

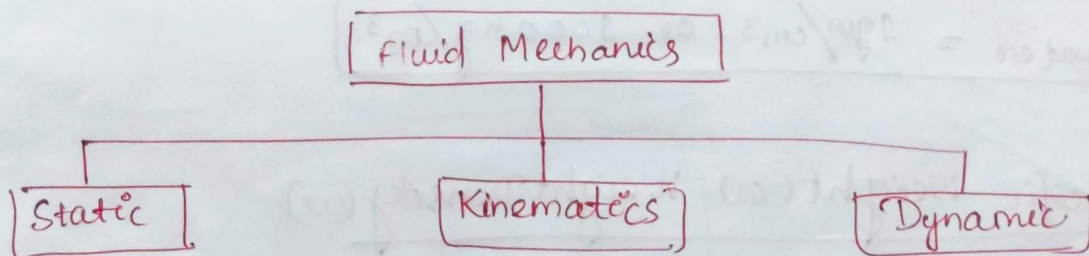
Chapter - 01

Properties of Fluid

Fluid :- A fluid is a gas or liquid that flows to assume the shape of the container in which it is placed.

Fluid Mechanics :- Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as ⁱⁿ motion.

→ Thus this branch of science deals with the static, kinematics & dynamic aspect of fluids.



Properties of Fluid

(1) Density or Mass Density (ρ) - It is defined as the ratio of the mass of a fluid to its volume.

$$\Rightarrow \text{Density / Mass Density } (\rho) = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

$$\therefore \rho = \frac{M}{V}$$

Unit $\Rightarrow$ Kg/m^3 [S.I.]

gm/mm^3 [CGS]

(or) gm/cm^3

$$\rho_{\text{water}} = 1 \text{ gm/cm}^3 \text{ or } 1000 \text{ kg/m}^3$$

(2) Specific Weight (or) Weight Density (w)

Sp. weight or weight density of a fluid is the ratio between the weight of a fluid to its volume.

$\Rightarrow$ This weight per unit volume of a fluid is called weight density.

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$w = \frac{\text{Mass of fluid} \times g}{\text{Volume}} = \rho \times g$$

$$\boxed{\therefore w = \rho g}$$

Specific weight

Unit :- Newton/m³ [S.I]

$\therefore$ Sp. weight or weight density of water = $9.81 \times 1000 \text{ N/m}^3$

(3) Specific Volume :-

Sp. Volume of a fluid is defined as the volume of a fluid occupied by a unit mass of a fluid.

$$\therefore \text{sp. volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}}$$

$$= \frac{\text{Vol}}{\text{Mass}}$$

$$= \frac{1}{\text{mass/vol.}}$$

$$\boxed{\text{Sp. Vol} = \frac{1}{\rho}}$$

Unit :- m³/kg [S.I]

(4) Specific Gravity (S)

It is defined as the ratio of the weight density (or Density) of a fluid to the weight density (or density) of a standard fluid.

$$\therefore \text{Sp. gravity (S) for liquids} = \frac{\text{Sp. weight of liquid}}{\text{Sp. weight of water}}$$

$$= \frac{W_{\text{liquid}}}{W_{\text{water}}}$$

$$= \frac{S_{\text{liquid}} \times g}{S_{\text{water}} \times g}$$

$$\boxed{S = \frac{S_{\text{liquid}}}{S_{\text{water}}}}$$

$$\therefore \text{weight density of a liquid} = S \times \text{weight density of water}$$

$$= S \times 1000 \times 9.81 \text{ N/m}^3$$

$$\therefore \boxed{\text{Density of a liquid} = S \times S_{\text{water}}}$$

Q-1 Calculate the specific weight, density, & sp. gravity of one litre of a liquid which weighs 7N.

Given: Volume = 1 litre = $\frac{1}{1000} \text{ m}^3$ (or 1000 cm^3)

Weight = 7N

(i) Sp. weight (w) = $\frac{\text{Weight}}{\text{Volume}} = \frac{7\text{N}}{\left(\frac{1}{1000}\right) \text{ m}^3} = 7000 \text{ N/m}^3$ (Ans)

(ii) Density, $\rho = \frac{\text{Sp. weight}}{g} = \frac{w}{g} = \frac{7000}{9.81} = 713.5 \text{ kg/m}^3$ (Ans)

(iii) Sp. gravity, $S = \frac{\text{Density of liquid}}{\text{Density of water}}$

$= \frac{713.5}{1000}$

$= 0.7135$ (Ans)

Q-2 Calculate the density, specific weight of one litre of Petrol of specific gravity = 0.7.

Given:- Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{1000} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity, $S = 0.7$

$\therefore$ (i) Density $\rho = \text{Sp. gravity} \times \rho_{\text{water}}$

$= 0.7 \times 1000 \text{ kg/m}^3$

$= 700 \text{ kg/m}^3$ (Ans)

$$\begin{aligned}
 \text{(ii) Sp. weight } (w) &= \rho \times g \\
 &= 700 \times 9.81 \text{ N/m}^3 \\
 &= 6867 \text{ N/m}^3 \quad (\text{Ans})
 \end{aligned}$$

(iii) Weight $\Rightarrow W = ?$

• We know that $\text{Sp. weight} = \frac{\text{Weight}}{\text{Volume}}$

$$\Rightarrow 6867 = \frac{W}{0.001}$$

$$\Rightarrow W = 6867 \times 0.001 = 6.867 \text{ N (Ans)}$$

Viscosity

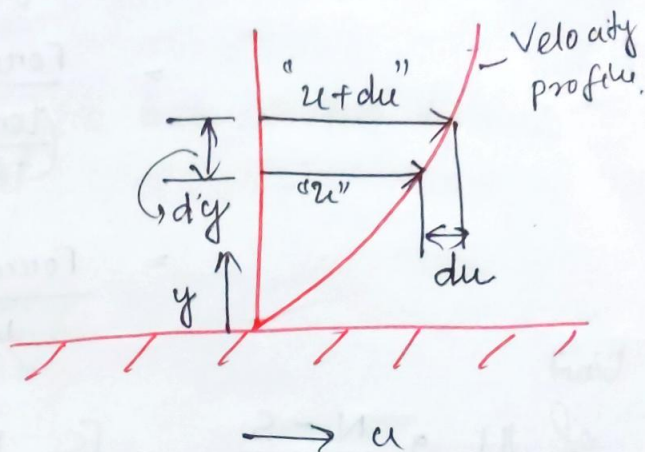
Defⁿ:- Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

* When two layers of a fluid, a distance ' dy ' apart move one over the other at different velocities $\rightarrow 'u'$ & ' $u+du$ '

$\Rightarrow$ The viscosity together with relative viscosity causes a shear stress acting between the fluid layers

⇒ The top layer causes a shear stress on the adjacent layer while the lower layer causes a shear stress on top layer.

⇒ This shear stress is proportional to the rate of change of velocity " u " w.r.t " y "



⇒ Shear stress " τ "

Mathematically " τ " $\propto \frac{du}{dy}$

$$\Rightarrow \tau = \mu \frac{du}{dy}$$

where " μ " is called the constant of proportionality

⇒ "Co-efficient of dynamic viscosity".
or only "viscosity"

$$\therefore \mu = \frac{\tau}{(du/dy)}$$

∴ Viscosity is also defined as the shear stress required to produce unit rate of shear strain.

Unit of viscosity :-

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

$$= \frac{\text{Force/Area}}{\left(\frac{\text{Length}}{\text{Time}}\right) \times \left(\frac{1}{\text{Length}}\right)}$$

$$= \frac{\text{Force/Length}^2}{1/\text{Time}} = \frac{\text{Force} \times \text{Time}}{\text{Length}^2}$$

Unit

$$\text{of } \mu = \frac{\text{N} \cdot \text{s}}{\text{m}^2} \quad [\text{S} \cdot \text{I}]$$

$$\frac{\text{dyne} \cdot \text{s}}{\text{cm}^2} \quad [\text{CGS}]$$

- The unit of viscosity in CGS is called "Poise"

$$\boxed{1 \text{ Poise} = \frac{\text{dyne} \cdot \text{s}}{\text{cm}^2}}$$

$$\bullet \quad \frac{1 \text{ kgf} \cdot \text{sec}}{\text{m}^2} = 9.81 \frac{\text{N} \cdot \text{sec}}{\text{m}^2}$$

$$1 \text{ N} = 1 \text{ kg} \times 1 \text{ m/sec}^2$$

$$= (1000 \text{ gm}) \times 100 \text{ cm}$$

$$= 9.81 \times 10^5 \frac{\text{dyne} \cdot \text{sec}}{\text{m}^2}$$

$$1 \text{ N} = 10^5 \text{ dyne}$$

$$= \frac{9.81 \times 10^5}{10^4} \cdot \frac{\text{dyne} \cdot \text{sec}}{\text{cm}^2}$$

$$\boxed{\frac{1 \text{ kgf} \cdot \text{sec}}{\text{m}^2} = 98.1 \text{ Poise}}$$

$$\boxed{1 \text{ Poise} = \frac{1}{10} \frac{\text{N-s}}{\text{m}^2}}$$

Kinematic viscosity : ν

It is defined as the ratio between the dynamic viscosity & density of fluid

$$\nu = \frac{\text{Viscosity}}{\text{Density}}$$

$$\boxed{\nu = \frac{\mu}{\rho}}$$

Unit of ν

$$\frac{\left(\frac{\text{Force} \times \text{Time}}{\text{Length}^2} \right)}{\left(\frac{\text{Mass}}{\text{Length}^3} \right)}$$

$$= \frac{\text{Mass} \times \text{Acc.} \times \text{Time}}{\text{Length}} \cdot \left(\frac{\text{Length}}{\text{Mass}} \right)$$

$$= \frac{(\text{Length})^2}{\text{Time}}$$

$$\text{m}^2/\text{sec}$$

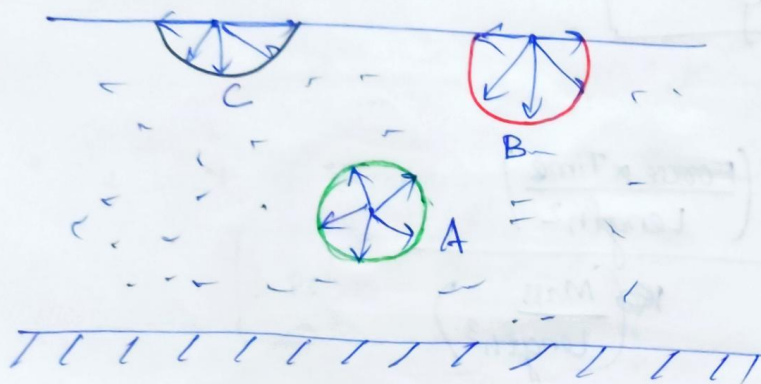
$$\text{cm}^2/\text{sec} = 1 \text{ stoke} = 10^{-4} \text{ m}^2/\text{sec}$$

Surface Tension

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas (or) on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

→ " σ "

→ Units → N/m or kgf/m



Net force on 'A' = 0

'B' = downward (unbalanced)

'C' = " "

⇒ All the molecules on the free surface experience a downward force.

⇒ Thus the free surface of the liquid acts like a very thin film under tension of the surface of liquid act as though it is an elastic membrane

under tension.

Surface Tension on Liquid Droplet

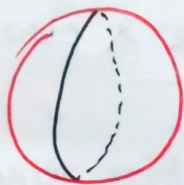
Consider a small spherical droplet of a liquid of radius " r "

Let σ = surface tension of the liquid

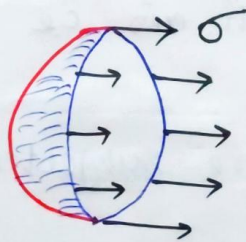
P = pressure intensity inside the droplet

d = Dia of droplet

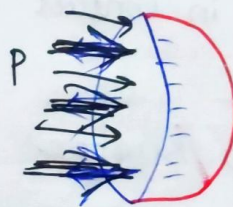
= Cut the droplet into two halves



(1)



(2)



(3)

- The forces action on one half ^{fig (2)} will be tensile force due to surface tension acting around the circumference of the cut portion.

$$\begin{aligned} \text{Force circumference} &= \sigma \times \text{Circumference} \\ &= \sigma \times \pi d \quad \text{--- (1)} \end{aligned}$$

- Pressure force on the area i.e. fig (3) = Pressure intensity inside $\times$ area

$$\begin{aligned} &= P \times \frac{\pi d^2}{4} \quad \text{--- (2)} \end{aligned}$$

Under Equilibrium Conditions, the two forces acting on the droplet ~~o~~ will be equal & opposite.

$$\therefore \text{eq}^n(1) = \text{eq}^n(2)$$

$$\Rightarrow P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\Rightarrow \boxed{P = \frac{4\sigma}{d}}$$

Surface tension on a Hollow Bubble :-

A Hollow bubble like a soap bubble in air, has two surfaces in contact with air i.e. one inside & other outside.

$\Rightarrow$ The two surfaces are subjected to surface tension

$$P \times \frac{\pi}{4} d^2 = 2 (\sigma \times \pi d)$$

$$\Rightarrow \boxed{P = \frac{8\sigma}{d}}$$

Surface Tension on Liquid Jet

Liquid jet of diameter 'd'
length 'L'

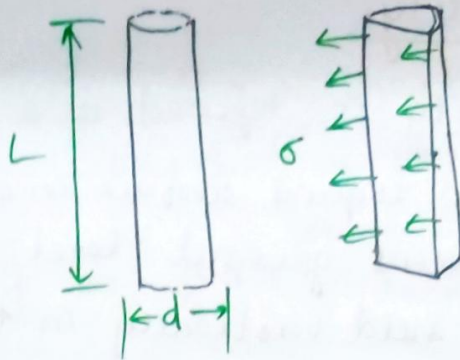
P = pressure intensity inside the liquid jet

σ = surface tension.

Considering Equilibrium conditions.

Force due to pressure,

$$= P \times \text{area of semi-jet} \\ = P \times L \times d \quad \text{--- (1)}$$



$$\text{Force due to surface tension} = \sigma \times 2L \quad \text{--- (2)}$$

$$\text{eqn (1)} = \text{eqn (2)}$$

$$\Rightarrow P \times L \times d = \sigma \times 2L$$

$$\Rightarrow P = \frac{\sigma \times 2L}{L \times d}$$

$$P = \frac{2\sigma}{d}$$

Q. Find the surface tension in a soap bubble of 40mm diameter when the inside pressure is 2.5 N/m^2 above atm. pressure.

Given

$$\text{Dia of bubble, } d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\text{For soap bubbles } P = \frac{8\sigma}{d}$$

$$\Rightarrow 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\Rightarrow \sigma = \frac{2.5 \times 40 \times 10^{-3}}{8 \times 2.5} = 0.0125 \text{ N/m (Ans)}$$

Capillarity

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

⇒ The rise of liquid surface is known as capillary rise. while the fall of the liquid surface is known as capillary depression.

⇒ Expressed in terms of "cm" or "mm" of liquid

⇒ Its value depends upon the sp. weight of the liquid, diameter of the tube & surface tension of the liquid.

Capillary Rise :-

Consider a glass tube of small diameter ^{of} (opened at both ends) & is inserted in a liquid. (water)

⇒ The liquid will rise in the tube above the level of the liquid.

Let, h = height of the liquid in the tube

∴ Under state of equilibrium

the weight of liquid of height " h " is balanced by the force at the surface of the liquid in the tube.

