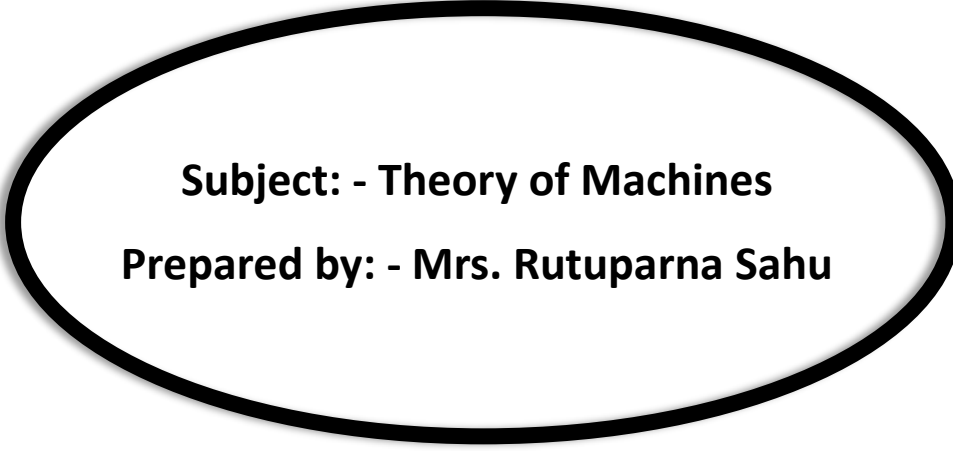




Subject: - Theory of Machines

Prepared by: - Mrs. Rutuparna Sahu

SYLLABUS

1. Simple Machines Mechanism

- 1.1 Link, kinematic chain, mechanism, machine
- 1.2 Inversion, four bar link mechanism and its inversion
- 1.3 lower pair and higher pair.
- 1.4 Cam and followers.

2. Friction

- 2.1 Friction between nut and screw for square thread, screw jack
- 2.2 Bearing and its classification, Description of roller, needle roller and ball bearings.
- 2.3 Torque transmission in flat pivot and conical pivot bearings.
- 2.4 Flat collar bearing of single and multiple clutches.
- 2.5 Torque transmission for single and multiple clutches.
- 2.6 Working of simple frictional brakes.
- 2.7 Working of Absorption type of dynamometer.

3. Power transmission

- 3.1 Concept of power transmission.
- 3.2 Types of drives, belt, gear and chain drive.
- 3.3 Computation of velocity ratio, length of belts Open and cross with and without slip.
- 3.4 Ratio of belt tensions, centrifugal tension and initial tension.
- 3.5 Power transmitted by the belt
- 3.6 Determine belt thickness and width for given permissible stress for open and crossed belts considering centrifugal tension.
- 3.7 V-belts and V-belts pulleys.
- 3.8 Concept of crowning of pulleys.
- 3.9 Gear drives and its terminology.
- 3.10 Gear trains; working principle of simple, compound, reverted and epicyclic gear trains.

4. Governors and Flywheel.

4.1 function of Governor.

4.2 classification of Governor.

4.3 Working of Watt, Porter, Proell and Hartnell Governors.

4.4 Conceptual Explanation of Sensitivity, Stability and Isochronisms.

4.5 function of Flywheel.

4.6 comparison between flywheel and Governor.

4.7 fluctuation of energy and coefficient of fluctuation of speed.



5. Balancing of Machine

5.1 Concept of static and dynamic balancing.

5.2 static balancing of rotating parts.

5.3 Principles of balancing of reciprocating parts.

5.4 causes and effect of unbalance.

5.5 difference b/w static and dynamic balancing.

6. Vibration of Machine Parts

6.1 Introduction to vibration and related terms (Amplitude, cycle, time period and frequency.)

6.2 classification of Vibration.

6.3 Basic concept of natural, forced and damped vibration.

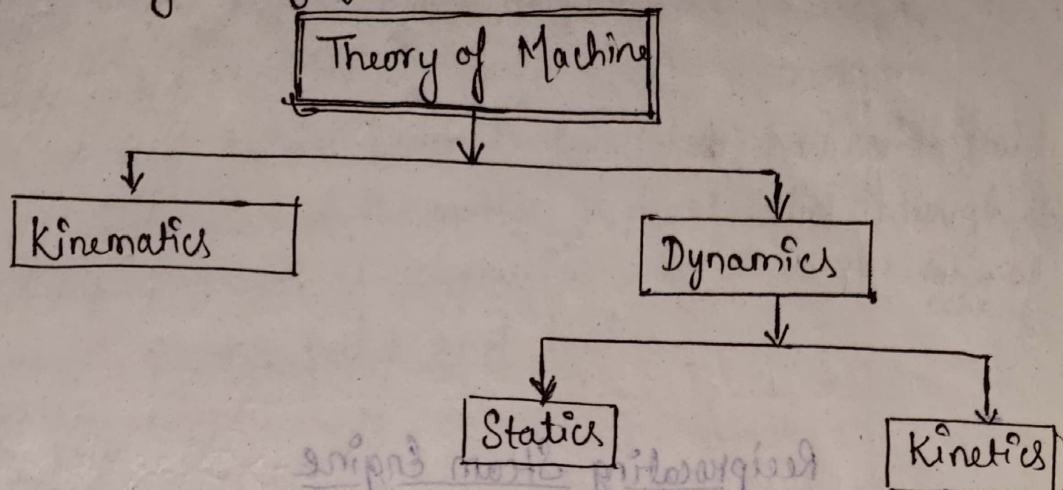
6.4 Torsional and longitudinal vibration.

6.5 Causes and remedies of vibration.

CHAPTERS TO BE COVERED FOR IA = 1, 2, 3

1. SIMPLE MECHANISM

classification of theory of Machine:



Kinematics:

Kinematics of machines is that branch of theory of machines that deals with the study of relative motion of parts of which the machines are constructed, neglecting consideration of forces producing it.

Dynamics:

Dynamics of machine is that branch of theory of machines that deals with the study of forces acting on different parts of the machine.

Statics:

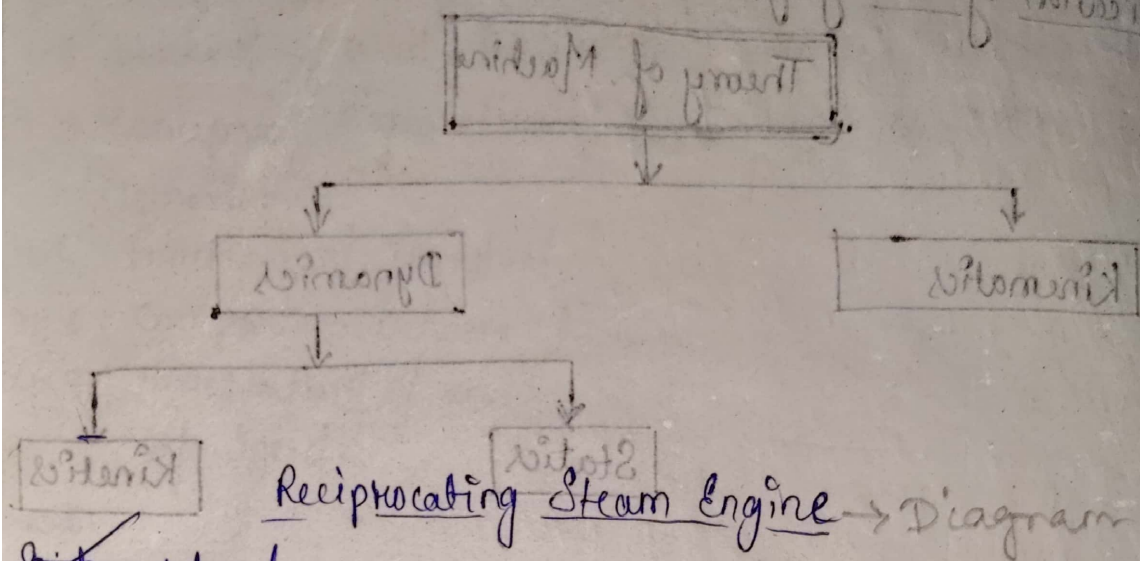
Statics deals with the study of forces acting on various parts of the machine (assumed to be without mass), when they are at rest.

Kinetics:

Kinetics deals with the study of forces which are produced, due to inertia of moving parts of a machine.

Kinetics deals with the study of forces which are produced by the combined effect of the mass and the motion of the various parts.

Kinematic Link



Each part of link

Each part of a machine, which moves relative to some ~~part~~ other part, is known as link or element.

Link in reciprocating engine

- link 1: Piston, piston rod and cross head.
- link 2: crank, crank shaft and flywheel.
- link 3: cylinder, engine frame and main bearings.

Characteristics of links

⇒ It should have relative motion.

⇒ It should be a resistant body.

Types of links

1. Rigid link:

a link which does not undergo any deformation while transmitting motion.

ex:- Deformation of a connecting rod, crank, etc is not appreciable.

2. Flexible link:

A flexible link is one which deforms partly in a manner not to affect the transmission of motion.

example:- belts, ropes, chains and wires.

3. Fluid link:

A fluid link is one which is formed by having a fluid in a ~~receptacle~~ ^{receptacle} and the motion is transmitted through the fluid by pressure or compression only as in the case of hydraulic presses, jacks and brakes.

Kinematic pair

- The two links or elements of a machine are if in contact with each other then, they are said to be a pair.
- When two elements or links are connected in such a way that their relative motion are constrained then they form a kinematic pair and the processes connecting them are called as pairing.

Types of Constrained motion

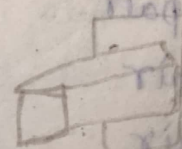
1. Completely constrained motion
2. Incompletely constrained motion
3. Successfully constrained motion

1. Completely constrained motion:

When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.

ex:- The piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction.

Square Rod in square hole



2. Incomplete constrained motion:

when the motion between a pair can take place in more than one direction, then the motion is called incompletely constrained. The change in the direction of impressed force may alter the direction of relative motion between the pair.

ex:- circular shaft in a circular hole may rotate or slide.

3. Successfully constrained motion:

when the motion between the elements, forming a pair is such that the constrained motion is not completely by itself, but by some other means, then the motion is said to be successfully constrained motion.

ex:- Shaft in a foot step bearing.

→ The shaft may rotate in the bearing or it may move upwards. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is successfully constrained.

Kinematic pair classification:

1. According to the type of relative motion b/w the elements
2. According to the type of contact b/w the elements.
3. According to the type of closure.

Kinematic pairs

Relative motion b/w elements	Contact b/w elements	Types of closure
i) Sliding pair	i) Lower pair	i) Self closed pair
ii) Turning pair	ii) Higher pair	ii) Force closed pair.
iii) Rolling pair		
iv) Screw pair		
v) Spherical pair		

The kinematic pairs may be classified to the following considerations:

1. According to the type of relative motion b/w the elements, kinematic pairs are classified into as follows;

a) Sliding pair :- when the two elements of ~~the~~^a pair are connected in such a way that one can only slide relative to the other, the pair is known as sliding pair.

ex:- ~~The~~ piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and guides in shaper, etc.

→ A little consideration will show, that a sliding pair has a completely constrained motion.

b) Rolling pair :- when the two elements of a pair are connected to each other in such a way that the rolls over another fixed link, the pair is known as rolling pair.

b) Turning pair :- when the two elements of a pair are connected to each other in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collar at both the ends fitted into a circular hole, the crankshaft in an internal bearing engine, lathe spindle supported in head stock etc are some examples.

d) Screw pair :- when two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair.

→ ~~The~~ Ex:- Lead screw of a lathe with nut and bolt with a nut.

e) Spherical pair :- when two elements of a pair are connected in such a way that one element turns about the other fixed element. This pair formed is called as spherical pair.

→ Ex:- Pen stand, car mirror, ball and socket, etc.

2. According to the type of contact b/w the elements written as follows:-

(a) Lower pair :- When the two elements of a pair have surface contact where relative motion takes place and the surface of one element slides ~~over~~ over the surface of the other, the pair formed is called as lower pair.

→ It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.

b) Higher pair :- When the two elements of a pair have line or point contact where relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair is known as higher pair.

→ A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

3. According to type of closure, the kinematic pairs are classified into following types:-

a) Self closed pair :- When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair.

→ The lower pairs are self closed pairs.

b) Force closed pair :- When the two elements of a pair are connected to each other mechanically but are kept in contact by the action of external forces, the pair is said to be a force closed pair.

→ Ex:- Cam and follower

Kinematic chain:

When kinematic pair are so coupled that the last link is joined to the first link to transmit a definite ~~complete~~ completely constrained motion, then it forms a kinematic chain.

$$L = 2p - 4 \quad \text{--- (i)}$$

$$J = \frac{3}{2}L - 2 \quad \text{--- (ii)}$$

where L = no. of ~~links~~ links

p = no. of pairs

J = no. of joints

Both the equation (i) and (ii) are applicable to kinematic chain with lower pairs. Also applicable to kinematic chain with higher pairs. Higher pair is equivalent to two lower pairs with an additional element or link.

Examples:-

① In this arrangement, link 3

no. of links $\therefore L = 3$

no. of pairs, $p = 3$

no. of joints, $J = 3$

we know, $L = 2p - 4$

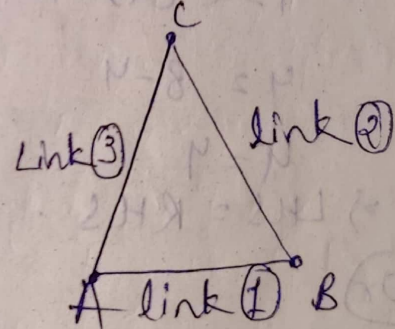
$$3 = 2(3) - 4$$

$$= 6 - 4$$

$$= 2$$

So, $LHS > RHS$

(or)



$$J = \frac{3}{2} (L - 2)$$

$$3 = \frac{3}{2} (8) - 2$$

$$= \frac{9}{2} - 2$$

$$= 4.5 - 2 = 2.5$$

So, $LHS > RHS$

Such type of chains are called as locked chain found in bridges, trusses, etc.

