$$\Rightarrow d_2 N_2 = d_1 N_1 \left(1 - \frac{s_1 + s_2}{100}\right)$$

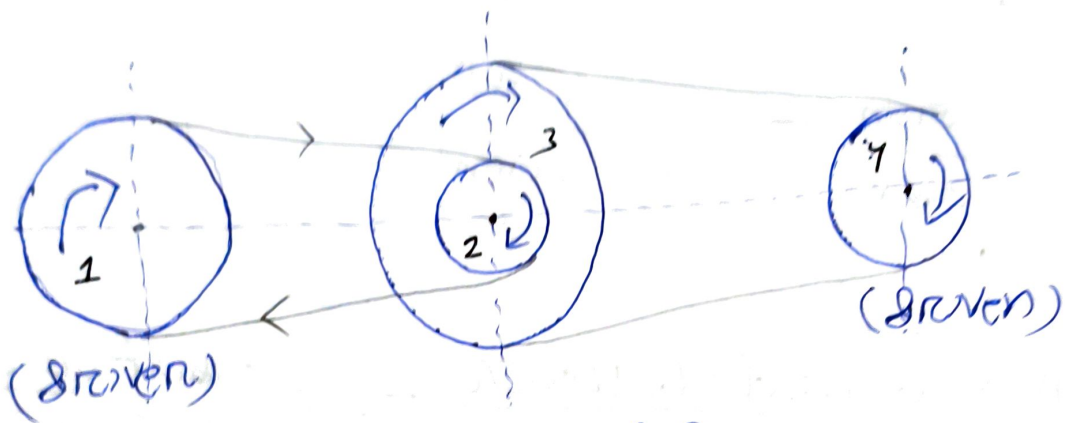
$$\Rightarrow d_2 N_2 = d_1 N_1 \left(1 - \frac{s}{100}\right)$$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2} \left[1 - \frac{s}{100}\right]}$$

* 96. belt thickness is considered

$$\boxed{\frac{N_2}{N_1} = \left(\frac{d_1 + t}{d_2 + t}\right) \left(1 - \frac{s}{100}\right)}$$

VELOCITY RATIO OF COMPOUND BELT DRIVE



→ considering pulley 1 & 2

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

$$\Rightarrow N_2 = \frac{d_1 N_1}{d_2} \quad \text{--- (1)}$$

→ considering pulley 3 & 4

$$\frac{N_4}{N_3} = \frac{d_3}{d_4}$$

$$\Rightarrow N_3 = \frac{N_4 d_4}{d_3} \quad \text{--- (2)}$$

→ Here pulley 2 & 3 are mounted on same shaft. so

$$\boxed{N_2 = N_3}$$

→ From eqn (1) & (2)