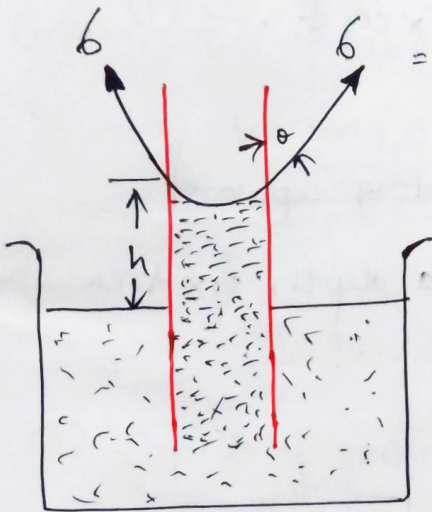
Equilibrium Condition

Weight of liquid of height 'h' = Force acting on the surface of liquid i.e. surface tension.

$$\Rightarrow (\text{Area of the tube} \times h) \times \rho \times g = \frac{\pi}{4} d^2 \times h \times \rho \times g \quad \text{--- (1)}$$

$\therefore$ vertical component of the surface tensile force
= $(\sigma \times \text{circumference}) \times \cos \theta$

$$= \sigma \times \pi d \times \cos \theta \quad \text{--- (2)}$$



Let σ = surface tension.

θ = Angle of contact
b/w liquid & glass tube.

$$\therefore \text{eqn (1)} = \text{eqn (2)}$$

$$\Rightarrow \frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos \theta$$

$$\Rightarrow h = \frac{4 \sigma \cos \theta}{\rho g d}$$

Capillary rise

Capillary Fall :-

If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid.

Let h = height of depression in tube

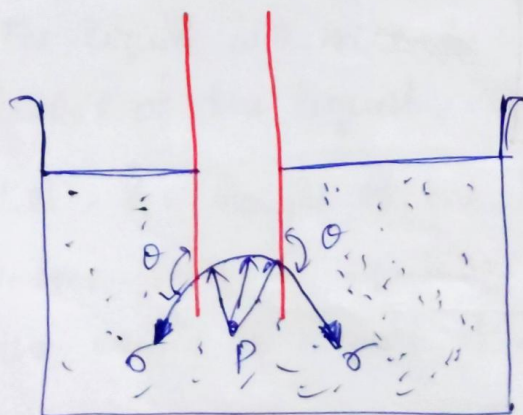
✓ Force ~~act~~ due to surface tension acting in the downward direction $\Rightarrow \sigma \times \pi d \times \cos \theta$ — (1)

✓ Force due to hydrostatic force acting upward
= intensity of pressure at a depth $h \times$ Area
= $P \times \frac{\pi}{4} d^2$

$$= \rho g h \times \frac{\pi}{4} d^2 \text{ — (2)}$$

$\therefore$ Equilibrium Condition, $\sigma \times \pi d \times \cos \theta = \rho g h \times \frac{\pi}{4} d^2$

$$\Rightarrow h = \frac{4\sigma \cos \theta}{\rho g d}$$



Value of θ "mercury" = 128°

Q Calculate capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water & (b) mercury

Take surface tension $\sigma = 0.0725 \text{ N/m}$ for water and $\sigma = 0.52 \text{ N/m}$ for mercury in contact with air.

The sp. gravity for mercury is given as 13.6 & angle of contact $= 130^\circ$

Given Dia of tube, $d = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m}$

Surface tension, $\sigma_{\text{water}} = 0.0725 \text{ N/m}$

$\sigma_{\text{mercury}} = 0.52 \text{ N/m}$

Sp. gravity = 13.6

Density $= 13.6 \times 1000 \text{ kg/m}^3$

(a) Capillary rise for water ($\theta = 0^\circ$)

$$h = \frac{4\sigma}{\rho g d} = \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}} = 0.0118 \text{ m} = 1.18 \text{ cm} \quad (\text{As})$$

(b) For mercury, Angle of contact $\theta = 130^\circ$

$$h = \frac{4\sigma \cos \theta}{\rho g d} = \frac{4 \times 0.52 \times \cos 130^\circ}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}} = -0.004 \text{ m} = -0.4 \text{ cm} \quad (\text{Ans})$$

Chapter - 2

Fluid Pressure & Its Measurements

Fluid Pressure

Fluid Pressure is the pressure at a point within a fluid arising due to the weight of the fluid.

Consider a small area " dA " in large mass of fluid
 $\Rightarrow$ Force exerted by the surrounding fluid on the area " dA " is always $\perp^r$ to the surface area " dA "
 $\hookrightarrow$ Let the $\perp^r$ force is " dF " (acting in normal dir n)

$\therefore \frac{dF}{dA}$ is 'intensity of pressure' or 'pressure'

$$\boxed{P = \frac{dF}{dA}}$$

$\therefore$ Pressure at a point in a fluid at rest

is $\boxed{P = \frac{dF}{dA}}$

If the force ' F ' is uniformly distributed over area ' A '

Pressure at any point is $\boxed{P = \frac{F}{A}}$

Unit of pressure $\Rightarrow \text{kgf/m}^2$ and kgf/cm^2 in MKS

Pascal Newton/m² or N/m² S.I
(N)(Pa)

KPa = Kilo Pascal = 1000 N/m^2

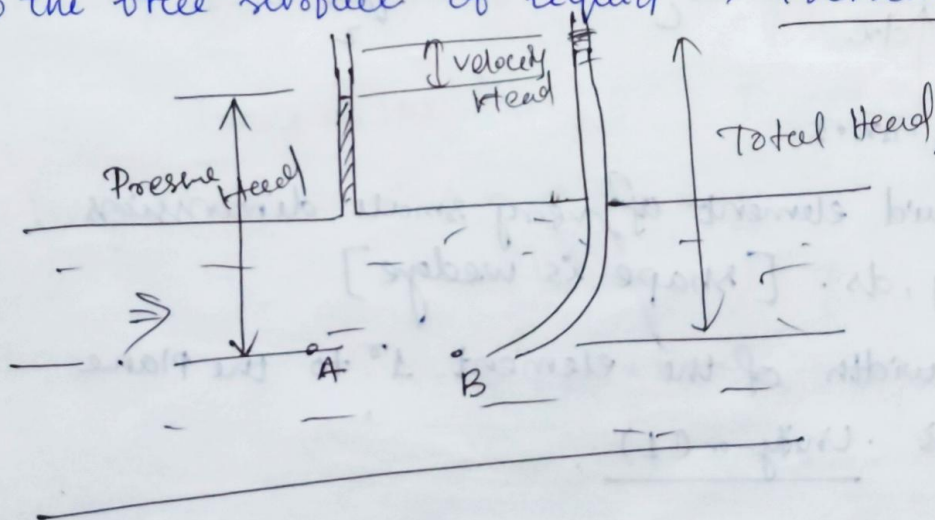
1 bar = 100 KPa = 10^5 N/m^2

Pressure Head :-

It is the height of a liquid column that corresponds to a particular pressure exerted by the column on the base of its container.

⇒ Pressure head represents the flow of energy of a column of fluid whose weight is equivalent to the pressure of the fluid

⇒ In static liquid, vertical distance from datum line to the free surface of liquid ⇒ Pressure Head



• PH in fluid mechanics is the pressure exerted by a liquid column on the base of the container.

⇒ It is represented as the height of the liquid column.

⇒ P.H is also called Static Head or static pressure head.

$\therefore$ Pressure Head, $\boxed{Z = \frac{P}{\rho g}}$

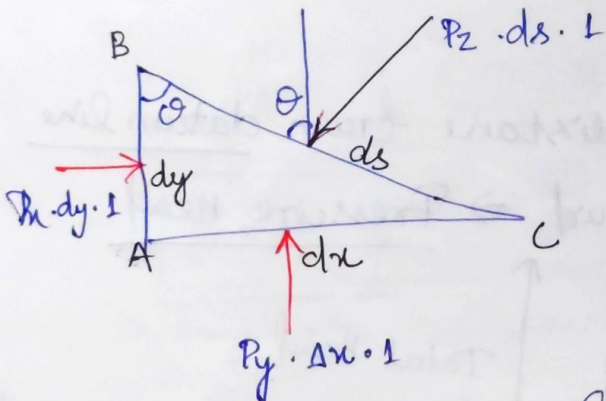
P = fluid pressure ie (Pressure at unit Area)

g =

ρ = density

2.2 Pascal's Law

It states that "the pressure or intensity of pressure at a point in a static fluid is equal in all directions".



Let a fluid element of very small dimensions ie dx, dy, ds . [shape is wedge]

Let the width of the element $\perp$ to the plane of paper is Unity or (1).

P_x Pressure acting on the face AB

P_y " " " AC

P_z " " " BC

$\angle ABC = \theta$

2.3 Concept of Atmospheric Pressure

- The pressure on a fluid is measured in 2 different systems.

(i) Absolute Pressure:- It is measured above the absolute zero (or) complete vacuum.

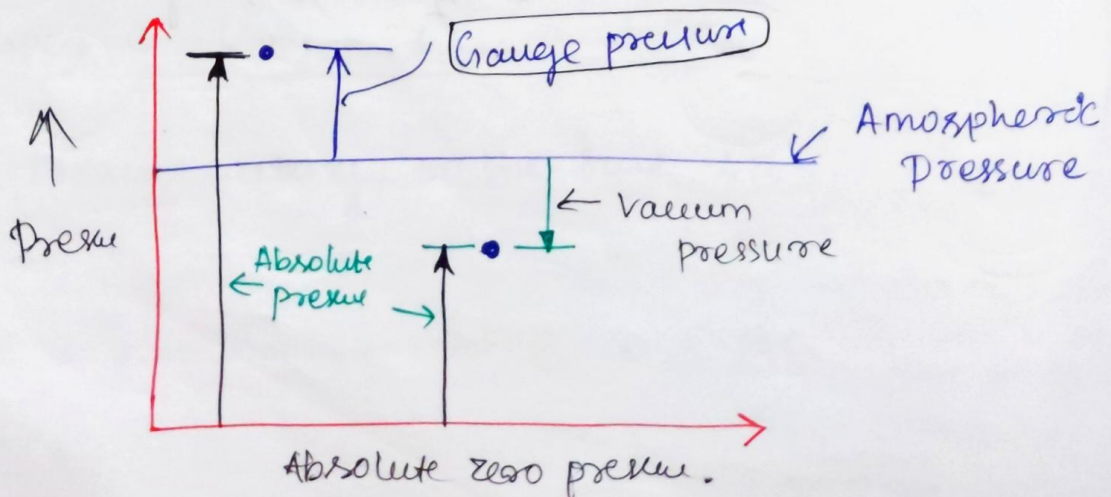
(ii) Gauge Pressure:- Pressure is measured above the atmospheric pressure.

(1) Absolute Pressure:- is defined as the pressure zero which is measured with reference to absolute vacuum pressure.

(2) Gauge pressure: is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum.

⇒ The atmospheric pressure on the scale is marked as Zero.

(3) Vacuum Pressure:- is defined as the pressure below the atmospheric pressure.



(i) $\text{Absolute Pressure} = P_{\text{atm}} + P_{\text{gauge}}$

(ii) $P_{\text{vacuum}} = P_{\text{atm}} - P_{\text{ab}}$

2.4 Pressure Measuring Instruments

* Manometers :- Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of fluid.

- Manometers
 - Simple
 - Differential

Simple Manometers :- A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured & other end remains open to atmosphere.

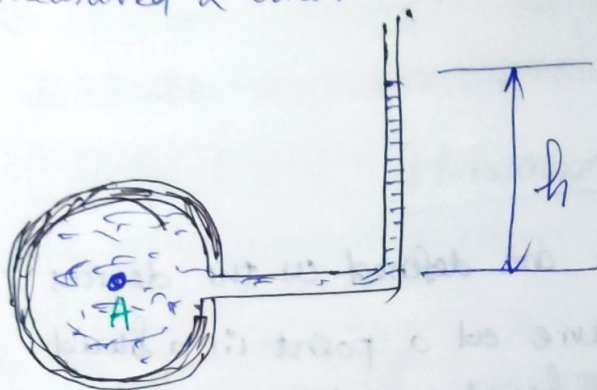
Simple Manometers

Piezometer

U-tube manometer

Single column manometer

(i) Piezometer : - It is the simplest form of manometers used for measuring gauge pressures. One end of the manometer is connected to the point where is to be measured & other end is open to the atmosphere.



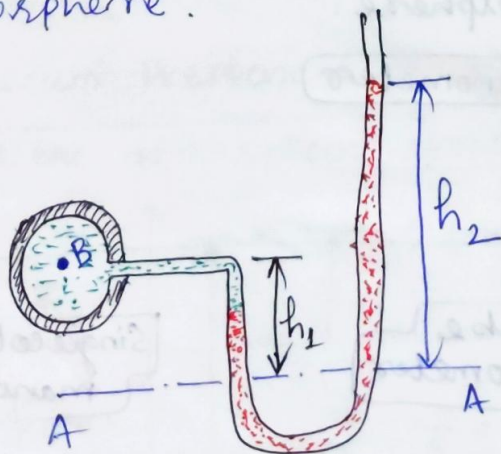
• The rise of liquid gives the pressure head at that point.

Pressure at point "A" is at height h of liquid.

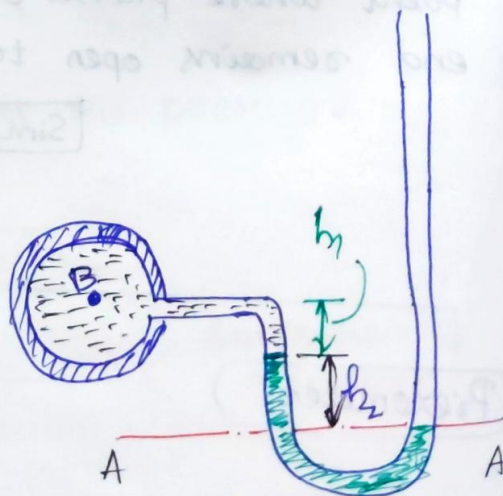
$$P_A = \rho g h \quad \text{N/m}^2$$

(ii) U-tube Manometer

It consists of glass tube bent in U shape, one end of which is connected to a point at which pressure is to be measured & other end remains open to the atmosphere.



(a) For gauge pressure



(b) For vacuum pressure.

(a) For Gauge Pressure

Let "B" be the point at which pressure is to be measured

A-A $\leftarrow$ datum line

$h_1 \leftarrow$ height of light liquid above the datum line

$h_2 \leftarrow$ height of heavy liquid " " "

$S_1 \leftarrow$ sp gravity of light liquid

$S_2 \leftarrow$ " " " heavy liquid.

$\rho_1 =$ Density of light liquid $= S_1 \times 1000$

$\rho_2 =$ " " heavy liquid $= S_2 \times 1000$

As pressure is same for the horizontal surface.

Pressure above A-A in left column $= P_B + \rho_1 \times g \times h_1$ — (1)

" " " " Right " $= \rho_2 \times g \times h_2$ — (2)

equating (1) & (2) $P_B + \rho_1 \times g \times h_1 = \rho_2 \times g \times h_2$

$$\Rightarrow \boxed{P_B = (\rho_2 g h_2 - \rho_1 g h_1)}$$

(b) For Vacuum Pressure

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + P_B$ — (1)

head in right column $= 0$ — (2)

eqⁿ (1) & (2) $\therefore \rho_2 g h_2 + \rho_1 g h_1 + P_B = 0$

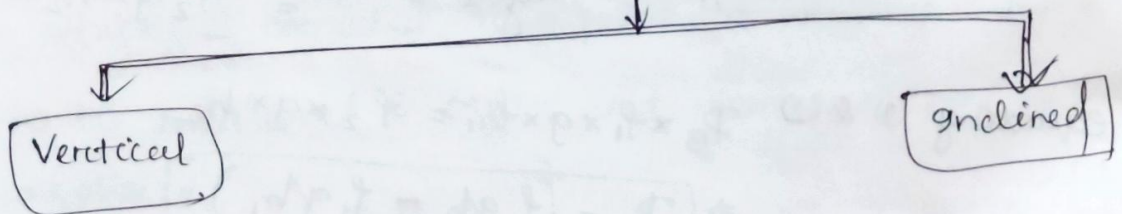
$$\Rightarrow \boxed{P_B = -(\rho_2 g h_2 + \rho_1 g h_1)}$$

Single column Manometer

Single Column manometer is a modified form of a U tube manometer in which reservoir, having a large cross sectional area, as compared to the area of the tube is connected to one of the limbs of the manometer.