② Here, $L = 4$

$$P = 4$$

$$J = 4$$

we know that,

$$L = 2P - 4$$

$$4 = 2(4) - 4$$

$$4 = 8 - 4$$

$$4 = 4$$

$$\Rightarrow LHS = RHS$$

③ Or

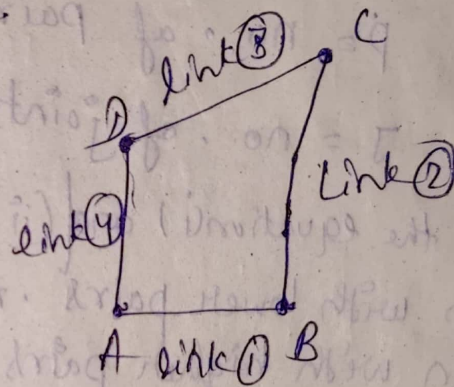
$$J = \frac{3}{2} (L) - 2 \Rightarrow \frac{3}{2} (4) - 2$$

$$4 = \frac{12}{2} - 2 = 6 - 2$$

$$4 = 4$$

$$LHS = RHS$$

Since $LHS = RHS$, this chain is called as constrained kinematic chain.



③

$$\text{Here, } L = 5$$

$$P = 5$$

$$J = 5$$

we know that,

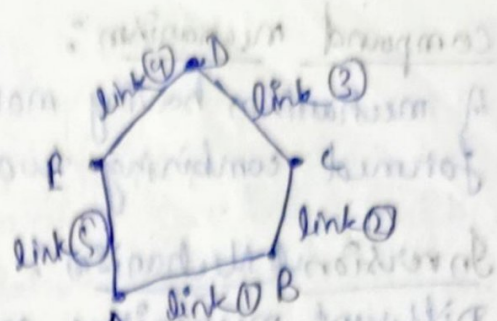
$$L = 2P - 4$$

$$5 = 2(5) - 4$$

$$= 10 - 4$$

$$5 = 6$$

$$LHS < RHS$$



or

$$J = \frac{3}{2} L - 2$$

$$\Rightarrow \frac{3}{2} (5) - 2 = 7.5 - 2 = 5.5$$

$$5 = 5.5$$

$$= 5.5$$

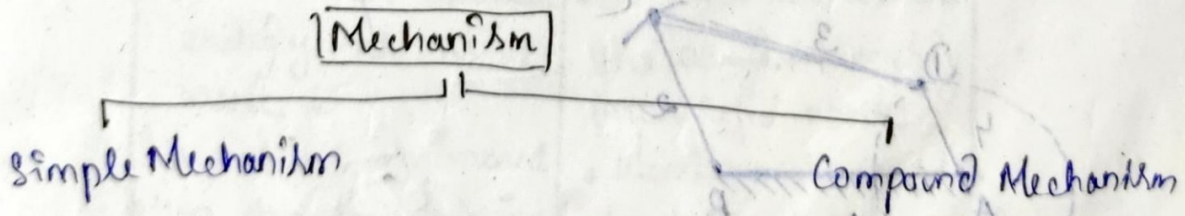
$$5 < 5.5$$

$$LHS < RHS$$

Such chain is called unconstrained chain. Here the relative motion is not completely constrained.

Mechanism:

When one element or link of a kinematic chain is fixed, the arrangement may be used for transmitting or transforming the motion. It is termed as mechanism.



Simple Mechanism

Mechanism having four links

ex:- coupling rod mechanism of a locomotive beam engine, etc.

Compound mechanism:

A mechanism having more than four links. It may be formed combining two or more simple mechanism.

Inversion Mechanism:

Different mechanisms can be obtained by fixing different pairs in a kinematic chain. It is known as inversion of the original kinematic chain.

Suppose number of links of a kinematic chain = L

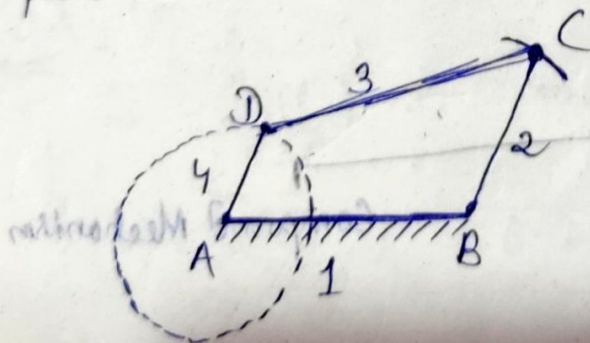
So L different mechanisms may be obtained by fixing each of the links in turn. Each mechanism is termed as inversion mechanism.

Types of kinematic chains (Most imp. kinematic chain)

1. Four bar chain or quadric cycle chain
2. Single slider crank chain
3. Double slider crank chain.

Four bar chain or Quadric cycle chain:

- The kinematic chain is a combination of four links or 4 ~~kinematic chain~~ pairs, such that the relative motion between the links or elements is completely constrained.
- Four links A, B, C and D each of them form a turning pair.



Graashof's Law

According to Graashof's Law for a four bar mechanism, the sum of longest and smallest link lengths should not be greater than the sum of the remaining two link lengths, if there is to be continuous relative motion between the two links.

Link description

In a four bar chain one of the links, in particular the shortest link will make a complete revolution relative to the other three links. If it satisfies the Graashof's Law. Such a link is known as crank or driver. AD (Link 4) is a crank. The link BC (Link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link CD (Link 3) which connects the crank and lever is called as connecting rod or coupler. The fixed link AB (Link 1) is known as frame of the mechanism.

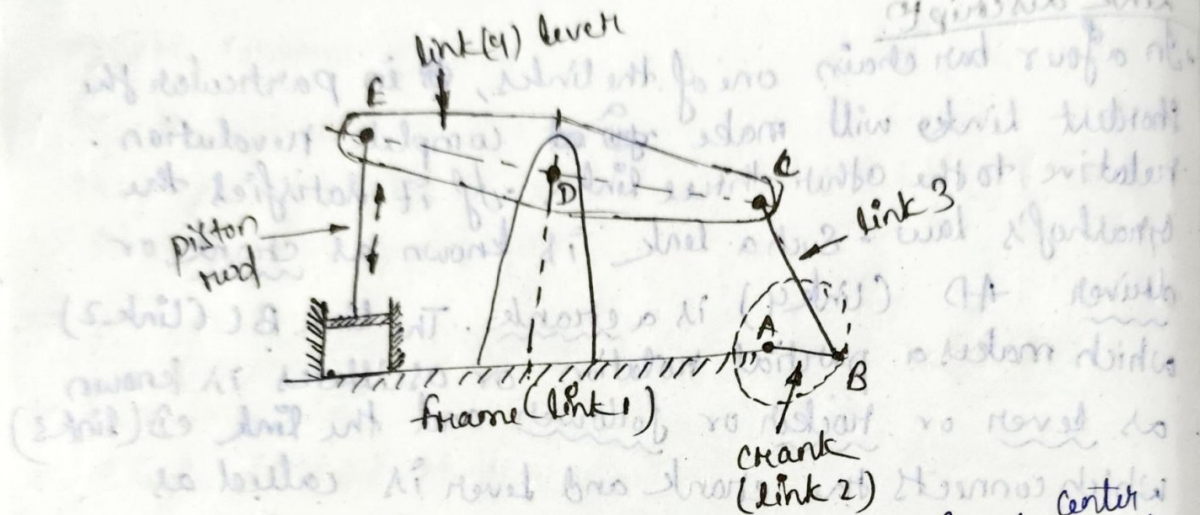
Difference b/w Machine and Structure.

<u>Machine</u>	<u>Structure.</u>
• There is relative motion between its members. • It converts available energy into useful work. • Members are meant to transmit motion and forces. • ex:- Car, lathe, etc	• There is no relative motion between them • It does not convert the available energy into useful work. • Members are meant to take up loads only. • Ex:- bridge, frames, buildings, etc

Inversion of four bar Machine :-

- 1) Beam engine
- 2) Coupling rod of locomotive (double crank mechanism)
- 3) Watt's indicator mechanism (Double lever mechanism)

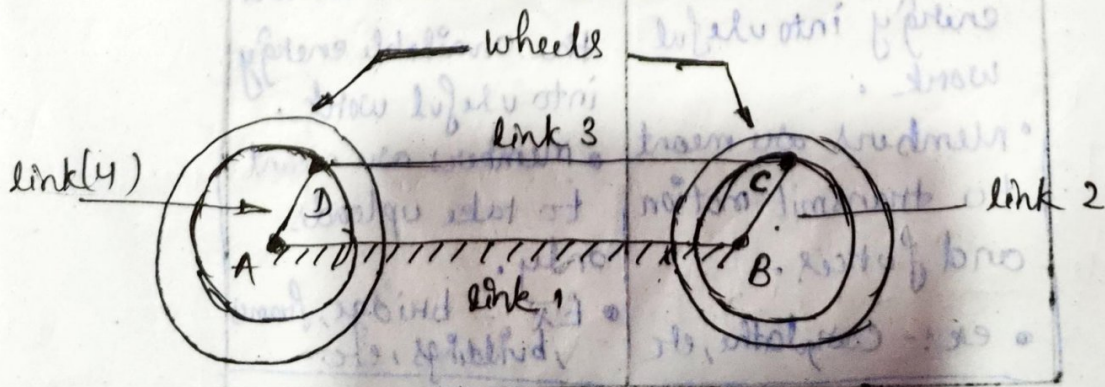
Beam Engine (Type of Steam Engine)



- Here when the crank rotates about the fixed center A, the lever oscillates about the fixed center D.
- The end E of CDE is connected to the piston which reciprocates as a result.

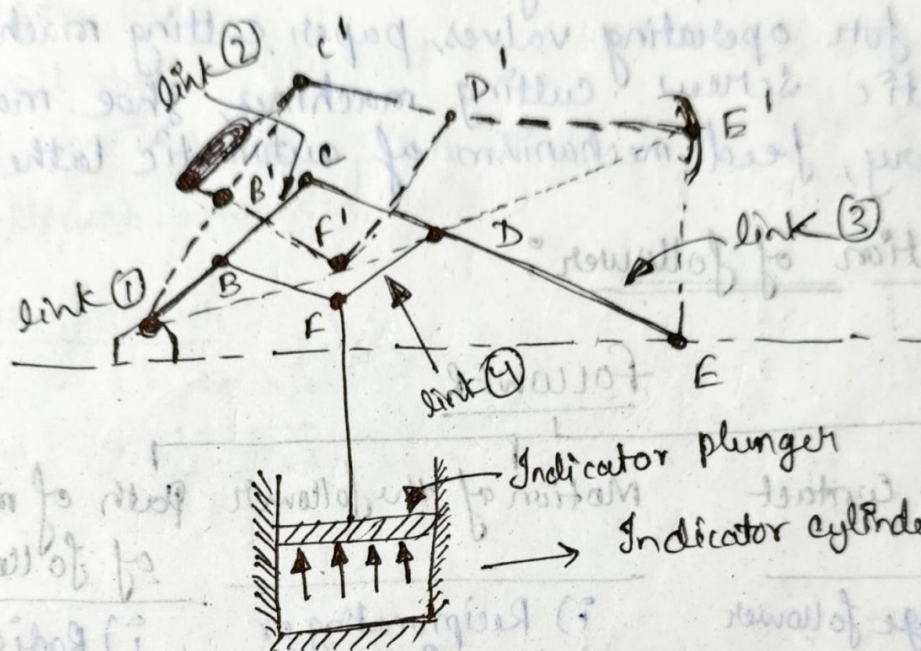
Rotary motion $\rightarrow$ Rectilinear motion $\rightarrow$ To power steamship.

Coupling rod of locomotive (Double crank mechanism).



- There $AD = BC$ (Both are cranks) connected to each other with their respective wheels.
- CD is the connecting rod.
- AB is fixed to maintain constant centre to centre distances between the wheels.
- This mechanism is meant for transmitting rotary motion from one wheel to another.
- As here ~~here~~ two cranks are used, so, it is also called as Double crank mechanism.

Watt's Indicator Mechanism:



• Four links

link 1 = fixed link

link 2 = AC

link 3 = CE

link 4 = ~~BDF~~ BFD

- The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator.
- For small displacement of the mechanism, the tracing point E, traces out approximately a straight line.

CAM and FOLLOWERS:

definition: A cam may be defined as a rotating, oscillating or reciprocating body which imparts a reciprocating or oscillating motion to another body called as follower.

→ The cam and follower constitute a higher pair.

Application:

used in clocks, printing machines, internal combustion engines for operating valves, paper cutting machines, automatic screw cutting machines, shoe making machinery, feed mechanism of automatic lathe.

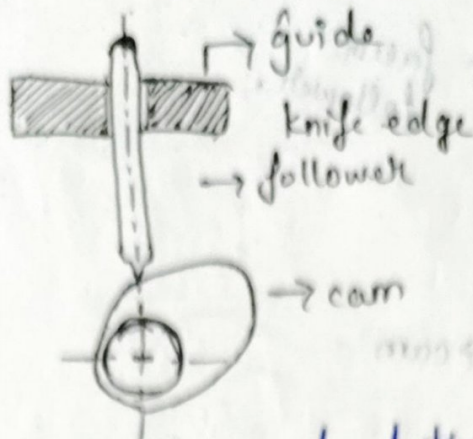
classification of follower:

<u>follower</u>		
<u>Surface in contact</u>	<u>Motion of the follower</u>	<u>Path of motion of follower</u>
i) Knife edge follower	i) Reciprocating or translating follower	i) Radial follower
ii) Roller follower	ii) Oscillating or rotating follower	ii) Offset follower
iii) Flat faced follower.		
iv) Spherical faced follower.		

Manufacturing of Cam

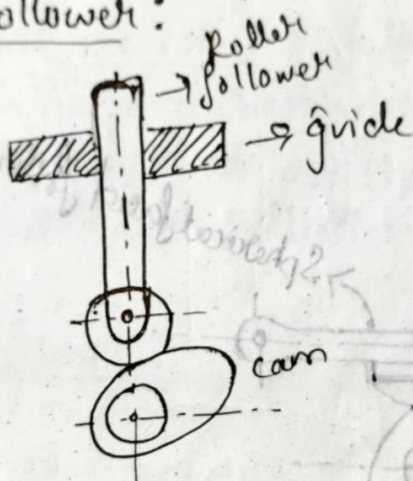
A cam is difficult to manufacture especially when it is to be produced in small quantities. When produced on a mass scale, it is manufactured by a punch press, by die casting or by milling from the master cam.