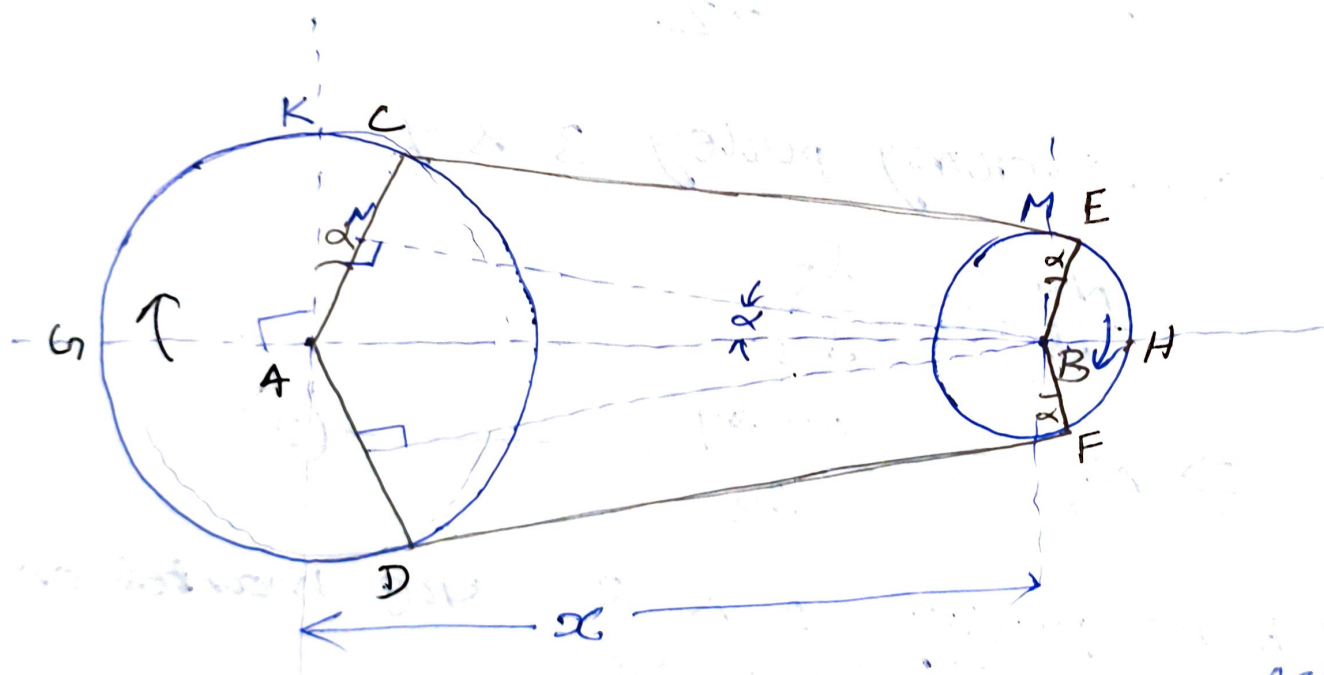
$$\frac{d_1 N_1}{d_2} = \frac{N_4 d_4}{d_3}$$

$$\Rightarrow \boxed{\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}}$$

$$\boxed{\frac{\text{even}}{088} = \frac{088 \times 088}{\text{even} \times \text{even}}}$$

$$\boxed{\frac{\text{speed of last follower}}{\text{speed of 1st driver}} = \frac{\text{product of dia. of driver}}{\text{product of dia. of follower}}}$$

LENGTH OF BELT IN OPEN BELT DRIVE



Let x = distance between the centre of two pulleys.

r_1 = radius of larger pulley.

r_2 = radius of smaller pulley.

L = Total Length of belt.

→ Belt leaves the Larger pulley at CSD and smaller pulley at ESF .

→ From B draw BN parallel to EC but EC is tangent at ' C '. Hence AC is $\perp$ to EC .

→ NOW $\angle ACE = 90^\circ$
 $\angle ANB = 90^\circ$

Let $\angle ABN = \alpha$ radian.

Then $\angle BAN = 90 - \alpha$.

But $\angle BAK = 90^\circ$

hence $\angle KAC = \alpha$.

→ NOW $AN = AC - CN$
 $= r_1 - r_2$

$$BN = \sqrt{AB^2 - AN^2}$$
$$= \sqrt{x^2 - (r_1 - r_2)^2}$$

$$\rightarrow BN = CE = \sqrt{x^2 - (r_1 - r_2)^2}$$

FROM $\triangle ABN$

$$\sin \alpha = \frac{AN}{AB} = \frac{r_1 - r_2}{x}$$

$$\sin \alpha \approx \alpha$$

$$\boxed{\alpha = \frac{r_1 - r_2}{x}}$$

NOW

$$L = \text{ARC DBC} + CE + \text{ARC EHF} + FD$$

$$= 2[\text{ARC DC} + CE + \text{ARC EH}]$$

$$= 2\left[\pi_1\left(\frac{\pi}{2} + \alpha\right) + \sqrt{x^2 - (\pi_1 - \pi_2)^2} + \pi_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$L = 2\left[\frac{\pi}{2}(\pi_1 + \pi_2) + \alpha(\pi_1 - \pi_2) + \sqrt{x^2 - (\pi_1 - \pi_2)^2}\right]$$