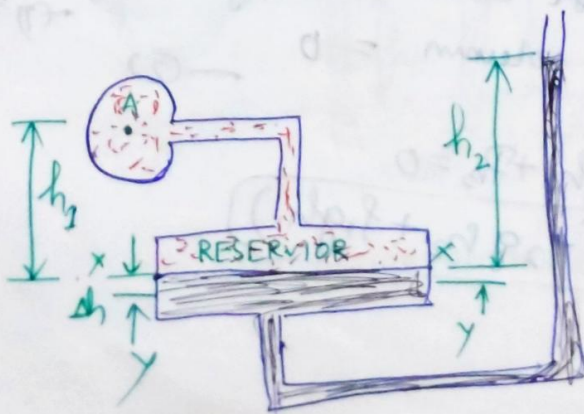
- ➔ Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected & hence the pressure is given by the height of liquid in the other limb. (← may be vertical or inclined)

Single Column Manometer



Vertical Single Column

Let $x-x$ be the datum line in the reservoir



Δh = Fall of heavy liquid in reservoir

h_2 = Rise of Heavy liquid

h_1 = Height of centre of pipe above $x-x$

P_A = Pressure at 'A'

A = cross-sectional area of the reservoir

a = cross-sectional area of the right limb.

S_1 = sp. gr of liquid in pipe.

S_2 = sp. gra of heavy liquid in reservoir & right limb

ρ_1 = density of liquid in pipe

ρ_2 = density of liquid in reservoir.

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$A \times \Delta h = a \times h_2$$

$$\Rightarrow \Delta h = \frac{a \times h_2}{A}$$

Now consider Y-Y datum line, Pressure in right limb above Y-Y = $\rho_2 g (\Delta h + h_2)$ — (1)

Pressure in left limb above Y-Y, = $\rho_1 g (\Delta h + h_1) + P_A$

$$\rho_2 g (\Delta h + h_2) = \rho_1 g (\Delta h + h_1) + P_A$$

$$\Rightarrow P_A = \rho_2 g (\Delta h + h_2) - \rho_1 g (\Delta h + h_1)$$

$$\therefore \Delta h = \frac{a \times h_2}{A}$$

$$\therefore P_A = \frac{a \times h_2}{A} [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

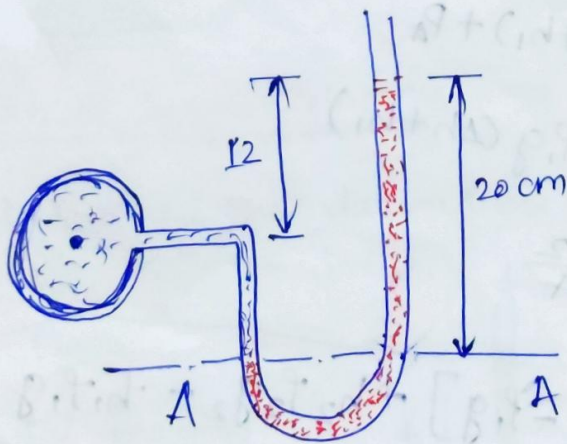
As Area $A \gg a$ so "a" is neglected $\Rightarrow \frac{a}{A}$ is negl.

$$\therefore P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

Q - The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gravity 0.9 is flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Given; sp gravity of fluid $S_1 = 0.9$
 $\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000$
 $= 0.9 \times 1000 = 900 \text{ kg/m}^3$
 Sp. gravity of mercury, $S_2 = 13.6$
 $\rho_2 = 13.6 \times 1000 = 13600 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 20 \text{ cm} = 0.2 \text{ m}$
 Height of fluid from A-A, $h_1 = 20 - 12 = 8 \text{ cm}$
 $= 0.08 \text{ m}$.



Let P = pressure of fluid in pipe

Equating the pressure above A-A

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

$$\Rightarrow P = \rho_2 g h_2 - \rho_1 g h_1$$

$$= 13600 \times 9.81 \times 0.2 - 900 \times 9.81 \times 0.08$$

$$= 2597.7 \text{ N/m}^2$$

Q A simple U tube manometer containing mercury is connected to a pipe in which a fluid of sp.gr. 0.8 & having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40cm & the height of fluid in the left from the centre of pipe is 15cm

$$\text{Sp. gr. of fluid } \gamma_1 = 0.8$$

$$\text{" " of mercury, } \gamma_2 = 13.6$$

$$\rho_1 = \text{density of fluid} = 0.8 \times 1000$$

$$\rho_2 = \text{" " mercury} = 13.6 \times 1000$$

$$\text{Difference of mercury level, } h_2 = 40\text{cm} \\ = 0.4\text{m}$$

$$\text{Height of liquid in left limb, } h_1 = 15\text{cm} \\ = 0.15\text{m}$$

$$\text{Let pressure in pipe, } = P$$

$\therefore$ Equating pressure above datum line A-A

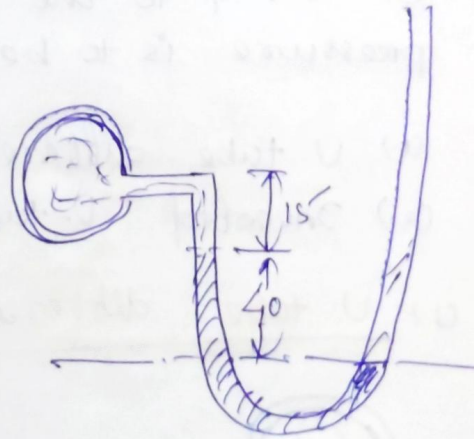
$$\rho_2 g h_2 + \rho_1 g h_1 + P = 0$$

$$\Rightarrow P = - [\rho_2 g h_2 + \rho_1 g h_1]$$

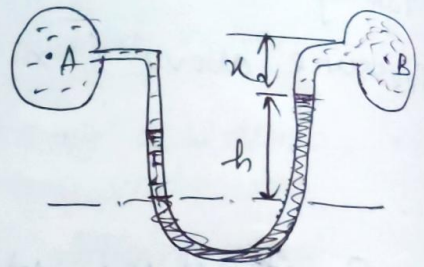
$$=$$

$$= -54543.6 \text{ N/m}^2$$

$$= -5.454 \text{ N/cm}^2$$



Q. A pipe contains an oil of sp. gr 0.9. A differential manometer connected at the two points A & B shows a difference in mercury level as 15 cm. Find the difference of pressure at the two points.



Given

$$\text{Sp gr. oil } S_1 = 0.9$$

$$\rho_1 = 0.9 \times 1000 \text{ kg/m}^3$$

$$\text{Difference in mercury level } h = 15 \text{ cm} = 0.15 \text{ m}$$

$$\text{Sp gr. of mercury, } S_m = 13.6$$

$$\rho_m = 13.6 \times 1000 \text{ kg/m}^3$$

The difference of pressure is given by

$$P_A - P_B = gh (\rho_m - \rho_1)$$

$$= 9.81 \times 0.15 \{ [13.6 - 0.9] \times 1000 \}$$

$$= 18688 \text{ N/m}^2$$

Q. A differential manometer is connected at the two points A & B of two parallel pipes as shown in fig. The pipe A contains a liquid of sp. gr = 1.5 while pipe B contains a liquid of sp. gr = 0.9. The pressures at A & B are 1 kgf/cm^2 and 1.80 kgf/cm^2 respectively. Find the difference in mercury level in the differential manometer.

$$\text{Given Sp gr. of liquid at A, } S_1 = 1.5$$

$$\text{Sp gr. of " at B, } S_2 = 0.9$$

$$\begin{aligned} \text{Pressure at A, } P_A &= 1 \text{ kgf/cm}^2 = 1 \times 10^4 \text{ kgf/m}^2 \\ &= 9.81 \times 10^4 \text{ N/m}^2 \end{aligned}$$

Pressure at pt B, $P_B = 1.8 \text{ kgf/cm}^2$
 $= 1.8 \times 10^4 \text{ kgf/m}^2$
 $= 1.8 \times 9.81 \times 10^4 \text{ N/m}^2$

$\rho_{\text{mercury}} = 13.6 \times 1000$

Taking x-x as datum line

Pressure above x-x line in Left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2+3) + P_A$$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure ~~at~~ above x-x in the right limb

$$= 900 \times 9.81 \times (h+2) + P_B$$

Equating the two pressure, we get

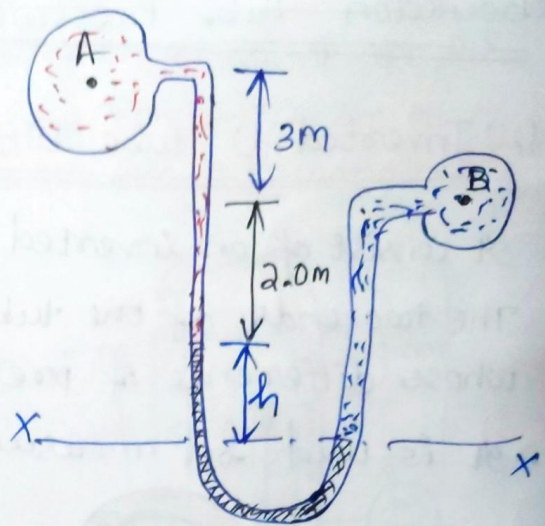
$$13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4 = 900 \times 9.81 (h+2) + 1.8 \times 10^4 \times 9.81$$

$$\Rightarrow 13.6h + 17.5 = 0.9h + 1.8 + 1.8$$

$$= 0.9h + 19.8$$

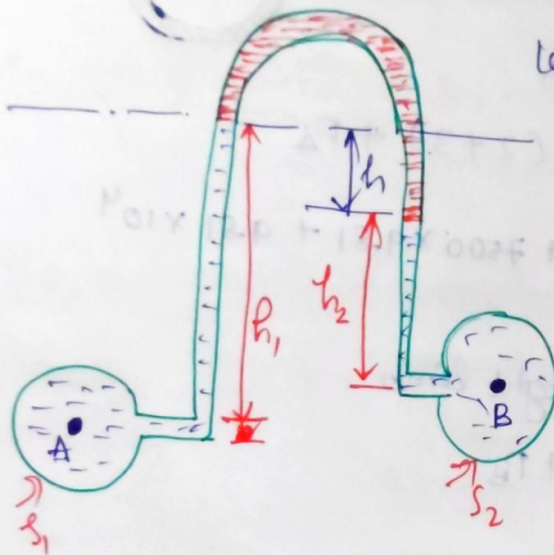
$$\Rightarrow (13.6 - 0.9)h = 19.8 - 17.5$$

$$\Rightarrow h = \frac{2.3}{12.7} = 0.181 \text{ m} = 18.1 \text{ cm (Ans.)}$$



(ii) Inverted U-tube Differential Manometers

- It consists of an inverted U-tube containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured.
- It is used for measuring difference of low pressure.



Let $P_A > P_B$

h_1 = Height of liquid in left limb below the datum line x-x

h_2 = Height of liquid in right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = " " " at B

ρ_s = Density of light liquid

P_A = Pressure at pt 'A'

P_B = " " 'B'

Taking x-x as datum line, Then the pressure in the left limb below x-x = $P_A - \rho_1 g h_1$ — (1)

Pressure in the right limb below x-x

$$= P_B - \rho_2 g h_2 - \rho_s g h \quad \text{--- (2)}$$

$\therefore$ Equating the two pressure

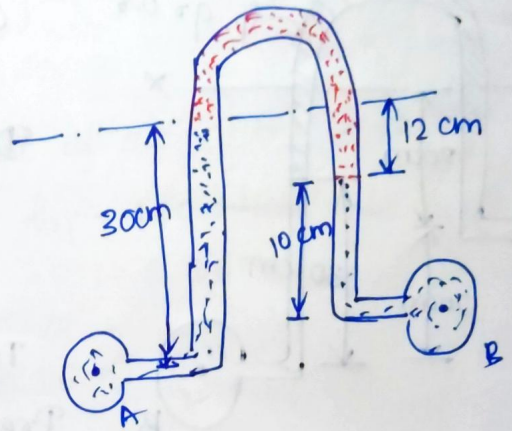
$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_s g h$$

$$\Rightarrow [P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_s g h]$$

Q. Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr 0.8 is connected. The Pressure head in the Pipe A is 2m of water, find the Pressure in the Pipe B for the manometer reading as

Given Pressure Head at A = $\frac{P_A}{\rho g} = 2\text{m}$

$$\begin{aligned}\Rightarrow P_A &= \rho g \times 2 = \\ &= 1000 \times 9.81 \times 2 \\ &= 19620 \text{ N/m}^2\end{aligned}$$



Taking x-x as datum line

$$\begin{aligned}\text{Pressure at below x-x in left limb} &= P_A - \rho g h_1 \\ &= 19620 - 1000 \times 9.81 \times 0.3 \\ &= 16677 \text{ N/m}^2\end{aligned}$$

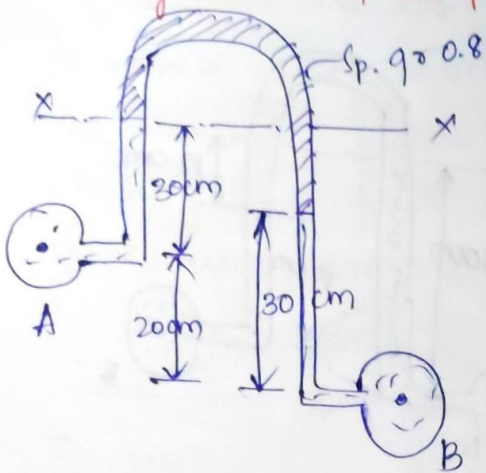
Pressure below x-x in right limb

$$\begin{aligned}&= P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12 \\ &= P_B - 981 - 941.76 \\ &= P_B - 1922.76\end{aligned}$$

$$\therefore \text{Equating two Pressure} \quad 16677 = P_B - 1922.76$$

$$\Rightarrow P_B = 18599.76 \text{ N/m}^2$$

Q. An inverted differential manometer is connected to two pipes A & B which convey water. The fluid in manometer is oil sp. gravity 0.8. For the manometer readings shown in figure. Find the pressure difference b/w A & B



Given Sp gr of oil = 0.8

$$\rho_o = 800 \text{ kg/m}^3$$

Difference of oil in the two limbs

$$= (30 + 20) - 30$$

$$= 20 \text{ cm}$$

Taking datum x-x

Pressure below x-x in Left limb

$$P_A - 1000 \times 9.81 \times 0.3 = P_A - 2943 \quad \text{--- (1)}$$

Pressure below x-x in Right

$$\text{limb}, = P_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$$

$$= P_B - 2943 - 1569.6$$

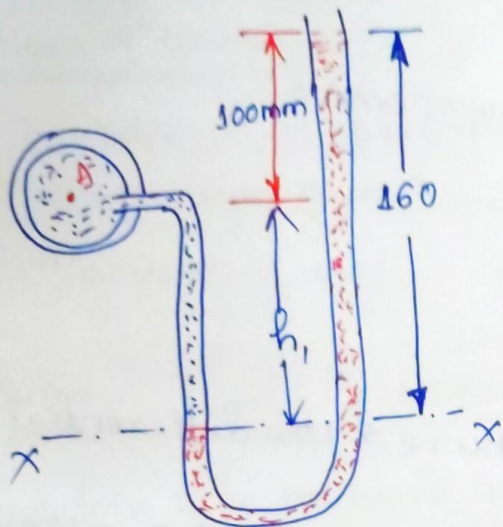
$$= P_B - 4512.6$$

$\therefore$ Equating the two Pressure

$$P_A - 2943 = P_B - 4512.6$$

$$\Rightarrow P_A - P_B = 1569.6 \text{ N/m}^2 \text{ (Ans)}$$

Q. A U-tube manometer is used to measure the pressure of oil of sp. gr 0.85 flowing in a pipe line. Its left end is connected to the pipe & the right limb is open to atmosphere. The centre of the pipe is 100mm below the level of mercury the right limb. If the difference of mercury level in the two limbs is 160 mm, determine the absolute pressure of the oil in the Pipe.