Knife edge follower:



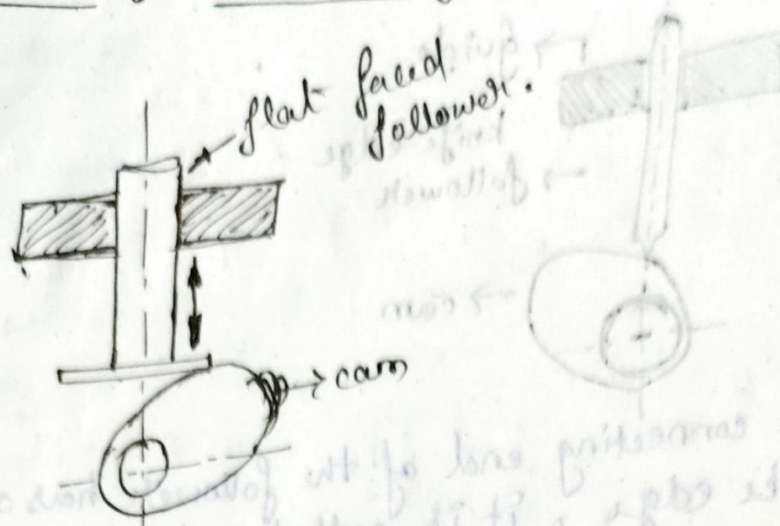
- When the connecting end of the follower has a sharp knife like edge, it is called as knife edge follower.
- Seldom used.
- Small area of contact results in excessive wear.
- A considerable side thrust exists between the follower and the guide.

Roller follower:

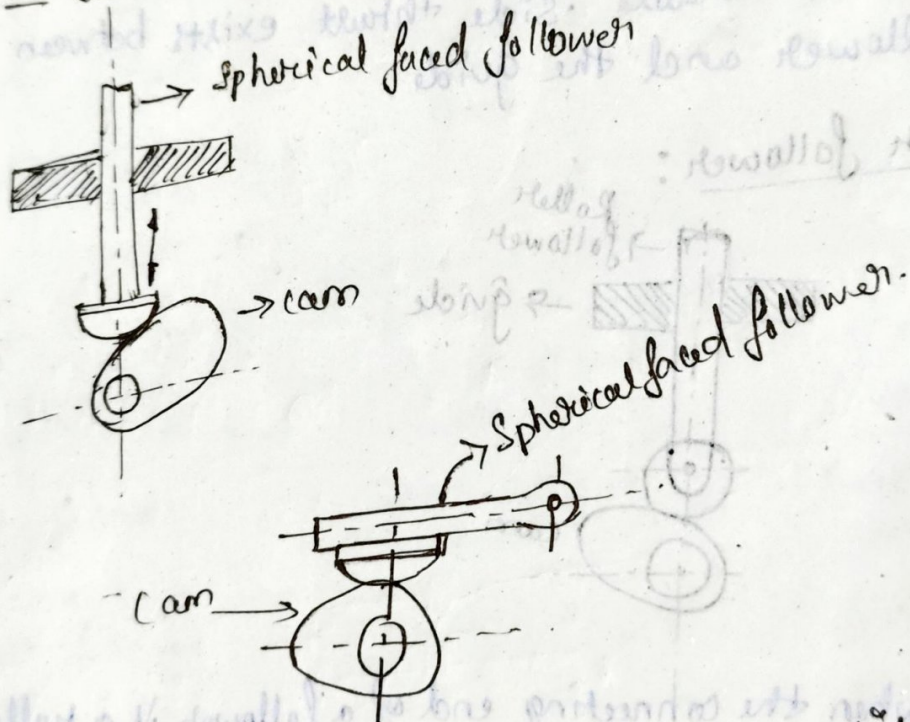


- When the connecting end of a follower is a roller, it is called a roller follower.
- Since rolling motion between the contact surfaces, the rate wear is generally reduced.
- Also side thrust exist between the follower and guide.
- Extensively used (where more space is available).
- Application - Stationary gas and oil engines, aircraft engines.

Flat faced follower or Mushroom follower :-



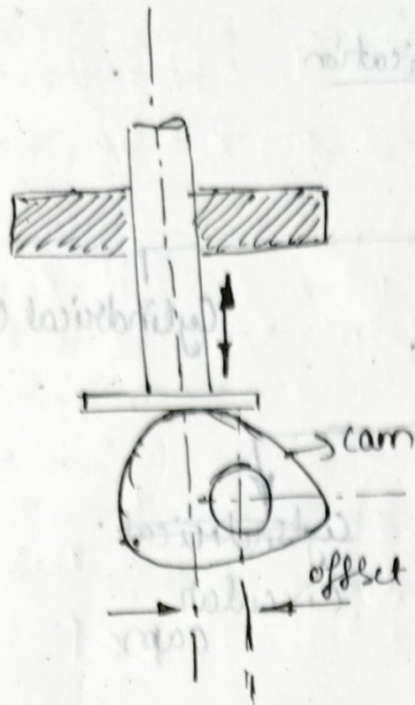
Spherical faced follower :-



Note:

When a flat faced follower is used in automobile engine, high surface stresses are produced as to minimize these stresses, the flat end is machined to spherical shape.

Offset follower :-



Reciprocating or translating follower:-

when the follower reciprocates in guide as the cam rotates uniformly, it is known as reciprocating or translating follower.

Oscillating or rotating follower:-

when the uniform rotary motion of the cam is connected to the predetermined oscillatory motion of the follower, it is called as oscillating or rotating follower.

Radial follower:-

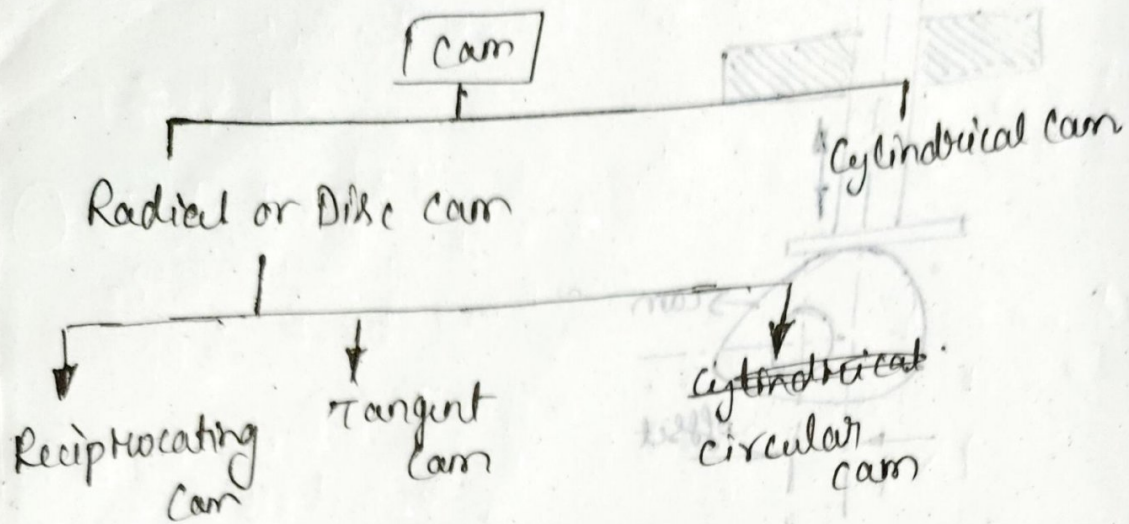
when the motion of the follower is along the axis passing through the center of the cam, it is known as radial follower.

Offset follower:-

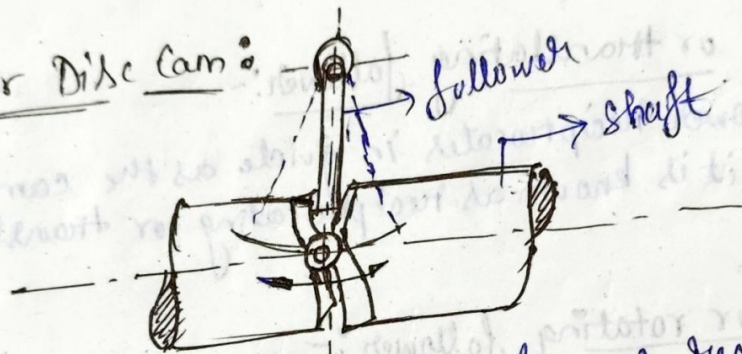
• when the motion of the follower is along an axis away from the axis of ~~cam center~~, it is called as ~~of the center of cam~~

offset follower.

Cam classification

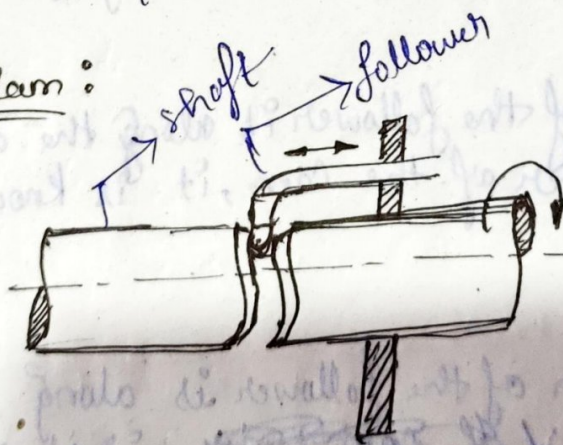


Radial or Disc cam:



In a radial or disc cam, the follower reciprocates or oscillates in a direction perpendicular to the axis of cam rotation or perpendicular to the cam shaft.

Cylindrical cam:



In cylindrical cam, the follower reciprocates or oscillates in a direction parallel to the cam axis or cam shaft.

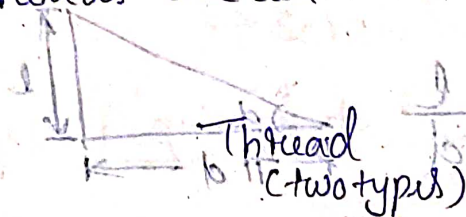
- The ~~for~~ follower rides in a groove at its cylindrical surface.

2. FRICTION

Friction between nut and screw for square thread, screw jack

- Term
- Screw thread is a temporary fastening / joint.
 - Screw thread found in bolts and nuts.
 - ~~Screw~~ Threads are obtained by ~~cutting~~ cutting a continuous helical groove on a cylindrical surface.
 - Threads are cut on the outer surface of solid rod → external thread.

- Threads are cut on the inner surface of solid rod → internal thread.



V thread Square thread

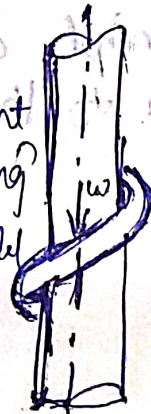
- It is stronger and have more friction resistance

→ Used in screw jack

- Used in nut, bolt

Terms related with screw

- ① Helix :- Curve traced by a particle while moving along a screw thread.
- ② Pitch :- Distance measured between the point on one thread to the corresponding point on the another thread parallel to the axis of screw.
- ③ Depth of thread :- It is the distance between crest and root of the thread



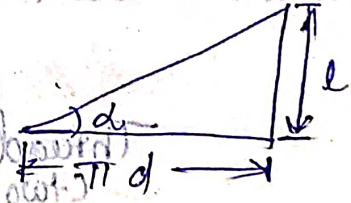
④ Lead :- It is the distance that the screw thread advances axially for one rotation of the nut.

⑤ Single threaded screw :- when lead is equal to pitch; then in that case the screw is called as single threaded screw.

⑥ Multi threaded screw :- when more than one thread is cut in one lead distance; then the screw is called as multi threaded screw. $L < P$

⑦ Helix angle :- (α)

$$\tan \alpha = \frac{\text{Lead of the screw}}{\text{circumference}} = \frac{l}{\pi d}$$



$$= \frac{P}{\pi d} \quad (\text{For single threaded screw, as } l = P)$$

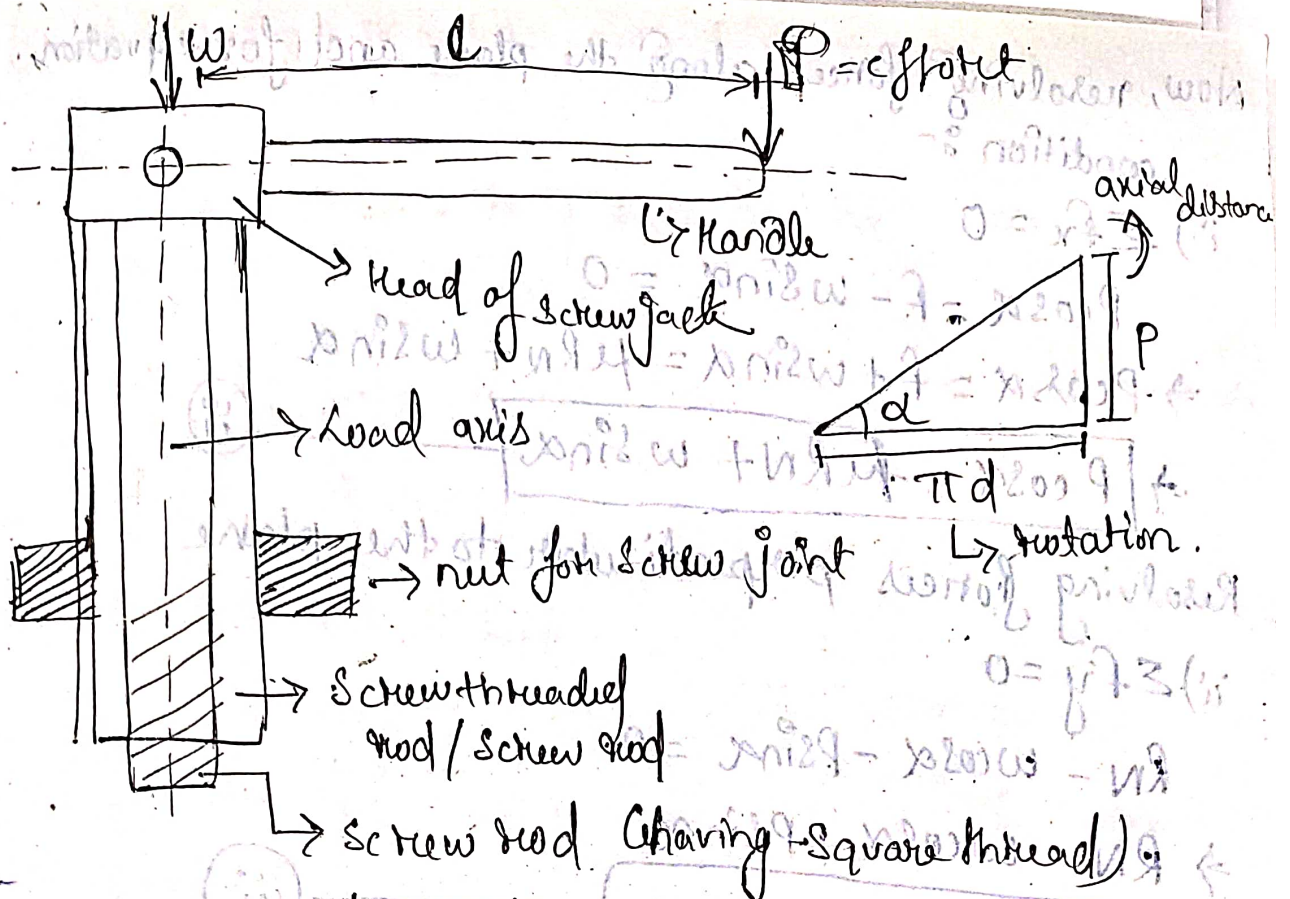
d = mean dia of screw.

