<u>LECTURE NOTES ON</u>

STRENGTH OF MATERIAL

PREPARED BY PRABIR KUMAR SWAIN

DEPARTMENT OF MECHANICAL ENGINEERING

B.O.S.E Cuttack

Two dimensional stress system

3.1. Determination of normal stress, shear stress and resultant stress on oblique plane.

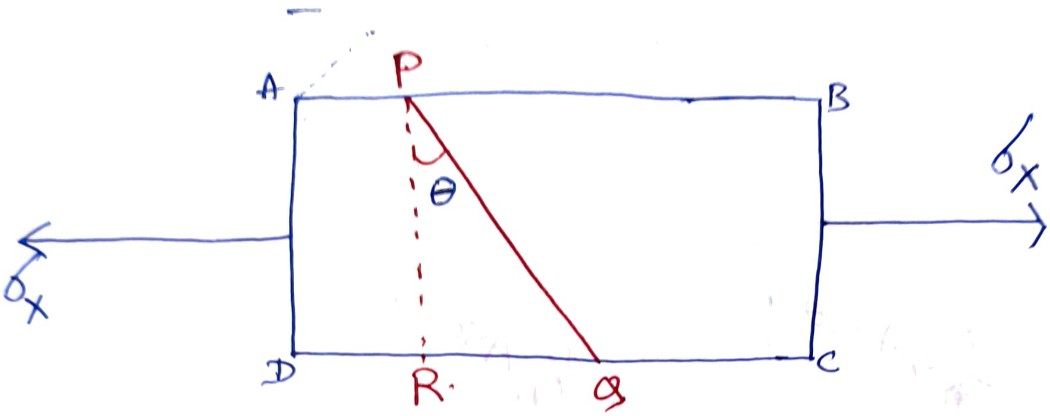
3.2 Location of principal plane and computation of principal stress.

3.3 Location of principal plane and computation of principal stress and Maximum shear stress using Mohr's

{CHAPTER-3}

STRESS ON OBLIQUE PLANE UNDER UNIAXIAL LOADING

(P.K. SWAIN)

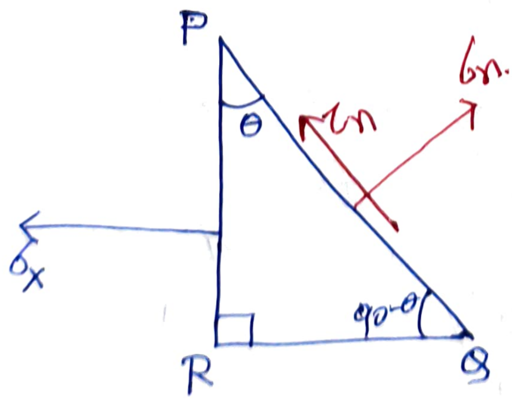


PQ = oblique plane.

σ_x = stress along longitudinal direction or along x-axis.

θ = angle of PQ plane with vertical in anticlock-wise sense

considering equilibrium of region PQR.



σ_n = normal stress on oblique plane

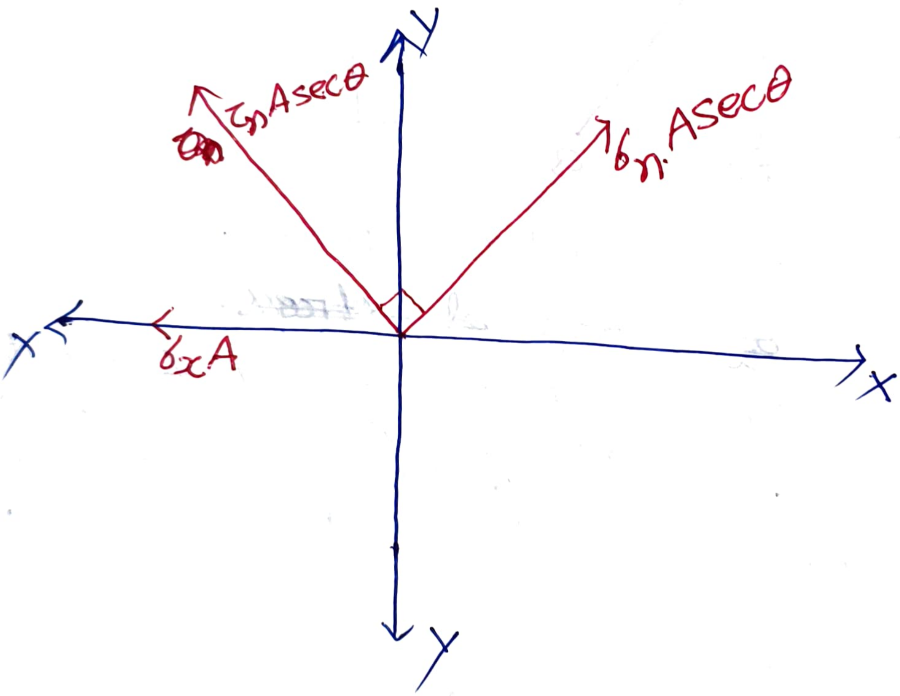
τ_n = shear stress on oblique plane

Let cross-sectional area of bar = PR = A, solving

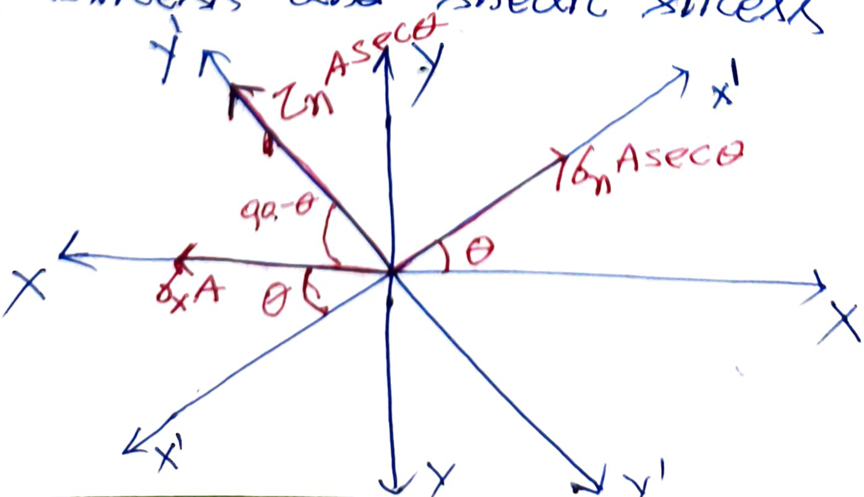
Area of QR = $A \tan\theta$

Area of PQ = $A \sec\theta$ (area of oblique plane).

→ Let us show all the above forces on a body on X-Y axis.



→ Rotate X and Y axis by 'θ' in counter clockwise direction so that they match with normal stress and shear stress of inclined plane.



→ Resolving all the forces along $x'-x'$ axis.

$$\sigma_n A \sec\theta = \sigma_x A \cos\theta$$

$$\Rightarrow \sigma_n = \frac{\sigma_x A \cos\theta}{A \sec\theta}$$

$$\Rightarrow \boxed{\sigma_n = \sigma_x \cos^2\theta} \text{ or } \boxed{\frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 2\theta} \quad **$$

→ Resolving forces along $y'-y'$ axis.

$$\tau_n A \sec\theta + \sigma_x A \sin\theta = 0$$

$$\Rightarrow \tau_n A \sec\theta = -\sigma_x A \sin\theta$$

$$\Rightarrow \tau_n = -\frac{\sigma_x \sin\theta}{\sec\theta}$$

$$\Rightarrow \tau_n = -\sigma_x \sin\theta \cdot \cos\theta$$

$$\Rightarrow \tau_n = -\frac{\sigma_x}{2} \cdot 2\sin\theta \cdot \cos\theta$$

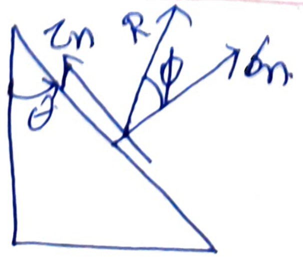
$$\Rightarrow \boxed{\tau_n = -\frac{\sigma_x}{2}\sin 2\theta} \quad **$$

↳ τ_n -ve sign means shear stress acts in counter clockwise direction.