Specific gr. of oil $S_o = 0.85$
 Density of oil, $\rho_o = 0.85 \times 1000$
 $= 850 \text{ kg/m}^3$

Sp. gra. of mercury, $S_m = 13.6$
 $\rho_m = 13600 \text{ kg/m}^3$

Height of oil in the left limb.

$$h_1 = 160 - 100$$

$$= 0.06 \text{ m}$$

Difference of mercury level

$$h_2 = 160 \text{ mm} = 0.16 \text{ m}$$

Absolute Pressure of oil

let h_1 = Gauge pressure in the pipe in terms of head of H_2O

P = Gauge pressure in terms of KN/m^2

Taking $x-x$ as datum

Pressure in left limb, $P_A + \rho_o g h_1$ — (1)

" " Right limb, $\rho_m g \times h_2$ — (2)

Equating eqn (1) & (2) $\therefore P_A + \rho_o g h_1 = \rho_m g h_2$

$$\Rightarrow P_A + 850 \times 9.81 \times 0.06$$

$$= 13600 \times 9.81 \times 0.16$$

$$P_{atm} = 101325 \text{ Pa}$$

$$= 101.325 \text{ kPa}$$

$$\Rightarrow P_A = 21346.56 - 500.31$$

$$= 20846.25 \text{ N/m}^2$$

$$= 20.84 \text{ kN/m}^2$$

Absolute pressure of oil

in the tube, $P_{abs} = P_{atm} + P_{gauge}$

$$= 100 + 20.84 = 120.84 \text{ kPa}$$

Advantages

- 1- Easy to fabricate & relatively inexpensive.
- 2- Good accuracy
- 3- High sensitivity
- 4- Require little maintenance
- 5- Not affected by vibration.
- 6- Specially suitable for low pressure & low differential pressure.
- 7- It is easy to change the sensitivity by affecting a change in the quality of manometric liquid in the manometer.

Disadvantages

- 1- Usually bulky & large in size.
- 2- Being fragile, get broken easily.
- 3- Readings of the manometers are affected by changes in temperature, altitude & gravity.
- 4- A capillary effect is created due to surface tension of manometric fluid.

Mechanical Gauges

Mechanical gauges are used for measuring medium & high pressure.

↳ Elastic pressure gauge.

• Thus they employ different forms of elastic systems such as tubes, diaphragms or bellows etc. to measure the pressure.

→ The elastic deformation of these elements is used to show the effect of pressure.

Types of gauge

- 1 - Bourdon tube pressure gauge
- 2 - Diaphragm gauge
- 3 - Vacuum gauge.

Bourdon Tube Pressure Gauge

Bourdon tube pressure gauge is used for measuring high as well as low pressure.

- ⇒ In Bourdon tube, the pressure element consists of a metal tube of approximately elliptical cross-section
- ⇒ This tube is bent in the form of a segment of a circle & responds to pressure changes
- ⇒ When one end of the tube which is attached to the gauge case is connected to the source of pressure, the internal pressure causes the tube to ~~be~~ expand, where by circumferential stress, i.e. hoop tension is set up
- ⇒ The free end of the tube moves & is in turn connected by suitable ~~level~~ levers to a rock, which engages with a small pinion mounted on the same spindle as the pointer.

→ Thus the pressure applied to the tube causes the rack & pinion to move.

→ The pressure is indicated by the pointer over a dial which can be graduated on a suitable scale.

→ The Bourdon tubes are generally made of bronze or nickel steel. The former is generally used for low pressures & the latter for high pressure.

(i) Compound Bourdon tube - used for measuring pressures both above & below atmospheric pressure.

(ii) Double Bourdon tube - used where vibrations are encountered.

Hydrostatics

3.1 Definition of hydrostatic pressure :-

Hydrostatic Pressure :- is defined as 'the pressure exerted by a fluid at equilibrium at any point of time due to the force of gravity.'

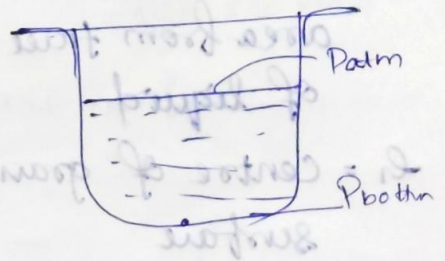
⇒ Hydrostatic pressure is proportional to the depth measured from the surface as the weight of the fluid increases when a downward force is applied.

⇒ H.P is the pressure present within a fluid when it is at rest.

⇒ It is equal in all directions.

⇒ It acts at a right angle or normal to any surface in contact with the fluid.

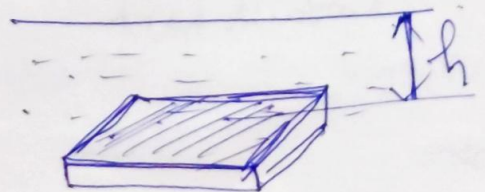
$$H.P = \rho g h$$
$$\Delta P = \rho g h$$



3.2 Total Pressure

Total Pressure is defined as the force exerted by static fluid on a surface (curved or plane) when the fluid comes in contact with the surface.

This force always acts normal to the surface.



Centre of Pressure

C.P is defined as the point of application of the total pressure on the surface.

- 1 - Horizontal plane surface
- 2 - Vertical plane surface

2 types of submerged surface.

Vertical Plane Surface Submerged in Liquid

Consider a plane vertical surface of arbitrary shape immersed in a liquid.

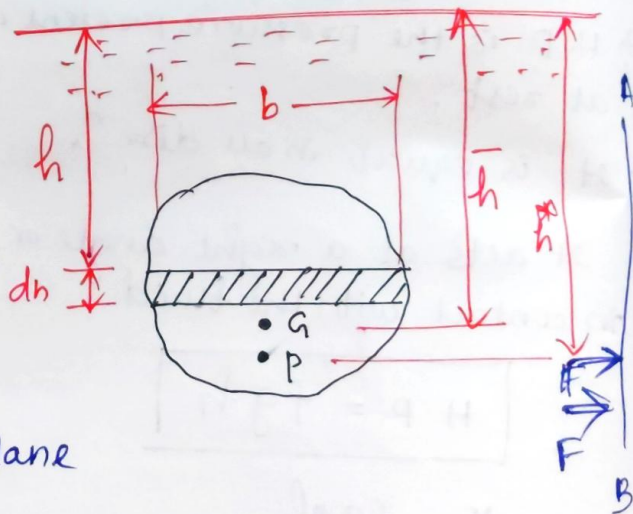
Let A = Total Area of the surface

h = Distance of C.G of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.



The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips.

• The force on small strip is then calculated & the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh & width ' b ' at a depth of ' h ' from free surface of liquid.

∴ Pressure intensity on the strip, $P = \rho gh$

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = P \times \text{Area}$
 $= \rho gh \times b \times dh$

∴ Total Pressure force on the whole surface

$$F = \int dF = \int \rho gh \times b \times dh$$
$$= \rho g \int b \times h \times dh \quad \text{--- (1)}$$

$$\text{But } \int b \times h \times dh = \int \bar{h} \times dA$$

= Moment of surface area about the free surface of liquid

= Area of surface $\times$ Distance of C.G. from free surface

$$= A \times \bar{h}$$

From eqn (1)

$$\boxed{F = \rho g A \bar{h}}$$

∴ Total Pressure Force on the wall
Whole surface $F = \rho g A h$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

$$g = 9.81 \text{ m/s}^2$$

Unit = Newton

(b) Centre of Pressure :- Centre of pressure is calculated by using the ^(h*) "Principle of Moments" which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

- The Resultant force F acting at P , at a distance h^* from free surface of the liquid as shown in Fig.

Hence Moment of the force F about free surface of the liquid = $F \times h^*$ (1)

Moment of Force dF , acting on a strip about free surface of liquid = $dF \times h$

$$= \rho g h \times b \times dh \times h$$

$$\therefore dF = \rho g h \times b \times dh$$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h$$

$$= \rho g \int b h^2 dh$$

$$= \rho g \int b h^2 dh$$

$$= \rho g \int h^2 dA$$

But $\int h^2 dA = \int b h^2 dh$

= Moment of inertia of the surface about free surface of liquid

$$= I_0$$

$\therefore$ Sum of moments about free surface

$$= \rho g I_0 \quad \text{--- (2)}$$

Equating eqn (1) & eqn (2)

$$F \times h^* = \rho g I_0$$

But $F = \rho g A \bar{h}$

$$\therefore \rho g A \bar{h} \times h^* = \rho g I_0$$

$$\Rightarrow h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}} \quad \text{--- (3)}$$

But by the Theorem of Parallel axis we have

$$I_0 = I_G + A \times \bar{h}^2$$

$\therefore I_G$ = Moment of inertia of area about an axis passing through the C.G of the area & parallel to the free surface of liquid.

Q.3)

$$I_h^* = \frac{I_0}{A \bar{h}}$$

$$= \frac{I_0 + A \bar{h}^2}{A \bar{h}}$$

$$I_h^* = \frac{I_0}{A \bar{h}} + \bar{h}$$

* The first moment of area is the distribution of the area of a shape around a rotational axis.

⇒ It is used to find the centroid of an area.

⇒ Unit ⇒ cubic meter.

$$\boxed{\text{Area} \times \text{distance}}$$

* The second moment of area or second area moment is the dispersion of points of a shape in an arbitrary axis.

⇒ The 2nd moment of area is also known as the moment of inertia of a shape.

⇒ The 2nd moment of area is a measure of the 'efficiency' of a cross-sectional shape to resist bending caused by loading.

⇒ Unit ⇒ mm⁴

$$\boxed{\text{Area} \times \text{distance}^2}$$

Parallel axis theorem states that "The moment of inertia of a body about an axis parallel to the body passing through its centre is equal to the sum of moment of inertia of the body about the axis passing through the centre & product of the mass of the body times the square of the distance between the two axes.

$$I_0 = I_{CG} + Mh^2$$

$$I_0 = I_{CG} + Ah^2$$

$$\therefore h^* = \frac{I_G}{Ah} + \bar{h}$$

⇒ Centre of pressure (h^*) lies below the centre of gravity of the vertical surface

⇒ The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

$$\begin{aligned}\text{Total pressure (F)} &= \rho g \bar{h} A \\ &= 900 \times 9.81 \times 8 \times 1.33 \\ &= 9597.6 \text{ N (Ans)}\end{aligned}$$

Centre of pressure from free surface of oil

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$= \frac{7.11}{8 \times 1.33} + 1.33$$

$$h^* = 1.99 \text{ m (Ans)}$$

$\therefore I_G = \text{Moment of inertia of triangular res about C.G}$

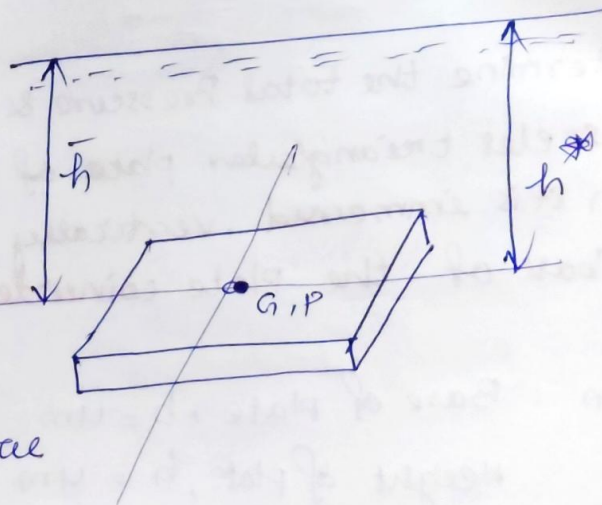
$$= \frac{bh^3}{36} = \frac{4 \times 4^3}{36}$$

$$= 7.11 \text{ m}^4$$

Horizontal Plane Surface Submerged in Liquid

Consider a plane horizontal surface immersed in a static fluid.

As every point of the surface is at the same depth from the free surface of the liquid the pressure intensity will be equal on the entire surface & equal to $\boxed{p = \rho gh}$



Where $h = \text{depth of surface}$

Let $A = \text{Total area of surface}$

$F = \text{Total Pressure force on the surface}$

$$= p \times \text{Area}$$

$$= \rho gh \times A$$

$$= \rho g \bar{h} \times A$$

$h =$ Depth of CG from free surface of liquid $= h$

$h_p =$ Depth of centre of pressure from free surface $= h$

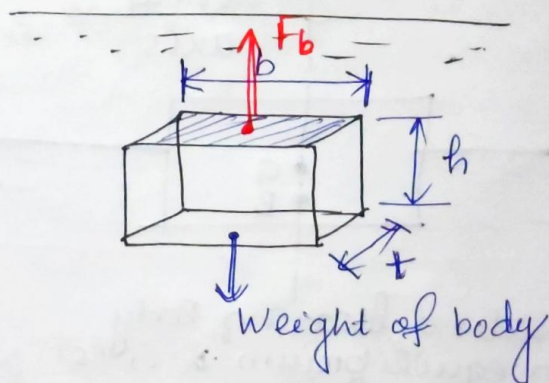
Buoyancy:- When a body is immersed in a fluid, an upward force is exerted by the fluid on the body.

This upward force is equal to the weight of the fluid displaced by the body & is called the force of ~~the~~ buoyancy or simply buoyancy.

$F_b =$ Force of Buoyancy
or Upward force

$W_f =$ weight of fluid
displaced by the body

$$F_b = W_f$$



• **Centre of Buoyancy**:- It is defined as the point, through which the force of buoyancy is supposed to act.

⇒ As the force of buoyancy is a vertical force & is equal to weight of fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

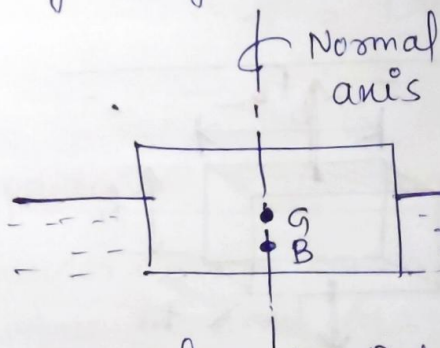
Archimedes Principle

The upward buoyant force that is exerted on a body immersed in a fluid, whether partially or fully submerged is equal to the weight of the fluid that the body displaces & acts in the upward direction at the center of mass of the displaced fluid.

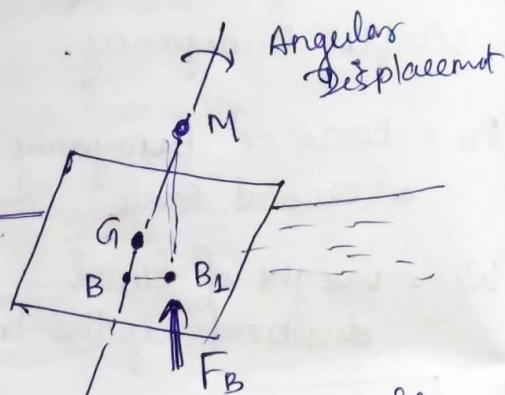
Meta Centre :-

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.

⇒ The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.



Consider a floating body in equilibrium & G is the centre of gravity
B centre of buoyancy
Both B & G lie on the normal axis



⇒ After applying a small angular displacement in clockwise direction, the point B & G shifted to right from normal axis i.e. B_1

The line of action of the force of buoyancy in this new position will intersect the normal axis of the body at some point 'M'

⇒ This point M is meta centre

Meta-centric Height :-

The distance between the meta centre of a floating body and the centre of gravity of the body is called meta-centric height.

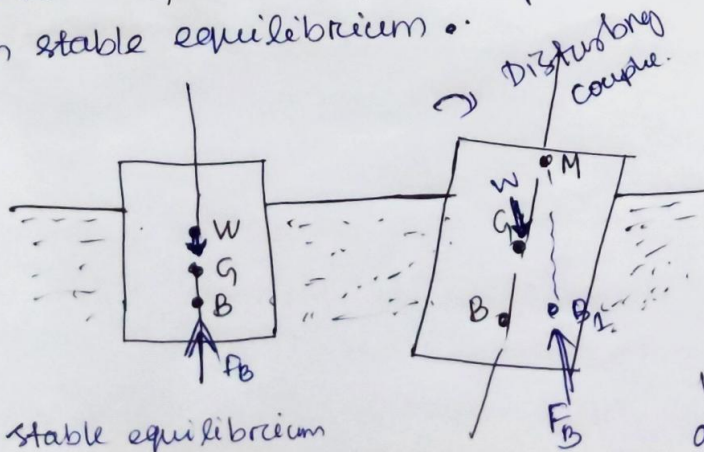
3.5 Concept of Floatation

A floating body is said to be stable if it comes back to its original position after a slight disturbance.