Screw Jack

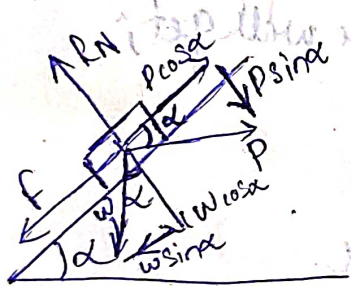
→ Screw jack is a device, which is used for lifting heavy loads, by applying comparatively a smaller effort at its handle.

→ The principle on which a screw jack works is similar to that of an inclined plane.





Torque Required to lift the load by a screw jack



P = Effort applied to lift the load along the plane

W = load to be lifted

f = friction ($f = \mu RN$)

RN = Normal reaction

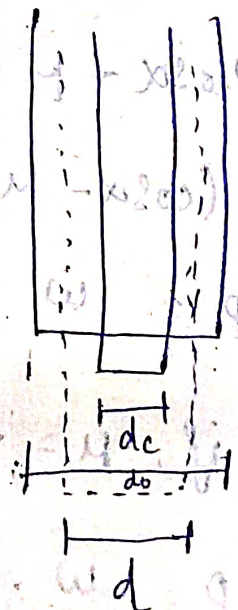
α = Helix angle.

Here, $\tan \alpha = \frac{p}{\pi d}$ (i)

where, p = pitch of screw thread (mm)

d = Mean dia of screw thread (mm)

$$d = \frac{d_o + d_c}{2}$$



d_c = core dia

d_o = Outer dia

Now, resolving forces along the plane and for equation condition :-

$$i) \sum F_x = 0$$

$$P \cos \alpha = f - w \sin \alpha = 0$$

$$\Rightarrow P \cos \alpha = f + w \sin \alpha = \mu R_N + w \sin \alpha$$

$$\Rightarrow \boxed{P \cos \alpha = \mu R_N + w \sin \alpha} \quad (ii)$$

Resolving forces perpendicular to the plane

$$ii) \sum F_y = 0$$

$$R_N - w \cos \alpha - P \sin \alpha = 0$$

$$\Rightarrow R_N = w \cos \alpha + P \sin \alpha$$

$$\Rightarrow \boxed{R_N = P \sin \alpha + w \cos \alpha} \quad (iii)$$

Putting the value of R_N in Eqⁿ (ii), we will get;

$$P \cos \alpha = \mu (P \sin \alpha + w \cos \alpha) + w \sin \alpha$$

$$P \cos \alpha = \mu P \sin \alpha + \mu w \cos \alpha + w \sin \alpha$$

$$P \cos \alpha - \mu P \sin \alpha = w \sin \alpha + \mu w \cos \alpha$$

$$P (\cos \alpha - \mu \sin \alpha) = w (\sin \alpha + \mu \cos \alpha)$$

$$P = \frac{w (\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

i.e., $\mu = \tan \phi$, then,

$$P = w \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

$$= w \times \frac{\sin \alpha + \frac{\sin \phi}{\cos \phi} \cos \alpha}{\cos \alpha - \frac{\sin \phi}{\cos \phi} \sin \alpha}$$

$$\frac{\cos \alpha - \frac{\sin \phi}{\cos \phi} \sin \alpha}{\cos \phi}$$

$$P = W \times \frac{\sin \alpha + \cos \phi + \sin \phi \cos \alpha}{\cos \phi}$$

$$\frac{\cos \alpha + \cos \phi + \sin \phi \sin \alpha}{\cos \phi}$$

multiplying $\cos \phi$ in both the denominator and numerator, we may get;

$$P = W \times \frac{\sin \alpha \cdot \cos \phi + \sin \phi \cos \alpha \times \cos \phi}{\cos \phi}$$

$$\frac{\cos \alpha \cdot \cos \phi + \sin \phi \cdot \sin \alpha \times \cos \phi}{\cos \phi}$$

$$P = W \times \frac{\sin \alpha \cdot \cos \phi + \sin \phi \cdot \cos \alpha}{\cos \alpha \cdot \cos \phi + \sin \phi \cdot \sin \alpha}$$

$$P = W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} = W \times \tan (\alpha + \phi)$$

$$P = W \times \tan (\alpha + \phi)$$

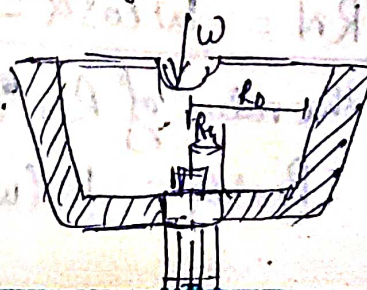
So, the torque required to raise the load is,

$$T_1 = P \times \frac{d}{2}$$

$$\Rightarrow T_1 = W \times \tan (\alpha + \phi) \times \frac{d}{2} \quad (a)$$

This is the amount of torque required to make the load rotate the screw.

Sometimes, the load will also rotate along with the thread. In order to avoid the rotation an arrangement is created called as cup.



In order to avoid the friction at the cup, torque required is,

$$T_2 = \mu \times W \times R_m \quad (b)$$

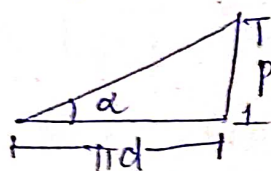
$R_m \Rightarrow$ mean radius

$$R_m = \frac{R_i + R_o}{2}$$

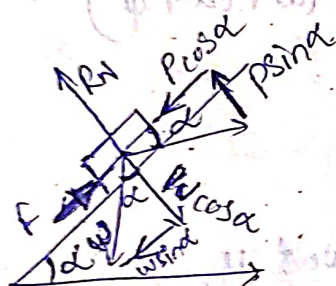
Therefore total torque required to raise the load is,

$$T = T_1 + T_2 \quad (c)$$

Torque required to lower the load by a Screw Jack



$$\tan \alpha = \frac{P}{\pi d} \quad (d)$$



$$f = \mu RN$$

Resolving forces along the plane:

$$\sum F_x = 0$$

$$P \cos \alpha + W \sin \alpha - f = 0$$

$$P \cos \alpha = f - W \sin \alpha$$

$$\Rightarrow P \cos \alpha = \mu RN - W \sin \alpha \quad (ii)$$

Resolving forces perpendicular to the plane:

$$\sum F_y = 0$$

$$RN + P \sin \alpha - W \cos \alpha = 0$$

$$RN = W \cos \alpha - P \sin \alpha \quad (iii)$$

Putting the value of RN in equation (ii), we will get,

$$P \cos \alpha = \mu (W \cos \alpha - P \sin \alpha) - W \sin \alpha$$

$$p \cos \alpha = \mu w \cos \alpha - \mu p \sin \alpha - w \sin \alpha$$

$$p \cos \alpha + \mu p \sin \alpha = \mu w \cos \alpha - w \sin \alpha$$

$$p (\cos \alpha + \mu \sin \alpha) = w (\mu \cos \alpha - \sin \alpha)$$

$$p = w \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

if $\mu = \tan \phi$, then,

$$p = w \frac{\tan \phi \cos \alpha - \sin \alpha}{\cos \alpha + \tan \phi \sin \alpha}$$

$$= w \times \frac{\frac{\sin \phi}{\cos \phi} \cos \alpha - \sin \alpha}{\cos \alpha + \frac{\sin \phi}{\cos \phi} \sin \alpha}$$

$$= w \times \frac{\sin \phi \cos \alpha - \cos \phi \sin \alpha}{\cos \phi \cos \alpha + \sin \phi \sin \alpha}$$

$$\frac{\sin \phi \cos \alpha - \cos \phi \sin \alpha}{\cos \phi \cos \alpha + \sin \phi \sin \alpha}$$

multiplying $\cos \phi$ to denominator and numerator, we get,

$$p = w \times \frac{\sin \phi \cos \alpha - \cos \phi \sin \alpha}{\cos \phi} \times \cos \phi$$

$$\frac{\cos \alpha \cos \phi + \sin \phi \sin \alpha}{\cos \phi} \times \cos \phi$$

$$= w \times \frac{\sin \phi \cos \alpha - \cos \phi \sin \alpha}{\cos \alpha \cos \phi + \sin \phi \sin \alpha}$$

$$= w \times \frac{\sin (\phi - \alpha)}{\cos (\phi - \alpha)}$$

$$P = w \times \tan(\phi - \alpha)$$

Then, torque required will be

$$T = P \times \frac{d}{2}$$

$$\Rightarrow \boxed{T = w \times \tan(\phi - \alpha) \times \frac{d}{2}}$$

*_c if $\alpha > \phi$, then,

$$\boxed{T = w \times \tan(\alpha - \phi) \times \frac{d}{2}}$$

Efficiency of a Screw Jack (η)

$\Rightarrow$ The η of a screw jack is defined as the ratio between ideal effort to the actual effort.

Ideal effort $\Rightarrow$ Effort need to lift the load neglecting the friction.

Actual effort $\Rightarrow$ Effort need to lift the load with consideration of friction.

When, friction between the screw and nut is not considered, ϕ will be equal to zero. So, now effort to raise a load will be;

$$P_0 = w \times \tan(\alpha + 0)$$

$$\Rightarrow \boxed{P_0 = w \tan \alpha}$$

$$\text{Actual effort} = \boxed{P = w \tan(\alpha + \phi)}$$

$$\Rightarrow \eta = \frac{P_0}{P} = \frac{w \tan \alpha}{w \tan(\alpha + \phi)} = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

when both screw friction and collar friction are taken into consideration;

$$\eta = \frac{T_o}{T_i} = \frac{P_o \times d/2}{P \times d/2 + \mu \times w \times r_m} = \eta$$

Maximum efficiency (η_{max}) of a screw jack

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\frac{\sin \alpha}{\cos \alpha}}{\frac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)}}$$

$$= \frac{\sin \alpha}{\cos \alpha} \times \frac{\cos(\alpha + \phi)}{\sin(\alpha + \phi)} = \frac{\sin \alpha \times \cos(\alpha + \phi)}{\cos \alpha \times \sin(\alpha + \phi)}$$

Multiplying '2' in both numerator and denominator,

$$= \frac{2 \sin \alpha \times \cos(\alpha + \phi)}{2 \cos \alpha \times \sin(\alpha + \phi)} \quad \text{--- (A)}$$

if we consider, $(\alpha + \phi) = A$ and, α as B , ~~then~~ and
As, we know,

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$$

$$\text{and, } \sin(A+B) - \sin(A-B) = 2 \cos A \sin B$$

$$\Rightarrow \underline{2 \sin \alpha \cos(\alpha + \phi)}$$

$$\sin(A+B) - \sin(A-B) = 2 \sin A \cos B$$

$$\text{as, } A = \alpha + \phi \text{ and } B = \alpha$$

$$\Rightarrow \sin(\alpha + \phi) - \sin(\alpha + \phi - \alpha)$$

$$\sin(\alpha + \phi + \alpha) - \sin(\alpha + \phi - \alpha)$$

$$\Rightarrow \sin(2\alpha + \phi) - \sin(\phi)$$

and similarly;

$$\Rightarrow \sin(A+B) + \sin(A-B) = 2 \cos A \sin B$$

$$\Rightarrow \sin(\alpha + \phi + \alpha) + \sin(\alpha + \phi - \alpha)$$

$$\Rightarrow \sin(2\alpha + \phi) + \sin(\phi)$$

Now, in place of η , Eqn (A) that is;

$$\frac{2 \sin \alpha \cos(\alpha + \phi)}{2 \cos \alpha \sin(\alpha + \phi)} = \frac{\sin(2\alpha + \phi) - \sin \phi}{\sin(2\alpha + \phi) + \sin \phi}$$

Therefore,

η will be maximum when $2\alpha + \phi$ is maximum.

$$\text{i.e. } \sin(2\alpha + \phi) = 1 \Rightarrow 2\alpha + \phi = 90^\circ$$

$$2\alpha + \phi = \pi$$

$$\Rightarrow 2\alpha = \pi - \phi$$

$$\alpha = \frac{\pi - \phi}{2}$$

$$\alpha = \frac{\pi}{2} - \frac{\phi}{2}$$

$$\Rightarrow \alpha = 45^\circ - \frac{\phi}{2}$$

if we substitute the value of 2α ; then

$$\eta = \frac{\sin 90 - \sin \phi}{\sin 90 + \sin \phi} \Rightarrow \eta = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$\eta = \frac{\sin 90 - \sin \phi}{\sin 90 + \sin \phi} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$\boxed{\eta_{\max} = \frac{1 - \sin \phi}{1 + \sin \phi}}$$

Q. The pitch of a 50mm mean diameter is a threaded screw of a screw jack is 12.5mm. The coefficient of friction between the screw and the nut is 0.13.

- Determine the torque required on the screw to raise a load of 25 kN.
- Determine the ratio of the torque required on the screw to raise the load to the torque required to lower the load and also the η of the machine.