→ Resultant stress on oblique plane is

$$\boxed{\sigma_R = \sqrt{\sigma_n^2 + \tau_n^2}} \quad **$$

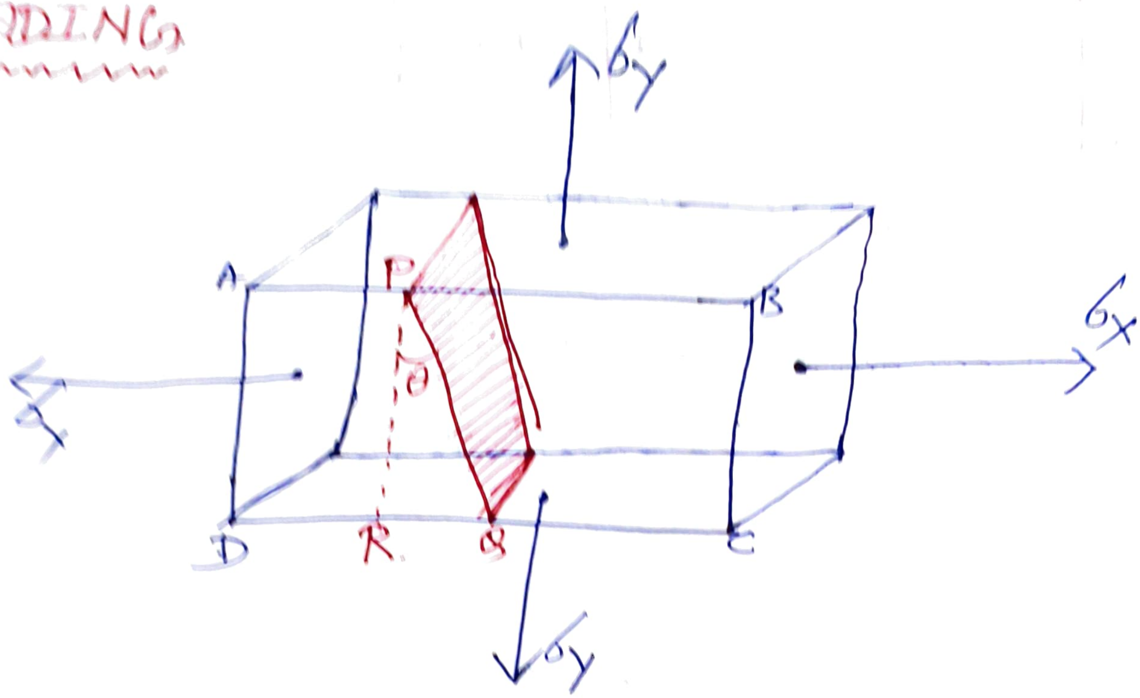
→

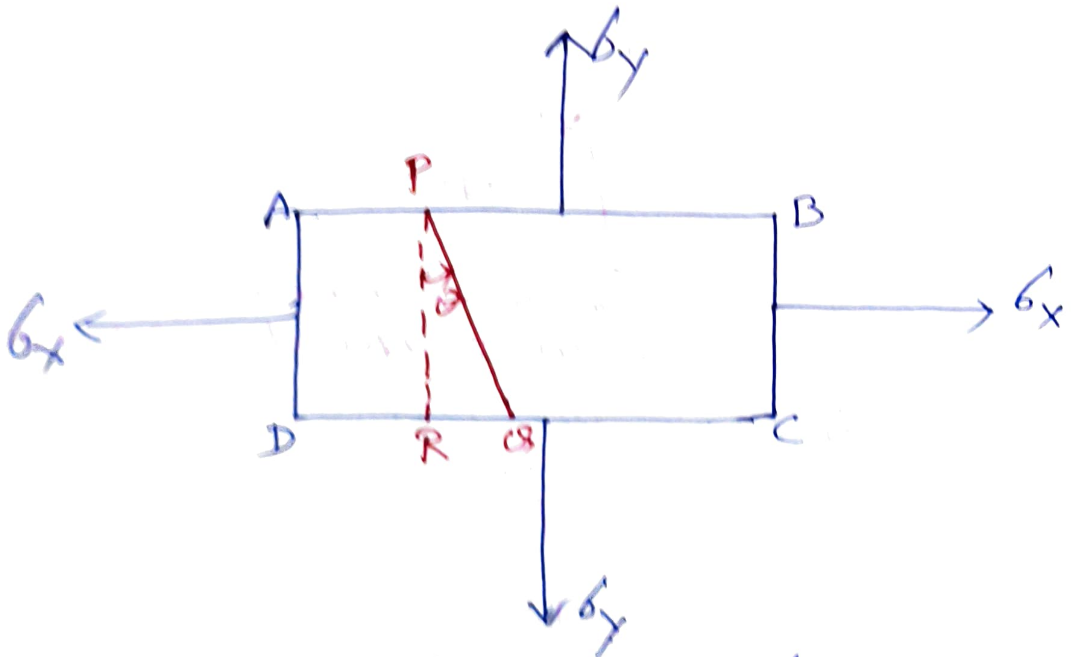


→ Direction of resultant stress on oblique = angle τ_n

$$\boxed{\tan\phi = \frac{\tau_n}{\sigma_n}}$$

STRESS ON OBLIQUE PLANE UNDER BIAXIAL LOADING

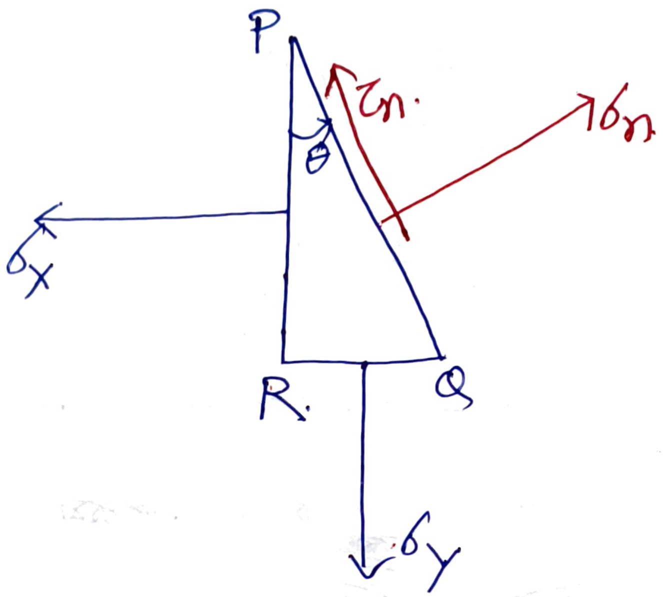




σ_x = stress along x-axis.
σ_y = stress along y-axis
PQ = oblique plane

= angle made by PQ plane with vertical in anticlockwise direction.