Stability of Floating Body :-

- The stability of a floating body is determined from the position of meta-centre (M).
- In case of a floating body, the weight of the body is equal to the weight of liquid.

Stable Equilibrium :- If the point M (meta-centre) is above G (centre of gravity), the floating body will be in stable equilibrium.

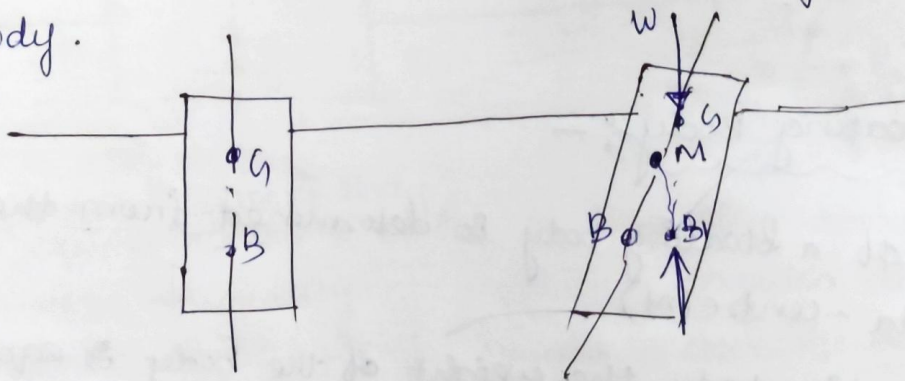


• If a slight angular displacement is given to the floating body in clockwise direction, the centre of buoyancy shifts from B to B_1 , such that the vertical line through B_1 cuts at M.

→ Then the buoyant force F_B through B , & weight w through G constitute a couple acting in the anti-clockwise direction & thus bringing the floating body in the original position.

Unstable Equilibrium:-

If the point M (meta centre) is below G (centre of gravity) the floating body will be in unstable equilibrium. The disturbing couple is acting in the clockwise direction. → The couple due to buoyant force F_B & w is also acting in clockwise direction & thus overturning the floating body.



If the point M is at the centre of gravity of the body the floating body will be in neutral equilibrium.

chapter - 04

Kinematics of Flow

4.1 Types of fluid flow :-

- (i) Steady & Unsteady Flow
- (ii) Uniform & non-uniform flow
- (iii) laminar & Turbulent flow
- (iv) Compressible and non-compressible flows.
- (v) Rotational and Irrotational flow.
- (vi) One, two and Three-dimensional flow...

* Kinematics :- is defined as that branch of science which deals with motion of particles without considering the forces causing the motion.

① Steady Flow :- It is defined as that type of flow in which the fluid characteristics like velocity, pressure, density -- etc. at a point don't change with time.

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0 \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0 \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

Unsteady Flow :- It is that type of flow in which, the velocity, pressure or density at a point changes with respect to time.

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

(ii) Uniform Flow :- Uniform flow is defined as that type of flow in which the velocity at any given time does not change w.r.t space i.e. length of direction of the flow)

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{const}} = 0$$

∂v = change in velocity
 ∂s = length of flow in the direction s

Non-Uniform flow :-

It is that type of flow in which the velocity at any given time changes w.r.t space.

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{const}} \neq 0$$

(iii) Laminar Flow :-

Laminar flow is defined as that type of flow in which the fluid particles move along well defined paths or stream line and all the stream-lines are straight and parallel.

- Thus the particles move in laminar or layers, gliding smoothly over the adjacent layer.

- This type of flow is called stream-line flow or viscous flow.

$$\text{If Reynold number, } R = \frac{\rho v D}{\eta} < 2000$$

Turbulent Flow :-

Turbulent flow is that type of flow in which the fluid particles move in zig-zag way.

→ Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are

responsible for high energy loss

- if $R > 4000$

(iv) Compressible Flow :-

Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density " ρ " is not constant for the fluid.

$$\rho \neq \text{constant}.$$

Non-Compressible Flow :-

Incompressible flow is that type of flow in which the density is constant for the fluid.

- The liquids are generally incompressible while gases are compressible.

$$\rho = \text{constant}.$$

(v) Rotational Flow :-

Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis.

Irrrotational flow :-

If the fluid particle while flowing along stream lines, do not rotate about their own axis then that type of is called irrotational flow.

(vi) One Dimensional flow :- It is that type of flow in which the flow parameters such as velocity is a function of time and one - space coordinate only.

$$u = f(x), \quad v = 0 \quad \& \quad w = 0$$

(i) Two-Dimensional Flow :- It is that type of flow in which the velocity is a function of time and two rectangular space-co-ordinates say x & y .

• For a steady two-dimensional flow the velocity is a function of two space co-ordinates only.

$$u = f_1(x, y) \quad v = f_2(x, y) \quad w = 0$$

(ii) Three Dimensional flow :- It is that type of flow in which the velocity is a function of time and three mutually perpendicular directions.

$$u = f_1(x, y, z)$$

$$v = f_2(x, y, z)$$

$$w = f_3(x, y, z)$$

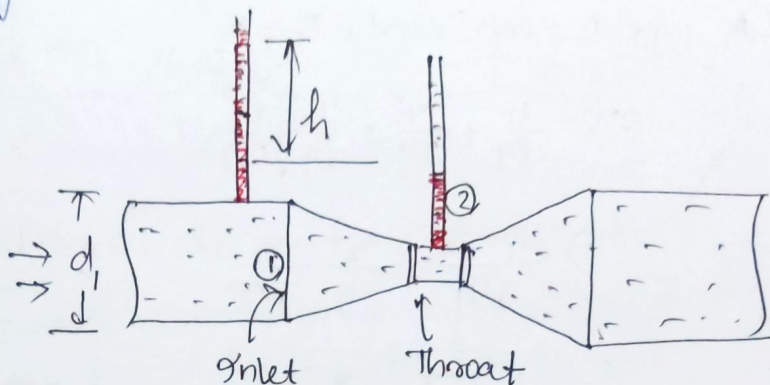
Practical Applications of Bernoulli's Equation

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved.

1 - Venturimeter

2 - Pitot - tube

(1) Venturimeters



→ A venturimeter is a

device used for measuring

the rate of flow of a fluid flowing through a pipe.

→ It consists of three parts

(i) A short converging part

(ii) Throat

(iii) Diverging part

Expression for rate of flow through venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in above figure.

Let d_1 = diameter in inlet . or at section (1)

P_1 = pressure at section (1)

v_1 = velocity of fluid at section (1)

a_1 = area at section (1) = $\frac{\pi d_1^2}{4}$

Similarly for a_2 , d_2 , P_2 & v_2 are corresponding values at section (2)

Applying Bernoulli's equation at section (1) & (2)

$$\therefore \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad \text{--- (1)}$$

As pipe is horizontal, $z_1 = z_2$

$$\therefore \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\therefore \frac{P_1 - P_2}{\rho g} = h \quad \text{head}$$

= pressure difference at sec (1) & (2)

$$\Rightarrow \boxed{h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}} \quad \text{--- (2)}$$

Now applying Continuity equation at section (1) & (2)

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow v_1 = \frac{a_2 v_2}{a_1}$$

Substituting the value of v_1 at eqn (2)

$$\therefore h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g} \left[1 - \frac{a_2^2}{a_1^2} \right]$$
$$= \frac{v_2^2}{2g} \left[\frac{a_1^2 - a_2^2}{a_1^2} \right]$$

$$\Rightarrow v_2^2 = 2gh \left[\frac{a_1^2}{a_1^2 - a_2^2} \right]$$

$$\Rightarrow v_2^2 = \sqrt{2gh} \sqrt{\frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$\therefore \boxed{v_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}}$$

$$\text{Discharge} = a_2 \times v_2 = a_2 \times \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{--- (3)}$$

Eqn ③ gives the discharge under ideal conditions & is called theoretical discharge. & Actual discharge will be less than theoretical.

$$\therefore Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

C_d = Coefficient of Venturimeter, < 1

Value of h given by Differential U-tube manometer

Case-I, Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe

Let S_h = sp. gravity of the heavier liquid
 S_o = sp. " " lighter " or liquid flowing through pipe

x = Difference of the heavier liquid columns in U-tube

$$h = x \left[\frac{S_h}{S_o} - 1 \right]$$

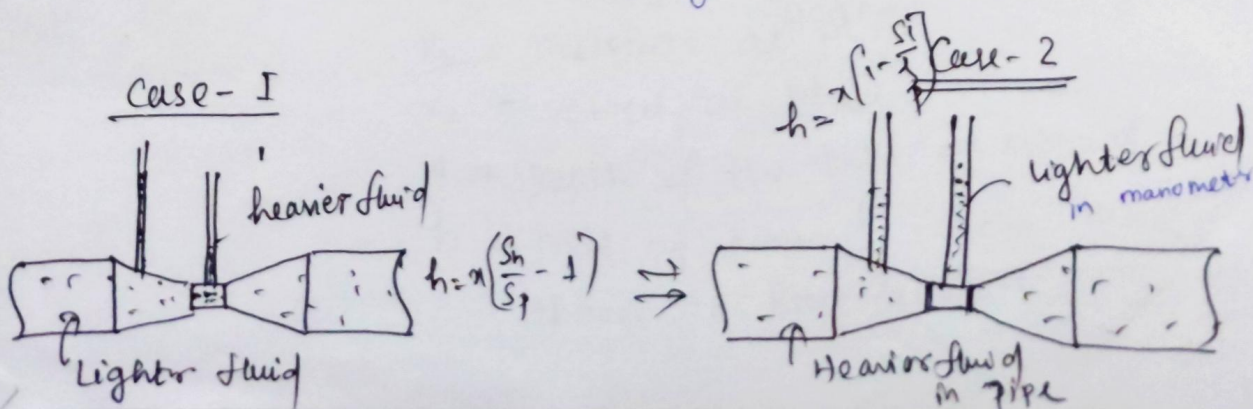
Case-II:- If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe.

$$h = x \left[1 - \frac{S_l}{S_o} \right]$$

S_l = sp gravity of lighter liquid in U-tube

S_o = sp. " of fluid flowing through pipe.

x = Difference of the lighter liquid columns in U-tube



Q. A horizontal venturimeter with inlet & throat diameters 30 cm & 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet & the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.

Given, $d_1 = 30 \text{ cm}$

$$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$$

As Dia. throat $d_2 = 15 \text{ cm}$

$$a_2 = \frac{\pi}{4} (d_2)^2 = \frac{\pi}{4} (15)^2 = 176.7 \text{ cm}^2$$

Coefficient of discharge, $C_d = 0.98$

Reading of differential manometer = $x = 20 \text{ cm}$ of mercury

$$\therefore \text{Difference of pressure head, } h = x \left[\frac{S_h}{S_p} - 1 \right]$$

$\swarrow$ Sp. Gr. of mercury = 13.6
 $\nwarrow$ Sp. Gr. of $H_2O = 1$

$$= 20 \left[\frac{13.6}{1} - 1 \right] = 20 \times 12.6 = 252 \text{ cm of } H_2O$$

The discharge through venturimeter

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= 125756 \text{ cm}^3/\text{s}$$

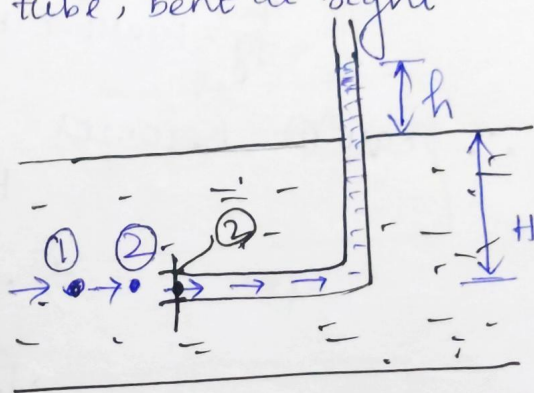
$$= \frac{125756}{1000} \text{ lit/s} = 125.756 \text{ lit/s}$$

Pitot Tube

It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

⇒ It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy.

⇒ Pitot tube consists of a glass tube, bent at right angle.



⇒ The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy.

⇒ The velocity is determined by measuring the rise of liquid in the tube.

* Consider two points ① & ② at same level in such a way that point ② is just at the inlet of the pitot tube & point ① is far away from the tube.

Let P_1 = Intensity of pressure at ①

V_1 = Velocity of flow at ①

P_2 = Pressure at pt ②

V_2 = velocity at pt ②

H = Depth of the tube in liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's Equation at pts ① & ②

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad \text{--- ①}$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{--- ②} \quad [\because z_1 = z_2]$$

$$\frac{P_1}{\rho g} = \text{pressure head at ①} = H \quad \because [v_1 = 0]$$

$$\frac{P_2}{\rho g} = \text{pressure head at 2} = (h + H)$$

$\therefore$ Equⁿ ② becomes

$$H + \frac{v_1^2}{2g} = (h + H)$$

$$\Rightarrow h = \frac{v_1^2}{2g}$$

$$\Rightarrow \boxed{v_1 = \sqrt{2gh}} \quad \leftarrow \text{Theoretical value}$$

$$\boxed{(v_1)_{\text{actual}} = C_v \sqrt{2gh}}$$

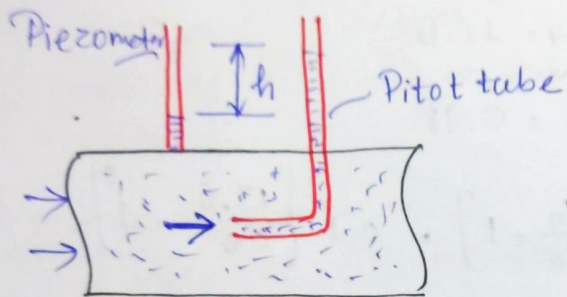
C_v = Co-efficient of pitot tube

$$\therefore \text{Velocity at any point} = C_v \sqrt{2gh}$$

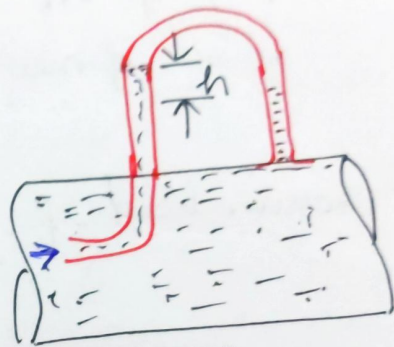
Velocity of flow in a pipe by Pitot Tube

Exo

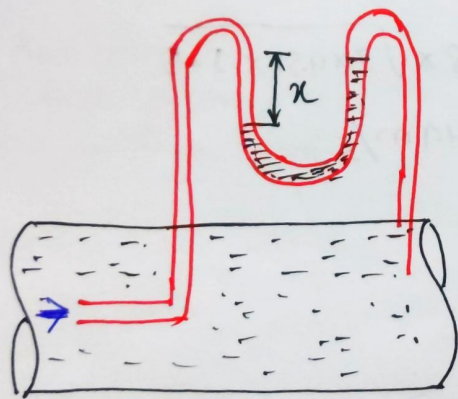
- (1) Pitot-tube along with a vertical piezometer tube
- (2) " " connected with piezometer tube as shown
- (3) " " & vertical piezometer tube connected with a differential U-tube manometer



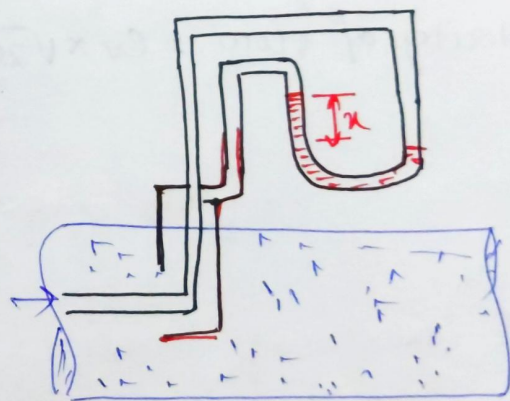
(1)



(2)



(3)



(4)

- (4) Pitot-static tube, which consists of two circular concentric tubes one inside the other with some annular space between.

$$h = x \left[\frac{S_g}{S_o} - 1 \right]$$

x = Difference of the levels of the manometer liquid.