→ exact Length

→ Approximate Length is

$$\sqrt{x^2 - (\pi_1 - \pi_2)^2} = x \sqrt{1 - \left(\frac{\pi_1 - \pi_2}{x}\right)^2}$$

$$= x \left[1 - \left(\frac{\pi_1 - \pi_2}{x}\right)^2\right]^{\frac{1}{2}}$$

$$= x \left[1 - \frac{1}{2} \left(\frac{\pi_1 - \pi_2}{x}\right)^2 + \dots\right]$$

by Binomial expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

$$\sum_{r=0}^n \frac{n!}{(n-r)!r!} x^r = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

$$= x \left[1 - \frac{(\pi_1 - \pi_2)^2}{2x^2}\right] \quad (\because \text{neglecting higher terms})$$

$$= x - \frac{(\pi_1 - \pi_2)^2}{2x}$$

$$L = 2\left[\frac{\pi}{2}(\pi_1 + \pi_2) + \alpha(\pi_1 - \pi_2) + x - \frac{(\pi_1 - \pi_2)^2}{2x}\right]$$

$$= \pi(\pi_1 + \pi_2) + 2\alpha(\pi_1 - \pi_2) + 2x - \frac{(\pi_1 - \pi_2)^2}{x}$$

$$\alpha = \frac{\pi_1 - \pi_2}{x}$$

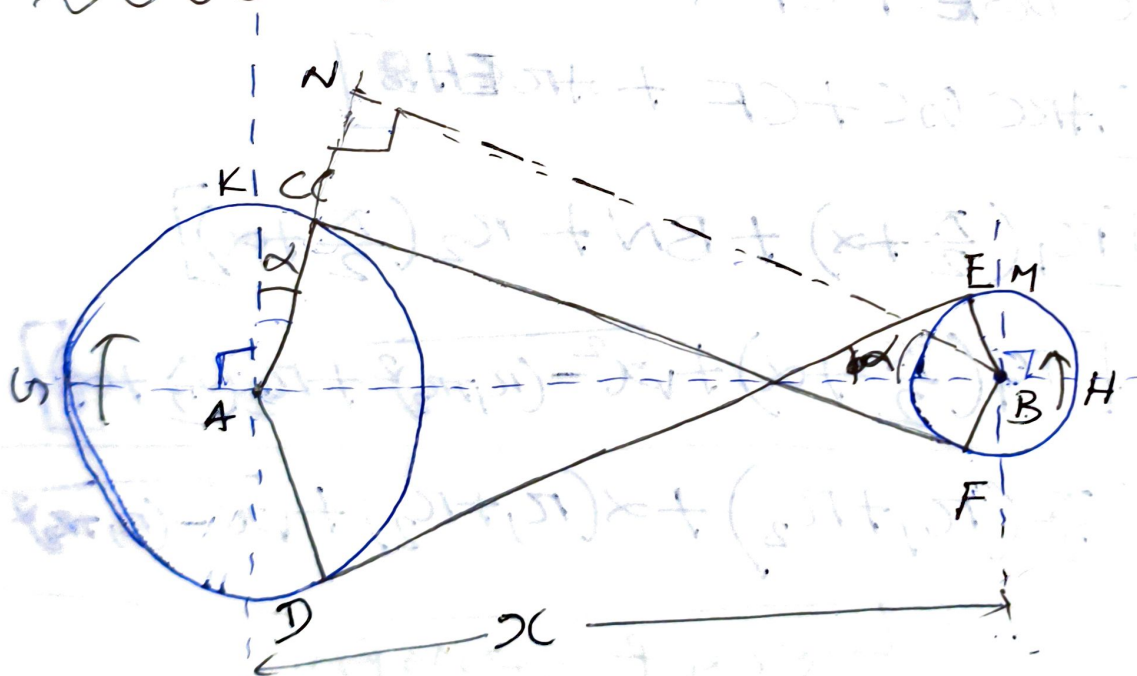
$$L = \pi(r_1 + r_2) + 2\left(\frac{r_1 - r_2}{x}\right)(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$\Rightarrow L = \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x.$$

$$\Rightarrow L = 2x + \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x}$$

So Length of open belt drive depends upon sum and difference of radius of two pulley.