→ Given that,

$$\text{dia} = 50 \text{ mm}$$

$$\text{pitch} = 12.5 \text{ mm}$$

$$\mu = 0.13$$

$$\mu = \tan \phi \Rightarrow \phi = 7.407^\circ$$

$$(i) P = 25 \times 10^3 \text{ N}$$

$$\tan \alpha = \frac{P}{\pi d} = \frac{12.5}{\pi \times 50} = 0.08$$

$$\tan \alpha = 0.08$$

$$\alpha = \tan^{-1}(0.08) = 4.574^\circ$$

$$\eta = \frac{P \times \frac{1}{2}}{P \times \frac{1}{2}} = 1$$

Force rolled due to load :- w with horizontal view

$$W = 25 \times 10^3 \text{ N} \quad \text{(given)}$$

$$P_1 = W \times \tan(\alpha + \phi)$$

$$= 25 \times 10^3 \times \tan(4.574 + 7.407)$$

$$= 5305.17 \text{ N}$$

(or)

$$P_1 = W \times \tan(\alpha + \phi) = W \times \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right]$$

$$= 25 \times 10^3 \times \left[\frac{0.08 + 0.13}{1 - 0.08 \times 0.13} \right] = 25 \times 10^3 \times \left[\frac{0.08 + 0.13}{1 - 0.08 \times 0.13} \right]$$

$$= 5305.25 \text{ N}$$

Torque required to raise the load :-

$$T_1 = P_1 \times \frac{d}{2} = 5305.25 \times \frac{50}{2}$$

$$= 5305.25 \times \frac{50}{2} = 132631.25 \text{ N/mm}$$

$$T_1 = 132631.25 \text{ N/mm}$$

Force required to lower the load :-

$$P_2 = W \times \tan(\phi - \alpha)$$

$$= 25 \times 10^3 \times \tan(7.407 - 4.574)$$

$$= 1237.138 \text{ N}$$

Torque required to lower the load :-

$$T_2 = P_2 \times \frac{d}{2} = 1237.138 \times \frac{50}{2}$$

$$T_2 = 30928.45 \text{ N/mm}$$

Ratio of torque required

$$\frac{T_1}{T_2} = \frac{132631.25}{30928.45} = 4.288$$

η of machine $\Rightarrow$

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{0.08}{\tan(4.574 + 7.407)}$$

$$= 0.377 = 37.69\%$$

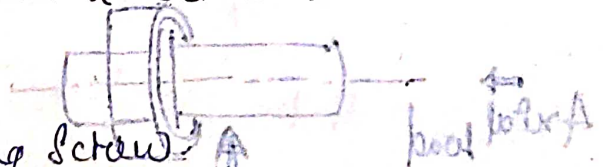
Overhauling and self locking screws

Effort required to lower the load $(P) = W \tan(\phi - \alpha)$

Torque required to lower the load $= (T \mp P) \times \frac{d}{2}$

* If $\phi < \alpha$, then the torque required to lower the load will be ~~positive~~ negative (-ve). Such a condition having screws are called as overhauling screws.

* If $\phi > \alpha$, then the torque required to lower the load will be positive (+ve). Such a screw is called as Self locking screws.



(η) efficiency of self locking screw

η of screw, $\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)}$

For self locking screw; $\alpha \leq \phi$

$$\eta \leq \frac{\tan \phi}{\tan(\phi + \phi)} \leq \frac{\tan \phi}{\tan 2\phi} = \frac{\tan \phi (1 - \tan^2 \phi)}{\tan 2\phi}$$

$$\Rightarrow \frac{\tan \phi}{2 \tan \phi} \Rightarrow \frac{\tan \phi \times 1 - \tan^2 \phi}{2 \tan \phi} \leq \frac{1}{2} = \frac{\tan^2 \phi}{2}$$

From the above equation it is clear that η of a self locking screw is less than $\frac{1}{2}$ or 50%.

★ If η is more than 50%, then the screw is said to be overhauling screw.

Bearing

- (i) Bearing is an element which supports another moving machine element.
- (ii) It allows the relative motion between the contact surfaces of the members while carrying a load.
- (iii) In order to avoid the frictional resistance and wear, lubricant is provided between the journal and the bearing.

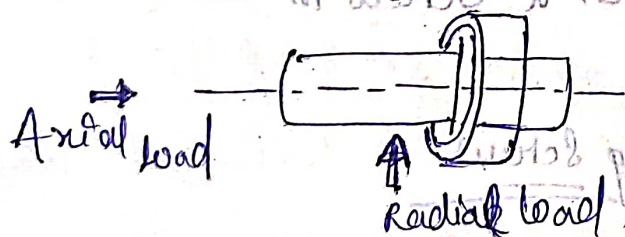
Classification of Bearing

① Depending on the direction of the load applied to be supported.

There are two types of bearing:-

① Radial bearing

② Thrust bearing



→ When the load acts in the perpendicular direction of the motion of the moving element, the bearing is known as radial bearing.

→ When the load acts along the axis of rotation of the moving element, then the bearing is called as thrust bearing.

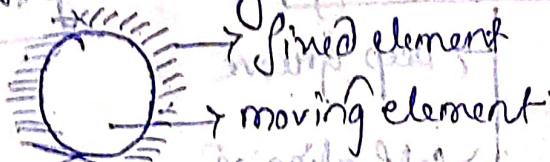
② Depending upon the nature of contact :-

There are two types of bearing :- Sliding bearing

Rolling contact bearing

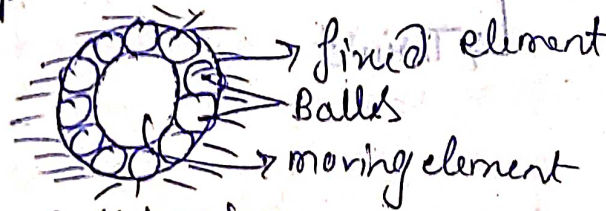
Sliding contact bearing

→ When the sliding motion takes place along the surfaces of contact between the moving element and the fixed element, the bearing is called as Sliding contact bearing.



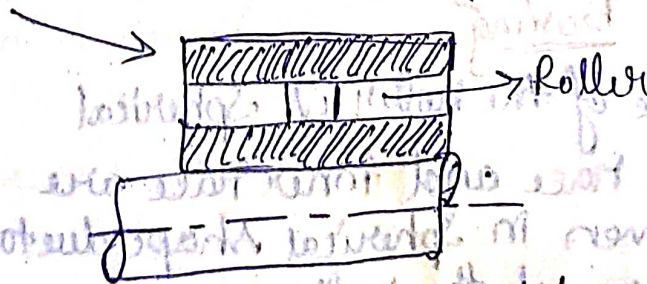
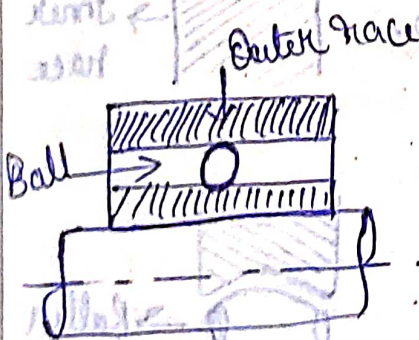
Rolling contact bearing

→ In case of rolling contact bearing ~~Steel~~ ball or rollers are interposed between the moving and fixed element.



Ball bearing

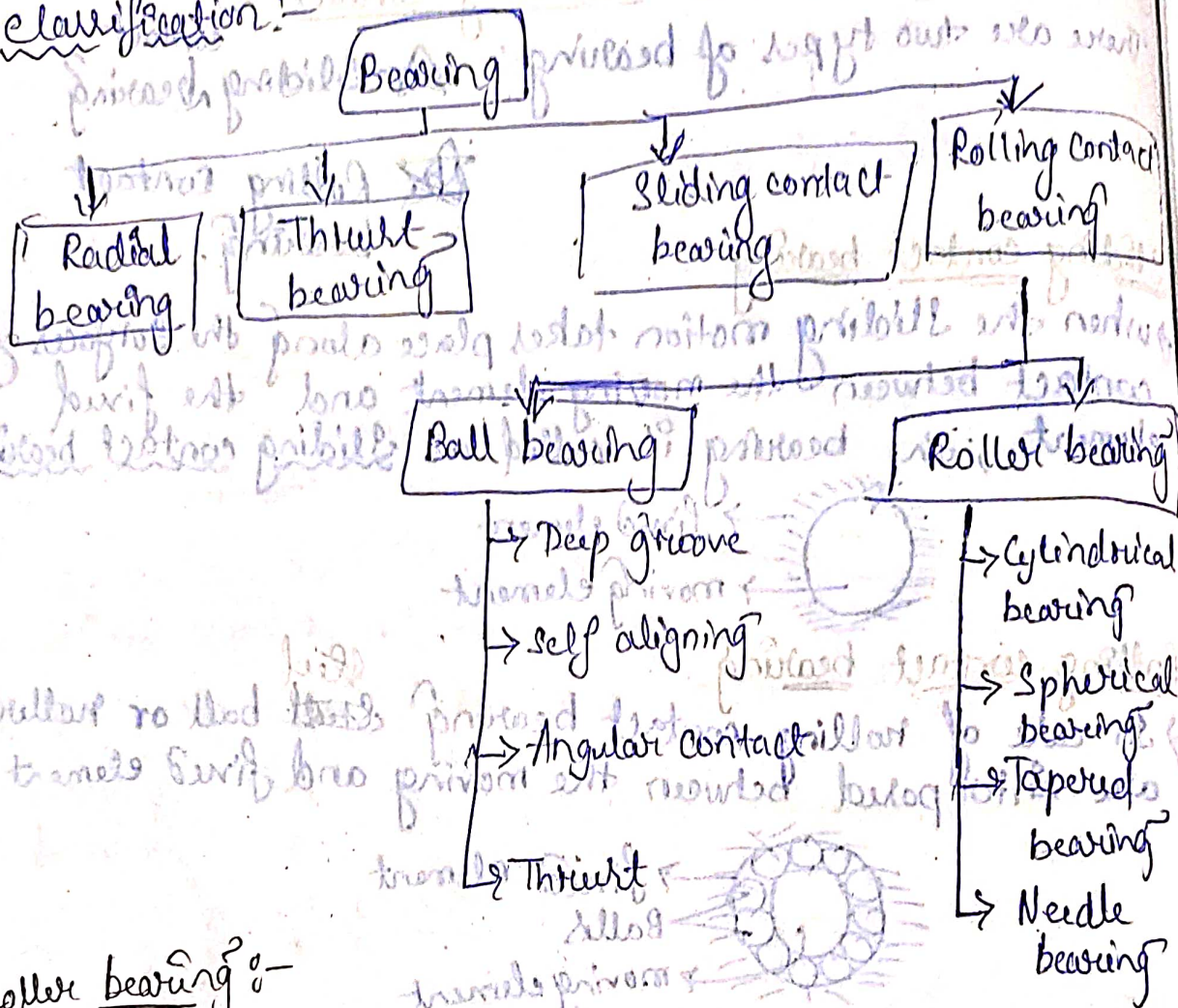
* Rolling contact bearing are again of two types :-
→ Ball bearing
→ Roller bearing



→ In case of ball bearing, point contact occurs and its load carrying capacity is less.

→ In case of roller bearing, line contact occurs and its load carrying capacity is more.

Classification:-



Roller bearing:-

⇒ Cylindrical bearing

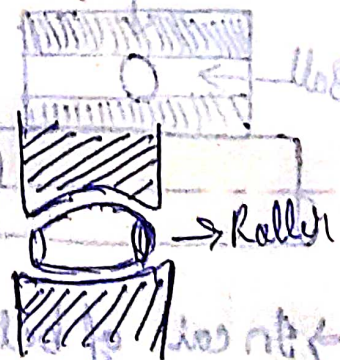
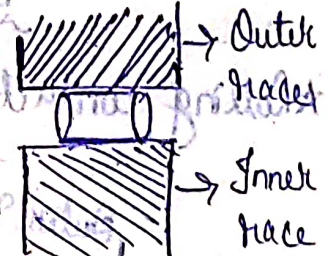
- The shape of the roller is cylindrical
- It carries only the radial load.
- It is used in the high speed application.

⇒ Spherical bearing

- The shape of the roller is Spherical
- The Outer race and inner race are also given in Spherical shape due to the shape of the roller.
- It can carry both radial load and Thrust (axial) load.

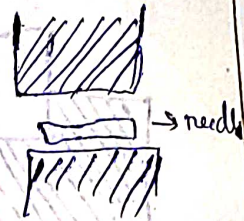
⇒ Tapered bearing

- The shape of the roller is ⁱⁿ taper shape.
- It can carry both radial and thrust load.



⇒ Needle bearing:

- The shape of the roller is just like the needle.
- The roller is present over major portion of the bearing, so the line of contact area increases. Therefore the load carrying capacity also increases.



Ball bearing:

⇒ Deep groove

- It can take heavy load but not upto the roller bearing.



- It is used in the high speed application.

- It can carry only radial load.

⇒ Angular contact

- The ball touches the inner race or outer race by making some angle.

- It can take both thrust load and radial load.



Angular contact

⇒ Self aligning

- The outer race is spherical.

- There are no grooves in the outer race.

- It can adjust by itself according to the deflection of the shaft.

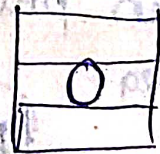
- It can bear only radial load.



⇒ Thrust bearing

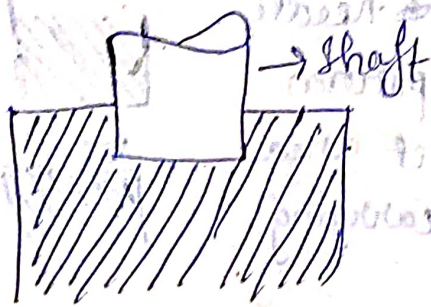
- It can only carry axial load.

- When the load is applied the balls will get rotated and distributed the load.

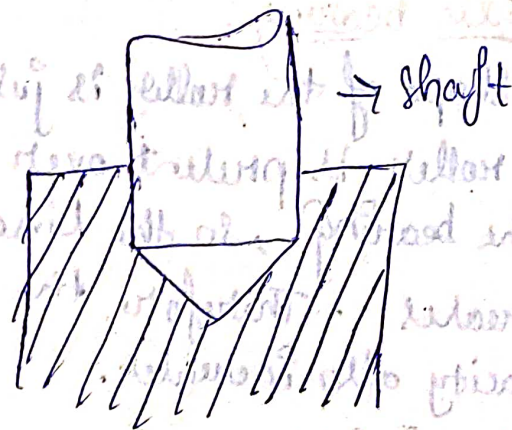


Torque transmission in flat pivot and Conical pivot :-

The bearing surfaces placed at the end of the shaft to take the axial thrust are known as pivot.



flat pivot



conical pivot

Torque transmission in flat pivot bearing:-

Case = I

uniform pressure:-

Let w = Load transmitted over the bearing surface

R = Radius of the shaft

P = Intensity of pressure per unit area of bearing surface.

μ = Coefficient of friction.

* When the shaft rotates in a flat pivot bearing, sliding friction along the surface of contact b/w the shaft and bearing will be developed.