→ considering equilibrium of region PQR

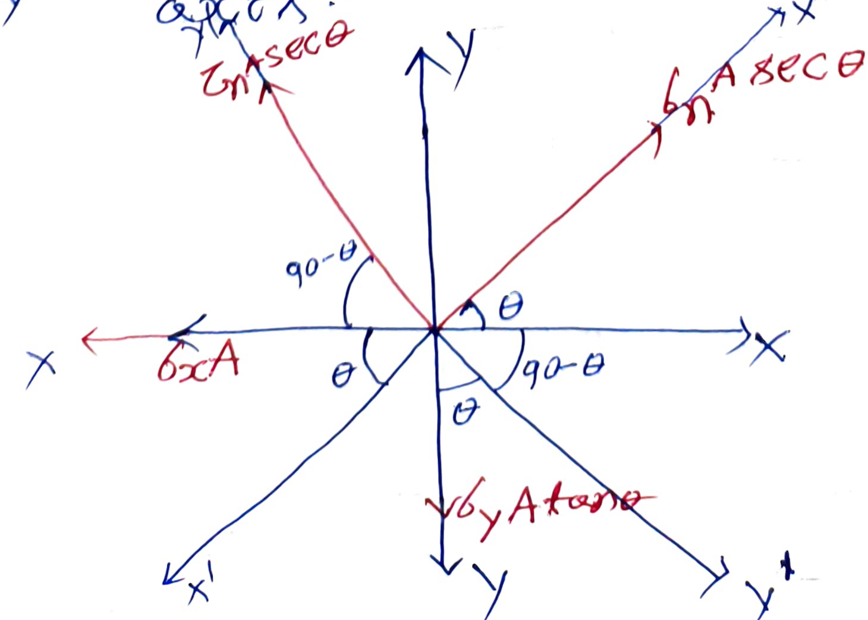


Let crossectional area of bar = PR = A

Area of QR = A $\tan\theta$

Area of PQ = A $\sec\theta$ ($\because$ area of oblique plane)

→ Let us show all the above forces on X-Y axis.



→ Resolving all the forces along $x'-x'$:

$$\sigma_n A \sec\theta = \sigma_x A \cos\theta + (\sigma_y A \tan\theta)\sin\theta$$

$$\Rightarrow \sigma_n \sec\theta = \sigma_x \cos\theta + \sigma_y \tan\theta . \sin\theta$$

$$\Rightarrow \sigma_n = \frac{\sigma_x \cos\theta + \sigma_y \tan\theta \sin\theta}{\sec\theta}$$

$$\Rightarrow \sigma_n = \sigma_x \frac{\cos\theta}{\sec\theta} + \frac{\sigma_y \sin\theta . \tan\theta}{\sec\theta}$$

$$\Rightarrow \sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin\theta . \cos\theta . \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta .$$

$$\Rightarrow \boxed{\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta .} *$$

Resultant σ

→ Resolving all the forces along $y'-y'$ axis

$$\tau_n A \sec\theta = -\sigma_x A \sin\theta + (\sigma_y A \tan\theta) \cos\theta$$

$$\Rightarrow \tau_n \sec\theta = -\sigma_x \sin\theta + \sigma_y \tan\theta . \cos\theta$$

$$\Rightarrow \tau_n = \frac{-\sigma_x \sin\theta + \sigma_y \tan\theta . \cos\theta}{\sec\theta}$$

$$\Rightarrow \tau_n = -\frac{\sigma_x \sin\theta}{\sec\theta} + \frac{\sigma_y \tan\theta . \cos\theta}{\sec\theta}$$

$$\Rightarrow \tau_n = -\sigma_x \sin\theta \cos\theta + \sigma_y \sin\theta . \cos\theta .$$

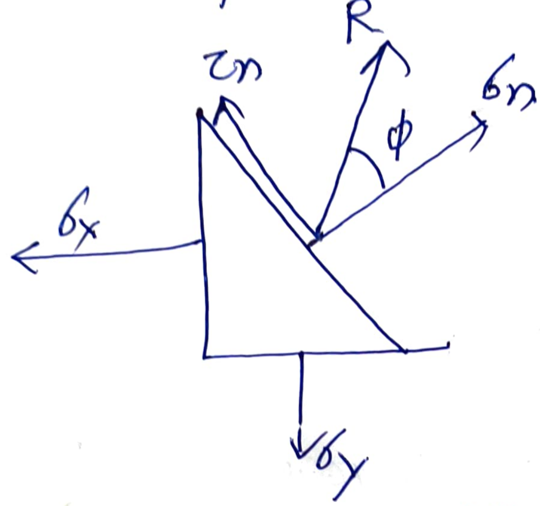
$$\Rightarrow \boxed{\tau_n = -\frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta .} **$$

→ Shear stress in counter clock wise sing contd.

Resultant stress on oblique plane.

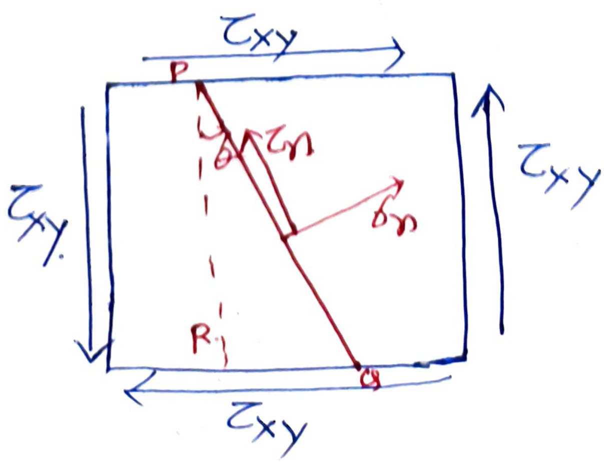
$$\boxed{\sigma_R = \sqrt{\sigma_n^2 + \tau_n^2}} \; *$$

→ Direction of resultant stress on oblique plane.



$$\boxed{\tan\phi = \frac{\tau_n}{\sigma_n}} \; **$$

STRESS ON OBLIQUE PLANE UNDER SHEAR LOADING.



6.P

→ τ_{xy} = shear stress.

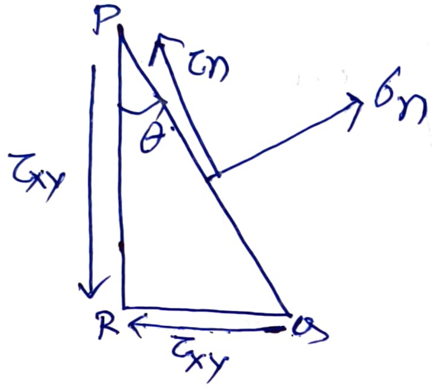
PQ = oblique plane.

θ = angle made by oblique plane with vertical in anticlockwise direction.

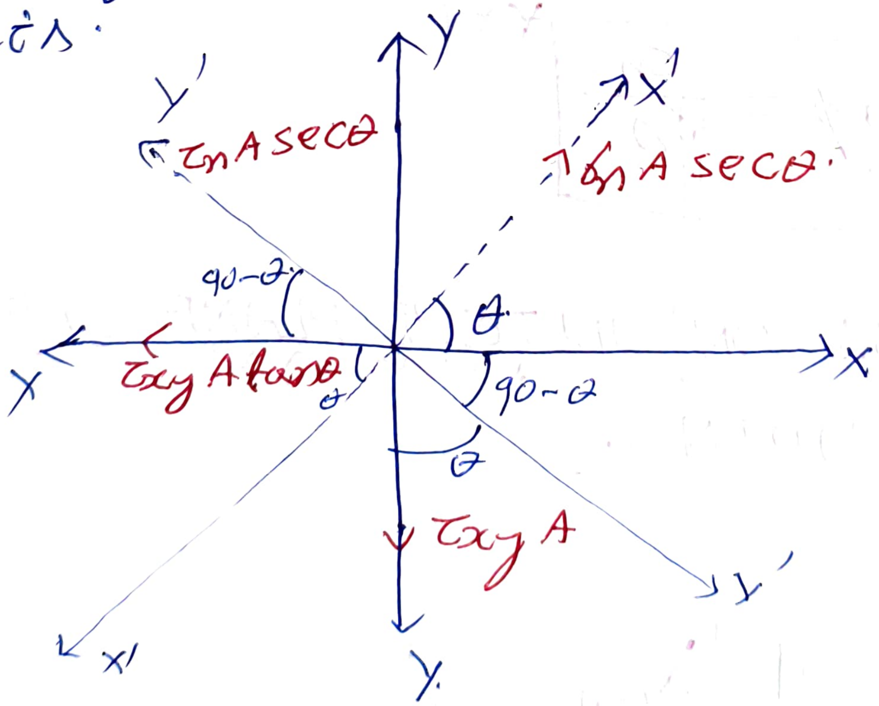
∴ A = crossectional area = PR

⇒ area of QR = A $\tan\theta$

area of PQ = A $\sec\theta$.