LENGTH OF BELT IN CROSS BELT DRIVE



$$\begin{aligned} AN &= AC + CN \\ &= r_1 + FB \\ &= r_1 + r_2 \end{aligned}$$

9th $\triangle ABN$

$$BN = \sqrt{AB^2 - AN^2}$$

$$= \sqrt{x^2 - (r_1 + r_2)^2}$$

$$\sin \alpha = \frac{AN}{AB} = \frac{r_1 + r_2}{x}$$

$$\sin \alpha \approx \alpha$$

$$\Rightarrow \boxed{\alpha = \frac{r_1 + r_2}{x}}$$

$$L = \text{ARC DORE} + CF + \text{ARC FHE} + ED$$

$$= 2[\text{ARC BC} + CF + \text{ARC EH}]$$

$$= 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + BN + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + \sqrt{x^2 - (r_1 + r_2)^2} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$\boxed{L = 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + \sqrt{x^2 - (r_1 + r_2)^2}\right]}$$

$\hookrightarrow$ exact Length.

$\rightarrow$ app. Length is

$$\begin{aligned}\sqrt{x^2 - (r_1 + r_2)^2} &= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2} \\ &= x\left[1 - \left(\frac{r_1 + r_2}{x}\right)^2\right]^{\frac{1}{2}}\end{aligned}$$

$$= x \left[1 - \frac{1}{2} \left(\frac{r_1 + r_2}{x} \right)^2 + \dots \right]$$

$$= x \left[1 - \frac{(r_1 + r_2)^2}{2x^2} \right] \quad \left(\text{By neglecting higher terms} \right)$$

$$= x - \frac{(r_1 + r_2)^2}{2x}$$

$$SO, \quad L = 2 \left[\frac{\pi}{2} (r_1 + r_2) + x (r_1 + r_2) + \sqrt{x^2 - (r_1 + r_2)^2} \right]$$

$$L = \pi (r_1 + r_2) + 2 \left(\frac{r_1 + r_2}{x} \right) (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi (r_1 + r_2) + \frac{2}{x} (r_1 + r_2)^2 + 2x - \frac{(r_1 + r_2)^2}{x}$$

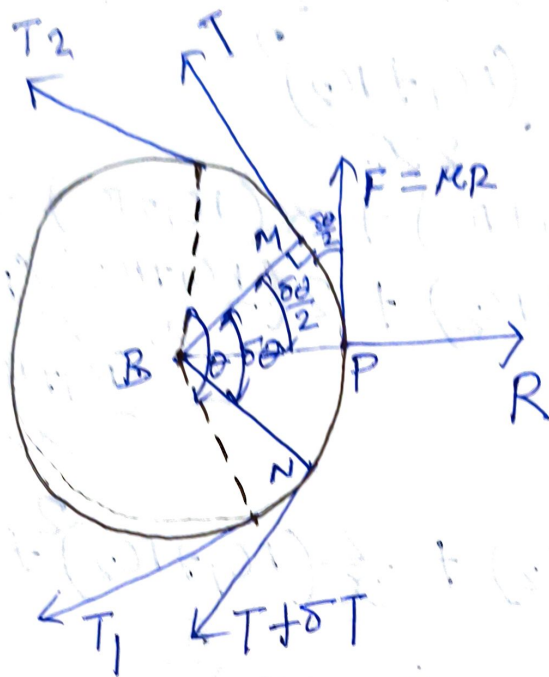
$$L = \pi (r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x$$

$$\Rightarrow L = 2x + \pi (r_1 + r_2) + \frac{(r_1 + r_2)^2}{x}$$

$\rightarrow$ app. Length

SO Length of cross belt drive depends upon only sum of radius of two pulley.