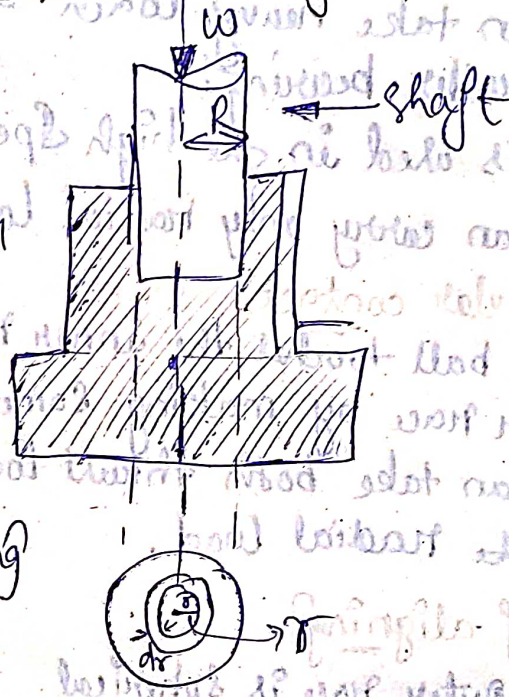
→ Uniform Pressure:-

when pressure is uniformly distributed over the bearing area.

$$P = \frac{w}{\pi R^2}$$

consider a ring of radius " r " with the thickness ' dr '

Area of the ring (dA) = $2\pi r \times dr$



POWER STATION ENGINE

load transmitted to the ring (dw) = $\mu \times d\alpha$

$$dw = \frac{W}{\pi R^2} \times 2\pi r \times dr$$

total ~~transmitted~~ frictional resistance on the ring;

$$f_r = \mu \times dw$$

$$f_r = \mu \times \frac{W}{\pi R^2} \times 2\pi r \times dr$$

frictional torque of the ring;

$$T_r = f_r \times r$$

$$= \mu \times \frac{W}{\pi R^2} \times 2\pi r \times dr \times r$$

$$T_r = \mu \times \frac{W}{\pi R^2} \times 2\pi r^2 dr$$

Integrating this equation within a limit from 0 to R for the total frictional torque of the flat pivot bearing

we will get,

$$\text{total torque } (T) = \int_0^R \mu \times \frac{W}{\pi R^2} \times 2\pi r^2 dr$$

$$T = \mu \times \frac{W}{\pi R^2} \times 2\pi \int_0^R r^2 dr = \mu \times \frac{W}{\pi R^2} \times 2\pi \left[\frac{r^3}{3} \right]_0^R$$

$$= \mu \times \frac{W}{\pi R^2} \times 2\pi \times \frac{R^3}{3} = \mu \times W \times \frac{2R}{3}$$

$$T = \frac{2}{3} \mu \cdot W \cdot R$$

Case = 2

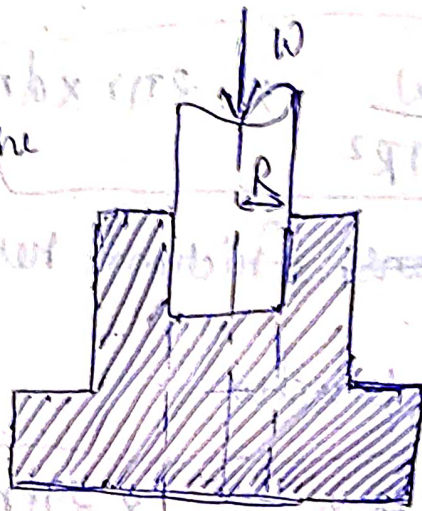
Uniform wear:-

Let w = load transmitted over the bearing. Screw

R = radius of shaft

P = intensity of pressure per unit area of bearing surface.

μ = coefficient of friction.



r = Radius

dr = thickness

When the shaft rotates in a flat pivot bearing, sliding friction along the surface of contact b/w the shaft and bearing will be produced.

$$P \cdot r = C$$

$$P = \frac{C}{r}$$

Considering a ring with radius " r " with thickness " dr ".

Area of the ring (dA) = $2\pi r \times dr$

load transmitted to the ring (dw) = $P \times dA$

$$dw = P \times dA$$

$$= \frac{C}{r} \times 2\pi r \times dr$$

$$dw = C \times 2\pi dr$$

total load $\Rightarrow$

$$w = \int_0^R dw = \int_0^R C \times 2\pi dr = C \times 2\pi \int_0^R dr$$

$$= C \times 2\pi \times R$$

$$W = C \times 2\pi \times R$$

$$T = \frac{1}{2} \times W \times R$$

$$C = \frac{W}{2\pi R}$$

frictional resistance on the ring.

$$F_r = \mu \times dw$$

$$= \mu \times C \times 2\pi \times dr$$

$$F_r = \mu \times 2\pi \times C \times dr$$

Now, substituting the value of 'C' in the eqⁿ;

$$F_r = \mu \times 2\pi \times \frac{W}{2\pi R} \times dr$$

$$F_r = \frac{\mu \times W \times dr}{R}$$

Frictional torque of the ring;

$$T_r = F_r \times r$$

$$\Rightarrow T_r = \frac{\mu \times W}{R} \times r \times dr$$

$$\Rightarrow T_r = \frac{\mu \times W \times r^2}{R}$$

Total torque required on the shaft with a limit of radius 'R' from 0 to R.

$$T = \int_0^R \frac{\mu \times W \times r}{R} dr = \frac{\mu \times W}{R} \int_0^R r dr$$

$$= \frac{\mu \times W}{R} \times \frac{r^2}{2} \Big|_0^R = \frac{\mu \times W}{R} \times \frac{R^2}{2}$$

$$\Rightarrow T = \frac{1}{2} \times \mu \times w \times R$$

Torque transmission in conical pivot bearing

The conical pivot bearing, supporting a shaft is carrying an axial load "w".

P_n = Intensity of pressure normal to the cone

α = Semi angle of the cone

μ = coefficient of friction.

Consider a small ring of radius "R" and thickness "dr".

Let "dl" be the length of the ring along the cone

$$dl = dr \sec \alpha$$

Area of the rings = $2\pi r \times dr \times \sec \alpha$

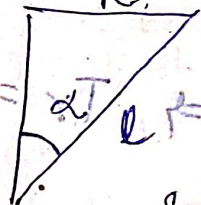
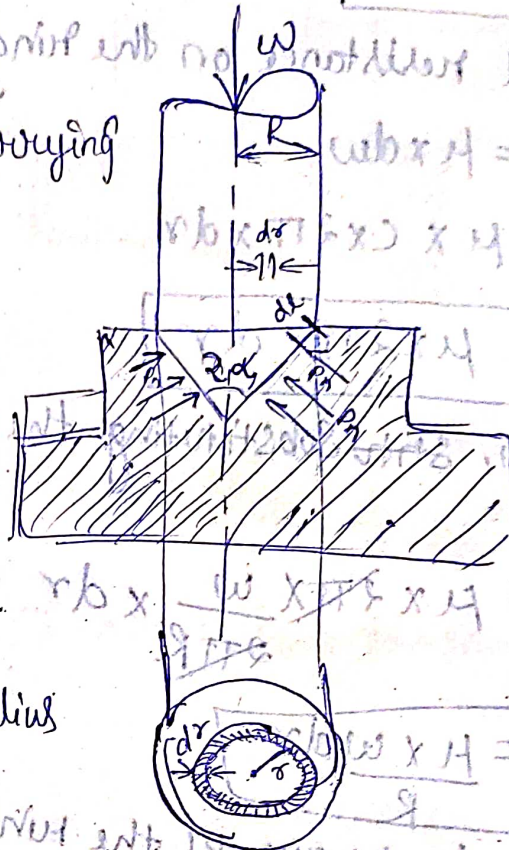
$$A = 2\pi r \times dr \times \sec \alpha$$

Case 1 :- Considering uniform pressure.

Normal load acting on the ring = $\sum w_n$

$$\sum w_n = P_n \times A \Rightarrow P_n \times 2\pi r \times dl$$

$$\sum w_n = P_n \times 2\pi r \times dr \times \sec \alpha$$



$$\sin \alpha = \frac{R}{dl}$$

$$\text{So, } \sec \alpha = \frac{dl}{R}$$

$$\frac{w \times A}{g} = \frac{w \times 2\pi r \times dl}{g} = \frac{w \times 2\pi r \times R}{g \times \sin \alpha}$$

$$\frac{w \times 2\pi r \times R}{g \times \sin \alpha} = \frac{w \times 2\pi r \times R}{g \times \frac{R}{dl}} = \frac{w \times 2\pi r \times dl}{g}$$

STAT-1

~~total load~~

The vertical load acting on the ring;

$$\delta W = \delta W_n \sin \alpha$$

$$\delta W = P_n \times 2\pi r \times dr \times \cos \alpha \times \sin \alpha$$

$$= P_n \times 2\pi r \times dr \times \frac{1}{\sin \alpha} \times \sin \alpha$$

$$\boxed{\delta W = P_n \times 2\pi r \, dr}$$

total vertical load transmitted on the bearing;

$$W = \int_0^R \delta W = \int_0^R P_n \times 2\pi r \, dr = P_n \times 2\pi \int_0^R r \, dr$$

$$= P_n \times 2\pi \left[\frac{r^2}{2} \right]_0^R$$

$$W = P_n \times \pi R^2 \Rightarrow \boxed{W = P_n \times \pi R^2}$$

frictional force acting on the ring;

$$f_r = \mu \times \delta W_n$$

$$= \mu \times P_n \times 2\pi r \, dr \times \cos \alpha$$

frictional torque on the ring;

$$T_r = f_r \times r = \mu \times P_n \times 2\pi r \, dr \times \cos \alpha \times r$$

$$T_r = \mu \times P_n \times 2\pi r^2 \, dr \times \cos \alpha$$

Total frictional torque acting on the bearing;

$$T = \int_0^R T_r = \int_0^R \mu \times P_n \times 2\pi r^2 \, dr \times \cos \alpha$$

$$= \mu \times P_n \times 2\pi \times \cos \alpha \int_0^R r^2 \, dr$$

$$= \mu \times P_n \times 2\pi \times \cos \alpha \times \left[\frac{r^3}{3} \right]_0^R$$

$$T = \mu \times P_n \times 2\pi R \cos \alpha \times \frac{R^3}{3}$$

$$P_n = \frac{W}{\pi R^2}$$

$$\Rightarrow T = \mu \times \frac{W}{\pi R^2} \times 2\pi R \cos \alpha \times \frac{R^3}{3}$$

$$= \mu \times W \times 2 \times \cos \alpha \times \frac{R}{3}$$

$$T = \frac{2}{3} \mu W R \cos \alpha$$

Case-2 Uniform wear

Rate of wear depends upon pressure and velocity,
So, rate of wear is proportional to $P \cdot V$ (velocity = $\pi \omega r$)

$$P \times V = C$$

$$P_n = \frac{C}{V}$$

$$\text{Area of ring} = 2\pi r \times dr$$

$$= 2\pi r \times dr \cos \alpha$$

$$\delta W_n = P_n \times 2\pi r \times dr \cos \alpha$$

$$\delta W = \delta W_n \times \sin \alpha$$

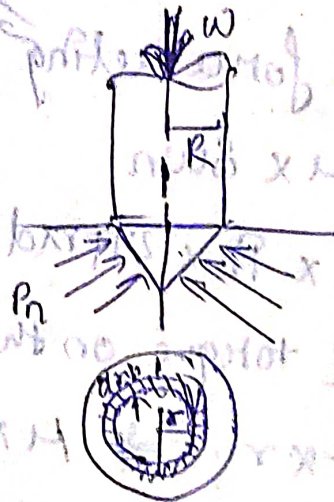
$$\Rightarrow \delta W = P_n \times 2\pi r \times dr \cos \alpha \times \sin \alpha$$

$$\delta W = P_n \times 2\pi r \times dr$$

$$W = \int \delta W = \int_0^R P_n \times 2\pi r \times dr$$

$$W = P_n \times 2\pi \int_0^R r \times dr \Rightarrow W = P_n \times 2\pi \times \frac{r^2}{2} \Big|_0^R$$

$$W = P_n \times 2\pi \times \frac{R^2}{2}$$



$$P_n = \frac{C}{r}$$

$$\text{So, } w = \frac{C}{r} \times 2\pi r \times \cancel{\frac{r^2}{2}} \Big|_0^R \Rightarrow \cancel{\frac{1}{2} \pi C r^2} \Big|_0^R$$

$$w = \pi C R$$

$$w = \int_0^R P_n \times 2\pi r dr = \int_0^R \frac{C}{r} \times 2\pi r dr$$

$$w = 2\pi C \int_0^R dr = 2\pi C R$$

$$\boxed{w = 2\pi C R} \Rightarrow \boxed{C = \frac{w}{2\pi R}}$$

frictional force acting on the ring;

$$f_n = \mu \times \delta w_n$$

$$\boxed{f_n = \mu \times P_n \times 2\pi r dr \cos \alpha}$$

frictional torque on the ring;

$$T_r = f_n \times r \Rightarrow \mu \times P_n \times 2\pi r dr \cos \alpha \times r$$

$$\boxed{T_r = \mu \times P_n \times 2\pi r^2 dr \cos \alpha}$$

Total torque on the shaft or bearing surface with a limit from 0 to R will be;

$$T = \int_0^R \mu \times P_n \times 2\pi r^2 dr \cos \alpha$$

$$= \int_0^R \mu \times \frac{C}{r} \times 2\pi r^2 dr \cos \alpha$$

$$= \int_0^R \mu \times C \times 2\pi r dr \cos \alpha = \cancel{\frac{1}{2} \pi C R^2}$$

$$= \mu \times 2\pi C \cos \alpha \int_0^R r dr = \mu \times 2\pi C \cos \alpha \times \frac{r^2}{2} \Big|_0^R$$

$$= \mu \times 2\pi C \cos \alpha \times \frac{R^2}{2}$$

$$= \mu \times 2\pi R \times \cos \alpha \times \frac{R^2}{2}$$

$$= \mu \times 2\pi \times \frac{w}{2\pi R} \times \cos \alpha \times \frac{R^2}{2}$$

$$= \mu \times w \times R \cos \alpha$$

$$T = \frac{1}{2} \mu w R \cos \alpha$$

Q. A vertical shaft of 150 mm in diameter rotating at 100 rpm resting on the flat and foot step bearing. The shaft carries the vertical load of 20 kN assuming uniform pressure distribution. Calculate the power lost in friction take $\mu = 0.05$.