Q- Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot tube is 100mm. Take co-efficient of tube 0.98 & sp.gr of oil = 0.8

Given Diff. of mercury level, $h = 100 \text{ mm} = 0.1 \text{ m}$

Sp. gr of oil $S_o = 0.8$

" " of mercury $S_g = 13.6$

$C_v = 0.98$

$$\begin{aligned} \text{Diff. of pressure head, } h &= h \left[\frac{S_g}{S_o} - 1 \right] = 0.1 \left[\frac{13.6}{0.8} - 1 \right] \\ &= 1.6 \text{ m of oil} \end{aligned}$$

$$\begin{aligned} \therefore \text{Velocity of flow} &= C_v \times \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times 1.6} \\ &= 5.49 \text{ m/s} \end{aligned}$$

Orifice, Notches & Weirs

Orifice

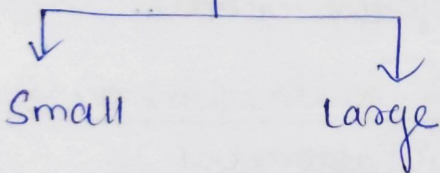
Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc) on the side or at the bottom of a tank, through which a fluid is flowing.

* A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank or vessel containing the fluid.

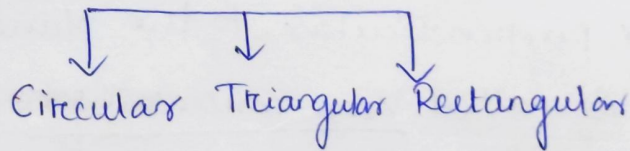
* Orifice as well as mouthpieces are used for measuring the rate of flow of fluid.

Classification of Orifice

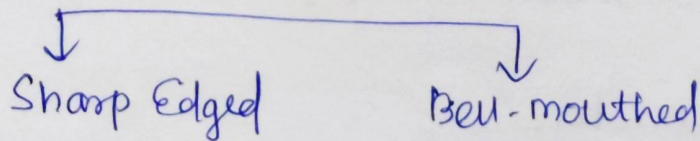
According to
size of orifice



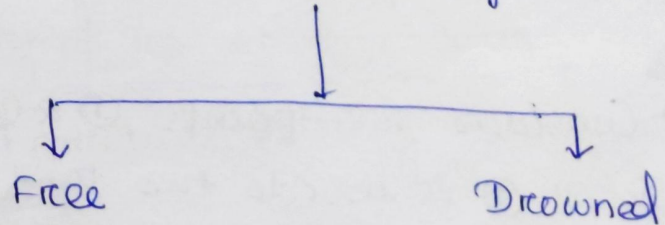
Cross-sectional
Area.



Shape of Upstream Edge
of the Orifice.



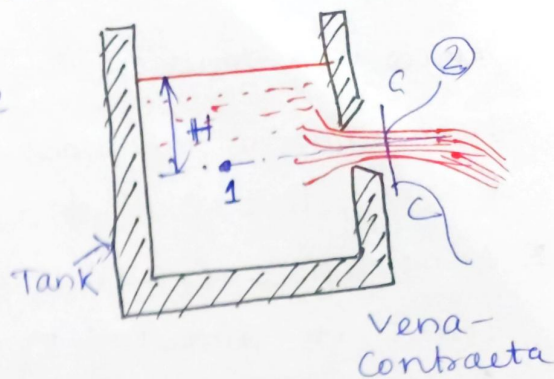
Nature of discharge



Flow Through an Orifice

Consider a tank fitted with a circular orifice in one of its sides

- Let H be the head of the liquid above the centre of the orifice.
- The liquid flowing through the orifice forms a jet of liquid whose area of cross section is less than that of orifice



- The area of jet of fluid goes on decreasing & at a section C-C the area is minimum.
- This section is approximately at a distance of half of diameter of the orifice.
- At this section, the streamlines are straight & parallel to each other & perpendicular to the plane of the orifice.
- This section is called Vena-contracta.
- Beyond this section, the jet diverges & is attracted in the downward direction by the gravity.

• Consider two points ① & ② as shown in fig.

Point ① is inside the tank

Point ② is at the Vena-contracta.

* Let the flow is steady & at a constant head H .

Applying Bernoulli's eqn at ① & ②.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 \quad \text{--- (1)}$$

$$\text{But } Z_1 = Z_2$$

$\therefore$ eqn ① becomes $\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$

$\Rightarrow H + 0 = 0 + \frac{v_2^2}{2g}$

$\therefore v_1 \ll v_2$ as area of tank is very large as compared to the area of the jet of liquid.

$v_2^2 = 2gH$

$\Rightarrow \boxed{v_2 = \sqrt{2gH}}$ Theoretical

Hydraulic Co-efficients

The hydraulic co-efficients are

1 - Co-efficient of velocity C_v

2 - Co-efficient of contraction C_c

3 - Co-efficient of discharge C_d

(1) Co-efficient of velocity (C_v)

It is defined as the ratio between the actual velocity of a jet of liquid at vena contracta & the theoretical velocity of jet.

$C_v = \frac{\text{Actual velocity of Jet at Vena-Contracta}}{\text{Theoretical velocity}}$

$= \frac{v \leftarrow \text{Actual}}{\sqrt{2gH} \leftarrow \text{Theoretical}}$

$\boxed{0.95 < C_v < 0.99}$

$C_v \rightarrow 0.95 \text{ to } 0.99$

$\rightarrow$ depending on the shape, size of orifice,

$C_v = 0.98$ [for sharp-edged orifice]

(2) Coefficient of Contraction C_c

It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice.

$$C_c = \frac{\text{Area of jet at Vena Contracta}}{\text{Area of orifice}}$$

$$C_c = \frac{a_c}{a}$$

⇒ $C_c \rightarrow 0.61 \text{ to } 0.69$ [Depending on shape & size of the orifice]

$$C_c \rightarrow 0.64$$

(3) Co-efficient of Discharge C_d :-

It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice.

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical Area}}$$

$$C_d = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual Area}}{\text{Theoretical area}}$$

$$C_d = C_v \times C_c$$

*

Q. The head of water over an orifice of diameter 40mm is 10m. Find the actual discharge & actual velocity of the jet at Vena-Contraeta. Take $C_d = 0.6$ & $C_v = 0.98$.

Given, Head $H = 10\text{m}$

Dia. of orifice $d = 40\text{mm} = 0.04\text{m}$

$$\text{Area}, a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.04)^2 = 0.001256\text{m}^2$$

$$C_d = 0.6, C_v = 0.98$$

$$\text{(i) } \frac{\text{Actual Discharge}}{\text{Theoretical discharge}} = 0.6$$

Theoretical velocity
↓

$$\begin{aligned}\text{But Theoretical discharge} &= V_{th} \times \text{Area of orifice} \\ &= \sqrt{2gH} \\ &= \sqrt{2 \times 9.81 \times 10} \\ &= 14\text{m/s}\end{aligned}$$

$$\therefore \text{Theoretical discharge} = 14 \times 0.001256 = 0.01758 \frac{\text{m}^3}{\text{s}}$$

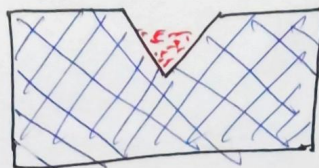
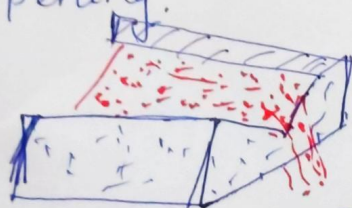
$$\begin{aligned}\therefore \text{Actual Discharge} &= 0.6 \times \text{Theoretical discharge} \\ &= 0.6 \times 0.01758 \\ &= 0.01054 \frac{\text{m}^3}{\text{s}}\end{aligned}$$

$$\text{(ii) } \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = 0.98$$

$$\begin{aligned}\Rightarrow \text{Actual velocity} &= 0.98 \times \text{Theoretical velocity} \\ &= 0.98 \times 14 \\ &= 13.72 \text{m/s}\end{aligned}$$

Notches :-

- * A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank.
- ⇒ It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.



Notch

Weir :- A weir is a concrete or masonry structure placed in an open channel over which the flow occurs, It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel.

⇒ The notch is of small size while the weir is of a bigger size.

⇒ The notch is generally made of metallic plate while weir is made of concrete.

Weir

(1) According to the shape of the opening

- (i) Rectangular weir
- (ii) Triangular weir
- (iii) Trapezoidal weir

(2) According to the shape of the crest

- (i) Sharp-crested weir
- (ii) Broad-crested weir
- (iii) Narrow crested weir
- (iv) Ogee-shaped weir.

(3) According to the effect on sides on the emergency nappe.

- (i) Weir with end contraction
- (ii) Weir without end contraction.

Nappe or Vein: - The sheet of water flowing through a notch or over a weir is called Nappe or Vein

2. Crest or sill: - The bottom edge of a notch or a top of a weir over which the water flows is known as the sill or crest.

Classification of Notches & Weirs

(1) According to the shape of the opening

(a) Rectangular Notch

(b) Triangular notch

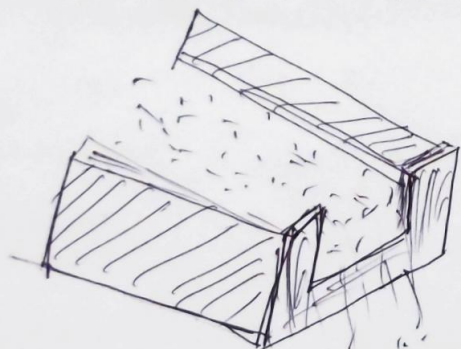
(c) Trapezoidal notch

(d) Stepped notch

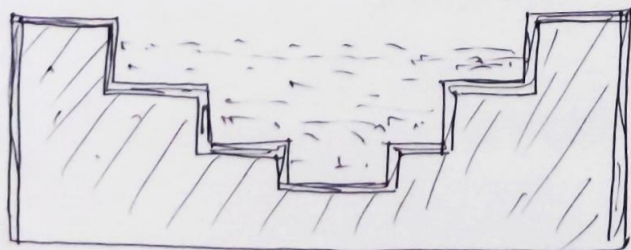
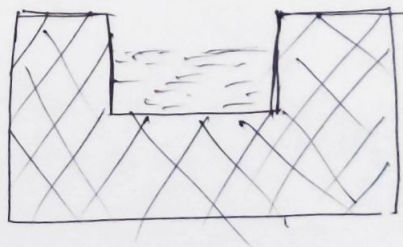
(2) According to the effect of the sides on the nappe

(a) Notch with end contraction

(b) Notch without end contraction or suppressed notch



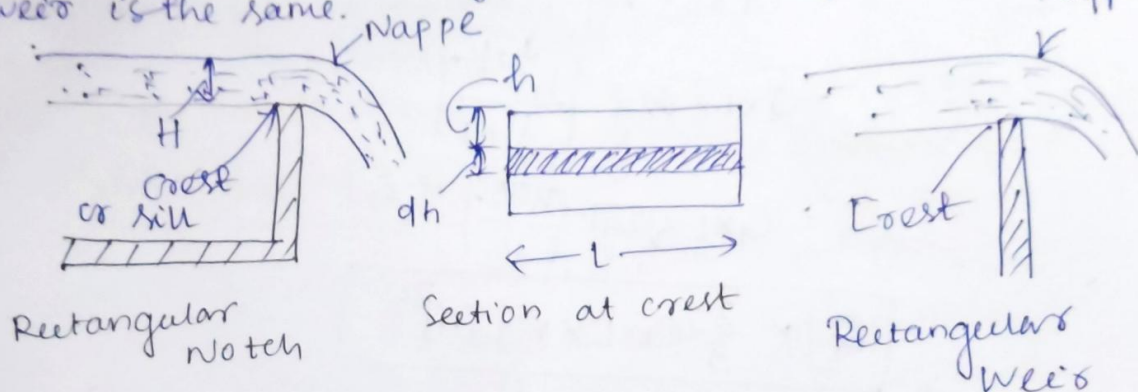
Rectangular notch



Trapezoidal notch
stepped

Discharge over a Rectangular Notch or Weir

The expression for discharge over a rectangular notch or weir is the same.



Consider a rectangular notch or weir provided in a channel carrying water as shown.

Let H = Head of water over the crest
 L = length of the notch or weir.

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh & length L at a depth h from the free surface of water as shown.

The area of strip = $L \times dh$

& theoretical velocity of water flowing through strip = $\sqrt{2gh}$

The discharge dQ , through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \quad \text{--- (i)}$$

Where C_d = Co-efficient of discharge

The total discharge Q , for the whole notch or weir is determined by integrating equation (i) between the limits 0 & H

$$\begin{aligned}
 Q &= \int_0^H C_d \cdot L \cdot \sqrt{2gh} \, dh \\
 &= C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} \, dh \\
 &= C_d \times L \times \sqrt{2g} \left[\frac{h^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right] \\
 &= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]
 \end{aligned}$$

$$\boxed{Q = \frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2}}$$

→ Discharge over a rectangular notch or weir.

Q-1 Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300mm. Take $C_d = 0.60$.

Given, Length of the notch, $L = 2.0\text{m}$

Head over notch, $H = 300\text{mm} = 0.3\text{m}$

$C_d = 0.60$.

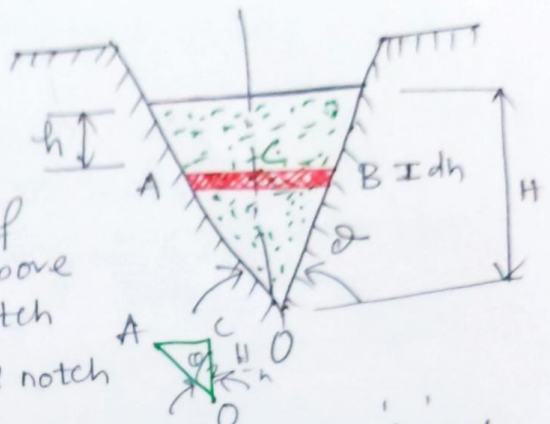
$$\begin{aligned}
 \text{Discharge, } Q &= \frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2} \\
 &= \frac{2}{3} \times 0.6 \times 2 \times \sqrt{2 \times 9.81} \times (0.3)^{3/2} \\
 &= 3.5435 \times 0.164 = 0.582\text{m}^3/\text{s}
 \end{aligned}$$

Discharge over a triangular notch or weir

The expression for the discharge over a triangular notch or weir is the same.