→ showing all the forces along X & Y axis.



→ Resolving the forces along X'−X' axis.

$$\sigma_n A\sec\theta = \tau_{xy} A \tan\theta \cdot \cos\theta + \tau_{xy} A \sin\theta$$

$$\Rightarrow \sigma_n \sec\theta = \tau_{xy} \tan\theta \cdot \cos\theta + \tau_{xy} \sin\theta$$

$$\sigma_n = \frac{\tau_{xy}\tan\theta\cdot\cos\theta + \tau_{xy}\sin\theta}{\sec\theta}$$

$$\Rightarrow \sigma_n = \frac{\tau_{xy}\tan\theta\cdot\cos\theta}{\sec\theta} + \frac{\tau_{xy}\sin\theta}{\sec\theta}$$

$$\Rightarrow \sigma_n = \tau_{xy}\sin\theta\cos\theta + \tau_{xy}\sin\theta\cdot\cos\theta.$$

$$\Rightarrow \sigma_n = 2\tau_{xy}\sin\theta\cos\theta.$$

$$\Rightarrow \boxed{\sigma_n = \tau_{xy}\sin 2\theta}\ \times\times.$$

→ Resolving forces along y'-y' direction.

$$\tau_n A\sec\theta = -\tau_{xy}A\tan\theta\cdot\sin\theta + \tau_{xy}A\cos\theta$$

$$\Rightarrow \tau_n = \frac{-\tau_{xy}\tan\theta\cdot\sin\theta + \tau_{xy}\cos\theta}{\sec\theta}$$

$$\Rightarrow \tau_n = -\tau_{xy}\sin^2\theta + \tau_{xy}\cos^2\theta$$

$$\Rightarrow \tau_n = \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

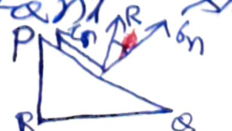
$$\Rightarrow \boxed{\tau_n = \tau_{xy}\cos 2\theta}\ \times\times.$$

→ Resultant stress on oblique plane

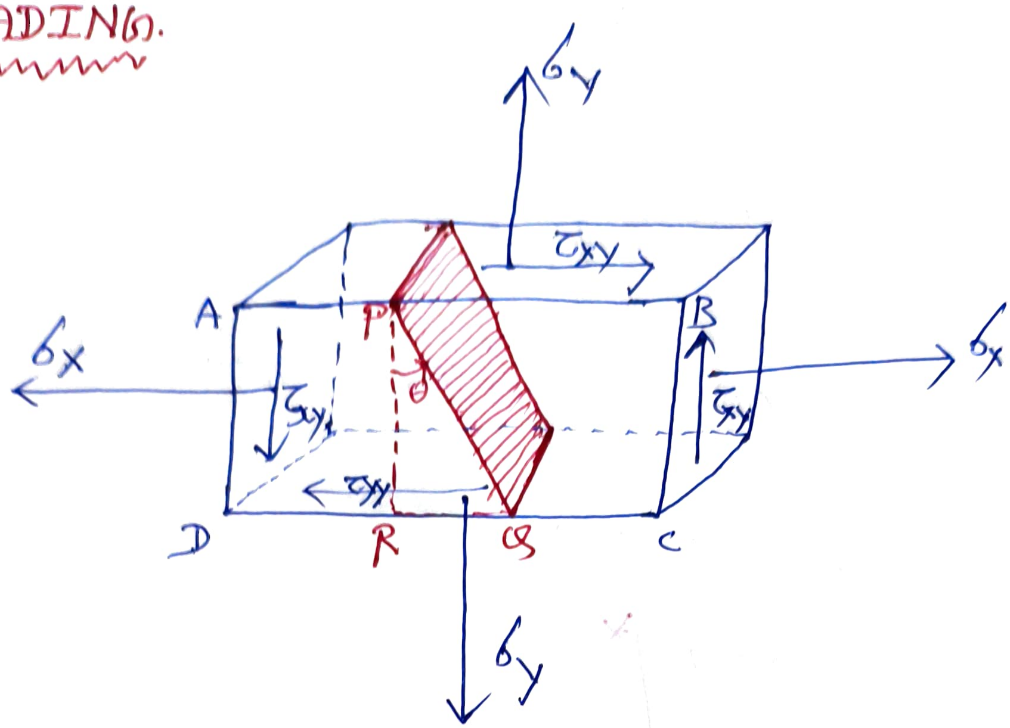
$$\boxed{R = \sqrt{\sigma_n^2 + \tau_n^2}}$$

→ Direction of resultant stress on oblique plane

$$\boxed{\tan\phi = \frac{\tau_n}{\sigma_n}}$$



STRESS ON OBLIQUE PLANE UNDER COMBINED LOADING.



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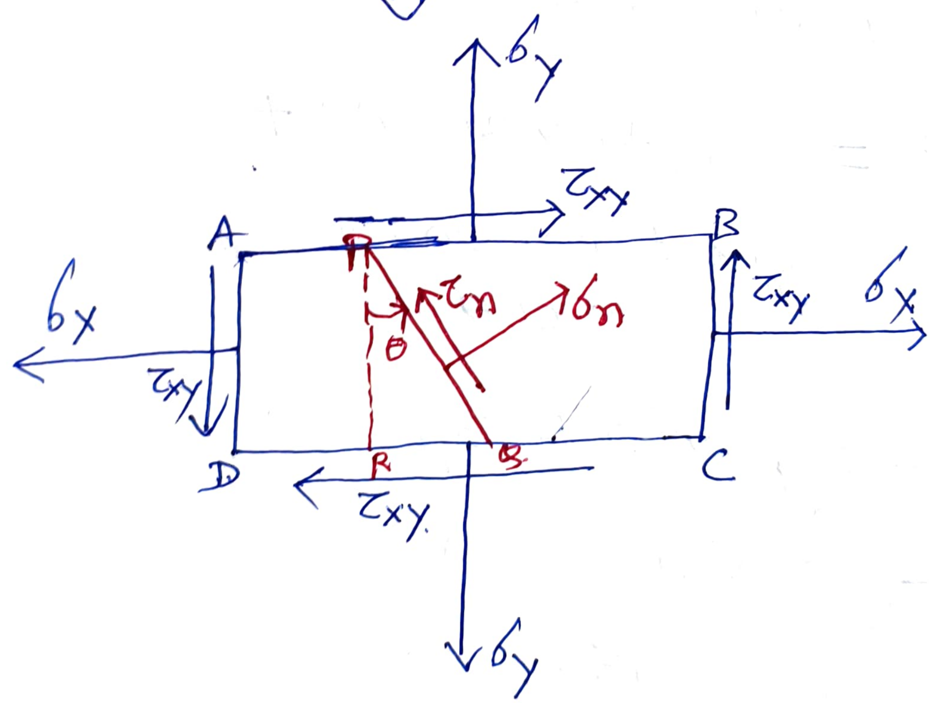


σ_x = stress along x-axis.

σ_y = stress along y-axis.

τ_{xy} = shear stress

σ_n = normal stress on oblique plane.

τ_n = shear stress on oblique plane.

θ = inclined plane or oblique plane making an angle 'θ' with vertical plane in anticlock-wise direction.

Then

$$\boxed{\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy} \sin 2\theta}$$

or

$$= \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy}\sin 2\theta$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} - \frac{\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} + \frac{\sigma_x}{2}\cos 2\theta - \frac{\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\boxed{\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta}$$

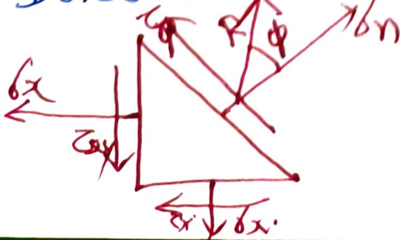
Then

$$\boxed{\tau_n = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}$$

→ Then Resultant stress on oblique plane.