RATIO OF BELT TENSION



Let T_1 = Tension in the belt on tight side

T_2 = Tension in the belt on slack side

θ = angle of contact

μ = coefficient of friction b/w the belt & pulley.

→ let us consider a section of belt 'MN' subtending an angle ' θ ' at the centre of pulley B.

→ The forces acting on element 'MN' to keep equilibrium are.

- (i) Tension ' T ' on the belt at ' M ' acting tangentially
- (ii) Tension ' $T + \delta T$ ' on the belt at ' N ' acting tangentially.
- (iii) Normal reaction ' R ' acting radially outwards at ' P ', where ' P ' is the middle point of MN .
- (iv) Frictional force $F = \mu R$ acting at right angles to ' R ' and in the opposite direction of the motion of pulley.

Now $\angle PBM = \frac{\delta\theta}{2}$ and $\angle PBN = \frac{\delta\theta}{2}$

→ Resolving all forces in horizontal direction

$$R = T \cos(90 - \frac{\delta\theta}{2}) + (T + \delta T) \cos(90 - \frac{\delta\theta}{2})$$

$$= T \sin \frac{\delta\theta}{2} + (T + \delta T) \sin \frac{\delta\theta}{2}$$

since $\frac{\delta\theta}{2}$ is very small.

$$= T \frac{\delta\theta}{2} + (T + \delta T) \frac{\delta\theta}{2}$$

$$= T \delta\theta + (\delta T \times \frac{\delta\theta}{2})$$

$$\Rightarrow R = T \delta\theta \quad \text{————— (1)}$$

→ Now resolving all forces vertically

$$F = (T + \delta T) \cos \frac{\delta \theta}{2} - T \cos \frac{\delta \theta}{2}$$

$$= T + \delta T - T$$

$$\Rightarrow F = \delta T$$

$$\Rightarrow r_R = \delta T$$

$$\Rightarrow \boxed{R = \frac{\delta T}{r}} \quad \text{--- (ii)}$$

From eqn (i) & (ii)

$$T \times \delta \theta = \frac{\delta T}{r}$$

$$\Rightarrow \frac{\delta T}{T} = r \delta \theta$$

integrating above equation

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \int r \delta \theta$$

$$\Rightarrow \log \frac{T_1}{T_2} = r \theta$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = e^{r \theta}}$$

NOTE

→ For open belt drive, smaller angle of contact is taken i.e. angle of lap on smaller pulley i.e.

$$\theta = 180 - 2\alpha$$

→ For cross belt drive, angle of lap is same for both pulley i.e.

$$\theta = 180 + 2\alpha$$

CENTRIFUGAL TENSION ON BELT

Let V = velocity of belt in m/sec.

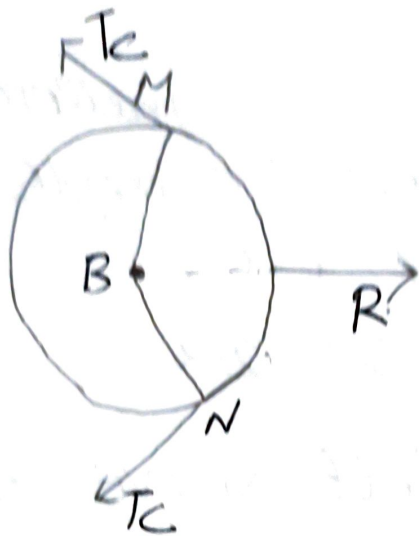
r = radius of pulley over which belt runs.

M = mass of ~~belt per~~ elemental length of belt MN

m = mass of belt per meter length.