It is derived as: Let H = Head of water above V-notch

θ = angle of notch



Consider a horizontal strip of water of thickness dh at a depth of h from the free surface of water, as shown in figure. $\therefore$ We have $\tan \frac{\theta}{2} = \frac{AC}{OC}$

$$= \frac{AC}{(H-h)}$$

$$\therefore AC = (H-h) \tan \frac{\theta}{2}$$

$$\text{Width of strip} = AB = 2AC = 2(H-h) \tan \frac{\theta}{2}$$

$$\therefore \text{Area of strip} = 2(H-h) \tan \frac{\theta}{2} \times dh$$

$$\text{The theoretical velocity of water through strip} = \sqrt{2gh}$$

$\therefore$ Discharge, through the strip

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times [2(H-h) \tan \frac{\theta}{2} \times dh] \times \sqrt{2gh}$$

$$= 2C_d (H-h) \tan \frac{\theta}{2} \sqrt{2gh} \times dh$$

$$\therefore \text{Total discharge } Q = \int_0^H$$

$$\begin{aligned}
 \therefore \text{Total Discharge, } Q &= \int_0^H 2 C_d (H-h) \tan \frac{\theta}{2} \sqrt{2gh} \times dh \\
 &= 2 C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H (H-h) h^{1/2} dh \\
 &= 2 C_d \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H H h^{1/2} - h^{3/2} dh \\
 &= 2 C_d \tan \frac{\theta}{2} \sqrt{2g} \left[\frac{H h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H \\
 &= 2 C_d \tan \frac{\theta}{2} \sqrt{2g} \cdot \left[\frac{2}{3} H^{3/2} - \frac{2}{5} h^{5/2} \right]_0^H \\
 &= 2 C_d \tan \frac{\theta}{2} \sqrt{2g} \left[\frac{2}{3} H^{3/2} \cdot H - \frac{2}{5} H^{5/2} \right] \\
 &= 2 C_d \tan \frac{\theta}{2} \sqrt{2g} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2} \right] \\
 &= 2 C_d \tan \frac{\theta}{2} \sqrt{2g} \left[\frac{4}{15} H^{5/2} \right] \\
 &= \frac{8}{15} C_d \tan \frac{\theta}{2} \sqrt{2g} H^{5/2}
 \end{aligned}$$

For a right angled v-notch, if $C_d = 0.6$

$$\theta = 90^\circ \therefore \tan \frac{\theta}{2} = 1$$

$$\therefore \text{Discharge, } Q = \frac{8}{15} \times 0.6 \times 1 \sqrt{2 \times 9.81} \times H^{5/2}$$

$$Q = 1.417 H^{5/2}$$

Q-1 Find the discharge over a triangular notch of angle 60° when the head over the v-notch is 0.3m. Assume $C_d = 0.6$.

Given: Angle of v-notch, $\theta = 60^\circ$

Head over notch, $H = 0.3\text{m}$

$$C_d = 0.6$$

Discharge, Q over a v-notch is given by $Q = \frac{8}{15} C_d \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$

$$= \frac{8}{15} \times 0.6 \times \tan \frac{60}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$Q = 0.8182 \times 0.0493 = 0.040 \text{ m}^3/\text{s}$$

Flow Through Pipe

6.1 Definition of Pipe

A pipe is a tubular section or hollow cylinder, usually but not necessarily of circular cross-section, used mainly to convey substance which can flow - liquids & gases (fluids).

Laminar flow:- When the Reynolds number is less than 2000 for pipe flow, the flow is known as laminar flow.

$$R < 2000$$
$$R = \frac{\rho V L}{\mu}$$

Turbulent flow:- When the Reynolds number is more than 4000 $\rightarrow$ Turbulent flow.

$$R > 4000$$

Loss of Energy in Pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

Energy Losses

Major Energy Loss

This is due to friction & it is calculated by the following formulae

- (a) Darcy - Weisbach formula
- (b) Chezy's formula

Minor Energy Loss

This is due to

- (a) Sudden expansion of pipe
- (b) Sudden contraction of pipe
- (c) Bend in pipe
- (d) Pipe fittings
- (e) An obstruction in pipe.

Loss of Energy Due to Friction

(a) Darcy - Weisbach Formula :- The loss of head (or energy) in pipes due to friction is calculated from Darcy - Weisbach eqn which has been derived as

$$h_f = \frac{4fLv^2}{2gd}$$

h_f = loss of head due to friction

f = co-efficient of friction which is a function of Reynold's numbers

$$= \frac{16}{Re} \text{ for } Re < 2000 \text{ (viscous flow)}$$

$$= \frac{0.079}{Re} \text{ for } Re > 4000 \text{ to } 10^6$$

L = length of pipe

v = mean velocity of flow.

d = diameter of pipe

Q. Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula

(ii) Chezy's formula for which $C = 60$

Take ν for water = 0.01 stoke.

Ans

Dia of pipe, $d = 300 \text{ mm} = 0.3 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Velocity of flow, $V = 3 \text{ m/s}$

Chezy's Constant, $C = 60$

Kinematic viscosity, $\nu = 0.01 \text{ stoke}$

$$= 0.01 \text{ cm}^2/\text{s}$$

$$= 0.01 \times 10^{-4} \text{ m}^2/\text{s}$$

(i) Darcy's Formula

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{2 g d}$$

f = co-efficient of friction in a function of Reynolds number Re

$$Re = \frac{V \times d \times \rho}{\mu} = \frac{V \times d}{\nu} = \frac{V \times d}{\left(\frac{\mu}{\rho}\right)} = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 9 \times 10^5$$

$$f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$$

$$\therefore \text{Head Lost, } h_f = \frac{4 \times f \times L \times V^2}{2 \times g \times d} = \frac{4 \times 0.00256 \times 50 \times 3^2}{2 \times 9.81 \times 0.3} = 0.7828 \text{ m} \quad \left(\overset{h_f}{\text{(Ans)}} \right)$$

(ii) Chezy's Formula

$$V = C \sqrt{m i}$$

where $C = 60$

$$m = \frac{d}{4} = \frac{0.3}{4} = 0.075 \text{ m}$$

$$\text{But } i = \frac{h_f}{L} = \frac{h_f}{50}$$

$$\Rightarrow 0.0333 = \frac{h_f}{50}$$

$$\Rightarrow \boxed{h_f = 1.665 \text{ m}} \quad (\text{Ans})$$

$$\Rightarrow 3 = 60 \times \sqrt{0.075 \times i}$$

$$\Rightarrow i = \left(\frac{3}{60}\right)^2 \times \frac{1}{0.075} = 0.0333$$

(b) Chezy's Formula For Loss of head due to friction in pipes

$$h_f = \frac{f'}{fg} \times \frac{P}{A} \times L \times v^2 \quad \text{--- (1)}$$

Where h_f = loss of head due to friction.

A = Area of cross-section of pipe

v = mean velocity of flow.

P = wetted perimeter of pipe.

L = Length of pipe.

$\frac{A}{P} = \left[\frac{\text{Area of flow}}{\text{Perimeter (wetted)}} \right]$ is called hydraulic mean depth or "radius".

& denoted as "m"

$$\therefore \text{Hydraulic mean depth, } m = \frac{A}{P} = \frac{\frac{\pi d^2}{4}}{\pi d} = \frac{d}{4}$$

$$\text{Substituting } \frac{A}{P} = m \text{ or } \frac{P}{A} = \frac{1}{m}$$

$$\therefore \text{eqn (1) becomes } h_f = \frac{f'}{fg} \times L \times v^2 \times \frac{1}{m}$$

$$\Rightarrow v^2 = h_f \times \frac{fg}{f'} \times m \times \frac{1}{L}$$

$$\Rightarrow v^2 = \frac{fg}{f'} \times m \times \frac{h_f}{L}$$

$$\Rightarrow v = \sqrt{\frac{fg}{f'} \times m \times \frac{h_f}{L}}$$

$$\Rightarrow v = \sqrt{\frac{fg}{f'}} \sqrt{m \times \frac{h_f}{L}}$$

Let $\sqrt{\frac{fg}{f'}} = c$ where c is a constant known as Chezy's constant & $\frac{hf}{L} = i$ where i is loss of head per unit length of pipe.

Substituting the values of $\sqrt{\frac{fg}{f'}}$ & $\sqrt{\frac{hf}{L}}$

$$V = C\sqrt{mi}$$

Chezy's formula

$$m = \frac{d}{4}$$

Q-2 Find the diameter of a pipe of length 2000m when the rate of flow of water through the pipe is 200 litres/s & the head lost due to friction is 4m. Take the value of $c = 50$ in Chezy's formulae.

Given: Length of pipe, $L = 2000\text{m}$

Discharge, $Q = 200 \text{ litres/s} = 0.2 \text{ m}^3/\text{s}$

Head lost due to friction, $h_f = 4\text{m}$

Value of Chezy's constant, $c = 50$

Let the diameter of pipe = d

• velocity of flow, $v = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{\left(\frac{\pi}{4}d^2\right)} = \frac{0.2}{\left(\frac{\pi}{4}d^2\right)} = \frac{0.2 \times 4}{\pi d^2}$

• Hydraulic mean path, $m = \frac{d}{4}$

• Loss of head per unit length, $i = \frac{h_f}{L} = \frac{4}{2000} = 0.002$

Chzy's formula is given by $V = C\sqrt{mi}$
By substituting the values of v , m , i & C

$$\therefore \frac{0.2 \times 4}{\pi d^2} = 50 \sqrt{\frac{d}{4} \times 0.002}$$

$$\Rightarrow \sqrt{\frac{d}{4} \times 0.002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{0.00509}{d^2}$$

$$\Rightarrow \frac{d}{4} \times 0.002 = \left(\frac{0.00509}{d^2} \right)^2$$

$$\Rightarrow d^5 = \frac{4 \times 0.0000259}{0.002} = 0.0518$$

$$\Rightarrow d = \sqrt[5]{0.0518} = 0.553 \text{ m} = 553 \text{ mm (Ans)}$$

Minor Energy Losses :-

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy.

The minor loss of energy (or head) includes the following cases

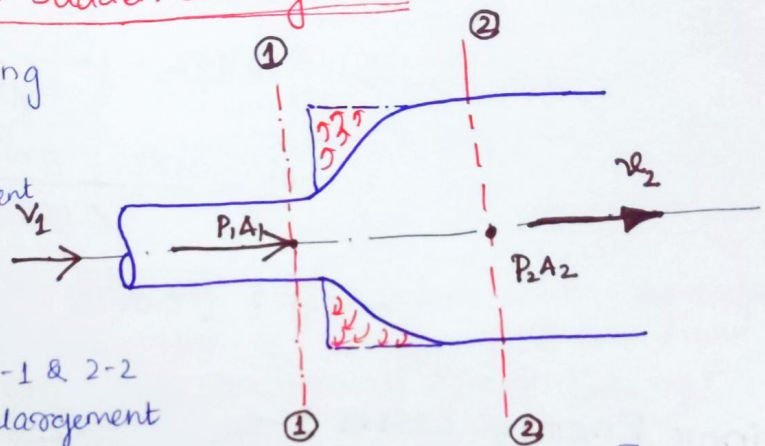
- (1) Loss of head due to sudden enlargement.
- (2) Loss of head due to sudden contraction
- (3) Loss of head at the entrance of a pipe
- (4) Loss of head at the exit of a pipe.
- (5) Loss of head due to an obstruction in a pipe.
- (6) Loss of head due to bend in the pipe.
- (7) Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction & hence they are called minor losses and even may be neglected without serious error.

* But in case of a short pipe, these losses are comparable with the loss of head due to friction.

(1) Loss of Head due to Sudden Enlargement.

Consider a liquid flowing through a pipe which has sudden enlargement



- Consider two sections 1-1 & 2-2 before & after the enlargement

Let P_1 = Pressure intensity at section 1-1

v_1 = velocity of flow at section 1-1

A_1 = Area of pipe at section 1-1

Similarly P_2 , v_2 & A_2 are corresponding values at section 2-2

⇒ Due to sudden change of diameter of the pipe from D_1 to D_2 the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary.

⇒ Thus the flow separates from the boundary & turbulent eddies are formed.

⇒ The loss of head (or energy) takes place due to the formation of these eddies.

Let P' = pressure intensity of the liquid eddies on the area $(A_2 - A_1)$
 h_e = loss of head due to sudden enlargement

Applying Bernoulli's equation at section 1-1 & 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{loss of head due to sudden enlargement.}$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e \quad [\because z_1 = z_2]$$

$$\Rightarrow h_e = \left[\frac{P_1}{\rho g} - \frac{P_2}{\rho g} \right] + \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right] \quad \text{--- (1)}$$

Consider the control volume of liquid between section 1-1 & 2-2
 Then the force acting on the liquid in the control volume in the direction of flow is given by

$$F_x = P_1 A_1 + P' (A_2 - A_1) - P_2 A_2$$

$$= P_1 A_1 + P_1 (A_2 - A_1) - P_2 A_2$$

$$= P_1 A_2 - P_2 A_2$$

$$F_x = (P_1 - P_2) A_2 \quad \text{--- (2)}$$

$\left[\because \text{But experimentally found that } P' = P_1 \right]$

Momentum of liquid/sec at section 1-1 = mass $\times$ velocity

$$= (\rho A_1 V_1) \times V_1 = \rho A_1 V_1^2$$

Momentum of liquid/sec at section 2-2 = $m \times v$

$$= \rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$$

$\therefore$ change of momentum = $\rho A_2 V_2^2 - \rho A_1 V_1^2$

$$= \rho A_2 V_2^2 - \rho \left(\frac{A_2 V_2}{V_1} \right) V_1^2$$

$$= \rho A_2 V_2^2 - \rho A_2 V_2 V_1$$

$$= \rho A_2 [V_2^2 - V_2 V_1] \quad \text{--- (3)}$$

$\left[\text{Continuity Eqn} \right]$
 $A_1 V_1 = A_2 V_2$
 $\Rightarrow A_1 = \frac{A_2 V_2}{V_1}$

Net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum or
or change of momentum/sec.

$$(P_1 - P_2) A_2 = \rho A_2 [v_2^2 - v_1 v_2]$$

$$\Rightarrow \frac{P_1 - P_2}{\rho} = v_2^2 - v_1 v_2$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1 v_2}{g}$$

$$\Rightarrow h_e = \frac{v_2^2 - v_1 v_2}{g}$$

Substituting the value of $\frac{P_1 - P_2}{\rho g}$ in equation (A)

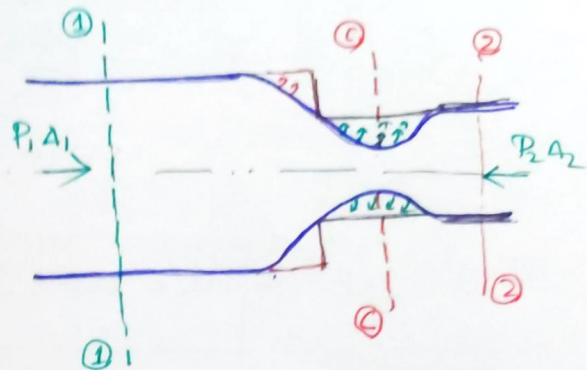
$$h_e = \frac{v_2^2 - v_1 v_2}{g} + \frac{v_1^2 - v_2^2}{2g}$$

$$= \frac{2v_2^2 - 2v_1 v_2 + v_1^2 - v_2^2}{2g} = \frac{v_1^2 + v_2^2 - 2v_1 v_2}{2g}$$

$$\boxed{h_e = \frac{(v_1 - v_2)^2}{2g}} \rightarrow \text{Loss of head due to enlargement.}$$

(2) Loss of Head due to Sudden Contraction :-

- Consider a liquid flowing in a pipe which has a sudden contraction in area
- Consider two sections 1-1 & 2-2 before and after contraction.



- ⇒ As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing & becomes minimum at a section c-c
- ⇒ This section c-c is called Vena-contracta.
- ⇒ The loss of head due to sudden contraction is actually due to sudden enlargement from Vena-contracta to smaller pipe.

Let A_c = area of flow at section c-c

V_c = Velocity of flow at section c-c

A_2 = Area of flow at section 2-2

V_2 = Velocity of flow at section 2-2

h_c = Loss of head due to sudden contraction

Now h_c = actual loss of head due to enlargement from section 2-2

$$\& \text{ given by } = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g} \left[\frac{V_c}{V_2} - 1 \right] \quad \text{--- (1)}$$

$$A_c V_c = A_2 V_2 \quad [\because \text{Continuity Eqn}]$$

$$\Rightarrow \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c/A_2)} = \frac{1}{C_c}$$

$$\Rightarrow \boxed{\frac{V_c}{V_2} = \frac{1}{C_c}}$$

Substituting the value of $\frac{v_c}{v_2}$ in eqn ①

$$h_c = \frac{v_2^2}{2g} \left[\frac{1}{C_c} - 1 \right]$$

$$= \frac{K v_2^2}{2g}$$

$$\therefore K = \left[\frac{1}{C_c} - 1 \right]^2$$

$$h_c = 0.375 \frac{v_2^2}{2g}$$

$$\left[\text{if } C_c = 0.62 \text{ (assumed)} \right]$$
$$K = \left[\frac{1}{0.62} - 1 \right]^2 = 0.375$$

* If the value of C_c is not given then the head loss due to contraction is

$$h_c = 0.5 \frac{v_2^2}{2g}$$

Q Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s.