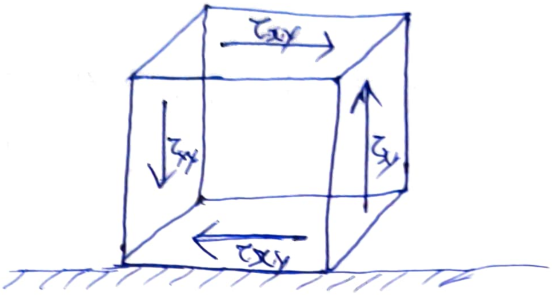
$$\boxed{\sigma_R = \sqrt{\sigma_n^2 + \tau_n^2}}$$

→ Direction of resultant stress on oblique plane.



$$\boxed{\tan\phi = \frac{\tau_n}{\sigma_n}}$$

SHEAR STRESS



* shear stress acting on +ve plane along +ve direction taken as +ve.

+ve plane

Front plane
Top plane
right plane.

Angle

* angle measured from vertical plane in anticlock wise direction taken as +ve
* angle measured from vertical plane in clock -wise direction taken as -ve

θ = +ve.

θ = -ve.

τ_n = shear ...

INCIPAL PLANE

→ There will be an oblique plane at which shear stress is zero, ~~called~~ such a plane is known as principal plane.

PRINCIPAL STRESS

→ Normal stress acting on principal plane is known as principal stress.

→ There are two types of principal stre

(a) Major principal stress.

(b) Minor principal stress.

Major principal stress

→ Maximum value of normal stress acting on principal plane is known as major principal stress.

Minor principal stress

→ Minimum value of normal stress acting on principal plane is known as minor principal stress.

LOCATION OF PRINCIPAL PLANE

We know

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

For maximum value of "σ_n"

$$\frac{d\sigma_n}{d\theta} = 0$$

$$\Rightarrow -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta\cdot 2 + \tau_{xy}\cos 2\theta\cdot 2 = 0$$

$$\Rightarrow \cancel{+}\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta\cdot\cancel{2} = \cancel{+}\tau_{xy}\cos 2\theta\cdot\cancel{2}$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)}$$

$$\Rightarrow \boxed{\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)}} \;**$$

→ Location of principal plane.

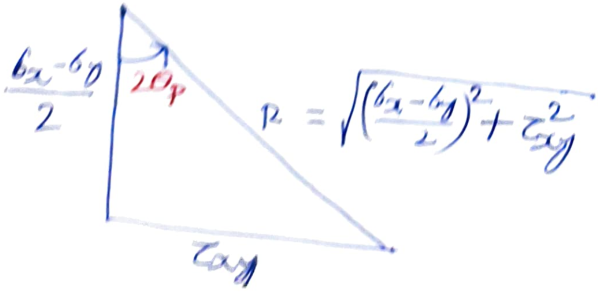
θ_p = principal plane

$$\boxed{\tan(180+2\theta_p) = \tan 2\theta_p} \;*$$

$2\theta_p = 2\theta_p + 180$

$\Rightarrow \boxed{\theta_p = \theta_p + 90}$

* Two principal plane are separated by 90°



ie

τ_n = shear ......

plane.

putting $\theta = \theta_p$ in σ_n eqn, we get

$$\boxed{\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \quad **$$

→ Major principal stress.

By putting $\theta = 90 + \theta_p$ in σ_n eqn, we get

$$\boxed{\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \quad **$$

Minor principal stress.

~~so principal stress is.~~

→ Maximum value of shear stress, on shear plane is

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad ***$$

or

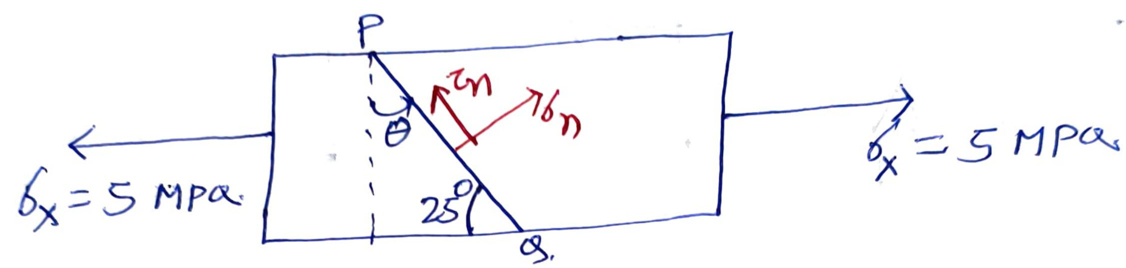
$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} \quad *$$

→ Normal stress on shear plane.

$$\boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}} \quad ***$$

A wooden bar is subjected to a tensile stress of 5 MPa, What will be the values of normal and shear stresses across a section, which makes an angle of 25° with the direction of tensile stress.

Ans



Here

$$\sigma_x = 5 \text{ MPa}$$

$$\theta = 90^0 - 25^0 = 65^0$$

We know

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_x}{2} \cos 2\theta$$

$$= \frac{5}{2} + \frac{5}{2} \cos(2 \times 65)$$

$$= 0.89 \text{ MPa}$$

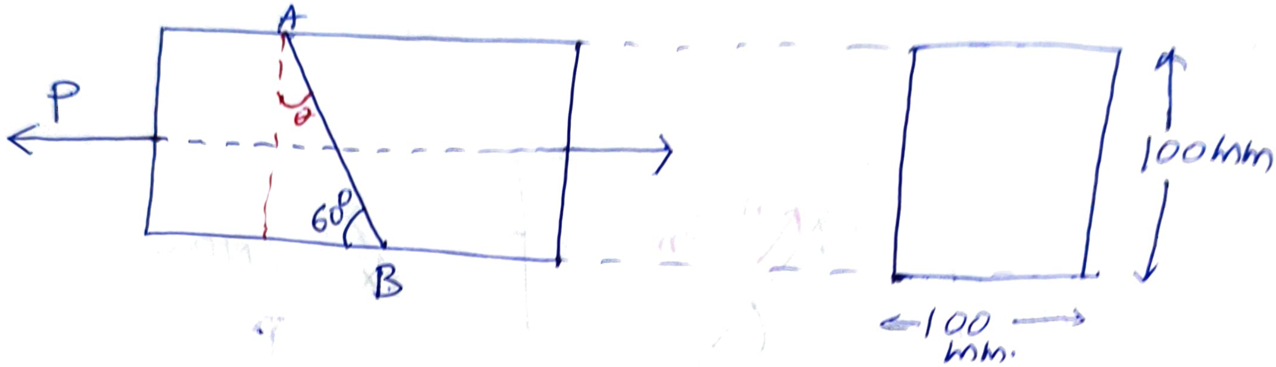
$$\tau_n = -\frac{\sigma_x}{2} \sin 2\theta$$

$$= -\frac{5}{2} \sin(2 \times 65)$$

$$= -1.915 \text{ MPa (counter clockwise)}$$

∴ The value of normal stress on inclined plane is 0.89 MPa and shear stress is 1.915 MPa in counter clockwise direction.

Q Two wooden pieces 100 mm x 100 mm in cross are joined together along line AB as ; & Find the maximum force 'P' which can applied if the shear stress along the joint is 1·3 MPa.