$$\Rightarrow N = 100 \text{ rpm}$$

$$\text{dia} = 150 \text{ mm}, r = \frac{150}{2} = 75 \text{ mm}$$

$$W = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

$$\mu = 0.05$$

$$T = \frac{2}{3} \mu \cdot w \cdot R$$

$$= \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 75$$

$$= 50,000$$

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 100 \times 50,000}{60} = 523598.776$$

$$P = 523.59 \text{ kW}$$

Q. A conical pivot bearing supports a vertical shaft of 200 mm diameter. It is subjected to a load of 30 kN. The angle of the cone is 120° and $\mu = 0.025$. Find out the power lost in friction, when the speed is 40 rpm assuming both uniform pressure and uniform

$$\Rightarrow W = 30 \times 10^3 \text{ N}$$

$$D = 200 \text{ mm} \Rightarrow r = 100 \text{ mm}$$

$$\mu = 0.025$$

$$N = 140 \text{ rpm}$$

$$2\alpha = \frac{1.20^\circ}{2} \Rightarrow \alpha = 60^\circ$$

for uniform pressure;

$$T = \frac{2}{3} \mu \times W \times R \cos \alpha$$

$$T = \frac{2}{3} \times 0.025 \times 30 \times 10^3 \times 100 \times \cos(60^\circ)$$

$$T = 577.35$$

$$P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 140 \times 577.35}{60} = 2418.398$$

$$P = 2418 \text{ kW}$$

$$T = \frac{2}{3} \times 0.025 \times 30 \times 10^3 \times 100 \times \cos(60^\circ)$$

$$T = 57735.027$$

$$P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 140 \times 57735.027}{60} = 846439.704$$

$$P = 241.840 \text{ kW}$$

$$P = 846.44 \text{ kW}$$

for uniform wear;

$$T = \frac{1}{2} \mu \times W \times R \cos(\alpha) = \frac{1}{2} \times 0.025 \times 30 \times 10^3 \times 100 \times \cos(60^\circ)$$

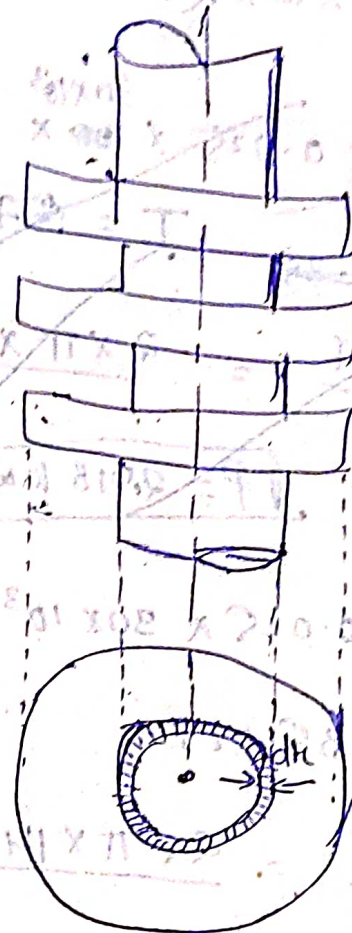
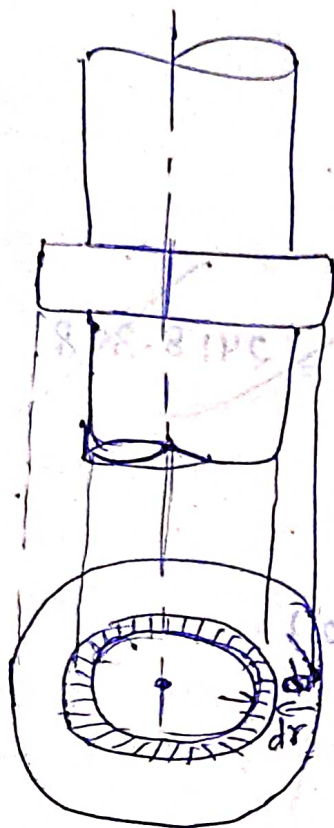
$$= 43301.270$$

$$P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 140 \times 43301.270}{60} = 634829.775$$

$$P = 634.83 \text{ kW}$$

Flat collar bearing for single and Multiple types:-

When the bearing surfaces are placed along the length of the shaft, the bearing is known as collar bearing.



consider a single flat collar bearing supporting a shaft.
Let r_1 = external radius, r_2 = internal radius or the radius of the shaft.

Also consider a ring of thickness dr , and let the load acting on the axis of the shaft be w .

$$\text{Area of the bearing surface} = \pi(r_1^2 - r_2^2)$$

Case-1 Uniform Pressure.

considering uniform pressure :-

when the pressure is uniformly distributed ;

$$p = \frac{w}{\pi(r_1^2 - r_2^2)}$$

$$\text{Area of the ring} = 2\pi r dr$$

$$\text{load transmitted by ring} = (\delta w) = \rho \times \delta A$$

$$\delta w = \rho \times 2\pi r dr$$

frictional resistance transmitted by ring;

$$(f_r) = \mu \times \delta w$$

$$= \mu \times \rho \times 2\pi r dr$$

frictional torque transmitted by the ring with radius "r" =

$$T_r = f_r \times r \Rightarrow \mu \times \rho \times 2\pi r dr \times r$$

$$= \mu \times \rho \times 2\pi r^2 dr$$

$$= \mu \times \frac{w}{\pi (r_1^2 - r_2^2)} \times 2\pi r^2 dr$$

$$= \mu \times \frac{w}{(r_1^2 - r_2^2)} \times 2r^2 dr$$

$$= \frac{2 \mu w \times r^2 dr}{(r_1^2 - r_2^2)}$$

$$\text{Total torque transmitted} = T = \int_{r_2}^{r_1} T_r$$

$$T = \int_{r_2}^{r_1} \frac{2 \mu w \times r^2 dr}{(r_1^2 - r_2^2)} = \frac{2 \mu w}{r_1^2 - r_2^2} \int_{r_2}^{r_1} r^2 dr$$

$$= \frac{2 \mu w}{r_1^2 - r_2^2} \times \frac{r^3}{3} \Big|_{r_2}^{r_1}$$

$$T = \frac{2}{3} \mu w \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

for multiple collar bearing :-

Multiple collar bearing torque transmitted (T)

$$= \frac{2}{3} \mu W \left[\frac{n_1^3 - n_2^3}{n_1^2 - n_2^2} \right]$$

In this case pressure = $\frac{W}{n \pi (n_1^2 - n_2^2)}$

Clutch

~~A clutch is a mechanism~~

Case-2

Uniform wear

consider a uniform wear theory! -

$$P \cdot r = C$$

$$P = \frac{C}{r}$$

$$\delta A = 2\pi r dr$$

$$\delta W = P \times 2\pi r dr$$

$$= \frac{C}{r} \times 2\pi r dr$$

$$\delta W = 2\pi C dr$$

$$W = \int_{n_2}^{n_1} \delta W$$

$$= \int_{n_2}^{n_1} 2\pi C dr$$

$$= 2\pi C \int_{n_2}^{n_1} dr = 2\pi C (n_1 - n_2)$$

$$2\pi C (n_1 - n_2) = W$$

$$\therefore C = \frac{W}{2\pi (n_1 - n_2)}$$

frictional torque $\Rightarrow$

$$(F_{fr}) = \mu \cdot \Delta w = \mu \times p \times 2\pi r dr$$

$$= \mu \times 2\pi r C dr = \mu \times 2\pi \frac{w}{(r_1 - r_2)} r dr$$

$$= \mu \times \frac{w}{(r_1 - r_2)} r^2 dr$$

Total frictional torque $\Rightarrow$

$$T_H = F_{fr} \times r \Rightarrow T_H = \mu \times 2\pi C r^2 dr$$

$$= \mu \times \frac{w}{(r_1 - r_2)} r^2 dr$$

Total torque $= \int_{r_2}^{r_1} T_H = \int_{r_2}^{r_1} \frac{\mu w r^2}{(r_1 - r_2)} dr$

$$= \frac{\mu w}{r_1 - r_2} \left[\frac{r^3}{3} \right]_{r_2}^{r_1} = \frac{\mu w}{r_1 - r_2} \times \frac{r_1^3 - r_2^3}{3}$$

$$= \frac{\mu w}{r_1 - r_2} \times \frac{r_1^3 - r_2^3}{3}$$

$$= \frac{\mu w}{2} \times \frac{(r_1 + r_2)(r_1^2 - r_2^2)}{(r_1 - r_2)} = \frac{\mu w}{2} (r_1 + r_2)$$

$$\boxed{T = \frac{1}{2} \mu w (r_1 + r_2)}$$

for multiple collar bearings;

Multiple collar bearing torque will transmitted $\Rightarrow$

$$(T) = \frac{1}{2} \mu w (r_1 + r_2)$$

$$P_H = \frac{w}{n\pi (r_1^2 + r_2^2)}$$

Q. A thrust shaft of a ship has 6 collars of 600mm external diameter and 300mm of internal diameter. Total thrust from the propeller is 100kN. The coefficient of friction is 0.12 and the speed of rotation is 90 rpm. Find the power absorbed by the friction at the block (i) for uniform pressure, (ii) for uniform wear.

$$\Rightarrow D = 600 \text{ mm} \Rightarrow R = 300 \text{ mm} = 30 \text{ cm}$$

$$d = 300 \text{ mm} \Rightarrow r = 150 \text{ mm} = 15 \text{ cm}$$

$$W = 100 \text{ kN} = 100 \times 10^3 \text{ N}$$

$$\mu = 0.12$$

$$N = 90 \text{ rpm}$$

(i) for uniform pressure,

$$T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= \frac{2}{3} \times 0.12 \times 100 \times 10^3 \left[\frac{300^3 - 150^3}{300^2 - 150^2} \right]$$

$$= 2800 \times 10^3 \text{ N/mm}$$

$$T = 2800 \text{ N/m}$$

$$\Rightarrow P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 90 \times 2800}{60}$$

$$P = 26389.328 \text{ W}$$

$$\boxed{P = 26.389 \text{ kW}}$$

(ii) for uniform wear,

$$T = \frac{1}{2} \mu W (r_1 + r_2) \Rightarrow \frac{1}{2} \times 0.12 \times 100 \times 10^3 (300 + 150)$$

$$= 2700 \times 10^3 \text{ N/mm} = 2700 \text{ N/m}$$

$$P = \frac{2\pi NT}{60} \Rightarrow P = 25446.9 \Rightarrow \boxed{P = 25.447 \text{ kW}}$$

Q. A shaft has a no. of collars into external diameter of the collar is 400mm and the diameter of the shaft is 250mm. If the intensity of pressure is 0.35 N/mm^2 and $\mu = 0.05$. Find out (i) Power absorbed for uniform pressure when the shaft carrying load of 150 kW. Turns at 105 rpm.

(ii) Find out the no. of collars.

Given:

$$D = 400 \text{ mm} \Rightarrow r_1 = 200 \text{ mm}$$

$$d = 250 \text{ mm} \Rightarrow r_2 = 125 \text{ mm}$$

$$p_r = 0.35 \text{ N/mm}^2$$

$$\mu = 0.05$$

$$(i) N = 105 \text{ rpm}$$

$$W = 150 \text{ kW} = 150 \times 10^3 \text{ N}$$

$$\tau = \frac{2}{3} \mu \times w \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right] \Rightarrow \frac{2}{3} \times 0.05 \times 150 \times 10^3 \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= \frac{2}{3} \times 0.05 \times 150 \times 10^3 \left[\frac{200^3 - 125^3}{200^2 - 125^2} \right] = 1240.385 \text{ N/m}^2$$

$$P = \frac{2\pi NT}{60} = 13638.745 \text{ W} = \boxed{P = 13.639 \text{ kW}}$$

$$(ii) \left[\frac{w}{n \pi (r_1^2 - r_2^2)} \right] = p_r$$

$$\Rightarrow 0.35 = \left[\frac{150 \times 10^3}{n \pi (200^2 - 125^2)} \right]$$

$$= n = \frac{150 \times 10^3}{0.35 \times \pi (200^2 - 125^2)} = 5.597 \approx 6$$

$$\cancel{n = 3.231} \quad n = 6$$

$$\Rightarrow \cancel{n = 4}$$

Clutch

- ⇒ A clutch is a mechanical device which is used to transmit rotating motion or torque from one shaft to another shaft when required.
- ⇒ Clutch provides a temporary connection between input and output shaft. Clutch is situated between the engine and gear box.
- ⇒ Using the clutch, we can engage and disengage power transmission from one shaft to another.
- ⇒ When we press the clutch pedal, the output shaft is disengaged from the power transmission. We generally press clutch pedal when we need to change gears. After changing gears, we release the clutch pedal and the power transmission is again engaged with output shaft.

$$\left[\frac{\epsilon_{s1} - \epsilon_{11}}{\epsilon_{s1} - \epsilon_{11}} \right] \times 0.01 \times 0.021 \times 20.0 \times \frac{c}{s} \left(\left[\frac{\epsilon_{s1} - \epsilon_{11}}{\epsilon_{s1} - \epsilon_{11}} \right] \times 0.01 \times \frac{c}{s} = T \right)$$

$$15.128.5 \times 10^3 \text{ N} = \left[\frac{\epsilon_{s1} - \epsilon_{11}}{\epsilon_{s1} - \epsilon_{11}} \right] \times 0.01 \times 0.021 \times 20.0 \times \frac{c}{s}$$

$$\boxed{W \times 13.03 \text{ kN} = 9} = W \times 13.03 \text{ kN} = \frac{T \times 1000}{0.03} = 9$$

$$T = \left(\frac{W}{(\epsilon_{s1} - \epsilon_{11}) \pi r} \right) \quad (1)$$

$$\left[\frac{\epsilon_{s1} - \epsilon_{11}}{(\epsilon_{s1} - \epsilon_{11}) \pi r} \right] = 28.0 \text{ N}$$

Let, T = Torque transmitted by the clutch

p = Intensity of axial pressure with which the contact surfaces are held together.

r_1 and r_2 = External and internal radii of friction faces.

μ = Coefficient of friction.
 consider an elementary ring of radius " r " and thickness " dr "
 we know that area of contact surface on friction surface,
 $= 2\pi r dr$

$\therefore$ Normal or axial force on the ring;

$$\delta W = \text{Pressure} \times \text{Area}$$

$$\delta W = p \times 2\pi r dr$$

Frictional force acting on the ring;

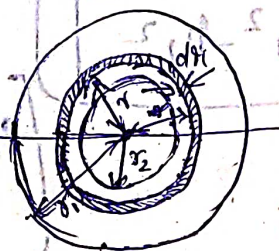
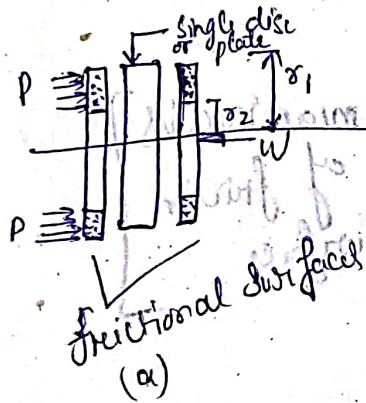
$$F_f = \mu \times \delta W$$

$$= \mu \times p \times 2\pi r dr$$

frictional torque on the ring;

$$T_f = F_f \times r = \mu \times p \times 2\pi r dr \times r$$

$$= \mu \times p \times 2\pi r^2 dr$$



(b)

forces on a single disc or plate clutch.