Given, Dia of smaller pipe, $D_1 = 200 \text{ mm} = 0.2 \text{ m}$

$$\text{Area, } A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.2)^2 = 0.03141 \text{ m}^2$$

Dia of large pipe, $D_2 = 400 \text{ mm} = 0.4 \text{ m}$

$$\therefore \text{Area, } A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (0.4)^2 = 0.12564 \text{ m}^2$$

Discharge, $Q = 250 \text{ litres/s} = 0.25 \text{ m}^3/\text{s}$

$$\text{velocity, } v_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96 \text{ m/s}$$

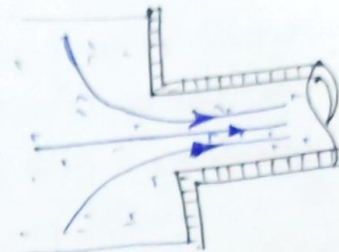
$$\text{Velocity, } v_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99 \text{ m/s}$$

Loss of head due to enlargement is given by eqn

$$h_e = \frac{(v_1 - v_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = 1.816 \text{ m of water.}$$

Loss of head at the Entrance of a pipe

→ This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir.



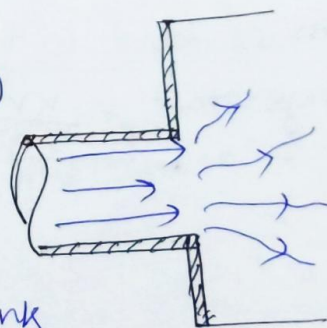
→ This loss depends on the form of entrance. This loss is similar to the loss of head due to sudden contraction.

$(h_{ent})_{\text{sharp edge}} > (h_{ent})_{\text{bell mouthed}}$
 $v = \text{velocity of liquid in pipe}$

$$h_i = 0.5 \frac{v^2}{2g}$$

Loss of head at the Exit of pipe

This is due to the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet (if outlet of the pipe is free) or it is in the tank or reservoir (if the outlet of the pipe is connected to the tank or reservoir).



$$h_o = \frac{v^2}{2g}$$

$v = \text{velocity at outlet of pipe}$

Loss of head Due to an Obstruction in a pipe :-

$$\text{Head loss due to obstruction} = \frac{(v_c - v)^2}{2g} = \frac{v^2}{2g} \left[\frac{A}{C_c(A-a)} - 1 \right]^2$$

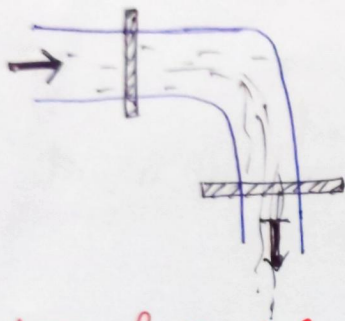
Loss of head due to Bend in Pipe :-

When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary & also formation of eddies takes place.

$$h_b = \frac{Kv^2}{2g}$$

K = co-efficient of bend
 v = velocity of flow

depends on - Angle of Bend
- Radius of curvature
- Dia. of pipe



Loss of Head in Various Pipe fittings :-

The loss of head in the various pipe fittings such as valve couplings etc.

$$h_{fp} = \frac{Kv^2}{2g}$$

v = velocity of flow

K = co-efficient of pipe fitting.

Hydraulic Gradient Line [HGL]

It is defined as the line which gives the sum of pressure head ($\frac{P}{\rho g}$) and datum head (z) of a flowing fluid in a pipe w.r.t some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head ($\frac{P}{\rho g}$) of a flowing fluid in a pipe from the centre of the pipe.

Total Energy Line :- [TEL]

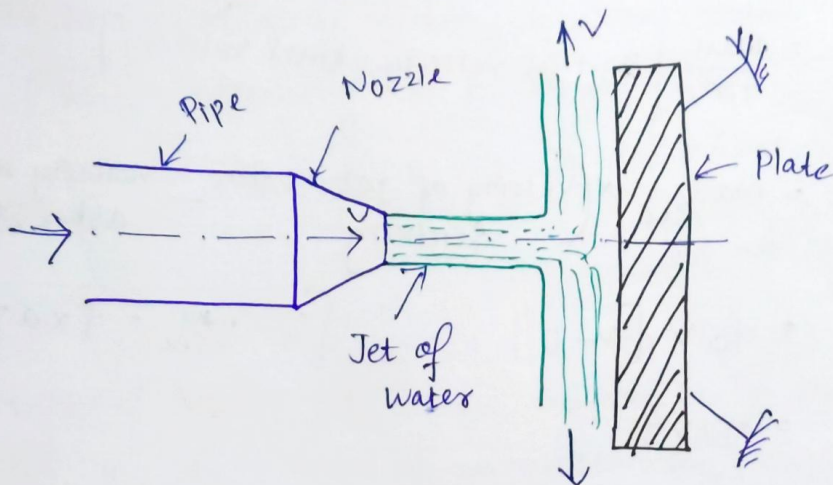
⇒ It is defined as the line which gives the sum of pressure head, datum head & kinetic head of a flowing fluid in a pipe w.r.t some reference line.

⇒ It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head & kinetic head from the centre of the pipe.

Chapter - 7 :- Impact of Jets

7.1 Impact of Jet on Fixed & Moving Vertical Flat Plates :-

(1) Impact of Jet on a Fixed Vertical Flat Plate :-



⇒ Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

Let v = velocity of the jet, d = diameter of the jet

a = area of cross-section of the jet $= \frac{\pi}{4} d^2$

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking, will get deflected through 90° . Hence the component of the velocity of jet in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet

$$F_x = \text{Rate of change of momentum in the direction of force} \\ = \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{[\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity}]}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

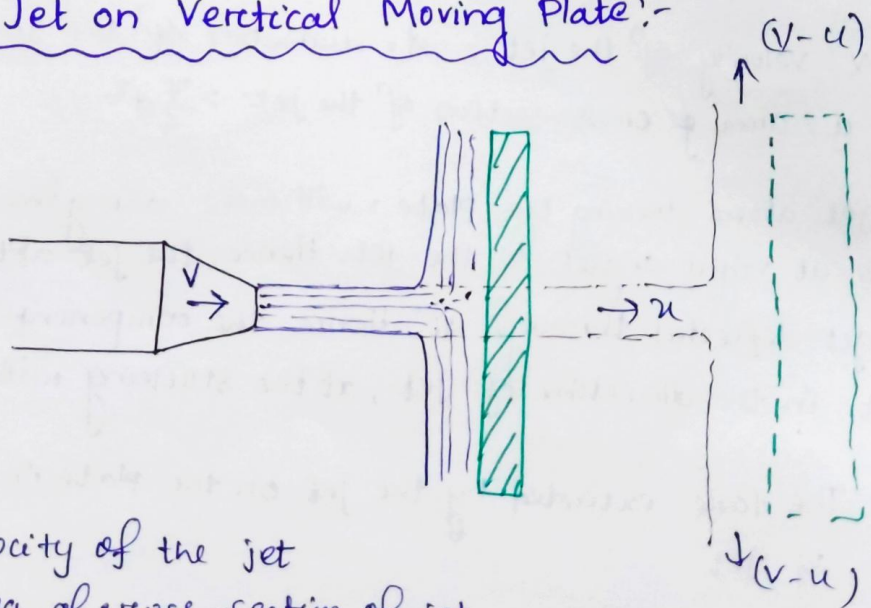
$$= \text{mass/sec} \times (\text{Velocity of jet before striking} - \text{velocity of jet after striking})$$

$$= \rho a v [v - 0] \quad \left\{ \because \text{mass/sec} = \rho \times a v \right\}$$

$$= \rho a v^2$$

(2) Force on Flat Vertical

Impact of Jet on Vertical Moving Plate:-



Let v = velocity of the jet
 a = area of cross-section of jet
 u = velocity of the flat plate

On this case, the jet does not strike the plate with velocity v , but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate.

∴ Hence relative velocity of the jet w.r.t plate $= (v - u)$

Mass of water striking the plate per sec

$$= \rho \times \text{Area of jet} \times \text{velocity with which jet strikes the plate.}$$

$$= \rho a (v - u)$$

∴ Force exerted by the jet on the moving plate in the direction of the jet $\Rightarrow F_x = \text{Mass of water striking per sec}$

$$\times \left[\text{Initial velocity with which water strikes} - \text{Final velocity} \right]$$

$$= \rho a (v - u) \cdot [(v - u) - 0]$$

$$\boxed{F_x = \rho a (v - u)^2}$$

⇒ Work done per second by the jet on the plate

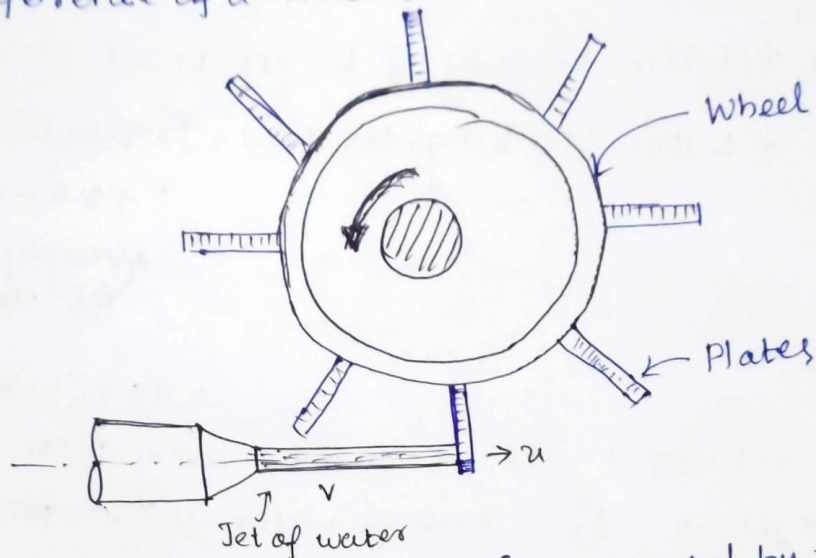
$$= \text{Force} \times \frac{\text{Distance in the direction of force}}{\text{Time}}$$

$$= F_x \times u$$

$$\boxed{W = \rho a (v - u)^2 \times u}$$

Force Exerted by a Jet of water on a Series of vanes :-

→ In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart.



→ The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving & the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd Plate.

→ Thus each plate appears successively before the jet & the jet and the jet exerts force on each plate.

→ The wheel starts moving at a constant speed.

v = velocity of Jet

d = diameter of jet

a = cross-sectional area of jet.

$$= \frac{\pi}{4} d^2$$

u = Velocity of vane

Mass of water coming out from the nozzle per second is always in contact with the plates, ~~with~~ when all the plates are considered. Hence mass of water per second striking the series of plates = $\rho a v$

The jet strikes the plate with a velocity = $(v-u)$

→ After striking, the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero.

∴ The force exerted by the jet in the direction of motion of plate, $F_x = \text{Mass per second} \left[\begin{matrix} \text{initial} \\ \text{velocity} \end{matrix} - \begin{matrix} \text{Final} \\ \text{velocity} \end{matrix} \right]$

$$= \rho a v [(v-u) - 0]$$

$$\boxed{F_x = \rho a v [v-u]}$$

∴ Work done by the jet on the series of plates per second
= Force $\times$ Distance per second in the direction of force

$$= F_x \times u$$

$$\boxed{W = F_x \times u = \rho a v [v-u] \times u}$$

∴ Kinetic energy of the jet per second = $\frac{1}{2} m v^2 = \frac{1}{2} (\rho a v) \times v^2$

$$\boxed{K.E = \frac{1}{2} \rho a v^3}$$

$$\therefore \text{Efficiency } \eta = \frac{\text{work done per second}}{\text{Kinetic Energy per second}} = \frac{\rho a v [v-u] \times u}{\frac{1}{2} \rho a v^3} = \frac{2u(v-u)}{v^2}$$

$$\therefore \left\{ \eta = \frac{2u[v-u]}{v^2} \right\}$$

Condition for Maximum Efficiency

$$\text{Efficiency, } \eta = \frac{2u(V-u)}{v^2}$$

$$\frac{d\eta}{du} = 0$$

$$\Rightarrow \frac{d \left[\frac{2u(V-u)}{v^2} \right]}{du} = 0$$

$$\Rightarrow \frac{d \left(\frac{2uV - 2u^2}{v^2} \right)}{du} = 0$$

$$\Rightarrow \frac{2V - 2 \times 2u}{v^2} = 0$$

$$\Rightarrow 2V - 4u =$$

$$\Rightarrow V = \frac{4u}{2} = 2u$$

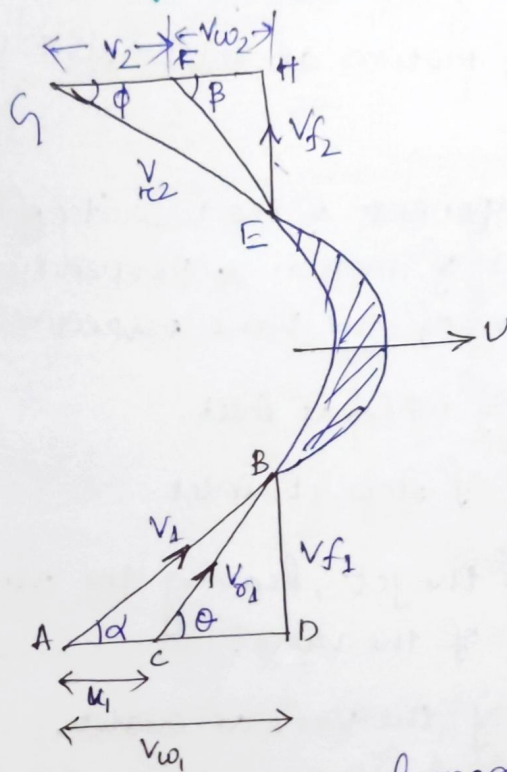
$$\Rightarrow \boxed{V = 2u}$$

$$\Rightarrow \boxed{u = \frac{V}{2}}$$

$$\therefore \text{Max}^n \text{ Efficiency } \eta_{\max} = \frac{2u[V-u]}{v^2} = \frac{2u(2u-u)}{4u^2}$$

$$= \frac{1}{2}$$
$$\boxed{\eta_{\max} = 50\%}$$

Impact of Jet on a Moving Curved Plate :-



⇒ As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero.

⇒ In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet w.r.t the plate.

⇒ As the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet & velocity of the plate at inlet.

Let V_1 = velocity of the jet at inlet

u_1 = velocity of plate (vane) at inlet

V_{b1} = Relative velocity of jet and plate at inlet

α = Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle.

θ = Angle made by the relative velocity (V_{r2}) with the direction of motion at inlet also called vane angle at inlet

V_{w1} & V_{f1} = The components of the velocity of the jet V_1 , in the direction of motion & perpendicular to the direction of motion of the vane respectively.

V_{w1} = Velocity of whirl at inlet.

V_{f1} = Velocity of flow at inlet.

V_2 = Velocity of the jet, leaving the vane or velocity of jet at outlet of the vane.

U_2 = Velocity of the vane at outlet.

V_{r2} = Relative velocity of the jet w.r.t the vane at outlet

ϕ = Angle made by the velocity V_2 with the direction of motion of vane & ~~perpendicular~~ at outlet & also called vane angle at outlet.

β = Angle made by the velocity V_2 with the direction of motion of the vane at inlet.

V_{w2} & V_{f2} = components of the velocity V_2 in the direction of motion of vane & perpendicular to the direction of motion of vane at outlet.

V_{w2} = velocity of whirl at outlet

V_{f2} = velocity of flow at outlet.