Ans

$$\theta = 90^\circ - 60^\circ = 30^\circ$$

We know

$$\tau_n = -\frac{\sigma_x}{2} \sin 2\theta$$

$$\Rightarrow 1\cdot 3 = \frac{\sigma_x}{2} \sin(2 \times 30)$$

$$\Rightarrow 1\cdot 3 = \frac{\sigma_x}{2} \sin 60$$

$$\Rightarrow \sigma_x = \frac{1\cdot 3}{2 \times \sin 60} = 3 \text{ MPa.}$$

$\therefore$ Axial force $= \sigma_x A$

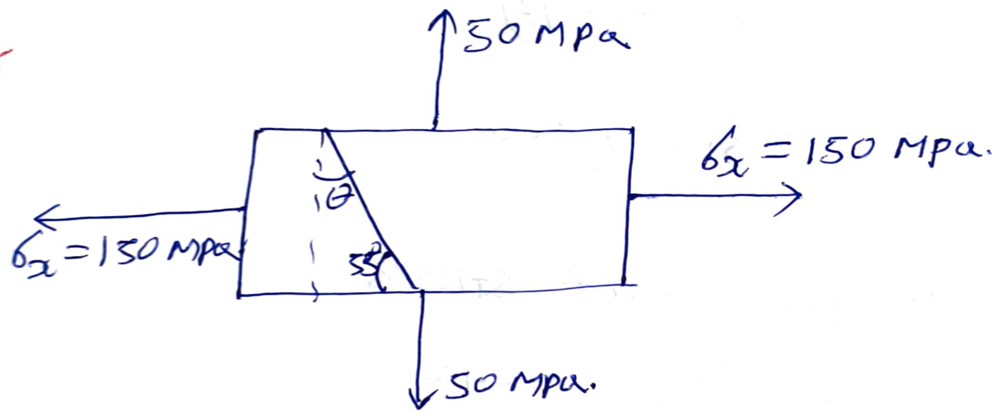
$= 3 \times 100 \times 100$

$= 30{,}000$ N

$= 30$ KN

The stress at point of a machine component are 150 Mpa and 50 Mpa both tensile. Find the intensities of normal, shear, resultant stress and direction of resultant stress on a plane inclined at an angle of 55° with the axis of major tensile stress. Also find the magnitude of maximum shear stress in the component.?

Ans



$\sigma_x = 150$ Mpa

$\sigma_y = 50$ Mpa.

$\theta = 90^\circ - 55^\circ = 35^\circ$

$\sigma_n = ?$

$\tau_n = ?$

$R = ?$

$\phi = ?$

$\tau_{max} = ?$

We know

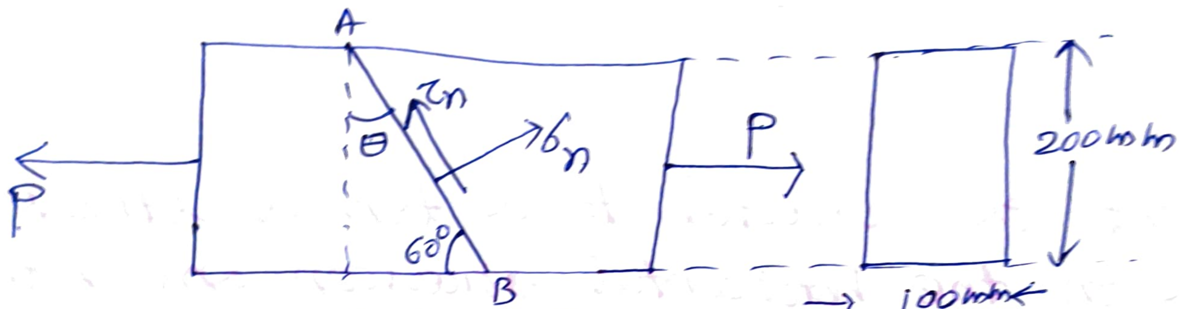
$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{150+50}{2}\right) + \left(\frac{150-50}{2}\right)\cos(2\times 35)$$

$$= 117.1 \text{ Mpa.}$$

tension member is formed by connecting wo wooden members 200 mm X 100 mm as shown in Fig. Determine safe value of force 'P', if permissible normal and shear stress in the joint are 0.5 MPa and 1.25 MPa res.

Ans



$\theta = 90^\circ - 60^\circ = 30^\circ$

$\sigma_n = 0.5$ MPa

$\tau_n = 1.25$ MPa

Area of cross section $= A = 200 \times 100 = 20{,}000 \text{ mm}^2$

$P = ?$

We know

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 2\theta$$

$$\Rightarrow 0.5 = \sigma_x\left(\frac{1+\cos(2\times30)}{2}\right)$$

$$\Rightarrow \sigma_x = 0.67 \text{ N/mm}^2 \text{ or MPa.}$$

We know

$$\tau_n = \left(\frac{\sigma_x}{2}\right)\sin 2\theta$$

$$\Rightarrow 1.25 = \frac{\sigma_x}{2} \times \sin(2\times30)$$

$$\Rightarrow \sigma_x = 2.89 \text{ N/mm}^2 \text{ or MPa.}$$

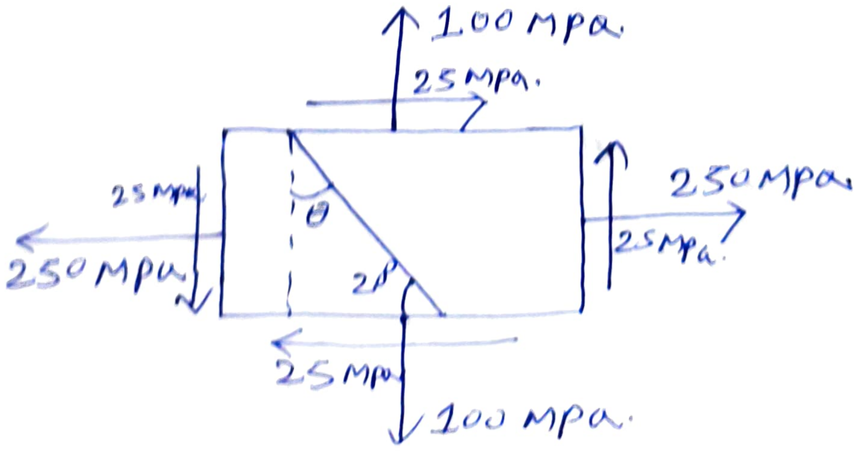
$\therefore$ So safe stress is ~~lowest~~ highest of two value i.e. 0.67 N/mm^2

∴ So safe value of force 'P' in x direction is

$$P = \sigma_x \cdot A$$
$$= 0.67 \times 20,000$$
$$= 13,400\ N$$
$$= 13.4\ kN$$

8. A point is subjected to a tensile stress of 250 Mpa in the horizontal direction and another tensile stress of 100 Mpa in the vertical direction. The point is also subjected to a simple shear stress of 25 Mpa, such that when it is associated with major tensile stress it tends to rotate the element in clockwise direction. What is the magnitude of the normal and shear stresses on a section at an angle of 20° with the major tensile stress?

Ans



$\sigma_x = 250$ Mpa.

$\sigma_y = 100$ Mpa.

$\tau_{xy} = 25$ Mpa.

$\theta = 90^\circ - 20^\circ = 70^\circ$

$\sigma_n = ?$

$\tau_n = ?$

we know

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta.$$

$$= \left(\frac{250+100}{2}\right) + \left(\frac{250-100}{2}\right)\cos(2\times 70) + 25\sin(200)$$

$= 101.48$ Mpa. (Tensile)

we know

$$\tau_n = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{250-100}{2}\right)\sin(2\times 70) + 25\cos(2\times 70)$$

$= -29.06$

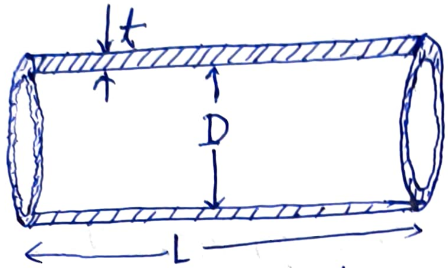
$= 29.06$ (counter clock wise)

THIN CYLINDER

(P.K. SWAIN)

→ A cylinder with wall thickness less than or equal to $\frac{1}{20}$th of internal diameter is called thin cylinder.

i.e.