Two cases can be considered:-

- i) Uniform pressure
- ii) Uniform wear

(i) Case 1: Uniform pressure
 when the intensity of pressure is distributed uniformly on the area of the friction face, then intensity of pressure,

$$\text{Pressure} = \frac{W}{\pi (r_1^2 - r_2^2)}$$

W = axial thrust

and as already discussed,

$$T_f = \mu \times p \times 2\pi r^2 dr$$

By integrating this value of frictional torque with
 limit from r_2 to r_1 ,
 $T = \int_{r_2}^{r_1} \mu \times P \times 2\pi r^2 dr$

$$T = \mu \times P \times 2\pi \int_{r_2}^{r_1} r^2 dr = \mu \times P \times 2\pi \times \left[\frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$T = \mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \times 2\pi \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= \mu W R$$



(as R = mean radius
 of friction
 surface)

$$\therefore \frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right] = R$$

Case 2: Uniform wear

Let p be the normal intensity of pressure
 at a distance ' r ' from the axis of clutch.
 Since the intensity of pressure varies
 inversely with the distance,

$$p \propto \frac{1}{r} \Rightarrow p r = C$$

$$\therefore p = \frac{C}{r}$$

Normal force on the ring;

$$\delta W = p r \times A$$

$$= p \times 2\pi r dr$$

$$= \frac{c}{r} \times 2\pi r dh (\sin + \cos) \times w \times h \times \frac{1}{2} \times$$

$$\delta w = 2\pi c dh$$

∴ total force acting on the ring;

$$W = \int_{h_2}^{h_1} \delta w \Rightarrow \int_{h_2}^{h_1} 2\pi c dh \Rightarrow 2\pi c \int_{h_2}^{h_1} dh$$

$$\Rightarrow 2\pi c \times \int_{h_2}^{h_1} 1 = 2\pi c (h_1 - h_2)$$

$$W = 2\pi c (h_1 - h_2)$$

$$c = \frac{W}{2\pi (h_1 - h_2)}$$

frictional torque on the ring;

$$T_h = f_h \times r = \mu \times \delta w \times r$$

$$= \mu \times 2\pi c dh \times r$$

$$\text{Total torque} = \int_{h_2}^{h_1} T_h = \int_{h_2}^{h_1} \mu \times 2\pi c r dh$$

$$T = \int_{h_2}^{h_1} \mu \times 2\pi \frac{W}{2\pi (h_1 - h_2)} r dh$$

$$T = \frac{\mu \times r \times W}{(h_1 - h_2)} \int_{h_2}^{h_1} dh$$

$$= \frac{\mu \times W}{(h_1 - h_2)} \times \frac{r^2}{2} \int_{h_2}^{h_1} \frac{1}{r} dh$$

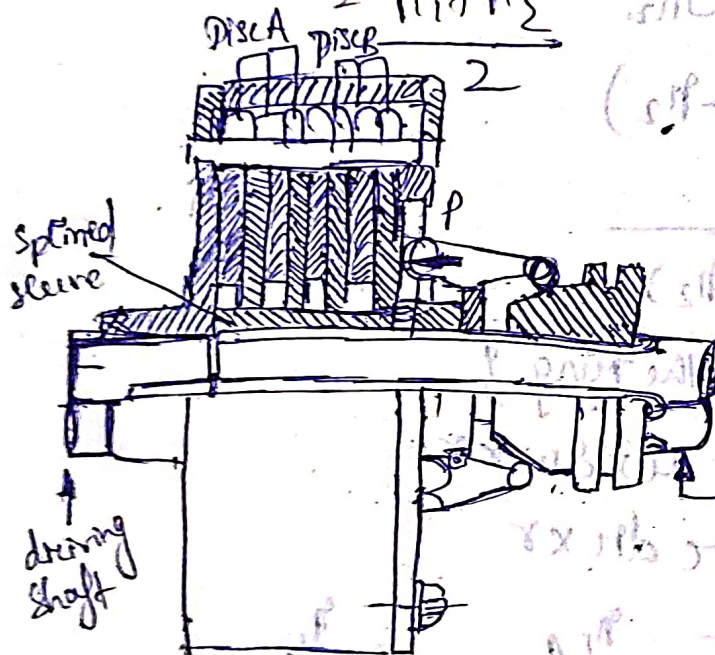
$$= \frac{\mu \times W}{(h_1 - h_2)} \times \frac{h_1^2 - h_2^2}{2}$$

$$\Rightarrow \frac{1}{2} \times \mu \times W \times \frac{(r_1 + r_2)(r_1 - r_2)}{(r_1 - r_2)} = \dots$$

$$T \Rightarrow \frac{1}{2} \times \mu \times W (r_1 + r_2)$$

$$\Rightarrow T = \mu \times W \times R$$

As, $R = \text{mean radius of the friction surface}$
 $= \frac{r_1 + r_2}{2}$



Multi disc clutch

Equations derived for torque transmitting capacity of the single plate clutch are modified to account for the number of pairs of contacting surfaces in the following way;

for the uniform pressure criterion;

$$T = \frac{2}{3} \mu W n \frac{(r_o^3 - r_i^3)}{(r_o^2 - r_i^2)}$$

for uniform wear;

$$T = \mu W n \left(\frac{r_o + r_i}{2} \right)$$

where $n = \text{no. of pairs of contact surfaces}$

Working of Simple frictional brakes

Brakes

→ A brake is a device by the means of which artificial frictional resistance is applied to a moving machine member in order to stop or retard the motion of machine.

→ In this process the brakes absorb either the kinetic energy or potential energy of the moving member.

→ Heat absorbed by the brakes is dissipated in the form of heat.

Types of brakes

- ① Hydraulic brake
- ② Electric brake
- ③ Mechanical brake

Mechanical brake

Radial brake

→ Force acting on the brake drum is in the radial direction.

- ex:- i) Block or shoe brake
ii) Band brake

Axial brake

→ Force acting on the brake drum is in the axial direction.

- ex:- i) Disc brake
ii) Cone brake

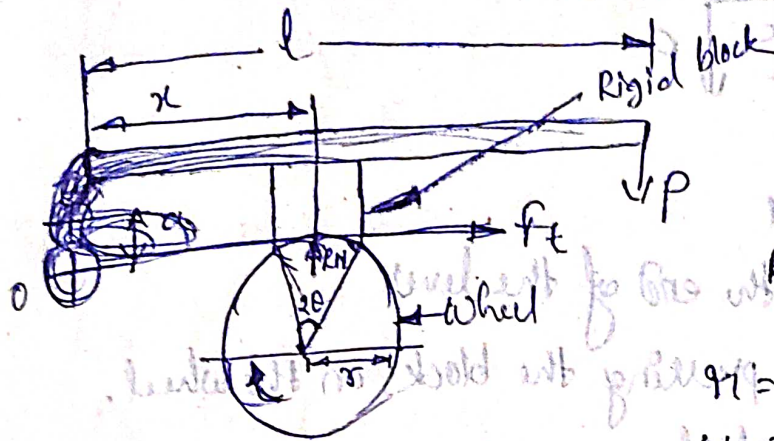
Single block or shoe brake

→ It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum.

→ This type of brake is used in trains

→ Here the friction b/w block and wheel causes a tangential braking force to act on the wheel, which retard the rotations of the wheel.

Case 1:- (When the line of action of tangential braking force (f_t) passes through fulcrum O of lever.)



P = force applied on the end of the lever

R_N = Normal force acting on the wheel.

r = radius of wheel

μ = coefficient of friction

2θ = Angle between surface of the block

f_t = Tangential braking force

T_B = Braking torque

when the angle is less than 60° ,

$$f_t = \mu \times R_N \quad \text{and} \quad T_B = f_t \times r = \mu \times R_N \times r$$

So taking moment about fulcrum "O":

$$R_N \times x = P \times l \quad \Rightarrow \quad \boxed{R_N = \frac{P \times l}{x}}$$

$$\begin{aligned} \therefore \text{Braking torque, } T_B &= f_t \times r \\ &= \mu \times R_N \times r \\ &= \mu \times \frac{P \times l}{x} \times r \end{aligned}$$

$$\boxed{T_B = \frac{\mu P l r}{x}}$$

Note:- if $2\theta > 60^\circ \Rightarrow$ using $\mu' = \frac{4\mu \sin \theta}{2\theta + \sin 2\theta}$

$$f_t = \mu' \times R_N$$

① Absorption dynamometer

Here the entire energy or power produced by the engine is absorbed by the dynamometer and transformed to heat.

ex:- Rope brake dynamometer

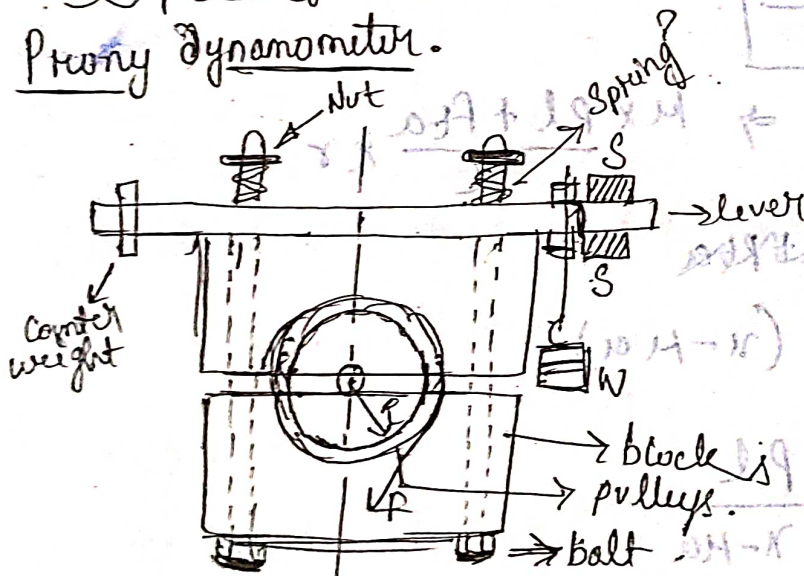
Prony brake dynamometer.

② Transmission Dynamometer

Here the power produced by the engine is transmitted through dynamometer to some other machines where the developed power is suitably measured.

Absorption dynamometer:-

Prony Dynamometer.



⇒ It consists of 2 blocks placed around a pulley fixed to the shaft of an engine where power is required to be measured.

⇒ Blocks are clamped by means of nuts and bolts.

⇒ A helical spring is provided between the nut and upper block to adjust the pressure on the pulley.

⇒ A lever is attached with the upper block and carries a weight "W" at its other end.

⇒ In order to balance the lever during unloading, a counter weight is placed at other end.

⇒ Two stops 'S' are provided to limit the motion of lever.

Let, W = weight at the end of the lever
 L = distance between W and center of pulley.
 f = frictional resistance between block and pulley.
 R = radius of the pulley.
 N = Speed of shaft in rpm.
 Moment of frictional resistance or torque $\Rightarrow f R N/m$
 $= W \times L$

Work done in 1 revolution $\Rightarrow$ torque $\times$ Angle turned
 in radian

$\Rightarrow T \times 2\pi \text{ N/m}$

Work done per minute $\Rightarrow T \times 2\pi N \text{ N/m}$

If Speed of pulley = N rpm.

So, Brake Power of engine $\Rightarrow B.P. = \frac{\text{Work done/min}}{60}$

$$B.P. = \frac{2\pi T N}{60} \text{ watts}$$

$$B.P. = \frac{W L \times 2\pi N}{60} \text{ watts}$$

Rope brake dynamometer

$\Rightarrow$ It is another type of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of

- * It consists of one, two, or more ropes wound around the fly wheel of the engine turn of a pulley fixed rigidly to the shaft of an engine.
- $\Rightarrow$ The upper end of the engine is attached to the spring balance while the lower end of the ropes is kept in position by applying a dead weight as

Shown in figure in order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of brake, the engine is made to run at a constant speed. The frictional torque due to the rope must be equal to the torque being transmitted by the engine.

Let, W = dead load in newtons.

S = Spring balance reading in newton

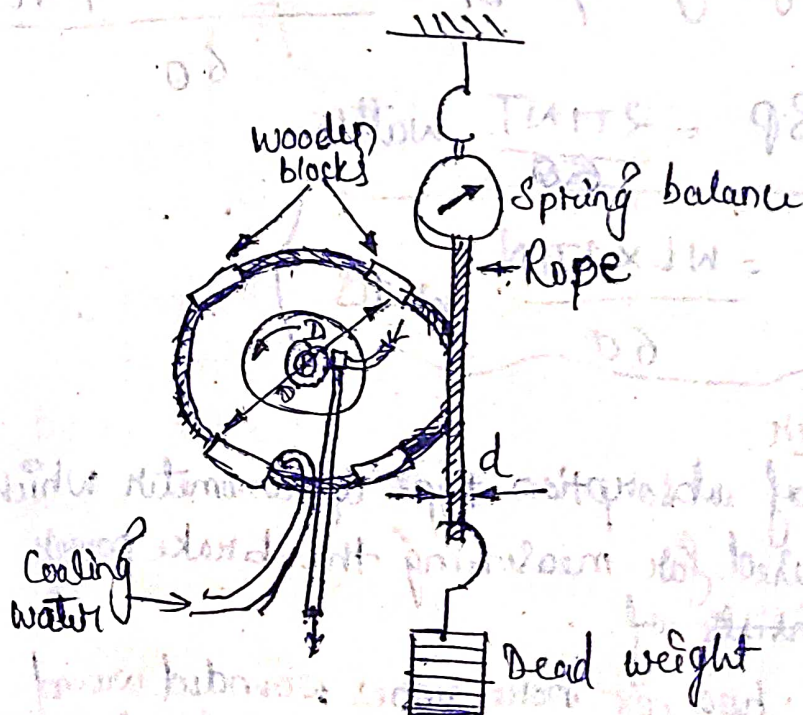
D = diameter of the wheel in meters.

d = diameter of the rope.

∴ Work done as per revolution;

$$\boxed{\cancel{W} = (W - S) \pi (D - d) N/m}$$

Work done as per minute $\cancel{= (W - S) \pi (D - d) N/m}$



Rope brake system

∴ Brake power of the engine:

$$B.P = \frac{\text{Work done per minute}}{60} = \frac{(W - S) \pi (D - d) N}{60}$$

if the diameter of rope (d) is neglected, then,

$$BP = \frac{(W-S) \pi D N}{60} \text{ watts}$$