T_c = centrifugal tension acting at M & N tangentially

R = centrifugal force acting radially outward.



→ Centrifugal force ' R ' acting radially outward is balanced by the component of ' T_c ' acting radially inward.

→ Now elemental length of belt $MN = r\delta\theta$

→ Mass of belt $MN = (\text{mass per unit length}) \times \text{length of } MN$
 $= m r \delta\theta$

→ Centrifugal force $= \frac{M v^2}{r}$
 $= \frac{m r \delta\theta v^2}{r}$

→ Resolving forces horizontally

$$T_c \sin \frac{\delta\theta}{2} + T_c \sin \frac{\delta\theta}{2} = R$$

$$\Rightarrow 2 T_c \sin \frac{\delta\theta}{2} = m r \delta\theta \frac{v^2}{r}$$

$$\Rightarrow \frac{2T_c \frac{\pi \theta}{2}}{\pi} = m \pi \frac{\pi \theta}{\pi} \frac{v^2}{\pi}$$

$$\Rightarrow \boxed{T_c = mv^2}$$

NOTE

→ T_c is independent of T_1 & T_2

→ When T_c is considered

$$T_{\max} = T_1 + T_c$$

$$T_{\min} = T_2 - T_c$$

→ Max^m Tension (T_m) in belt is

$$\boxed{T_m = 6 \times 8 \times t}$$

96

POWER TRANSMISSION IN BELT DRIVE

→ effective driving force = $T_1 - T_2$

$$\text{Work done/sec} = (T_1 - T_2) \times v$$

$$\Rightarrow \boxed{\text{Power} = \frac{(T_1 - T_2) v}{1000}}$$

→ Torque on driving pulley = $(T_1 - T_2) r_1$

→ Torque on driven pulley = $(T_1 - T_2) r_2$

MAXIMUM POWER TRANSMITED BY BELT

Let T_1 = Tension on tight side.

T_2 = Tension on slack side.

V = Linear velocity of belt

→ Then power transmitted by belt

$$= (T_1 - T_2)V$$

We know

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\Rightarrow T_2 = \frac{T_1}{e^{\mu\theta}} \quad \text{--- (i)}$$

→ substituting the value of T_2 in eqn

$$= \left(T_1 - \frac{T_1}{e^{\mu\theta}} \right) V$$

$$= T_1 \left(1 - \frac{1}{e^{\mu\theta}} \right) V$$

$$\text{Let } \left(1 - \frac{1}{e^{\mu\theta}} \right) = k$$

$$\Rightarrow T_1 k V$$

$$\rightarrow \text{Let } T_{\max} = T_1 + T_C$$

$$\Rightarrow T_1 = T_{\max} - T_C$$

$$\Rightarrow P = T_1 K V$$

$$= (T_{\max} - T_C) K V$$

$$= K(T_m - mv^2) V$$

$$= K(T_m V - mv^3)$$

$\rightarrow$ power is max. i.e.

$$\frac{dP}{dV} = 0$$

$$\Rightarrow K(T_m - 3mv^2) = 0$$

$$\Rightarrow T_m = 3mv^2$$

$$\Rightarrow \boxed{T_{\max} = 3T_C}$$

$$\rightarrow T_{\max} = T_1 + T_C$$

$$\Rightarrow T_{\max} = T_1 + \frac{T_{\max}}{3}$$

$$\Rightarrow T_{\max} - \frac{T_{\max}}{3} = T_1$$

$$\Rightarrow \boxed{T_1 = \frac{2}{3} T_{\max}}$$

$$\Rightarrow T_1 = \frac{2}{3} T_{max}$$

$$\Rightarrow T_1 = \frac{2}{3} (3T_C)$$

$$\Rightarrow \boxed{T_1 = 2T_C}$$

→ const for max power transmission

$$\textcircled{I} T_{max} = 3T_C$$

$$\textcircled{II} T_{max} = \frac{3}{2} T_1$$

① creep

② crowding of pulley.