D = internal diameter of cylinder

t = thickness of cylinder.

L = Length of cylinder.

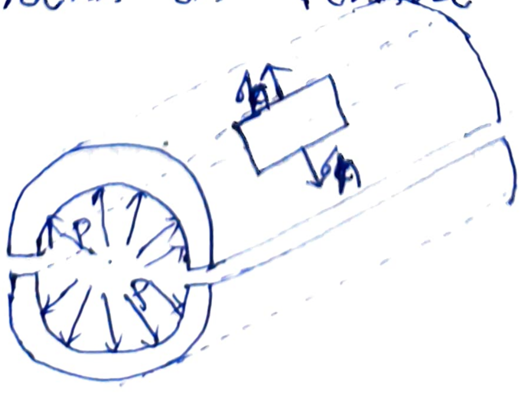
$$\boxed{t \leq \frac{1}{20}D}$$

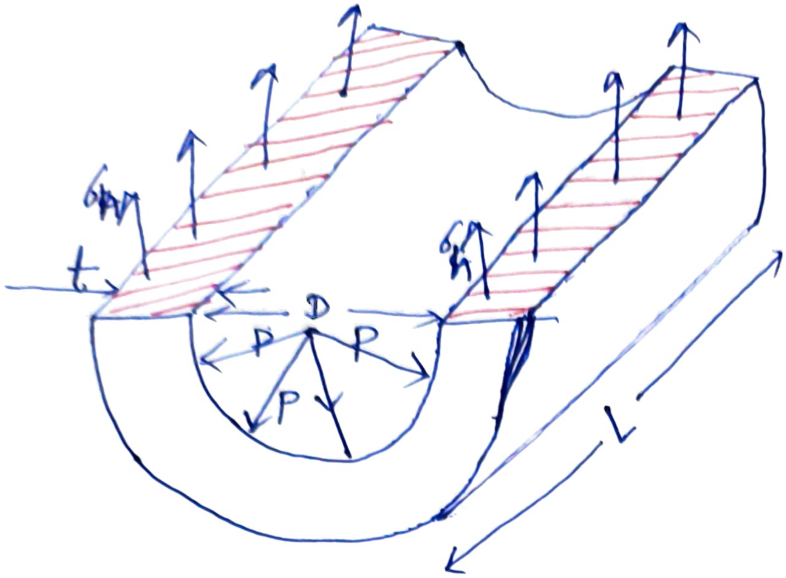
→ EX → oil tank, gas pipe, steam boiler etc

HOOP STRESS (σ_h)

→ Hoop stress is the stress induced in the hollow cylinder which acts tangentially to the circumference of the cylinder due to the internal fluid pressure.

→ Hoop stress is tensile in nature.





→ In thin cylinder, the pressure due to the fluid inside causes a brusting force on the cylinder walls due to which stress are induced in the cylinder.

→

Resisting force = brusting force

⇒ Resisting stress × resisting area = brusting pressure × projected area

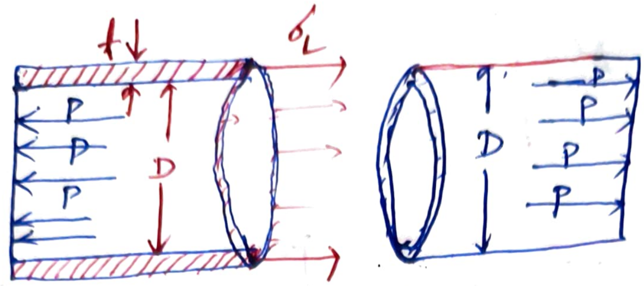
$$\Rightarrow (\sigma_h \times tL) + (\sigma_h \times tL) = P \times DL$$

$$\Rightarrow 2\sigma_h tL = PDL$$

$$\Rightarrow \boxed{\sigma_h = \frac{PD}{2t}} *$$

LONGITUDINAL STRESS (σ_L)

→ Longitudinal stress is the stress induced in the hollow cylinder in the wall of the cylinder along the longitudinal axis [redacted] due to internal fluid pressure when both ends of the cylinder are closed.



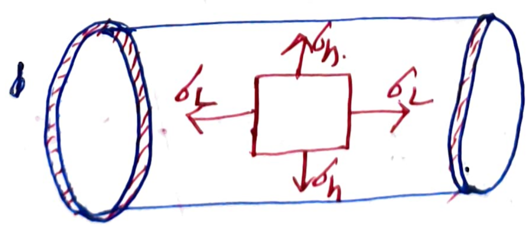
Resisting force = Brusting force.

$$\Rightarrow \sigma_L \times (\pi D t) = P \times \frac{\pi}{4} D^2$$

$$\Rightarrow \boxed{\sigma_L = \frac{PD}{4t}} \ *$$

HOOP STRAIN (ϵ_h)

→ It is the ratio of change in ~~circ~~ dimension of circumference of a ~~cir~~ cylinder to ~~the~~ original dimension of circumference due to internal fluid pressure call hoop strain



$$\epsilon_h = \frac{\sigma_h}{E} - \mu \frac{\sigma_L}{E}$$

$$\Rightarrow \epsilon_h = \frac{PD}{2tE} - \mu \frac{PD}{4tE}$$

$$\Rightarrow \boxed{\epsilon_h = \frac{PD}{4tE}\left[2-\mu\right]} **$$

$$\Downarrow$$

$$\boxed{\frac{\Delta D}{D} = \frac{\Delta R}{R} = \frac{PD}{4tE}\left[2-\mu\right]} * \rightarrow \text{Change in diameter}$$

LONGITUDINAL STRAIN (ϵ_L)

→ It is the ratio of change in length of cylinder to original length due to internal fluid pressure called longitudinal strain.

$$\epsilon_L = \frac{\sigma_L}{E} - \mu\frac{\sigma_h}{E}$$

$$\Rightarrow \epsilon_L = \frac{PD}{4tE} - \mu\frac{PD}{2tE}$$

$$\Rightarrow \boxed{\epsilon_L = \frac{PD}{4tE}\left[1-2\mu\right]} **$$

$$\Downarrow$$

$$\boxed{\frac{\Delta L}{L} = \frac{PD}{4tE}\left[1-2\mu\right]} \rightarrow \text{Change in length.}$$

VOLUMetric strain

→ It is the ratio of change in volume of a cylinder to original cylinder due to internal fluid pressure called Volumetric strain

We know

$$V = \frac{\pi}{4} D^2 \times L$$

Differentiating both the sides.

$$\delta V = \left(\frac{\pi}{4}\right) 2D\,\delta D\,L + \frac{\pi}{4} D^2\,\delta L$$

$$\Rightarrow \frac{\delta V}{V} = \frac{\frac{\pi}{4} 2D\,\delta D\,L}{V} + \frac{\frac{\pi}{4} D^2\,\delta L}{V}$$

$$\Rightarrow \epsilon_V = \frac{\cancel{\frac{\pi}{4}}\, 2\cancel{D}\,\delta D\,\cancel{L}}{\cancel{\frac{\pi}{4}}\, D^{\cancel{2}}\cancel{L}} + \frac{\cancel{\frac{\pi}{4}}\,\cancel{D^2}\,\delta L}{\cancel{\frac{\pi}{4}}\,\cancel{D^2}\,L}$$

$$\Rightarrow \epsilon_V = 2\frac{\delta D}{D} + \frac{\delta L}{L}$$

$$\Rightarrow \epsilon_V = 2\epsilon_h + \epsilon_L$$

$$\Rightarrow \epsilon_V = 2\frac{PD}{4tE}[2-\mu] + \frac{PD}{4tE}[1-2\mu]$$

$$\Rightarrow \epsilon_V = \frac{PD}{4tE}[4-2\mu+1-2\mu]$$

$$\Rightarrow \boxed{\epsilon_V = \frac{PD}{4tE}[5-4\mu]}$$ **

A cylinderical shell of 1.3 m diameter is made up of 18 mm thick plates. Find the circumferential and Longitudinal stress in the plate, if the boiler is subjected to an internal pressure of 2.4 MPa.

Ans

$t = 18\ mm$

$D = 1.3\ m = 1.3 \times 10^3\ mm$

$P = 2.4\ MPa$

$\sigma_h = ?$

$\sigma_L = ?$

We know

$$\sigma_h = \frac{PD}{2t}$$

$$= \frac{2.4 \times (1.3 \times 10^3)}{2 \times 18}$$

$$= 86.67\ MPa$$

$$\sigma_L = \frac{PD}{4t}$$

$$= \frac{2.4 \times (1.3 \times 10^3)}{4 \times 18}$$

$$= 43.335\ MPa$$

Q. A gas cylinder of internal diameter 40 mm is 5 mm thick. If the tensile stress in the material is not to exceed 30 MPa. Find the maximum pressure which can be allowed in the cylinder.

Ans

$D = 40\ mm$

$t = 5\ mm$

$\sigma_h = 30\ MPa.$

$P = ?$

we know:

$$\sigma_h = \frac{PD}{2t}$$

$$\Rightarrow 30 = \frac{P \times 40}{2 \times 5}$$

$$\Rightarrow P = \frac{30 \times 2 \times 5}{40} = 7.5\ MPa.$$

* A maximum pressure of 7.5 MPa ~~can~~ be applied on the cylinder without failure of the cylinder.

NOTE

$$\sigma_L = \frac{PD}{4t}$$

$$\Rightarrow 30 = \frac{P \times 40}{4 \times 5}$$

$$\Rightarrow P = \frac{30 \times 4 \times 5}{40} = 15\ Mpa.$$

* If we apply a pressure of 15 MPa on the cylinder. It will fail. ~~[illegible]~~ along circumferential direction bcz circumferential direction can resist 7.5 mpa stress which is smaller than 15 Mpa.

* But if we apply 7.5 Mpa pressure. It will not fail along ~~[illegible]~~ circumferential and longitudinal direction.

A thi... be ...

A thin cylindrical shell of 400 mm diameter is to be designed for an internal pressure of 2.4 MPa. Find the suitable thickness of shell, if ~~maximum tens~~ allowable circumferential stress is 50 MPa.

Ans

$D = 400$ mm

$P = 2.4$ MPa

$\sigma_h = 50$ MPa.

$t = ?$

we know.

$$\sigma_h = \frac{PD}{2t}$$

$$\Rightarrow 50 = \frac{2.4 \times 400}{2 \times t}$$

$$\Rightarrow t = \frac{2.4 \times 400}{2 \times 50} = 9.6 \text{ mm}$$

FACTOR OF SAFETY (FOS)

→ It is the ratio of maximum permissible stress to the working stress.

$$\boxed{FOS = \frac{\text{Maximum stress}}{\text{Working stress}}}$$

* maximum stress/strength of a material is fixed.

so $FOS \propto \frac{1}{\text{working stress}}$

$\Rightarrow FOS \propto \frac{1}{\left(\frac{F}{A}\right)}$

$\Rightarrow FOS \propto \frac{A}{F} \Rightarrow FOS \propto A \propto \frac{\pi}{4} D^2$ (for a circular bar)

Q. A cylinderical shell of 500 mm diameter required to withstand an internal pressure of 4 MPa. Find the minimum thickness of the shell, if maximum tensile strength in the plate material is 400 MPa. and efficiency of joint is 65%. Take factor of safety as 5.

Ans

$D = 500\,mm$

$P = 4\,MPa$

$t = ?$

$\eta = 65\% = 0.65$

$\sigma_{max} = 400\,MPa.$

$FOS = 5.$

We know

$$FOS = \frac{\text{maximum stress}}{\text{working stress}}$$

$$\Rightarrow 5 = \frac{400\,MPa}{\text{working stress}}$$

$$\Rightarrow \text{working stress} = \frac{400}{5} = 80\,MPa.$$

$$\Rightarrow \sigma = 80\,MPa.$$

→ For minimum thickness of cylinder, we will take hoop stress formula.

$$\sigma_h = \frac{PD}{2t\eta}$$

$$\Rightarrow 80 = \frac{4 \times 500}{2 \times t \times 0.65} \Rightarrow t = \frac{4 \times 500}{2 \times 80 \times 0.65} = 19.2\,mm$$

$\simeq 20\,mm$

A cylinderical thin drum 800 mm in diameter and 4 m long is made of 10 mm thick plate. If the drum is subjected to an internal pressure of 2.5 MPa. Determine its changes in diameter and length. Take E = 200 GPa and poisson's ratio as 0.25.

Ans

$D = 800\ mm$

$L = 4\ m = 4 \times 10^3\ mm$

$t = 10\ mm$

$P = 2.5\ MPa$

$E = 200\ GPa = 200 \times 10^3\ MPa$

$\mu = 0.25$

$\Delta D = ?$

$\Delta L = ?$

We know.

$$\epsilon_h = \frac{PD}{4tE}[2-\mu]$$

$$\Rightarrow \frac{\Delta D}{D} = \frac{PD}{4tE}[2-\mu]$$

$$\Rightarrow \frac{\Delta D}{D} = \frac{2.5 \times 800}{4 \times 10 \times 200 \times 10^3}[2-0.25]$$

$$\Rightarrow \frac{\Delta D}{D} = 0.0004375$$

$$\Rightarrow \Delta D = 0.0004375 \times D$$

$$\Rightarrow \Delta D = 0.0004375 \times 800$$

$$\Rightarrow \Delta D = 0.35\ mm$$

$\therefore$ change in diameter $= 0.35\ mm$

We know:

$$\epsilon_L = \frac{PD}{4tE}[1-2\mu]$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{PD}{4tE}[1-2\mu]$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{2.5\times 800}{4\times 10\times 200\times 10^3}[1-(2\times 0.25)]$$

$$\Rightarrow \frac{\Delta L}{L} = 0.000125$$

$$\Rightarrow \Delta L = 0.000125 \times L$$

$$\Rightarrow \Delta L = 0.000125 \times 4\times 10^3$$

$$\Rightarrow \Delta L = 0.5 \text{ mm}$$

$\therefore$ Change in Length of cylinder is 0.5 mm

Q
A cylindrical vessel 2m Long and 500 mm in diameter with 10 mm thick plates is subjected to an internal pressure of 3 Mpa. calculate the change in volume of the vessel. Take E = 200Gpa and $\mu = 0.3$ for vessel material.

Ans.

~~We know $\epsilon_V = \frac{PD}{4t}$~~

$L = 2m = 2\times 10^3 mm$

$D = 500 mm$

$t = 10 mm$

$P = 3 Mpa$

$E = 200\times 10^3 Mpa$

$\mu = 0.3$

We know

$$\epsilon_V = \frac{PD}{4tE}\left[5 - 4\mu\right]$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{PD}{4tE}\left[5 - 4\mu\right]$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3 \times 500}{4 \times 10 \times 200 \times 10^3}\left[5 - (4 \times 0.3)\right]$$

$$\Rightarrow \frac{\Delta V}{V} = 0.0007125$$

$$\Rightarrow \Delta V = 0.0007125 \times V$$

$$\Rightarrow \Delta V = 0.0007125 \times \frac{\pi}{4}(D^2) \times L$$

$$\Rightarrow \Delta V = 0.0007125 \times \frac{\pi}{4}(500)^2 \times 2 \times 10^3$$

$$\Rightarrow \Delta V = 279656.25 \text{ mm}^3$$

$$\Rightarrow \Delta V = 0.28 \text{ mm}^3$$

TORSION

7.0 Assumption of pure torsion.

7.1 The torsion equation for solid and hollow circular shaft.

7.2 Comparison between solid and hollow shaft subjected to pure torsion.